Alternatives to Inflation from Quantum Cosmology

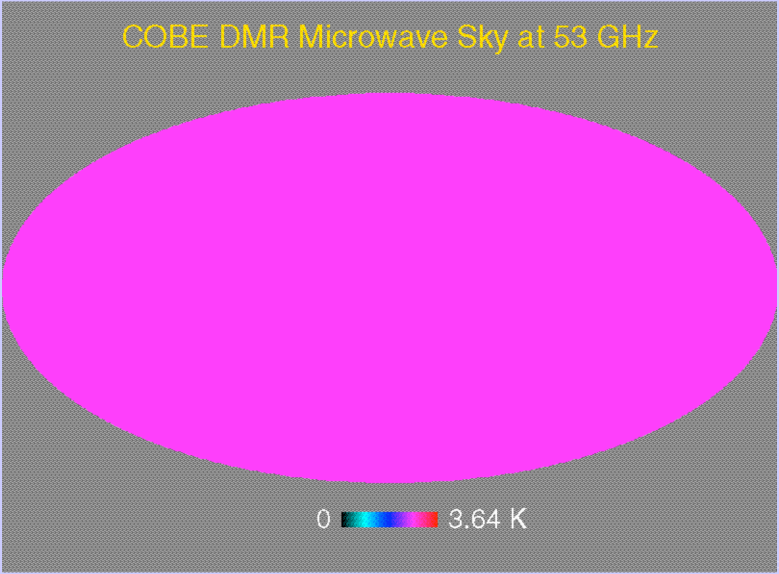
Robert Brandenberger
Physics Department, McGill University, Canada

Recent Progress in Quantum Cosmology Workshop
Yukawa Institute, Kyoto Univ., Nov. 8 2021

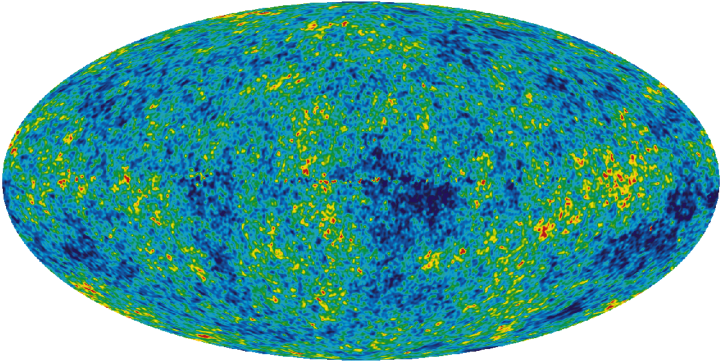
Outline

Plan

Isotropic CMB Background

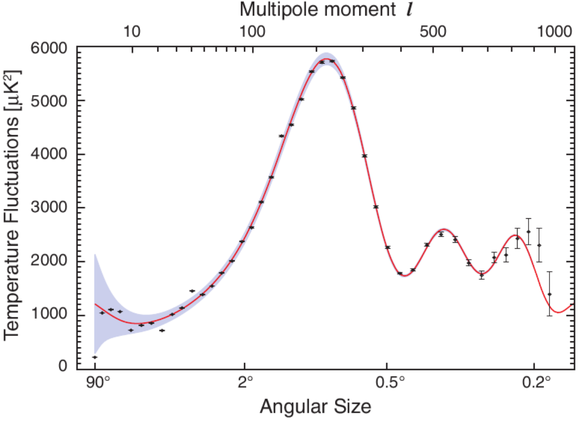


Map of the Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies



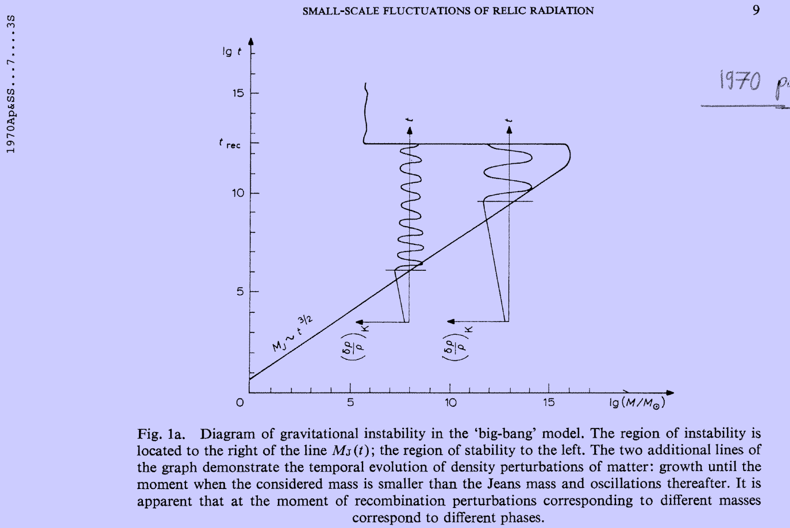
Credit: NASA/WMAP Science Team

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.
- → baryon acoustic oscillations in matter power spectrum.

Origin of Acoustinc Oscillations in the CMB Angular Power Spectrum



1970Ap&SS...7....3S

SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION 9

1970

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Challenge

How does one obtain such a spectrum?

- Inflationary Cosmology is the first scenario based on causal physics which yields such a spectrum.
- But it is not the only one.

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Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order to explain the isotropy of the CMB.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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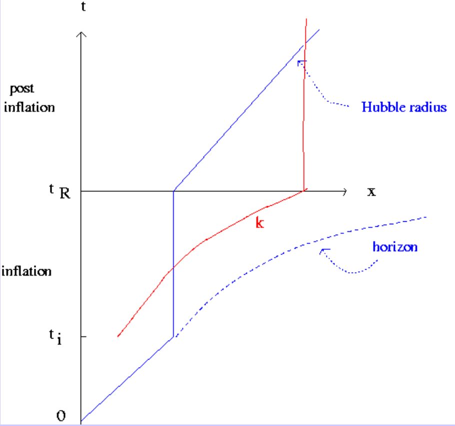
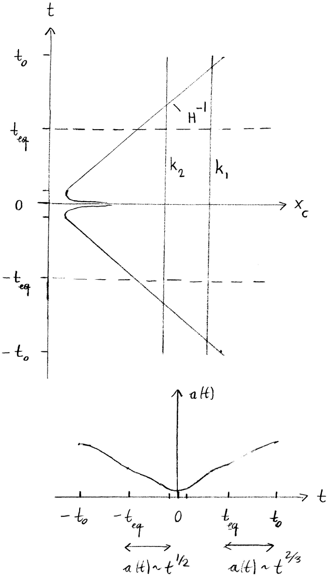
Inflation as a Solution

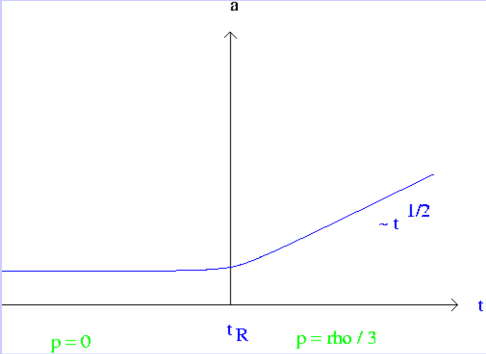


Bounce as a Solution



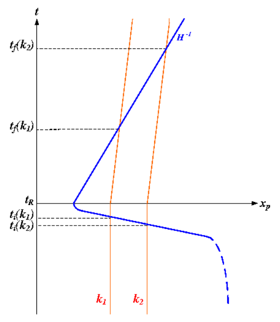
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Preview

- Quantum Gravity → **conceptual problems for inflationary cosmology**.
- These problems are less severe in alternatives to inflation.
- → motivates the search for realizations of alternative early universe scenarios from quantum gravity.
- Realization of **bouncing scenario** by means of an S-Brane [RB and Z. Wang, arXiv:2001.00638].
- Realization of **emergent scenario** from BFSS matrix model [S. Brahma, RB and S. Laliberte, arXiv:2108.11512].

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Plan

Idea of Cosmological Inflation

R. Brout, F. Englert and E. Gunzig (1978), A. Starobinsky (1980), A. Guth (1981), K. Sato (1981)

Assume the existence of a period of nearly exponential expansion of space during the early universe.

Period of inflation $t_i < t < t_R$.

Typical energy scale when inflation takes place: $E \sim 10^{16}\mathrm{GeV}$.

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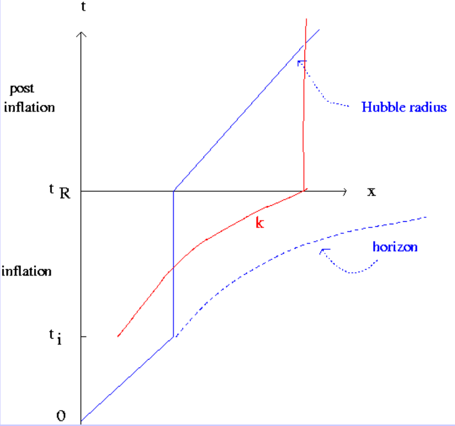
Space-Time Sketch of the Inflationary Scenario



Successes of Inflation

- Inflation renders the universe large, homogeneous and spatially flat, i.e. solution of the horizon and flatness problems of SBB cosmology
- Classical matter redshifts → matter vacuum remains
- Quantum vacuum fluctuations: seeds for the observed density fluctuations [Chibisov & Mukhanov, 1981] and gravitational waves (Starobinsky, 1978).
- **Approximately scale-invariant spectra** of density fluctuations and gravitational waves.
- **Small red tilt** of the spectrum of density fluctuations.
- **Prediction**: **Small red tilt** of the spectrum of gravitational waves.

Obtaining Inflation

- Assumption: Space-time described by General Relativity.
- $\rightarrow$ require matter with $p < -\frac{1}{3}\rho$.
- Consider scalar field φ as matter.
- In contrast to other matter fields, scalar fields have a **potential energy term** $V(\varphi)$.
- Potential energy has an equation of state $p = -\rho$.

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Obtaining Inflation II

- Other forms of energy have equations of state with $p > -\frac{1}{3}\rho$ which do not yield inflation.
- Thus one needs to ensure that potential energy dominated over other forms of energy!
- Require a slowly rolling scalar field:

$$\frac{V'}{V} \ll \frac{1}{m_{pl}} .$$

- Require rolling over large distances

$$\Delta\varphi > m_{pl} .$$

Initial Conditions for Inflation

Large field inflation is a local attractor in initial condition space (J. Kung and RB, 1990; RB, arXiv:1601.01918).

Fine tuning of initial conditions required for **small field inflation** (D. Goldwirth and T. Piran 1992).

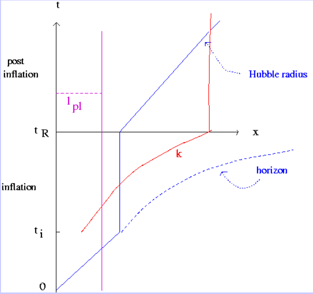
Note: Tuning of the initial spatial curvature required (also for large field inflation).

Problems of the Inflationary Scenario

- Singularity problem persists.
- How does one obtain inflation?
- Inflation takes place at energy scales close to the Planck scale.
- At this scale quantum effects of gravity should be important.
- **Effective field theory** analysis of inflationary cosmology is unable to handle this problem.

Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- → breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\frac{a(t_R)}{a(t_i)} l_{pl} < H(t_R)^{-1}$$

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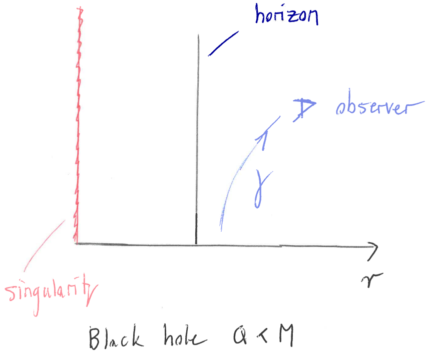
Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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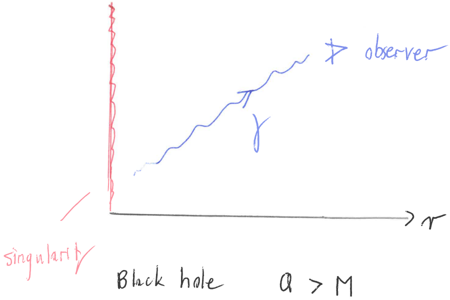

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- General Relativity allows for solutions with timelike singularities: super-extremal black holes.
- → Cauchy problem not well defined for observer external to black holes.
- Evolution non-unitary for external observer.
- Conjecture: ultraviolet physics → **external observer** shielded from the singularity and non-unitarity by horizon.

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Cosmological Version of the Censorship Conjecture

Translation

- Position space → momentum space.
- Singularity → trans-Planckian modes.
- Black Hole horizon → Hubble horizon.

Observer outside of Hubble horizon must be shielded from the trans-Planckian modes.

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Why Hubble Horizon?

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- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
- Demand: **classical region be insensitive to trans-Planckian region.**
- → no trans-Planckian modes ever exit Hubble horizon.

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- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all comoving wave numbers k
- Fixed k_{min}, time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
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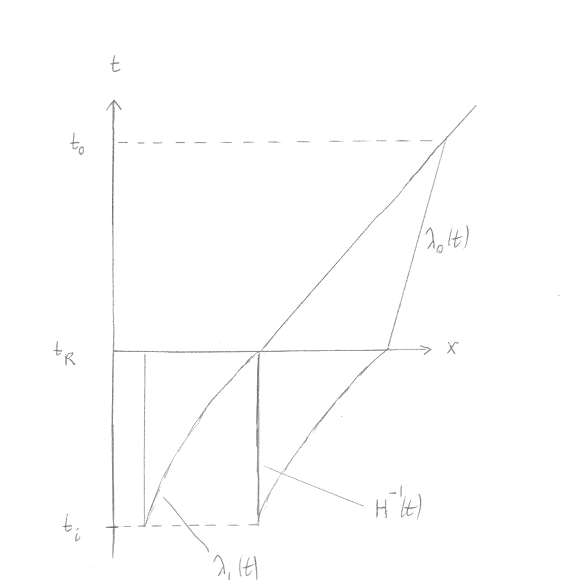
Application to Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106



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TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

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Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

→ upper bound on the **primordial tensor to scalar ratio** r:

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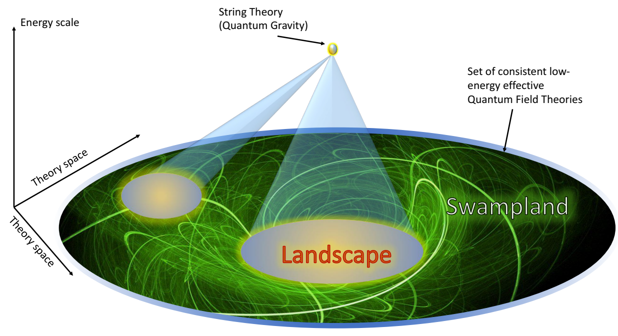
Implications for Dark Energy

Dark Energy cannot be a bare cosmological constant.

Constraints on Inflation from String Theory / Quantum Gravity

- There is a vast landscape of **effective field theories.**
- Any space-time dimension, and number of fields, any shape of the potential, any field range.
- Superstring theory is very **constraining.**
- Only a **small subset** of all EFTs is consistent with string theory.
- The rest lie in the swampland.


Energy scale
String Theory
(Quantum Gravity)
Set of consistent low-energy effective Quantum Field Theories
Theory space
Theory space
Swampland
Landscape

Swampland Conjectures

H. Ooguri and C. Vafa, hep-th/0605264; G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, arXiv:1806.08362; S. Garg and C. Krishnan, arXiv:1807.05193; H. Ooguri, E. Palti, G. Shiu and C. Vafa, arXiv:1810.05506.

What are conditions for habitable islands sticking out from the swamp?

- The effective field theory is only valid for $\Delta\varphi < d m_{pl}$ (field range condition).
- The potential of φ obeys (de Sitter conjecture)

$$\begin{aligned} \left|\frac{V'}{V}\right| m_{pl} &\geq c_1 \quad \text{or} \\ \frac{V''}{V} m_{pl}^2 &\leq -c_2 \end{aligned}$$

Note: d, c_1, c_2 constants of order 1.

Swamplandʼ Conjectures

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Consequences

- No canonical single field inflation.
- No bare positive cosmological constant.
- Dark Energy is not a bare cosmological constant.
- Quintessence dark energy is constrained (L. Heisenberg et al, arXiv:1808.02877).

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What about Alternatives to Inflation?

- Bouncing cosmologies are consistent with the TCC (as long as the energy scale at the bounce is lower than the Planck scale).
- Emergent cosmologies are consistent with the TCC.

Bottom Line

- TCC → limitations on EFT description of the very early universe.
- → improved description of the early universe needs to be based on physics beyond the usual EFT.

Plan

1. Introduction
2. Challenges for Inflationary Cosmology from Quantum Gravity
3. S-Brane Ekpyrosis: A Bouncing Scenario from Quantum Gravity
4. Models of Emergent Cosmology motivated by String Theory
    - String Gas Cosmology
    - Matrix Theory Cosmology
5. Conclusions

Ekpyrotic Scenario

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001) [hep-th/0103239]

- Ekpyrotic Scenario: **Bouncing cosmology** with a phase of very slow contraction.
- Among bouncing models the Ekpyrotic scenario has distinct advantages:
- Dilutes anisotropies.
- Creates spatial flatness.
- Attractor in initial condition space.
- Thus, the Ekpyrotic scenario shares the same nice features with **large field** inflation.

Ekpyrotic Bouncing Scenario

Ekpyrosis: Phase of **very slow contraction**:

$$a(t) \sim (-t)^p \quad (t < 0)$$

$$p \ll 1 \, .$$

In conformal time ($ad\tau = dt$):

$$a(\tau) \sim (-\tau)^q \quad (\tau < 0)$$

$$q = \frac{p}{1-p} \, .$$

This can be obtained using GR plus scalar field matter with a negative exponential potential

$$V(\varphi) = -V_0 e^{-\sqrt{2/p}\varphi/m_{pl}}$$

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Obtaining an Ekpyrotic Universe

$$V(\varphi) = -V_0 \exp(-\sqrt{2/p}\varphi/m_{pl}) \quad p \ll 1$$

$$a(t) \sim (-t)^p$$

$$w \simeq \frac{4}{3p}$$

$$\varphi(t) = \sqrt{2p} m_{pl} \log(-\sqrt{\frac{V_0}{m_{pl}^2 p(1-3p)}} t)$$

Ekpyrosis: Small Field and Large Slope

Consider $\delta t = H^{-1}$:

$$\varphi \sim p^{1/2} \log(p^{-1/2}) m_{pl} \ll m_{pl} \, .$$

Relative slope of the potential:

$$|\frac{V'}{V}| m_{pl} \sim p^{-1/2} \gg 1 \, .$$

Relative curvature of the potential:

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Note: **Ekpyrotic potentials are safe from the swampland constraints!**

H. Bernardo and R.B., arXiv:2104.00630

Challenges for the Ekpyrotic Scenario

Challenges for the Ekpyrotic Scenario:

- How is the bounce realized?
- How does one obtain a spectrum of almost scale-invariant cosmological perturbations?
- Are gravitational waves produced with a significant amplitude on cosmological scales?

Note: Previous realizations of Ekpyrosis

- Require extra/new matter fields to obtain a non-singular bounce.
- Require extra matter fields to obtain cosmological perturbations with an approximately scale-invariant spectrum.
- Predict a vacuum spectrum of gravitational waves.

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R.B. and Z. Wang, Phys.Rev.D 101 (2020) 6, 063522, arXiv:2001.00638;
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Our work:

- S-brane mediates a **continuous bounce.**
- **Gravitational waves** passing through the brane acquire a **scale-invariant spectrum**.
- If the S-brane has zero shear, then a **scale-invariant spectrum** of **curvature fluctuations** is generated.
- **Two consistency relations** among the four basic cosmological observables.

Motivation for an S-Brane

Idea: At the string scale a new tower of string states becomes comparable in mass to the usual low energy degrees of freedom;
→ they must be included in the low energy effective action.

The new term is localized on a space-like hypersurface: S-Brane.

Action including the S-Brane

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] \\
&\quad - \int d^4x\kappa\delta(\tau - \tau_B)\sqrt{\gamma}\,,
\end{aligned}
$$

$$
\kappa \equiv N\eta_s^3\,,
$$

Features of an S-Brane

- Vanishing $T^{\mu\nu}$ perpendicular to the surface $\rightarrow \ \rho = 0$
- Relativistic object $\rightarrow$ tension in space-like directions $\rightarrow \ p < 0$
- $\rightarrow$ **violation of the Dominant Energy Condition.**
- $\rightarrow$ it is possible to obtain a non-singular cosmology
- $\rightarrow$ it is possible to obtain a bouncing cosmology

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Background Evolution

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Matching conditions

- Continuity of the induced metric.
- Jump in the extrinsic curvature given by the tension of the S-brane.

$$\delta H \equiv \lim_{\epsilon \to 0} H(t_B + \epsilon) - H(t_B - \epsilon) = 4\pi G \kappa \, .$$

$$\delta H = \frac{2}{\sqrt{3}} \eta_s^2 m_{pl}^{-1} \, ,$$

$$N = \frac{4}{\sqrt{3}} \frac{m_{pl}}{\eta_s} \, .$$

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- Continuity of the induced metric.
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$$\delta H \equiv \lim_{\epsilon \to 0} H(t_B + \epsilon) - H(t_B - \epsilon) = 4\pi G \kappa \, .$$

$$\delta H = \frac{2}{\sqrt{3}} \eta_s^2 m_{pl}^{-1} \, ,$$

$$N = \frac{4}{\sqrt{3}} \frac{m_{pl}}{\eta_s} \, .$$

For sufficiently large tensions, the S-Brane mediates a transition between contraction and expansion.

Plan

1. Introduction
2. Challenges for Inflationary Cosmology from Quantum Gravity
3. S-Brane Ekpyrosis: A Bouncing Scenario from Quantum Gravity
4. Models of Emergent Cosmology motivated by String Theory
   - String Gas Cosmology
   - Matrix Theory Cosmology
5. Conclusions

Principles (String Gas Cosmology)

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
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T-Duality

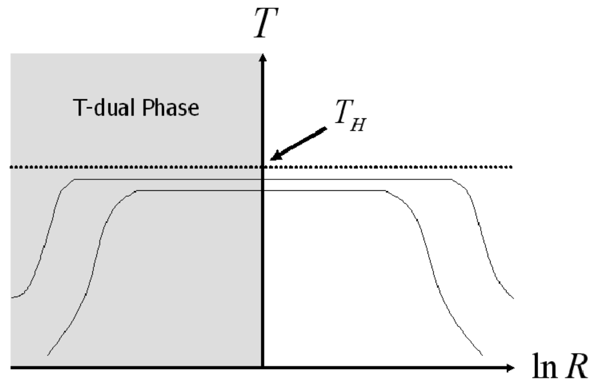
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

Adiabatic Considerations

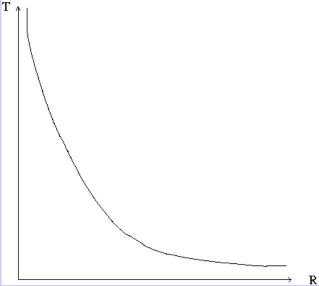
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology

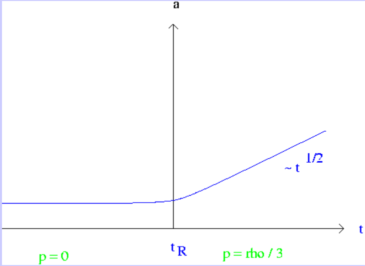


Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



Background for string gas cosmology



String Gas Cosmology and the Dimensionality of Space

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

- Winding modes prevent space from expanding.
- Momentum modes prevent space from shrinking.
- Expansion of space requires decay of winding modes.
- Since winding strings are **two-dimensional world sheets**, their interaction rate vanishes in more than four space-time dimensions.
- → only three spatial dimensions can become large.

Note: string gases can play a key role in the stabilization of both size and shape moduli.

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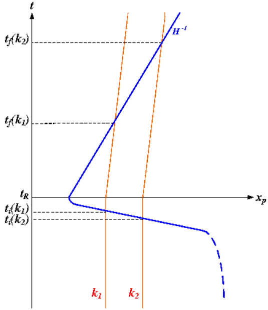
Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

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Note: The **matter correlation functions** are determined by the partition function of the system.

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \, .$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned} P_{\Phi}(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2 k^2 < (\delta M)^2 >_R \\ &= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \end{aligned}$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$\begin{aligned} P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\ &= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\ &\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H) \end{aligned}$$

Key ingredient for string thermodynamics

$$< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)$$

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Where do we stand?

- String Gas Cosmology appears to be an alternative to inflation for generating the structures we see in cosmological observations.
- String Gas Cosmology: nonsingular, solves the horizon problem.
- Achilles heel: how do we describe the emergent phase mathematically?

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Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of superstring theory: 10 dimensional superstring theory emerges in the $N \rightarrow \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

where

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is equivalent to the bosonic sector of the (Euclidean) IKKT matrix model.

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S = -\frac{1}{g^2}\mathrm{Tr}\Big(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\Big) ,$$

Partition function:

$$Z = \int dAd\psi e^{iS}$$

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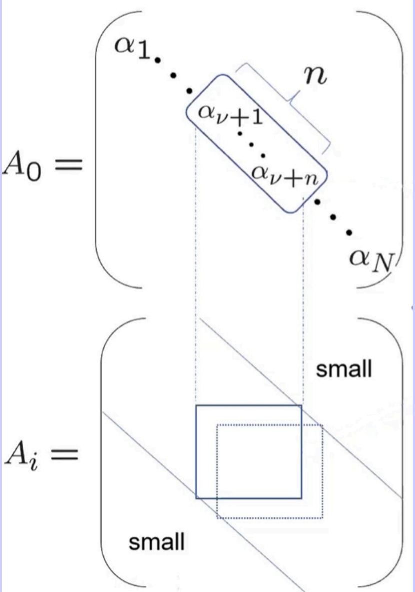
N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

- Work in the basis in which A_0 is diagonal.
- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- A_i matrices become block diagonal $\to$ emergent space, continuous in $N \to \infty$ limit.


$A_0 =$
α_1
n
$\alpha_{\nu+1}$
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α_N
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J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: X_i matrices become block diagonal → emergent space, continuous in $N \to \infty$ limit.
- Extent of space:

$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{X}_i)(t))^2 \right\rangle ,$$

- Local Lorentz invariance emerges in $N \to \infty$ limit.

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- In a thermal state there is spontaneous symmetry breaking: $SO(9) \rightarrow SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the BFSS-IKKT correspondence extends to the full model (not just the bosonic sector).
- Or else that the spontaneous symmetry breaking observed in the IKKT model also holds in the BFSS model.
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- → curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Results:

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

→ BFSS matrix model yields emergent space, emergent time and an emergent early universe phase, generating cosmological fluctuations and primordial gravitational waves consistent with observations..

Note: Horizon problem automatically solved.

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Open Problems

- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- How is the flatness problem addressed?
- Can we obtain sufficiently large spatial sections?
- What are the tilts of the spectra of curvature fluctuations and gravitational waves?
- ....

Plan

Conclusions I

- We have lots of data about the cosmos, and much more data is expected soon.
- Cosmological data can only be explained using new fundamental physics operating in the very early universe.
- Current paradigm: effective field theory (EFT) description of cosmological inflation.
- Alternatives to cosmological inflation exist, e.g. the Ekpyrotic Bounce.
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Conclusions II

- Challenge for Quantum Cosmology: need to go beyond an EFT analysis to describe the very early universe.
- Quantum Gravity → new ideas to obtain promising alternative scenarios.
- S-Brane (motivated from string theory) mediates continuous transition from contration to expansion and provides a realization of the **Ekpyrotic bouncing scenario**.
- Matrix Theory (nonperturbative candidate formulation of superstring theory) → emergent space, time and early universe cosmology.
- **Thermal fluctuations** in the high temperature state of matrix theory → **scale-invariant spectrum** of **curvature fluctuations** and **gravitational waves**.

Gravitational Waves during Ekpyrotic Contraction

Spatial metric including a gravitational wave travelling in x-direction:

$$\gamma_{ij} \, = \, a^2(\tau) \begin{pmatrix} 1 & 0 \\ 0 & 1 + h\epsilon_{ab} \end{pmatrix} \, ,$$

Eq. of motion in terms of the canonical variable $\tilde{h} \equiv ah$:

$$\tilde{h}'' \, + \, [k^2 - \frac{a''}{a}]\tilde{h} \, = \, 0 \, .$$

Dominant solution on super-Hubble scales (leading order in p)

$$\tilde{h}(\tau) \, \sim \, \tau^p \ \text{decreasing as } \tau \rightarrow 0.$$

Conclusion: An **initial vacuum spectrum** remains approximately vacuum with a slight **blue tilt.**

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Cosmological Perturbations in the Contracting Phase I

Metric including cosmological perturbations $\Phi(x, \tau)$:

$$ds^2 \,=\, a(\tau)^2 \left[(1 + 2\Phi) d\tau^2 - (1 - 2\Phi) d\mathbf{x}^2 \right] ,$$

Canonical variable

$$v \,=\, a\big(\delta\varphi + \frac{\dot{\varphi}_0}{H}\Phi\big) ,$$

Equation of motion in Fourier space (assuming equation of state of matter is constant)

$$v_k'' + \big(k^2 - \frac{a''}{a}\big) v_k \,=\, 0 ,$$

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Cosmological Perturbations in the Contracting Phase II

A second view:

$$u \equiv \frac{m_{pl}}{\mathcal{H}} a\Phi \,.$$

It obeys the mode equation

$$u_k'' + \left(k^2 - 2\mathcal{H}^2 - \frac{a''}{a}\right) u_k \, = \, 0 \,,$$

On super-Hubble scales this becomes

$$u_k'' - q(-\tau)^{-2} u_k \, = \, 0 \,,$$

growing mode with

$$u_k \, \sim \, (-\tau)^{-q} \, \sim \, a(\tau)^{-1} \,,$$

Conclusion: Vacuum initial spectrum is transformed to a **scale-invariant** spectrum with a slight **red tilt**.

Key Question

Which variable passes through the bounce continuously?

Some previous results:

- At the reheating transition in inflationary cosmology it is the variable v which is continuous, and Φ jumps (by a large factor).
- In models with a smooth bounce mediated by matter which violates the Null Energy Condition the variable v is continuous, and not Φ.
- For a space-like matching surface, most choices of the location of the surface lead to Φ being continuous (Durrer and Vernizzi, Phys. Rev. D **66**, 083503 (2002)).

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Gravitational Waves Passing Through the S-Brane

R.B. and Z. Wang, Phys.Rev.D 101 (2020) 6, 063522, arXiv:2001.00638.

$$\tilde{h}'' + \left[k^2 - \frac{a''}{a} - \kappa a m_{pl}^{-2}\delta(\tau - \tau_B)\right]\tilde{h} = 0\,.$$

Mathematical Aside

- Consider the equation

$$X''_k(\tau) + \left[k^2 + m\delta(\tau - \tau_B)\right] X_k(\tau) \; = \; 0 \, .$$

- Solutions: plane waves for $\tau < \tau_B$ and for $\tau > \tau_B$.
- Positive frequency solutions f_k and negative frequency ones f_k^*.
- **Bogoliubov mode mixing across the transition surface.**
- Pure positive frequency before τ_B can be written for $\tau > \tau_B$ as

$$X_k \; = \; \alpha_k f_k + \beta_k f_k^* \, ,$$

- where α_k and β_k are the Bogoliubov mode matching coefficients.

Mathematical Aside II

By integrating over time τ against a test function (a smooth function which decays exponentially at $\tau \to \pm\infty$) $f(\tau)$ it can be easily shown that

$$\beta_k = \frac{m}{k} .$$

This is the factor which transforms a vacuum spectrum into a scale-invariant one.

Gravitational Waves Passing Through the S-Brane

$$\tilde{h}'' + \left[k^2 - \frac{a''}{a} - \kappa a m_{pl}^{-2} \delta(\tau - \tau_B)\right] \tilde{h} = 0 \,.$$

- Spectrum before passage through the S-Brane is a vacuum spectrum with a small blue tilt.
- → Spectrum after passage through the S-brane is scale-invariant with a slight blue tilt!.
- Power spectrum of gravitational waves;

$$\mathcal{P}_h(k) \sim \frac{1}{2\pi^2} \kappa^2 m_{pl}^{-6} (k\tau_B)^{2q} \,.$$

Curvature Fluctuations Passing Through the S-Brane I

R.B. and Z. Wang, Phys. Rev. D 102 (2020) 2, 023516, arXiv:2004.06437.

Solution for Φ on super-Hubble scales in the contracting phase:

$$\Phi_-(k,\tau) \,=\, A_-(k)\frac{\mathcal{H}}{a^2} + B_-(k)\,,$$

Solution for Φ on super-Hubble scales in the expanding phase:

$$\Phi_+(k,\tau) \,=\, A_+(k)\frac{\mathcal{H}}{a^2} + B_+(k)\,.$$

Matching Conditions

- Continuity of the induced metric fluctuations.
- Extrinsic curvature jump given by the fluctuations of the tension of the S-brane.

Matching conditions for a zero shear S-brane (R. Durrer and F. Vernizzi, Phys. Rev. D **66**, 083503 (2002)):

$$\begin{aligned} A_+ &= \frac{\mathcal{H}_-}{\mathcal{H}_+}A_- + \frac{a_B^2}{\mathcal{H}_+}(B_- - B_+) \\ B_+ &= \left(\frac{\mathcal{H}_+(\mathcal{H}_-'/\mathcal{H}_- - \mathcal{H}_-) - \mathcal{H}_+' + {\mathcal{H}_+}^2}{2{\mathcal{H}_+}^2 - \mathcal{H}_+'}\right)\frac{\mathcal{H}_-}{a_B^2}A_- \\ &\quad + \left(1 + \frac{\mathcal{H}_-\mathcal{H}_+ - {\mathcal{H}_+}^2}{2{\mathcal{H}_+}^2 - \mathcal{H}_+'}\right)B_- \, , \end{aligned}$$

Curvature Fluctuations Passing Through the S-Brane II

Result of the matching:

$$B_+(k) \simeq -\frac{\mathcal{H}_+}{a_B^2}\frac{1}{3q}A_-(k)\,.$$

Using vacuum initial conditions to determine $A_-(k)$:

$$A_-(k) \simeq 2^{\mu}\Gamma(\mu)m_{pl}^{-1}k^{-3/2}(k\tau_B)^{-q}\,,$$

Power Spectrum of Cosmological Perturbations:

$$\mathcal{P}_\Phi(k) \simeq \frac{1}{2\pi^2}(k\tau_B)^{-2q}\Big(\frac{\mathcal{H}_+}{a_B^2 m_{pl}}\Big)^2\frac{1}{9q^2}2^{2\mu}\Gamma(\mu)^2\,.$$

Scale-invariant spectrum with a slight red tilt.

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Consistency relation for r

Comparing the results for the GW spectrum and the spectrum of cosmological perturbations yields

$$r \simeq 144(k_C\tau_B)^{4q}2^{-2\mu}\Gamma(\mu)^{-2}q^2 \, .$$

Since the value of q is given by the scalar tilt $q = (1 - n_s)/2$ we get

$$r \sim (1 - n_s)^2 \, .$$

Consistency relation for the tilts

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$$n_t \; = \; (1 - n_s) \, .$$

Recall:

$$P_h(k) \; \sim \; k^{n_t}$$

$$P_\Phi(k) \; \sim \; k^{n_s - 1}$$

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