Space and time in quantum cosmology

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Space and time in quantum gravity

General relativity determines dynamics and structure of space-time.

1. Background independence.

Space-time structure may be non-classical.

It may happen that no consistent quantum space-time structure exists, not even semiclassically.

2. Problem of time.

Time transformations and evolution.

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Hamiltonian constraint

Friedmann equation implies constraint

$$C = -6\pi G V p_V^2 + V \rho_{\mathrm{matter}} = 0$$

for “volume” $V = a^3$ and its momentum p_V in isotropic models.

→ How is evolution derived from Wheeler–DeWitt equation after quantization, replacing $p_V \propto \dot{a}$ with $-i\hbar\partial/\partial V$?

→ Can quantum corrections in C be consistent with larger algebra including diffeomorphism constraint?

Is there a consistent space-time line element

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \tilde{a}(\tau)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

where $\tilde{a}$ solves a quantum corrected Friedmann equation?

Examples in loop quantum gravity

1. Dressed-metric approach for cosmological inhomogeneity.
   [Agulló, Ashtekar, Nelson: PRL 109 (2012) 251301]

2. Partial Abelianization of spherically symmetric models.
   [Gambini, Pullin: PRL 110 (2013) 211301]

3. “Transfiguration” of black holes.
   [Ashtekar, Olmedo, Singh: PRL 121 (2018) 241301]

4. “Covariant polymerization” in spherically symmetric models.
   [Benítez, Gambini, Pullin: arXiv:2102.09501]

   Attempted justification of model of critical collapse.
   [Benítez, Gambini, Lehner, Liebling, Pullin: PRL 124 (2020) 071301]

None of these claims are based on consistent space-time structures.

Models of loop quantum gravity

Hamiltonian constraint quantized, but also modified:

$$C_{\rm mod} = -6\pi G V f(p_V)^2 + V\rho_{\rm matter} = 0$$

with bounded function $f(p_V)$ (holonomy modification).

Solutions: Energy density $\rho_{\rm matter} \propto f(p_V)^2$ bounded.

May avoid singularities and suggest new physical effects.

Open questions:

→ Interplay between modification and other quantum corrections, such as fluctuations?

→ Consistency with covariance conditions? Modification by $f(p_V)$ not of standard higher-curvature form.

→ Special way to deal with problem of time (deparameterization) may be restrictive.

Dressed-metric approach

[Agulló, Ashtekar, Nelson: PRL 109 (2012) 251301]

Treat cosmological inhomogeneity as two independent systems: Perturbations evolving on homogeneous background.

$\longrightarrow$ Insert $\tilde{a}(\tau)$ solving modified Friedmann equation in

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \tilde{a}(\tau)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

No obvious violation of covariance in homogeneous setting.

$\longrightarrow$ "Gauge-invariant" curvature perturbations available for inhomogeneity. [Bardeen: PRD 22 (1980) 1882]

Proposal:
Quantized perturbations subject to wave equations on modified background geometry $\mathrm{d}s^2 = \tilde{g}_{\alpha\beta}\mathrm{d}x^\alpha\mathrm{d}x^\beta$.

The metric's new clothes

Bardeen variables invariant under small coordinate changes.

[Stewart: CQG 7 (1990) 1169]

Relevant for perturbative inhomogeneity, but not sufficient:

Also need large background transformations.

Not independent (semidirect product): [MB: PRD 102 (2020) 023532]

$$\left[f(t)\frac{\partial}{\partial t},\xi^\alpha\frac{\partial}{\partial x^\alpha}\right]=f\dot{\xi}^\alpha\frac{\partial}{\partial x^\alpha}-\dot{f}\xi^0\frac{\partial}{\partial t}$$

Covariance requires precise algebra. Violated by independent treatment in dressed metric: implicitly assumes direct product.

Line element ${\rm d}s^2=\tilde{g}_{\alpha\beta}{\rm d}x^\alpha{\rm d}x^\beta$ based on modified metric components $\tilde{g}_{\alpha\beta}$ meaningless as a background for perturbations: $\tilde{g}_{\alpha\beta}$ and ${\rm d}x^\alpha{\rm d}x^\beta$ not subject to dual transformations.

Homogeneity in Schwarzschild geometry

Homogeneous geometry (Kantowski–Sachs)

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\mathrm{d}x^2 + b(t)^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)$$

realized in Schwarzschild interior: $t < 2M$ in

$$\mathrm{d}s^2 = -\frac{\mathrm{d}t^2}{2M/t-1} + (2M/t-1)\mathrm{d}x^2 + t^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)$$

$\longrightarrow$ Can apply minisuperspace quantization to interior.

$\longrightarrow$ Homogeneous timelike slices in exterior:

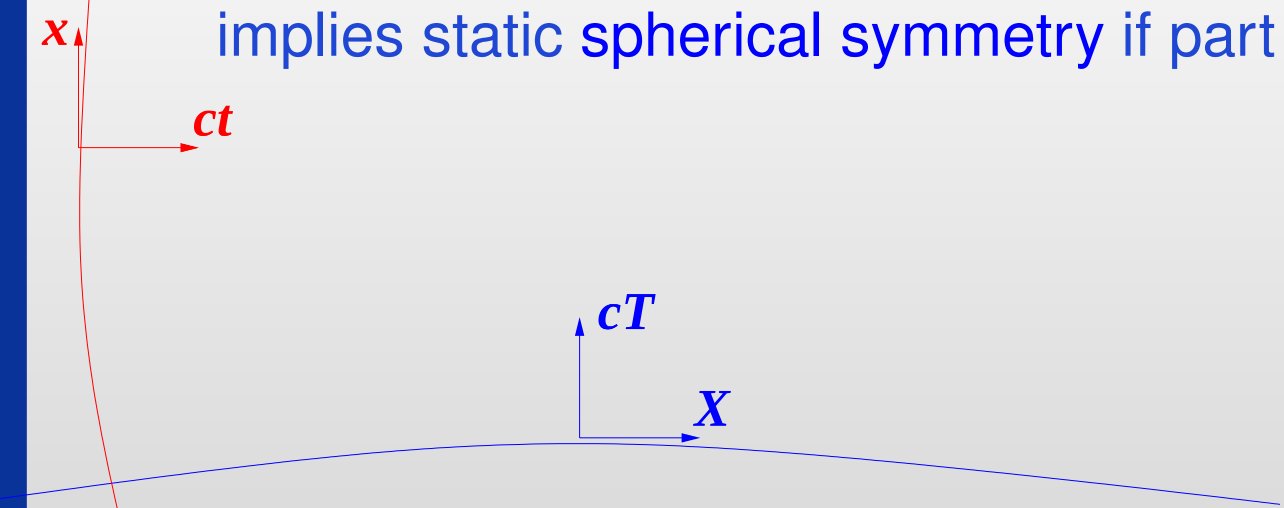
$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{1-2M/r} - (1-2M/r)\mathrm{d}t^2 + r^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right)$$

(Kantowski–Sachs after complex canonical transformation.)

[Ashtekar, Olmedo, Singh: PRL 121 (2018) 241301]

No local covariance

Timelike homogeneity remains intact with modified dynamics, implies static spherical symmetry if part of covariant theory.



All low-order local covariant $1+1$-dimensional theories known: generalized dilaton gravity.

[Kunstatter, Maeda, Taves: CQG 33 (2016) 105005]
[Takahashi, Kobayashi: CQG 36 (2019) 095003]

No holonomy modified dynamics of Kantowski–Sachs-style models can be part of a covariant space-time theory.

[MB: PRD 102 (2020) 046006]

Effective line element

[with Brahma, Reyes: PRD 92 (2015) 045043]

Meaningful line element $\mathrm{d}s^2 = \tilde{g}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta$ must be independent of coordinate choices:

$$g_{\alpha'\beta'} = \frac{\partial x^\alpha}{\partial x^{\alpha'}} \frac{\partial x^\beta}{\partial x^{\beta'}} g_{\alpha\beta}$$

if coordinates x^α transformed to $x^{\alpha'}$.

Canonical quantization does not modify x^α,
but alters equations for spatial metric q_{ij} in

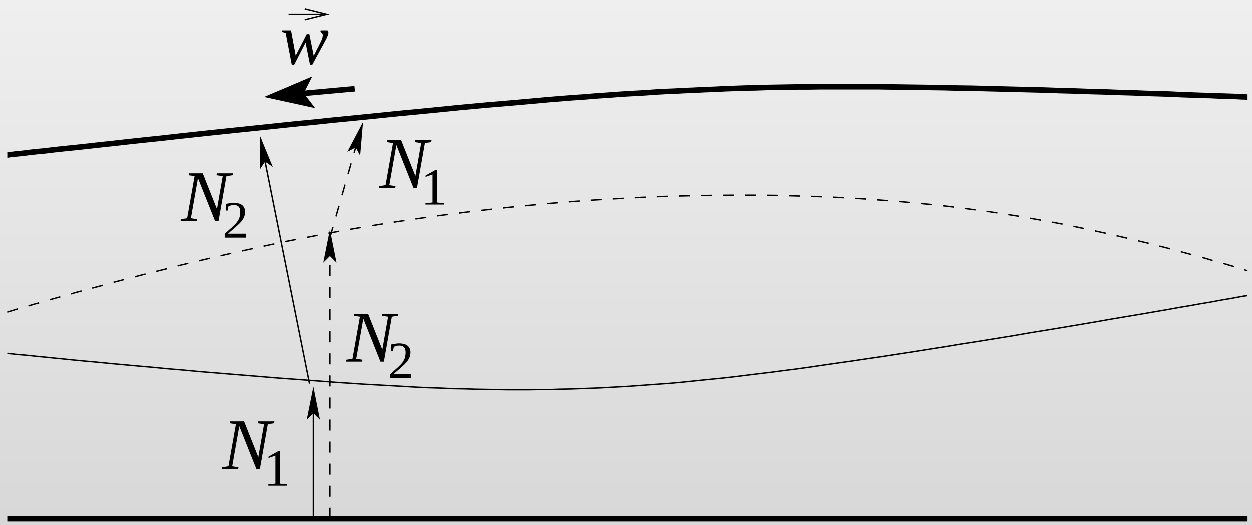
$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + q_{ij}(\mathrm{d}x^i + M^i\mathrm{d}t)(\mathrm{d}x^j + M^j\mathrm{d}t)$$

Modifications of lapse N and shift M^i follow more indirectly.

Hypersurface deformations

[Hojman, Kuchař, Teitelboim: Ann. Phys. 96 (1976) 88]

Non-linear coordinate changes — non-linear deformations.



Hypersurface-deformation brackets

$$
\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= -S((\vec{w}_2 \cdot \vec{\nabla})\vec{w}_1 - (\vec{w}_1 \cdot \vec{\nabla})\vec{w}_2) \\
[T(N), S(\vec{w})] &= -T(\vec{w} \cdot \vec{\nabla} N) \\
[T(N_1), T(N_2)] &= S(N_1 \vec{\nabla} N_2 - N_2 \vec{\nabla} N_1)
\end{aligned}
$$

represent general covariance in canonical/algebraic form.

Structure functions

Canonical realization of hypersurface deformations:

Hamiltonian and diffeomorphism constraints, $H[N]$ and $D[M^i]$.

$$
\begin{aligned}
\{D[M_1^i], D[M_2^j]\} &= D[[M_1, M_2]^i] \\
\{H[N], D[M^i]\} &= -H[M_1^i \nabla_i N] \\
\{H[N_1], H[N_2]\} &= D[q^{ij}(N_1 \nabla_j N_2 - N_2 \nabla_j N_1)]
\end{aligned}
$$

- → q^{ij} as well as D and H to be turned into operators.
- → Requires specific ordering/regularization/... for brackets to remain closed.
- → Even if closed, quantized structure functions may be quantum corrected. Non-Riemannian unless field redefinition leads to effective line element.

  [with Brahma, Yeom: PRD 98 (2018) 046015]

- → Effective equations not of higher-curvature form.

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Simple model

Scalar field $\phi(x)$, momentum $p(x)$, one spatial dimension.

$$H[N] = \int \mathrm{d}x N \left(f(p) - \frac{1}{4}(\phi')^2 - \frac{1}{2}\phi\phi'' \right) \quad , \quad D[w] = \int \mathrm{d}x w \phi p'$$

Spatial diffeomorphisms:

$$\delta_w \phi = \{\phi, D[w]\} = -(w\phi)' \quad , \quad \delta_w p = \{p, D[w]\} = -wp'$$

H-bracket:

$$\{H[N], H[M]\} = D[\beta(p)(N'M - NM')]$$

with $\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2$.

Lorentzian-type hypersurface deformations for $f(p) = p^2$.

Signature change

"Holonomy" modifications $f(p)$ bounded, $\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2$ negative around local maxima, such as

$$\{H[N], H[M]\} = D[-(N'M - NM')]$$

- $\rightarrow$ Gauge transformations from modified bracket consistent with effective line element of Euclidean signature.
- $\rightarrow$ Elliptic field equations. Consistent space(-time) structure but no deterministic evolution through high curvature.

Several gravity models available and being analyzed. [Brahma, Reyes; Barrau, Cailleteau, Grain, Mielczarek; Aruga, Ben Achour, Lamy, Liu, Noui]

Could generalize higher-curvature effective actions.

Status of covariance in loop models

→ Non-local effects could evade restrictions from generalized dilaton gravity.

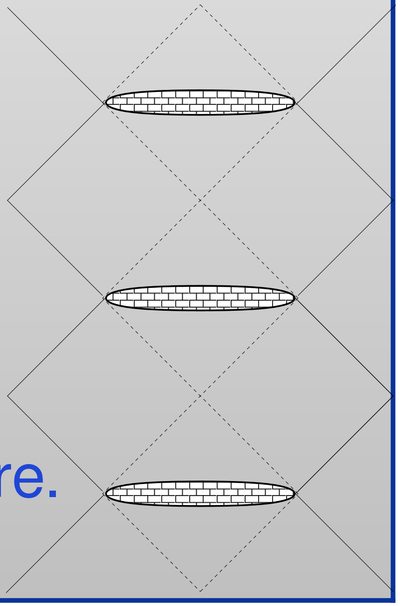
But modified minisuperspace dynamics implicitly assumes locality.

Non-locality often pathological.

→ Non-Riemannian space-time structure ($\beta \neq \pm 1$) may be consistent.
Unknown in general, other than perturbative inhomogeneity or spherical symmetry.

In some cases, Riemannian after field redefinition.

Have to deal with signature change at high curvature.



Problem of time

Friedmann constraint with scalar field ϕ, momentum p_ϕ:

$$\begin{aligned} C &= \frac{1}{2}\frac{p_\phi^2}{V} + \frac{1}{2}Vm^2\phi^2 - 6\pi G V p_V^2 \\ &= \left(-p_\phi + H(V,p_V,\phi)\right)\left(p_\phi + H(V,p_V,\phi)\right) \end{aligned}$$

implies relational Hamiltonian

$$H(V,p_V,\phi) = \pm\sqrt{12\pi G V^2(p_V^2 - m^2\phi^2)}$$

for evolution of (V, p_V) with respect to ϕ.

Simplifies for popular case of $m=0$ (deparameterization).

Not generic as fundamental field ϕ: no mass or self-interactions.

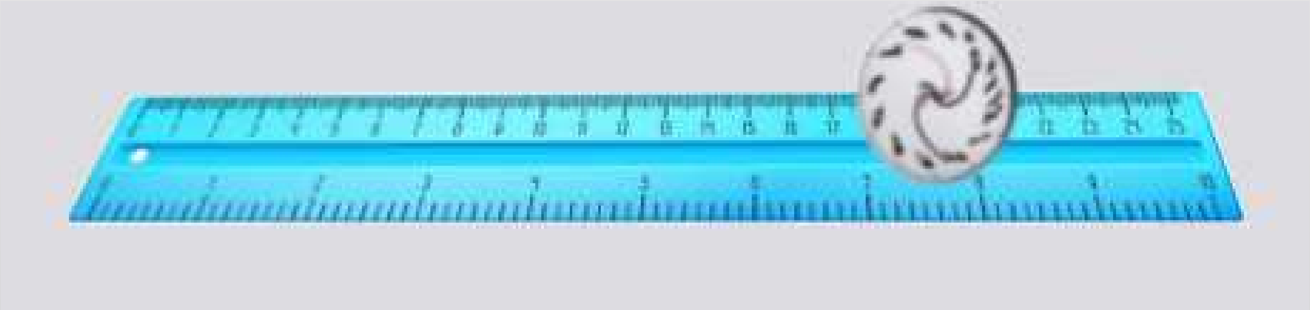
Unitarity if ϕ might run back and forth for $m \neq 0$?

Global evolution

[Amaral, MB: Ann. Phys. 388C (2018) 241]

Choosing single factor $C = \pm p_\phi + H = 0$ implies $\mathrm{d}\phi/\mathrm{d}t = \pm 1$, presupposes direction of ϕ.

Construct monotonic effective time τ by unwinding periodic clock ϕ if $m \neq 0$.



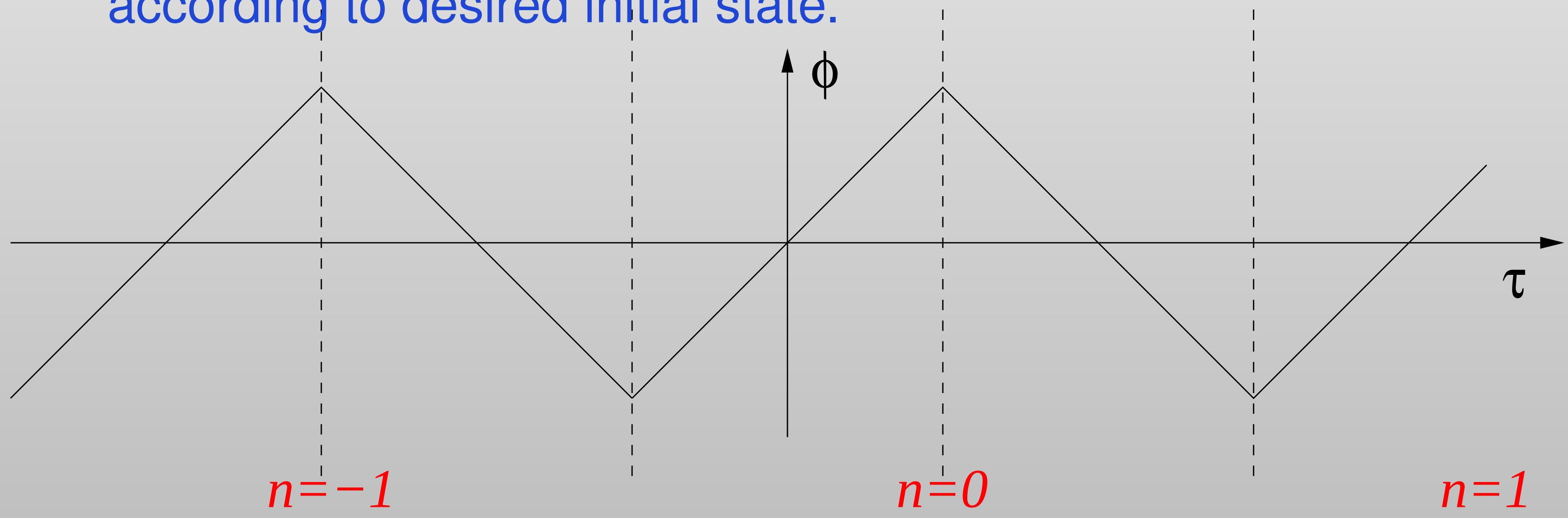
Schrödinger equation with respect to τ:

$$i\hbar \frac{\partial \psi}{\partial \tau} = i\hbar \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \frac{\partial \psi}{\partial \phi} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \hat{H} \psi$$

Stability for τ-Hamiltonian: $(\mathrm{d}\phi/\mathrm{d}\tau)H$ does not change sign.

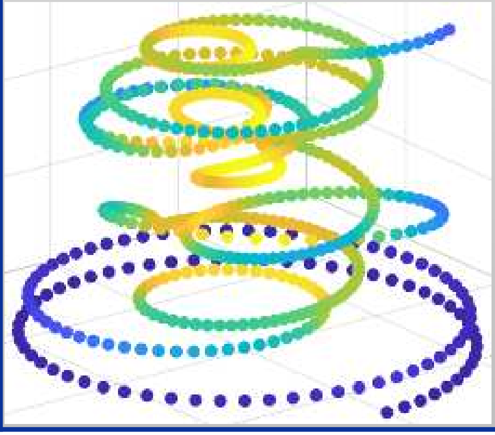
Unwinding time: $C = -p_\phi^2 - \lambda^2\phi^2 + H_0^2$

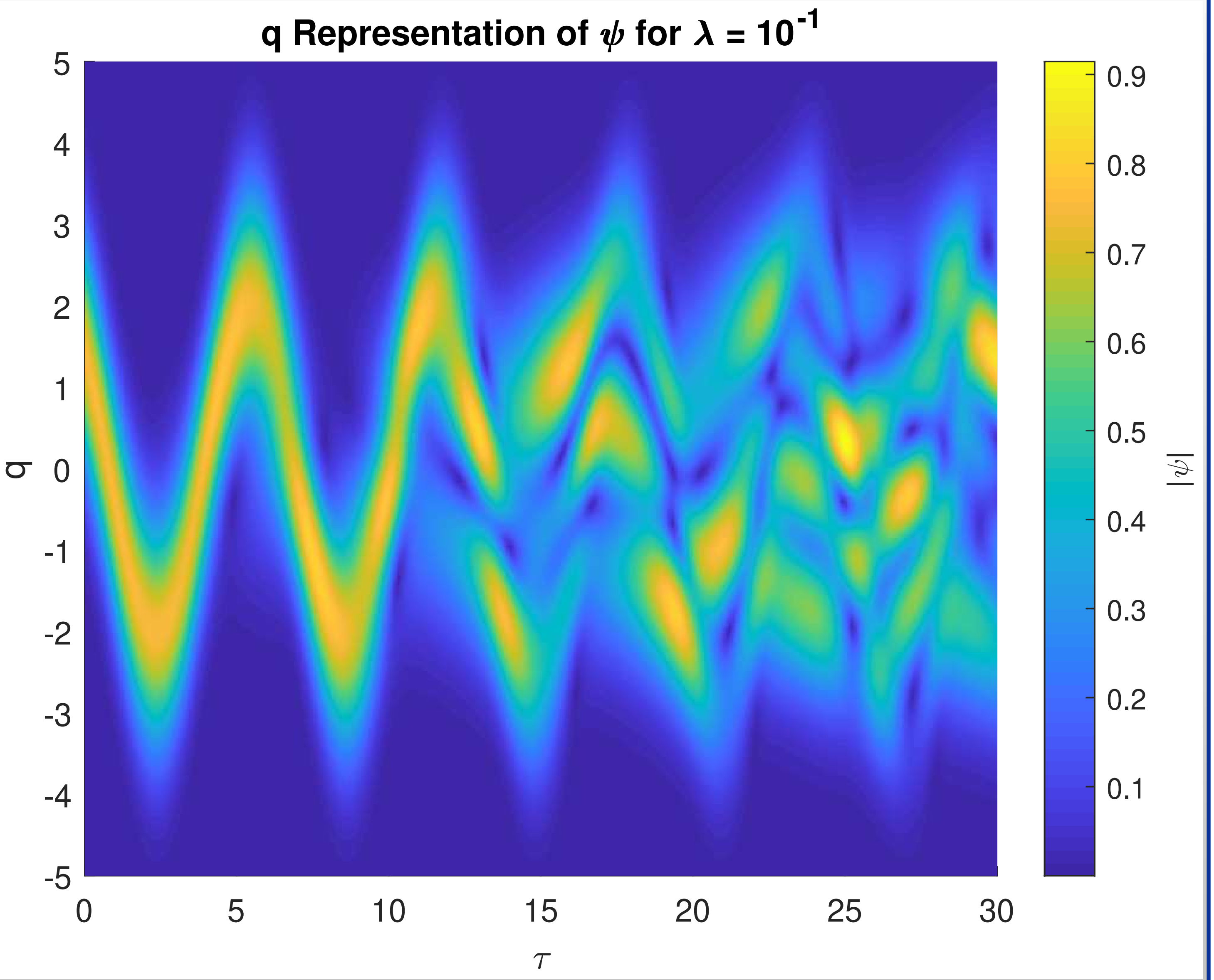
- Solve $i\hbar\partial\psi_k/\partial\phi = \pm\sqrt{E_k^2 - \lambda^2\phi^2}\psi_k$ where $\hat{H}_0\psi_k = E_k\psi_k$.
- For given clock Hamiltonian, construct piecewise linear $\phi(\tau)$ alternating between $\phi_+(\tau) = \tau + A_+$ and $\phi_-(\tau) = -\tau + A_-$.
- Insert $\phi(\tau)$ in $\phi_k(\phi)$. Alternate signs for constant $\mathrm{sgn}(\mathrm{d}\phi/\mathrm{d}\tau)H$, where $H = \pm\sqrt{H_0^2 - \lambda^2\phi^2}$.
- Obtain $\psi(\tau)$ as superposition of $\hat{H}_0$-stationary states $\psi_k(\tau)$ according to desired initial state.



Lost coherence: Harmonic oscillator

$$\hat{H}_0 = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$$

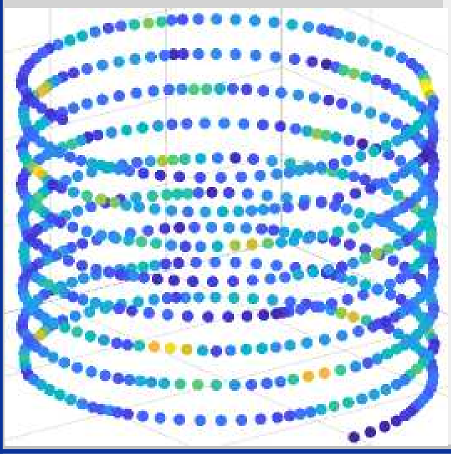


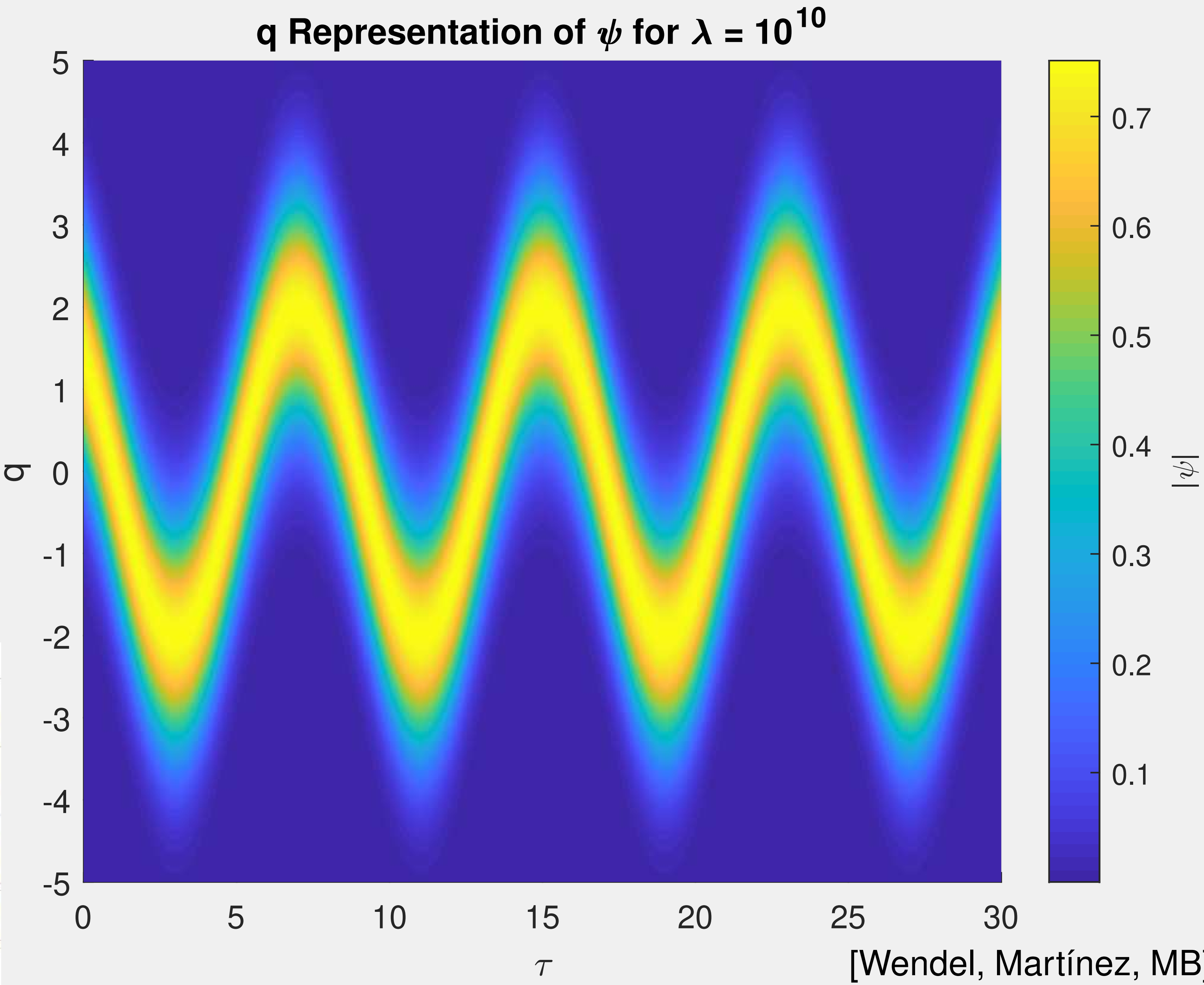


[Wendel, Martínez, MB: PRL 124 (2020) 241301]

Regained coherence: short clock period

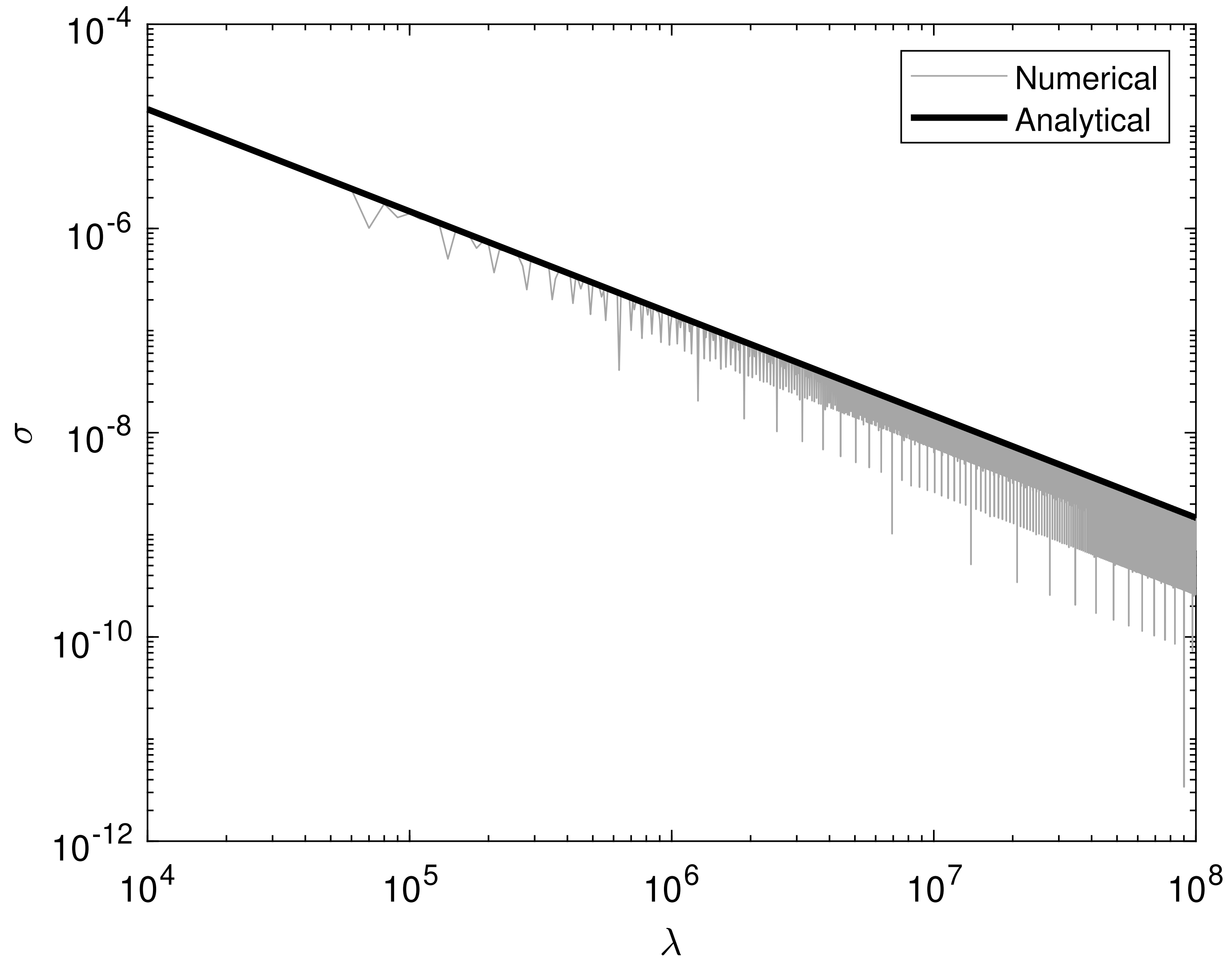
$\hat{H}_0 = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$





[Wendel, Martínez, MB]

Standard deviation of system periods



Upper bound

$T_{\mathrm{C}} \approx 9.7\sigma T_{\mathrm{S}}$ based on theory of fundamental clock.

If T_{S} can be measured with accuracy σ, then T_{C} cannot be greater than $9.7\sigma T_{\mathrm{S}}$.

Latest atomic clocks: $\sigma \approx 10^{-19}$ at system period of $T_{\mathrm{S}} \approx 2\,\mathrm{fs}$.

[Campbell et al. *Science* 358 (2017) 98]

Therefore,

$$T_{\mathrm{C}} < 2 \cdot 10^{-33}\,\mathrm{s} \approx 0.5 \cdot 10^{11} t_{\mathrm{P}}$$

Smallest direct measurement: Photon travel time across hydrogen molecule, $247 \cdot 10^{-21}\mathrm{s}$. [Grundmann *et al. Science* 370 (2020) 339]

Particle accelerators probe spatial distances of $10^{-20}\mathrm{m} \approx 10^{16}\ell_{\mathrm{P}}$. Corresponds to $\approx 10^{-28}\,\mathrm{s}$.

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Implications

- Deparameterization ($\lambda = 0$) approximates an oscillating clock as long as system period larger than clock period.
- New effects when system scales are Planckian.
- Background independence: Space-time structure to be derived. Inserting modified solutions in classical-type line elements not always consistent.
- Hypersurface deformations in space-time provide powerful algebraic means to analyze space-time without strong assumptions about geometry.
- No-go results in models of loop quantum gravity:
  Do not require specific form of modifications,
  use only qualitative features related to discreteness.
  Largely independent of specific approach.