The emergence of space-time in a matrix model formulation of superstring

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Yukawa Institute for Theoretical Physics, Kyoto University

Ref.) J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077 [arXiv:1904.05919 [hep-th]]
Anagnostopoulos-Azuma-Hatakeyama-Hirasawa-Ito-J.N.-Tsuchiya-Papadoudis, work in progress

0. Introduction

Recent excitement in Lorentzian Quantum Gravity

$$Z = \int \mathcal{D}g_{\mu\nu}\, e^{iS[g]} \qquad\qquad S = \int d^4x\sqrt{-g}\left(\frac{1}{G_\mathrm{N}}R - \Lambda\right)$$

Schematically, one deforms the integration contour as

$g_{\mu\nu}(x;\sigma) \in \mathbb{C}$ σ:deformation parameter

$$\frac{dg_{\mu\nu}(x;\sigma)}{d\sigma} = i\,\overline{\frac{\partial S}{\partial g_{\mu\nu}(x;\sigma)}} \qquad g_{\mu\nu}(x;\sigma=0) = g_{\mu\nu} \in \mathbb{R}$$

($\sigma \to \infty$ gives Lefschetz thimbles.) Picard-Lefschetz theory

Lorentzian QG can thus be made **well-defined**. (Cauchy's theorem).

mini-superspace models **(Lehners, Hartle, Hertog, Turok, …)**
simplicial Lorentzian QG based on Regge calculus **(Ding Jia 2110.05953)**

What about the non-renormalizability ?

QG is perturbatively **non-renormalizable.** $G_{\mathrm{N}} = [M^{-2}]$

This issue may be overcome in a nonperturbative approach such as simplicial QG. **(Renata Loll's talk)**

However, it could be that the non-renormalizability calls for a fully UV complete theory of QG such as superstring theory.

The idea of contour deformation enables us to define superstring theory in (9+1)-dimensions nonperturbatively.

The IKKT matrix model (type IIB matrix model)

a conjectured nonperturbative formulation of superstring theory

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

Ishibashi, Kawai, Kitazawa, Tsuchiya, ('96)

SO(9,1) symmetry

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

Lorentzian metric $\eta = \mathrm{diag}(-1, 1, \cdots, 1)$
is used to raise and lower indices.

Where is "space-time" ?

Ishibashi, Kawai, Kitazawa, Tsuchiya, ('96)

the IKKT action ⟵ super Yang-Mills action in 10d

$A_\mu\,,\ \Psi_\alpha$ Dimensional Reduction $A_\mu(x)\,,\ \Psi_\alpha(x)$

16 supersymmetries inherited from SYM

extra symmetries $\Psi_\alpha \mapsto \Psi_\alpha + \xi_\alpha \mathbb{1}$

enhanced to 32 supersymmetries (consistent with 10D SUGRA)

$$\{Q_\alpha, \bar{Q}_\beta\} = (\Gamma_\mu)_{\alpha\beta} P_\mu$$

translation $A_\mu \mapsto A_\mu + c_\mu \mathbb{1}$

The eigenvalues of A_μ are shifted by the translation.

space-time = the "eigenvalue distribution" of A_μ .

(non-commutative geometry)

The Euclidean IKKT model

"Wick rotation" : $A_0 = -iA_{10}$ $\quad F_{\mu\nu} = -i[A_\mu, A_\nu]$

Hermitian $\quad$ Hermitian

$$S_{\mathrm{b}} = \frac{1}{4g^2}\mathrm{tr}\,(F_{\mu\nu})^2$$ positive semi-definite

$$Z = \int dA\, e^{-S_{\mathrm{b}}}\, \mathrm{Pf}\mathcal{M}(A)$$

The integration converges.

Krauth-Nicolai-Staudacher ('98)
Austing-Wheater ('01)

SO(10) symmetry

SSB

SO(3)

Anagnostopoulos-Azuma-Ito-JN-Okubo-Papadoudis, JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]

Problems: The emergent space-time has Euclidean signature.
We do **not** obtain SO(4) symmetry.

Relevance of this model to our world is obscure…

The Lorentzian IKKT model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]

partition function

$$Z = \int dA\, d\Psi\, e^{i(S_{\rm b}+S_{\rm f})} = \int dA\, e^{iS_{\rm b}} \mathrm{Pf}\mathcal{M}(A)$$

polynomial in A

$$S_{\rm b} = \frac{1}{4g^2}\left\{ -2\,\mathrm{tr}(F_{0i})^2 + \mathrm{tr}(F_{ij})^2 \right\}$$

pure phase factor

The integration is **not** absolutely convergent.

This choice is motivated from the viewpoint of the worldsheet theory.

c.f.) $$S = \int d^2\xi \sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu,\Psi\}\right)$$

$$\xi_0 \equiv -i\xi_2$$

(The worldsheet coordinates should also be Wick-rotated.)

Plan of the talk

0. Introduction
1. How can one define the Lorentzian IKKT model ?
2. Numerical evidence for the emergence of space-time
3. Summary and discussions

1. How can one define the Lorentzian IKKT model ?

Applying the idea of contour deformation

(Yuhma Asano '19, private communication)

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi\sigma}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi\sigma}\tilde{A}_i \end{cases}$$

Hermitian

σ : deformation param. $(0 < \sigma \leq 1)$

$$e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})}$$

$$S(\tilde{A}) \sim 2 e^{i\frac{\pi}{2}(1-\sigma)} \mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-\sigma)} \mathrm{tr}\,(\tilde{F}_{ij})^2 \qquad \tilde{F}_{\mu\nu} = -i[\tilde{A}_\mu, \tilde{A}_\nu]$$

(real part) > 0 for $0 < \sigma \leq 1$

Lorentzian Euclidean

$\boxed{\mathrm{Re}\, S(\tilde{A}) \geq 0}$

well defined !

According to Cauchy's theorem,
$\langle \mathcal{O}(\mathrm{e}^{-i\frac{3}{8}\pi\sigma}\tilde{A}_0, \mathrm{e}^{i\frac{1}{8}\pi\sigma}\tilde{A}_i)\rangle_\sigma$ is σ-independent.

One can define the Lorentzian model by taking the $\sigma \to +0$ limit, but then

$$\langle \mathcal{O}(A_0, A_i)\rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}}\tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}}\tilde{A}_i)\rangle_{\mathrm{E}} \ .$$

Lorentzian IKKT thus defined is equivalent to Euclidean IKKT !

A new idea is definitely needed for the emergence of a real Lorentzian space-time!

Introducing a Lorentz-invariant mass term

$$S_{\mathrm{b}} = -\frac{1}{4}N\beta\,\mathrm{tr}\Big([A_\mu, A_\nu][A^\mu, A^\nu]\Big) - \frac{1}{2}N\gamma\,\mathrm{tr}\,(A_\mu A^\mu)$$

$$= \frac{1}{4}N\beta\left\{-2\mathrm{tr}(F_{0i})^2 + \mathrm{tr}(F_{ij})^2\right\} + \frac{1}{2}N\gamma\left\{\mathrm{tr}\,(A_0)^2 - \mathrm{tr}\,(A_i)^2\right\}$$

$\beta > 0\,,\ \ \gamma > 0$

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi\sigma}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi\sigma}\tilde{A}_i \end{cases}$$

$$e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})}$$

(real part) > 0 for $0 < \sigma \leq 1$

$$S(\tilde{A}) = \frac{1}{4}N\beta\left\{2e^{i\frac{\pi}{2}(1-\sigma)}\mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-\sigma)}\mathrm{tr}\,(\tilde{F}_{ij})^2\right\}$$
$$+\frac{1}{2}N\gamma\left\{e^{-i\frac{\pi}{2}(1+\frac{3\sigma}{2})}\mathrm{tr}\,(\tilde{A}_0)^2 + e^{i\frac{\pi}{2}(1+\frac{\sigma}{2})}\mathrm{tr}\,(\tilde{A}_i)^2\right\}$$

(real part) < 0 for $0 < \sigma \leq 1$

One cannot deform the contour in this case...

In particular, the equivalence to the Euclidean IKKT is lost !

A regularization

$$S_{\mathrm{b}} = \frac{1}{4} N\beta \left\{ -2\mathrm{tr}(F_{0i})^2 + \mathrm{tr}(F_{ij})^2 \right\} + \frac{1}{2} N\gamma \left\{ e^{i\epsilon} \mathrm{tr}\,(A_0)^2 - e^{-i\epsilon} \mathrm{tr}\,(A_i)^2 \right\}$$

This model can be made well defined by the contour deformation :

$$\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi\sigma} \tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi\sigma} \tilde{A}_i \end{cases} \quad \text{as far as } \frac{3}{4}\pi\sigma < \epsilon .$$

The $\sigma \to 0$ limit should be taken before the $\epsilon \to 0$ limit.

The emergence of Lorentzian space-time seems possible.

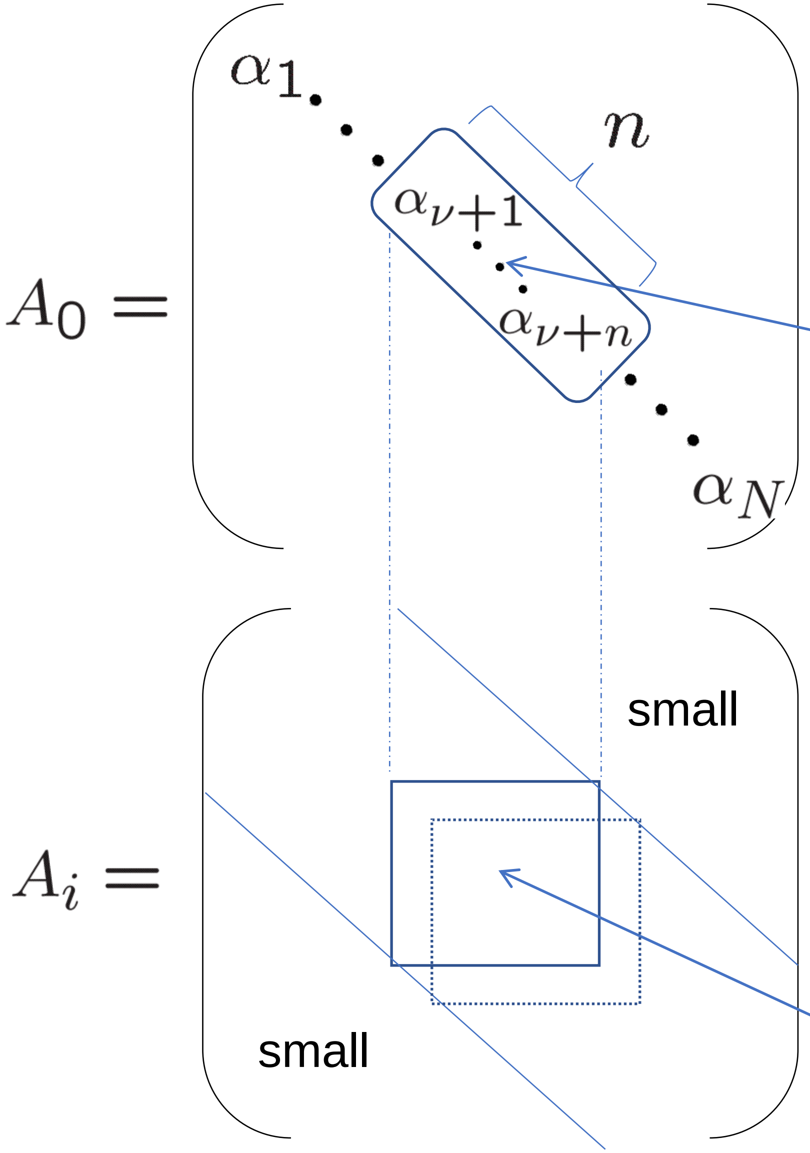
We provide numerical evidence for this
by performing <u>complex Langevin simulations</u>.

J.N. and Asato Tsuchiya, JHEP 1906 (2019) 077
[arXiv:1904.05919 [hep-th]]

2. Numerical evidence for the emergence of space-time

Extracting time-evolution from the Lorentzian model

Kim-J.N.-Tsuchiya PRL 108 (2012) 011601 [arXiv:1108.1540]



definition of "time"

$$\bar{\alpha}_\nu = \frac{1}{n}\sum_{i=1}^{n} \alpha_{\nu+i} \in \mathbf{C}$$

$$t_\rho = \sum_{\nu=1}^{\rho} |\bar{\alpha}_{\nu+1} - \bar{\alpha}_\nu|$$

A_i has a band diagonal structure

non-trivial dynamical property

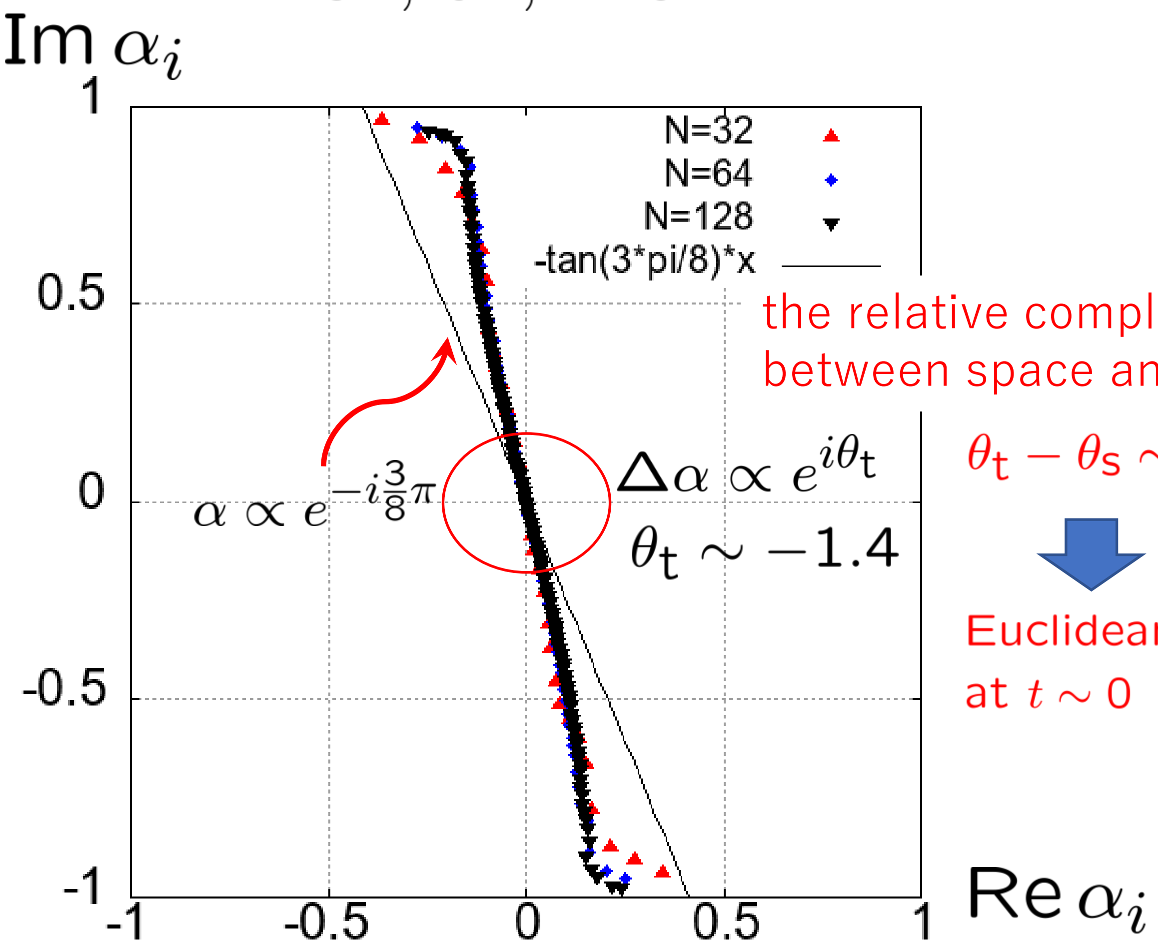
cf.) Klinkhamer, arXiv: 2105.05831 [hep-th]

the state of the universe $\bar{A}_i(t)$ at time t

Results for the bosonic model

$$\beta = 0.10625,\ \gamma = 1.0$$
$$N = 32,\ 64,\ 128$$

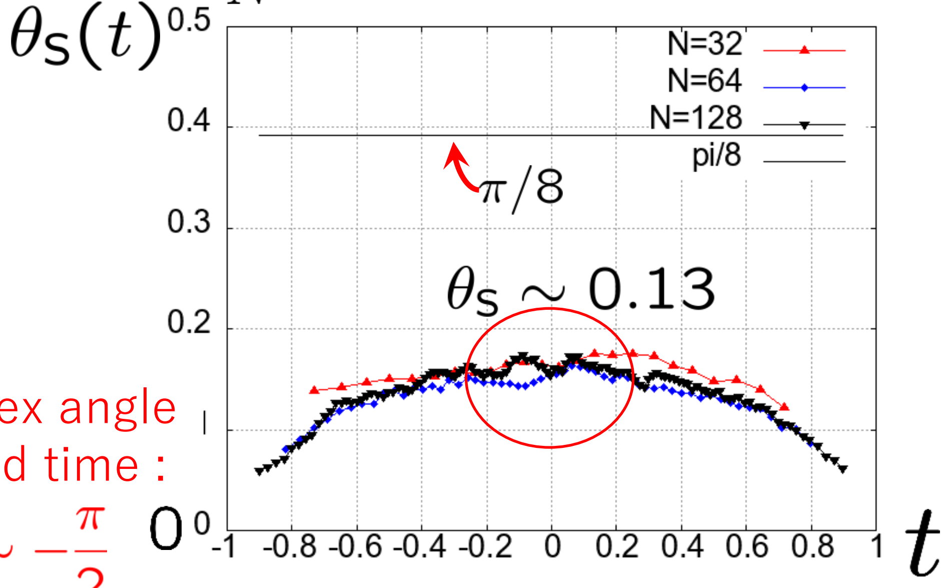
$$\frac{1}{N^2}\,\mathrm{tr}\,\{\bar{A}_i(t)\}^2 = R^2(t)\,e^{2i\theta_s(t)}$$

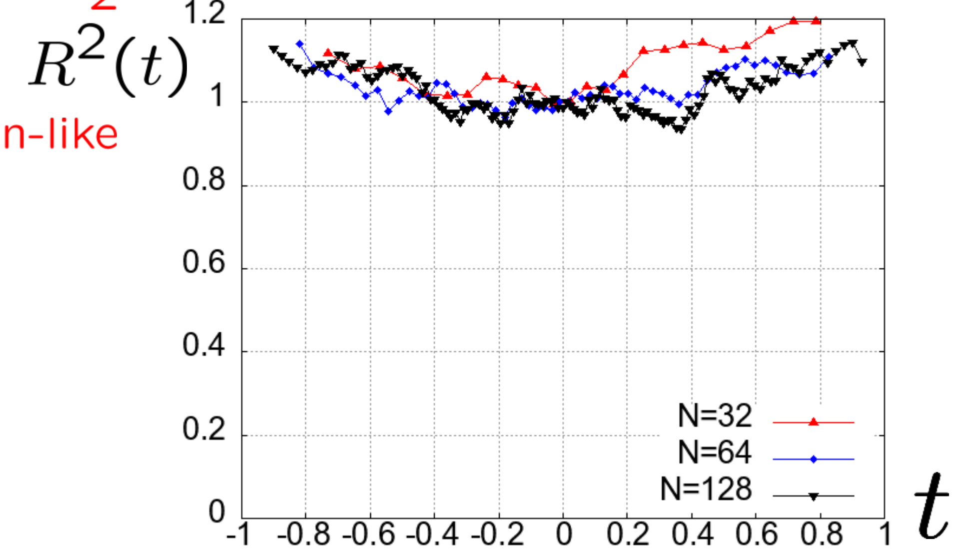


the relative complex angle between space and time :

$$\theta_t - \theta_s \sim -\frac{\pi}{2}$$

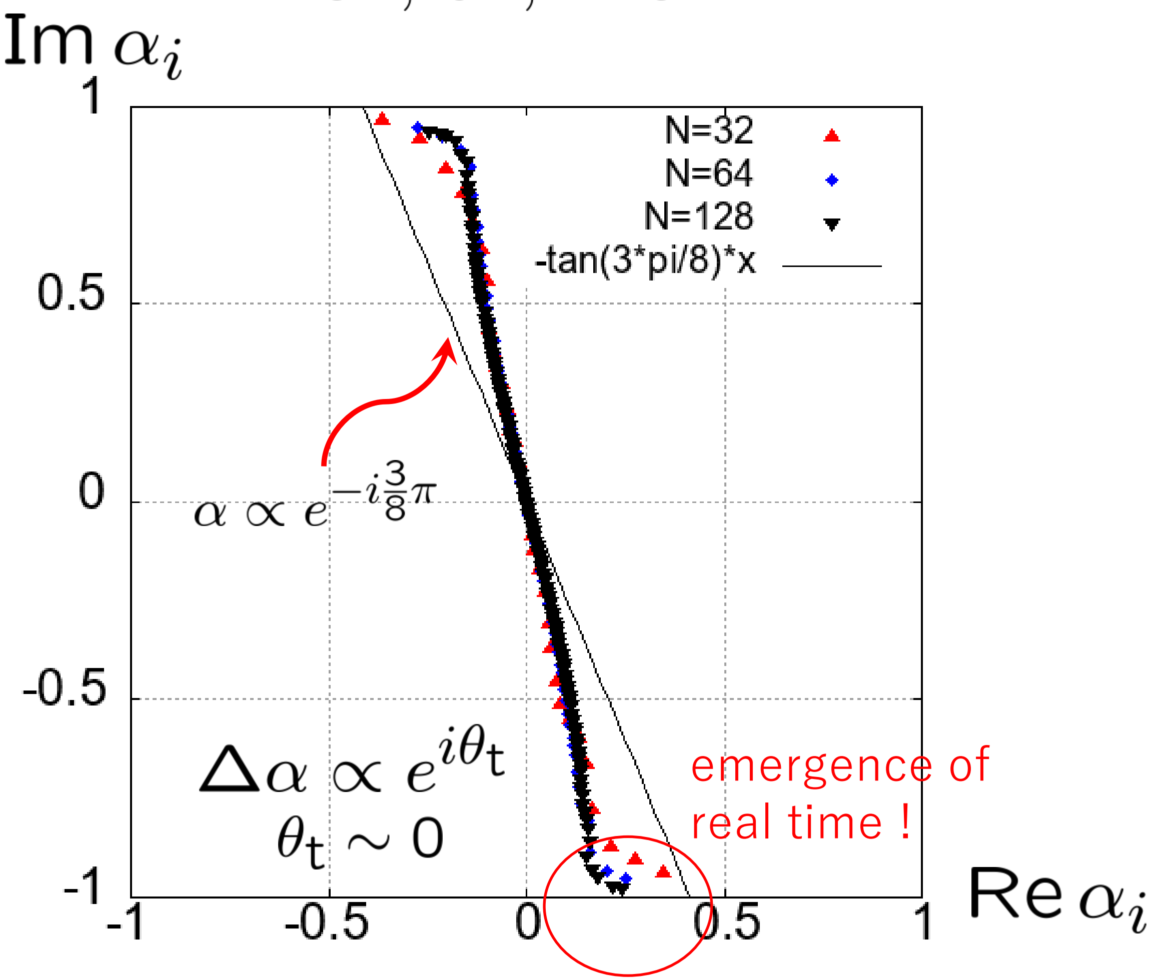
Euclidean-like at $t \sim 0$



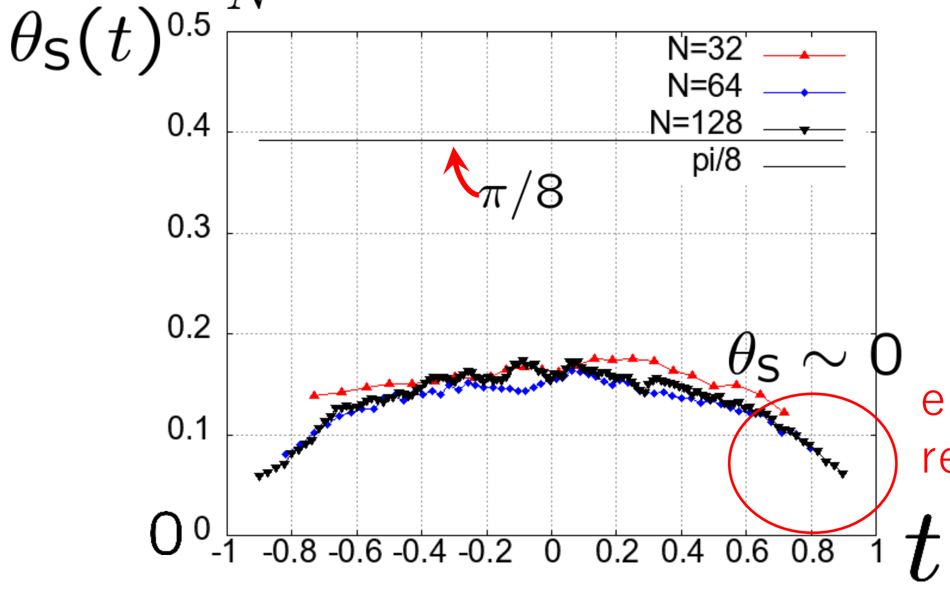


Results for the bosonic model

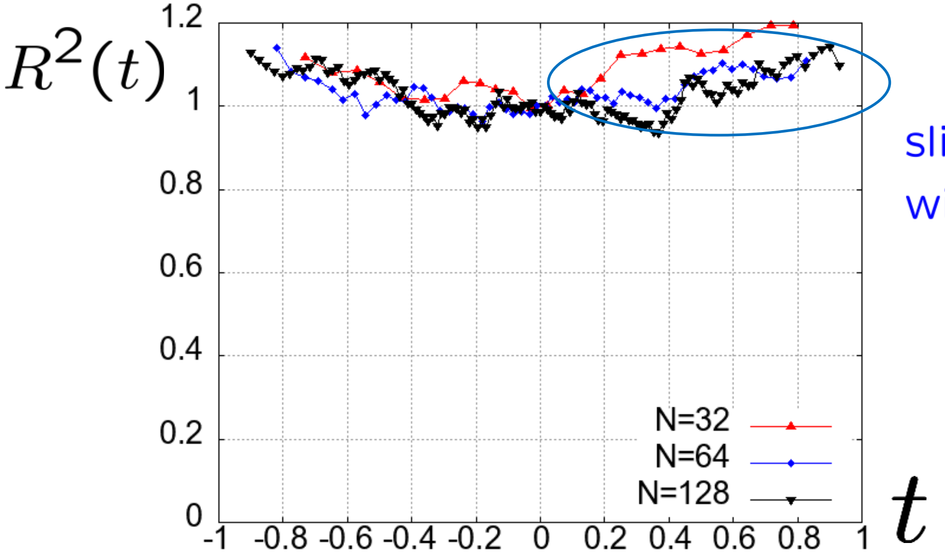
$\beta = 0.10625,\ \gamma = 1.0$
$N = 32,\ 64,\ 128$



$$\frac{1}{N^2}\,\mathrm{tr}\,\{\bar{A}_i(t)\}^2 = R^2(t)\, e^{2i\theta_s(t)}$$



emergence of real space !



slightly expanding with t

3. Summary and discussions

Summary

- IKKT model ('96)
  conjectured to be a nonperturbative formulation of superstring

- Euclidean IKKT : well defined as it is.
  SSB of SO(10) → SO(3) occurs due to the effects of fermions.

- Lorentzian IKKT can be made well defined by contour deformation,
  but then $\langle \mathcal{O}(A_0, A_i) \rangle_{\mathrm{L}} = \langle \mathcal{O}(\mathrm{e}^{-i\frac{3\pi}{8}}\tilde{A}_0, \mathrm{e}^{i\frac{\pi}{8}}\tilde{A}_i) \rangle_{\mathrm{E}}$.

- The emergence of real Lorentzian space-time requires a new idea.

- Introducing a Lorenz-inv. mass term looks promising.
  Contour deformation to the Euclidean model is no more possible.
  Simulations suggest the emergence of real Lorentzian space-time.

Some comments on the Lorentz inv. mass term

- The same mass term was used to obtain classical solutions of the Lorentzian IKKT model with expanding behavior.

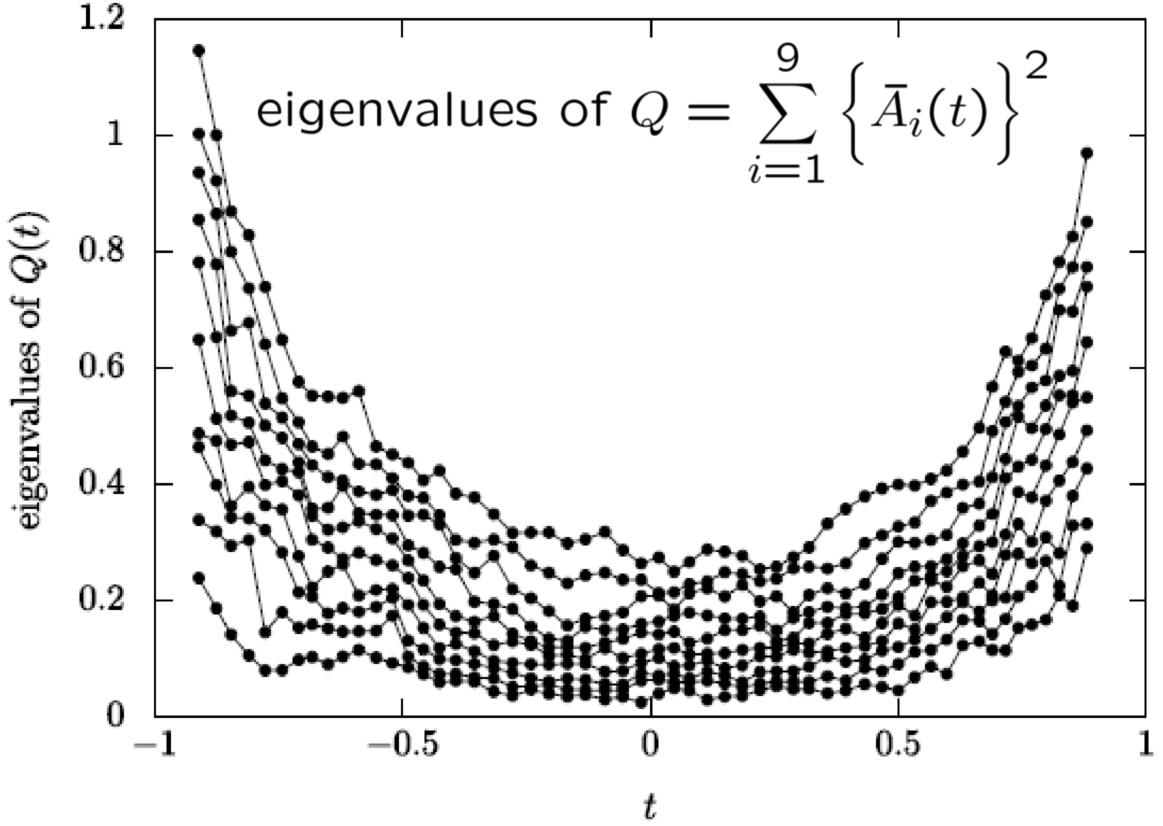
  Hatakeyama-Matsumoto-J.N.-Tsuchiya-Yosprakob, PTEP 2020 (2020) 4, 043B10, 1911.08132

$$[A^{\nu}, [A_{\nu}, A_{\mu}]] - \gamma A_{\mu} = 0$$

- The mass term with the right signature was crucial.

- Other interesting solutions are obtained with the mass term.

  Sperling-Steinacker,
  JHEP 07 (2019) 010, 1901.03522 [hep-th]



Discussions

- So far, bosonic model (the fermionic matrices are omitted). Scaling behavior is observed for $N = 32,\ 64,\ 128$.

- Does the emergent time extend to infinity in the $N \to \infty$ limit ? Does the emergent space become 3-dimensional and expand ?

- SUSY is expected to play a crucial role.

- The eigenvalues of A_μ in bosonic model attract each other strongly due to quantum effects. Such effects are largely suppressed by SUSY.

  Aoki, Iso, Kawai, Kitazawa, Tada ('99)

- The SSB of SO(9,1) to SO(3) may well occur at $t \sim 0$ due to fermions as in the case of the Euclidean model.

4. Backup slides

Introducing a Lorentz-invariant mass term (2)

$$
\begin{aligned}
S_{\mathrm{b}} &= -\frac{1}{4}N\,\mathrm{tr}\Big([A_\mu, A_\nu][A^\mu, A^\nu]\Big) + \frac{1}{2}Nm^2\mathrm{tr}\,(A_\mu A^\mu) \\
&= \frac{1}{4}N\beta\left\{ -2\mathrm{tr}(F_{0i})^2 + \mathrm{tr}(F_{ij})^2 \right\} + \frac{1}{2}Nm^2\left\{-\mathrm{tr}\,(A_0)^2 + \mathrm{tr}\,(A_i)^2\right\}
\end{aligned}
$$

$$
\begin{cases} A_0 = \mathrm{e}^{-i\frac{3}{8}\pi u}\tilde{A}_0 \\ A_i = \mathrm{e}^{i\frac{1}{8}\pi u}\tilde{A}_i \end{cases}
\qquad e^{iS_{\mathrm{b}}(A)} = e^{-S(\tilde{A})}
$$

$$
\begin{aligned}
S(\tilde{A}) &= \frac{1}{4}N\left\{2e^{i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{0i})^2 + e^{-i\frac{\pi}{2}(1-u)}\mathrm{tr}\,(\tilde{F}_{ij})^2\right\} \\
&\quad + \frac{1}{2}Nm^2\left\{e^{i\frac{\pi}{2}(1-\frac{3u}{2})}\mathrm{tr}\,(\tilde{A}_0)^2 + e^{-i\frac{\pi}{2}(1-\frac{u}{2})}\mathrm{tr}\,(\tilde{A}_i)^2\right\}
\end{aligned}
$$

(real part) > 0 for $0 < u \leq 1$

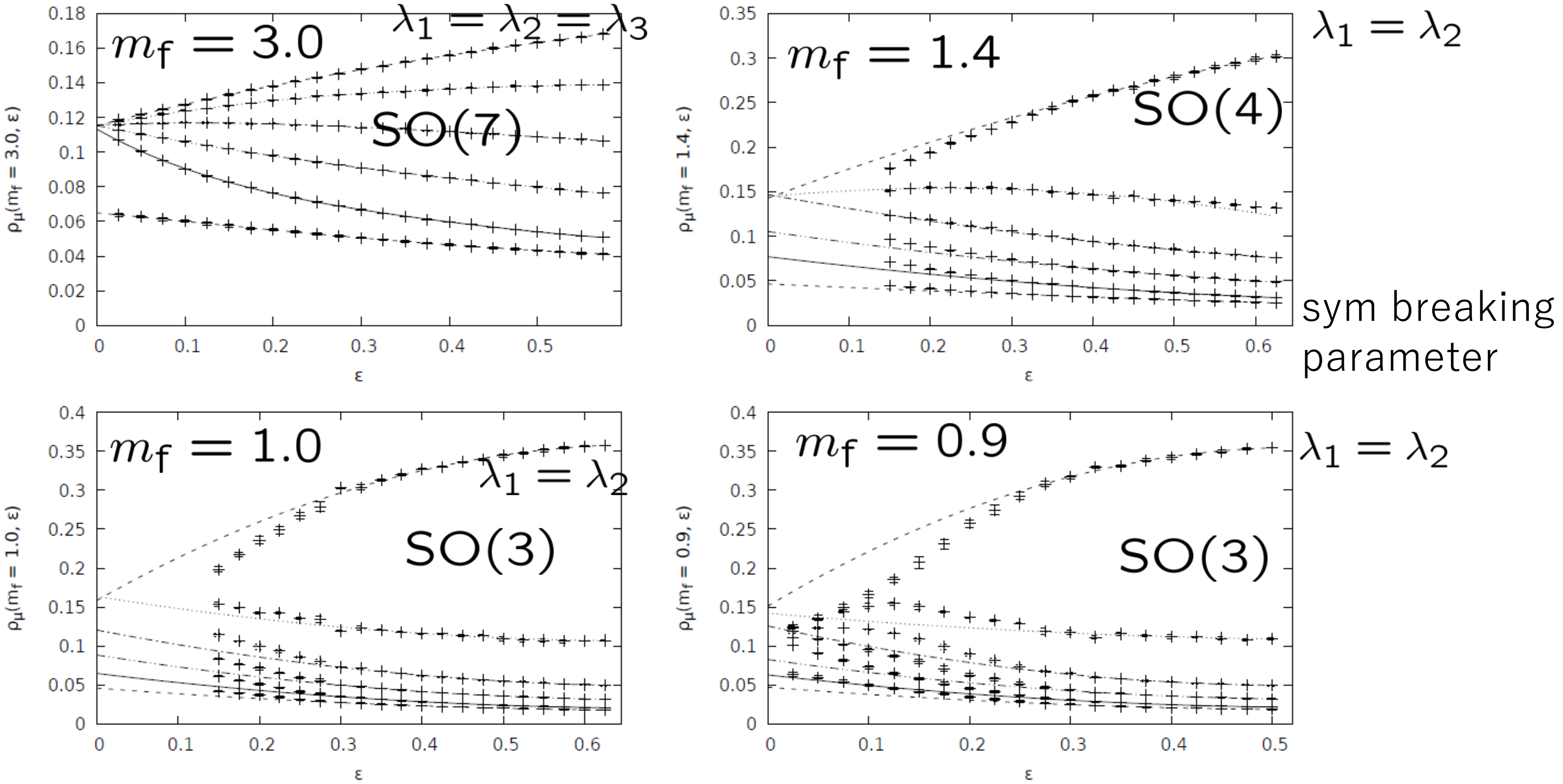
The situation does not seem to change drastically.

What happens if we flip the sign of the mass term ? $m^2 \mapsto -m^2$

Results for the Euclidean IKKT model $SO(10) \overset{SSB}{\rightarrow} SO(3)$

SSB of SO(10) observed by decreasing the deformation parameter m_f .

ten eigenvalues of $T_{\mu\nu} = \frac{1}{N}\text{tr}\left(A_\mu A_\nu\right)$



Anagnostopoulos-Azuma-Ito-JN-Okubo-Papadoudis,
JHEP 06 (2020) 069 , arXiv: 2002.07410 [hep-th]