De Sitter Space from first principles

YITP Workshop,
8 Nov 2021

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Preview

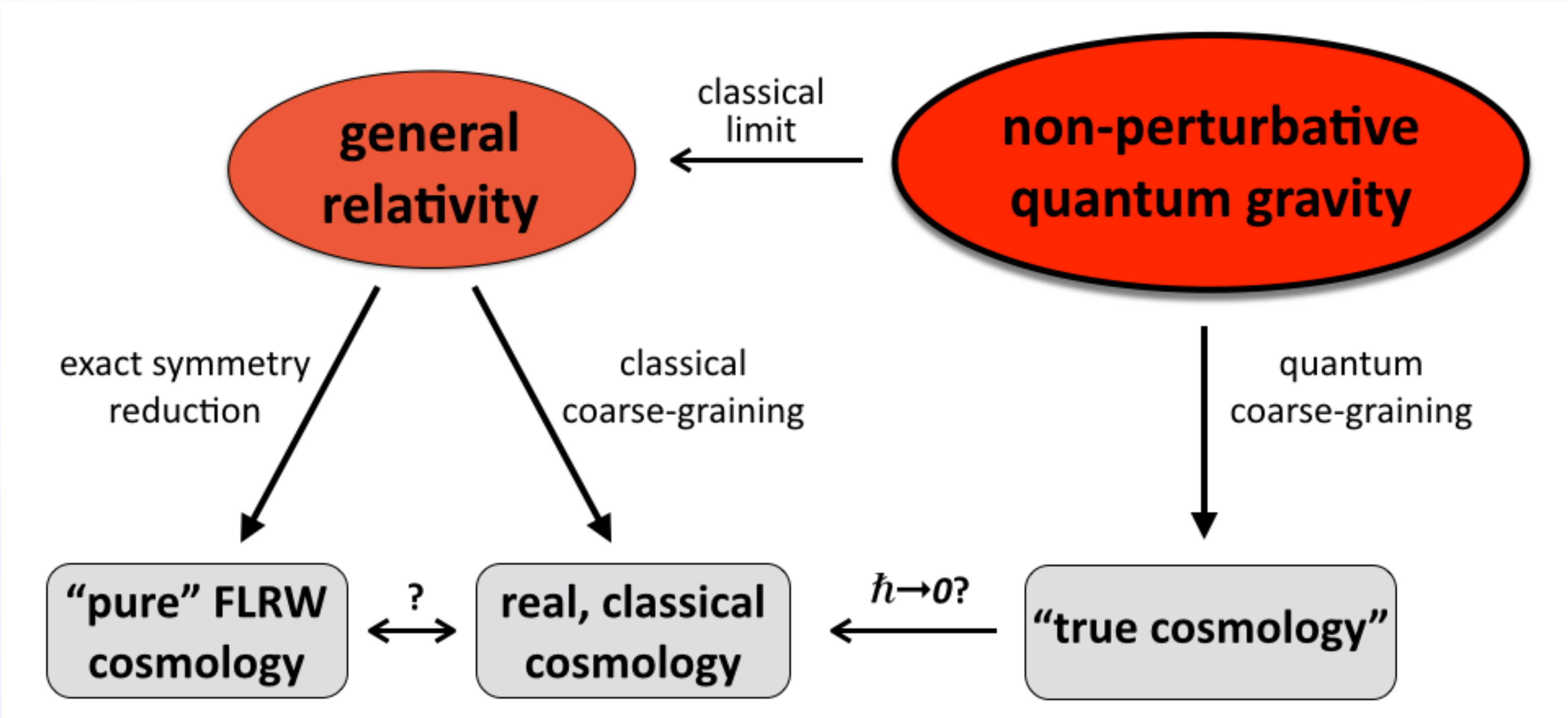
My talk will take a quantum-gravitational perspective on (quantum) cosmology, coming from a nonperturbative formulation of the *full* theory, *without* making any assumptions on symmetry (isotropy, homogeneity, or other). The key result, obtained in Causal Dynamical Triangulations (CDT) quantum gravity, is the dynamical "emergence" of a quantum universe with classical de Sitter properties, which one would like to relate to the physics of the early universe.

My talk today will discuss

1. intro & motivation
2. nonperturbative quantum gravity (we can calculate!);
3. observables as key to Planckian physics;
4. curvature and other evidence for "de Sitter";
5. outlook.

Quantum gravity perspective on cosmology

$[g_{\mu\nu}(x)]$ [microscopic d.o.f., all scales]



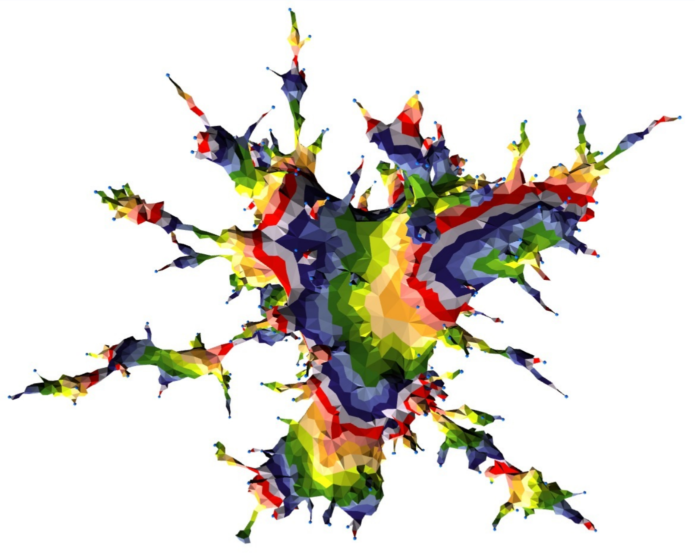
$[a(t)]$ [large-scale properties]

- FLRW spacetimes have no fundamental status in nature, but are (surprisingly?) robust effective models of gravity on large scales
- Are there implications of NPQG for cosmology that cannot be explained by classical GR alone? Relation with quantum cosmology?
- I will present some evidence from CDT for the "FLRW paradigm"

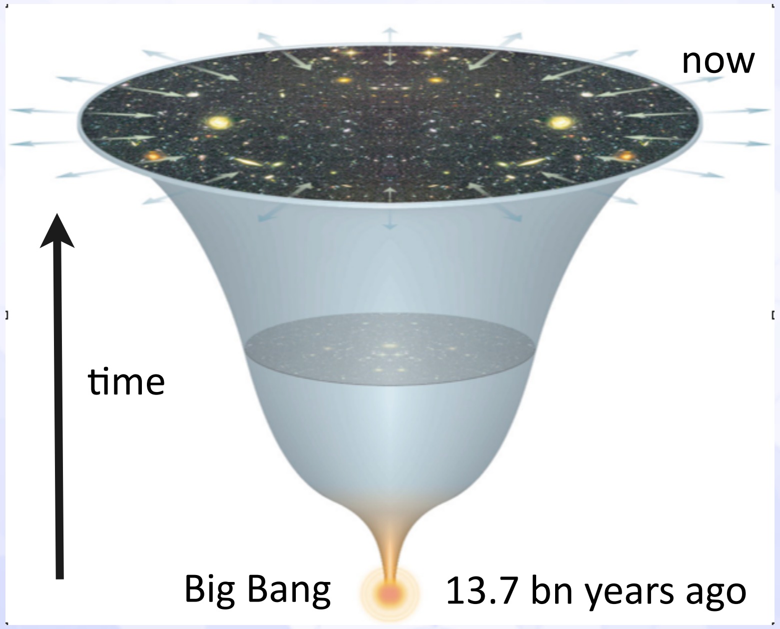
L. Glaser & RL, "CDT and Cosmology", Compt. Rend. Phys. 18 (2017) 265 [arXiv: 1703.08160]

Questions that quantum gravity should answer

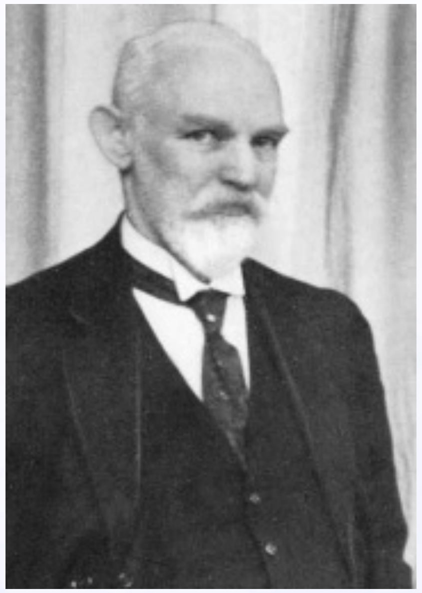
- What becomes of gravity and spacetime at the Planck scale ℓ_{Pl} : spacetime foam? wormholes? discrete "atoms of spacetime"?



quantum space, from a 2D toy model (©Budd)



- Are space and time fundamental or merely emergent on macroscopic scales?
- Was the very early universe created by some nonperturbative quantum dynamics?
- Is there a quantum-gravitational origin of structure formation?
- Can we predict observable, large-scale quantum gravity effects that are within the reach of upcoming gravitational wave detectors?



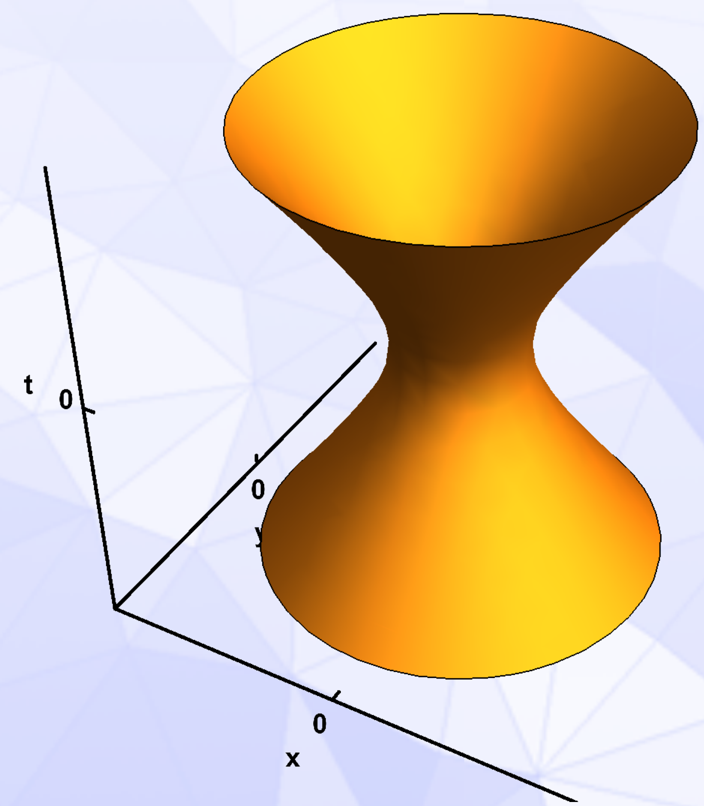
Willem de Sitter

Introducing de Sitter space

De Sitter space is a classical solution to the Einstein equations with a positive cosmological constant Λ, describing a constantly curved universe which expands exponentially in time. It is the Lorentzian analogue of a four-sphere, and has the metric

$$ds^2 = -dt^2 + c^2 \cosh^2(t/c)\, d\Omega^2_{(3)}, \;\; \Lambda = 3/c^2, \;\; d\Omega^2_{(3)} \sim \text{metric on unit } S^3$$

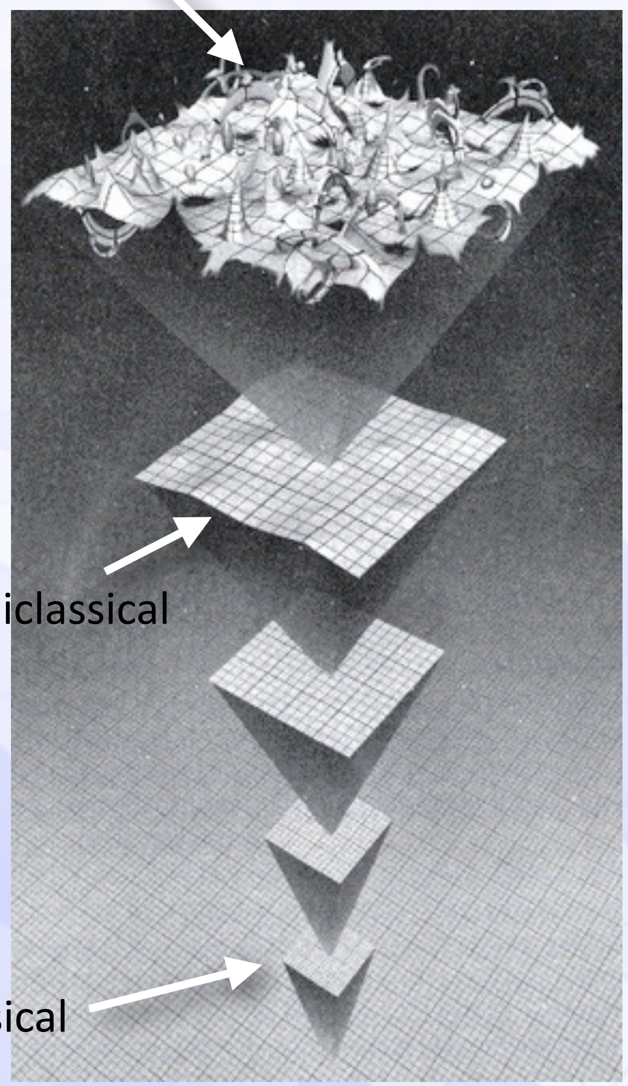
It is used to model the very early universe at the onset of inflation. In today's talk I will sketch how one may obtain a de Sitter-like universe *dynamically* from non-perturbative quantum gravity.



Classical vs quantum geometry in gravity

According to general relativity, zooming in on a piece of macroscopic, curved spacetime, it will on shorter length scales look more and more like flat Minkowski space, i.e. smooth, structureless.

spacetime ripples: gravitational wave



a piece of empty spacetime
under a quantum "microscope"

Our idea of quantum geometry is exactly the opposite: zooming in on a piece of spacetime, the closer we get to the Planck scale, the more it will resemble a "quantum foam". — Describable by $g^{eff}_{\mu\nu}(x)$? At any rate, <u>perturbation theory is *not* sufficient</u>. How to "coarse-grain"/renormalize in practice?

Nonperturbative QG in a nutshell

The longstanding problem of nonperturbative quantum gravity:

P1: We don't know *what* to compute and we don't know *how*.

Some underlying issues: complicated tensorial structure (curvature!), treatment of diffeomorphisms, renormalization compatible with diffeo-symmetry, background independence; covariant/path integral: action unbounded below, Wick rotation; canonical: complicated constraint algebra, Wheeler-DeWitt eqn intractible, problem of time, …

→ QG "radically different"? requires a radical strategy? must go beyond QFT (strings, loops, …)? UV completion with completely different d.o.f.? quantum theory must be modified? plenty of exotic/ speculative physics and controversies about the correct "approach"…

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However, in the meantime, **P1 is no longer true!**

We have blueprints of *how* to calculate:

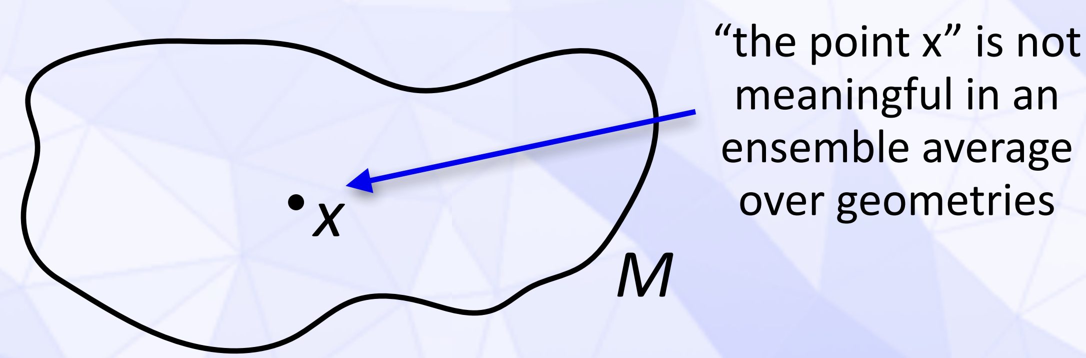
Nonperturbative, computational QFT methods developed over the last 20 years which allow us to explore the unknown and extremely rich quantum realm of *dynamical* geometry:

(i) put gravity on the lattice, but correctly: Dynamical Triangulations [builds on Regge calculus & mathematics of “random geometry”];

(ii) functional renormalization group: Asymptotic Safety.

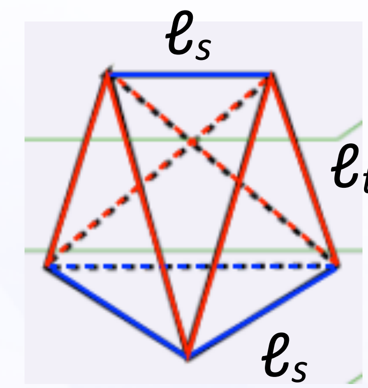
Reuter&Saueressig, CUP text book 2019

We have blueprints of *what* to calculate:



We use these tools to quantify Planckian dynamics in terms of diffeomorphism-invariant, quantum observables. In pure gravity, these are <u>nonlocal</u> quantities, e.g. spacetime averages of scalars like $\int_M d^4x \sqrt{g}\, R(x)$. We have identified a handful of such quantities.

Causal Dynamical Triangulations (CDT)



- a version of lattice gravity: use triangulated manifolds to regularize the gravitational path integral ("sum over histories"):

$$Z(G_{\mathrm{N}},\Lambda)=\int_{g\in\frac{Lor(M)}{Diff(M)}}\mathcal{D}g\;\mathrm{e}^{iS^{\mathrm{EH}}[g]}\;\rightarrow\;Z^{\mathrm{CDT}}(G_{\mathrm{N}},\Lambda)=\lim_{a\to 0}\sum_{\substack{inequiv.\\ causal\\ triang.\,T}}\frac{1}{C(T)}\,\mathrm{e}^{iS^{\mathrm{Regge}}[T]}$$

Newton's constant

Einstein-Hilbert action

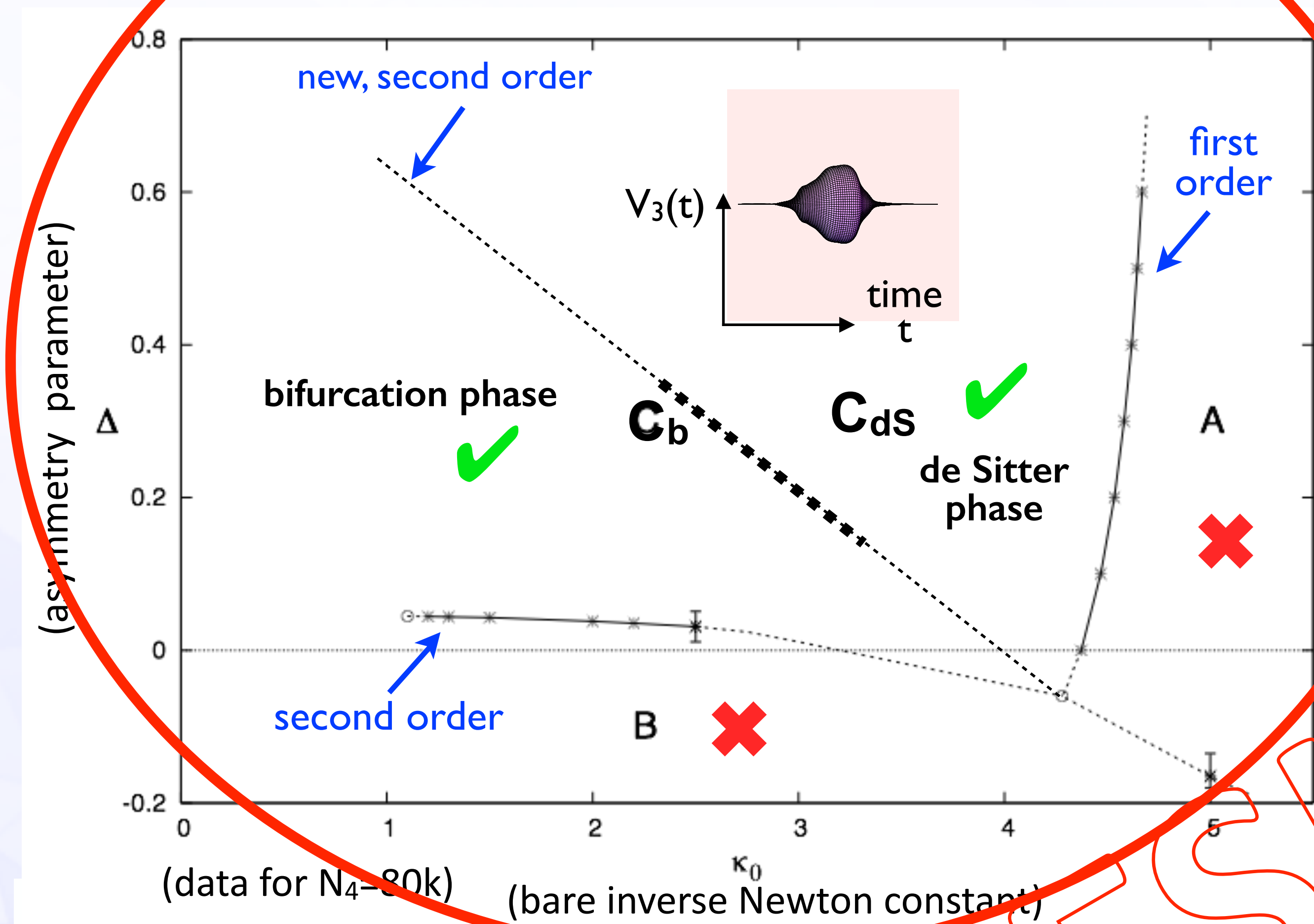
note continuum limit, a = edge length

discrete symmetries of T

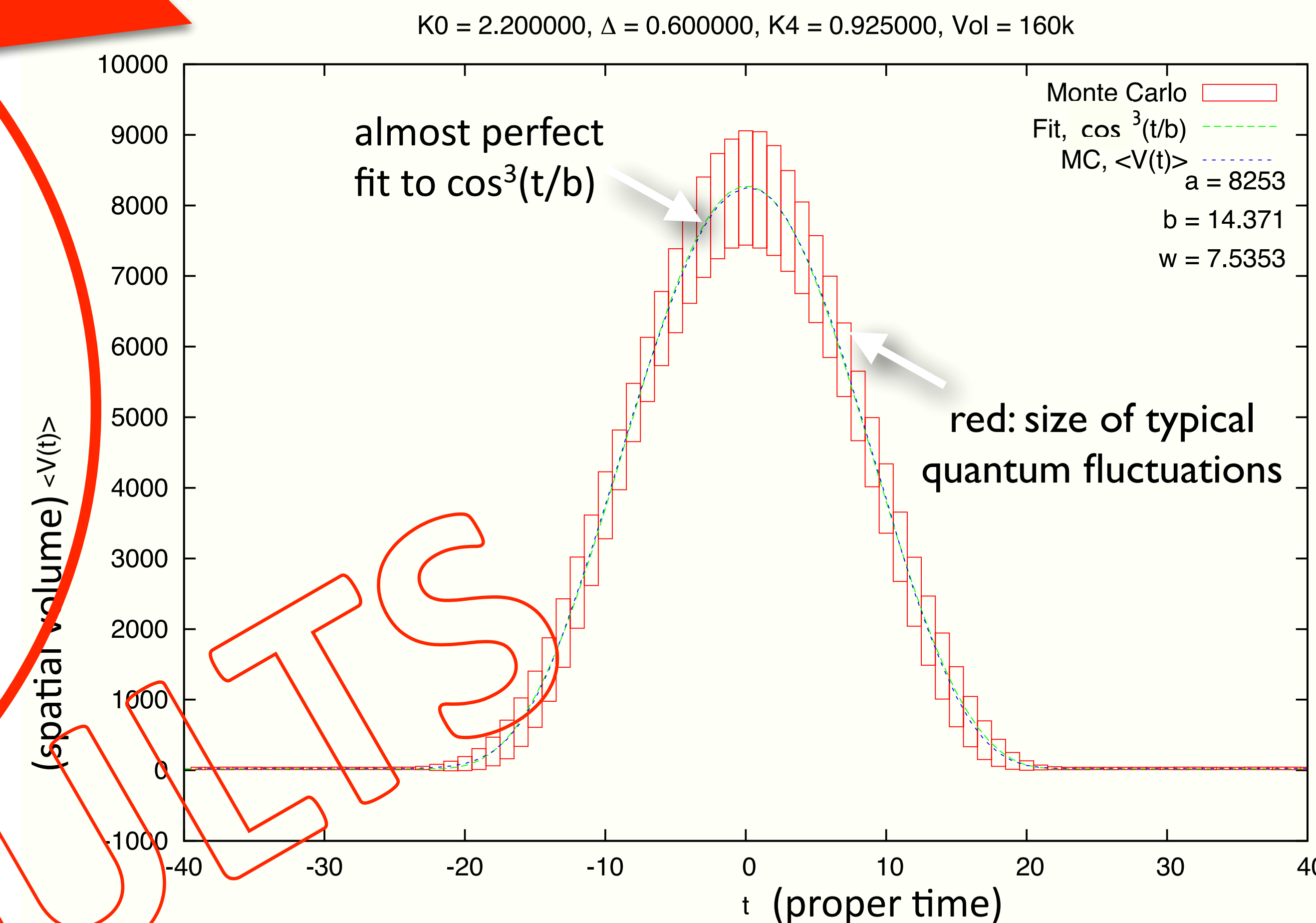
- following Regge 1961, geometries are described without coordinates, removing all diffeomorphism redundancy (QG's pain-in-the-neck)
- with the help of Monte Carlo simulations, one investigates the properties of this candidate theory by measuring observables, for universes of $\lesssim 20\ \ell_{Pl}$ across, with 'reasonable' results $\gtrsim 10\ \ell_{Pl}$

Ambjørn, Görlich, Jurkiewicz & RL, Phys. Rep. 519 (2012) 127 [arXiv: 1203.3591]
RL, CQG 37 (2020) 013002 [arXiv: 1905.08669]

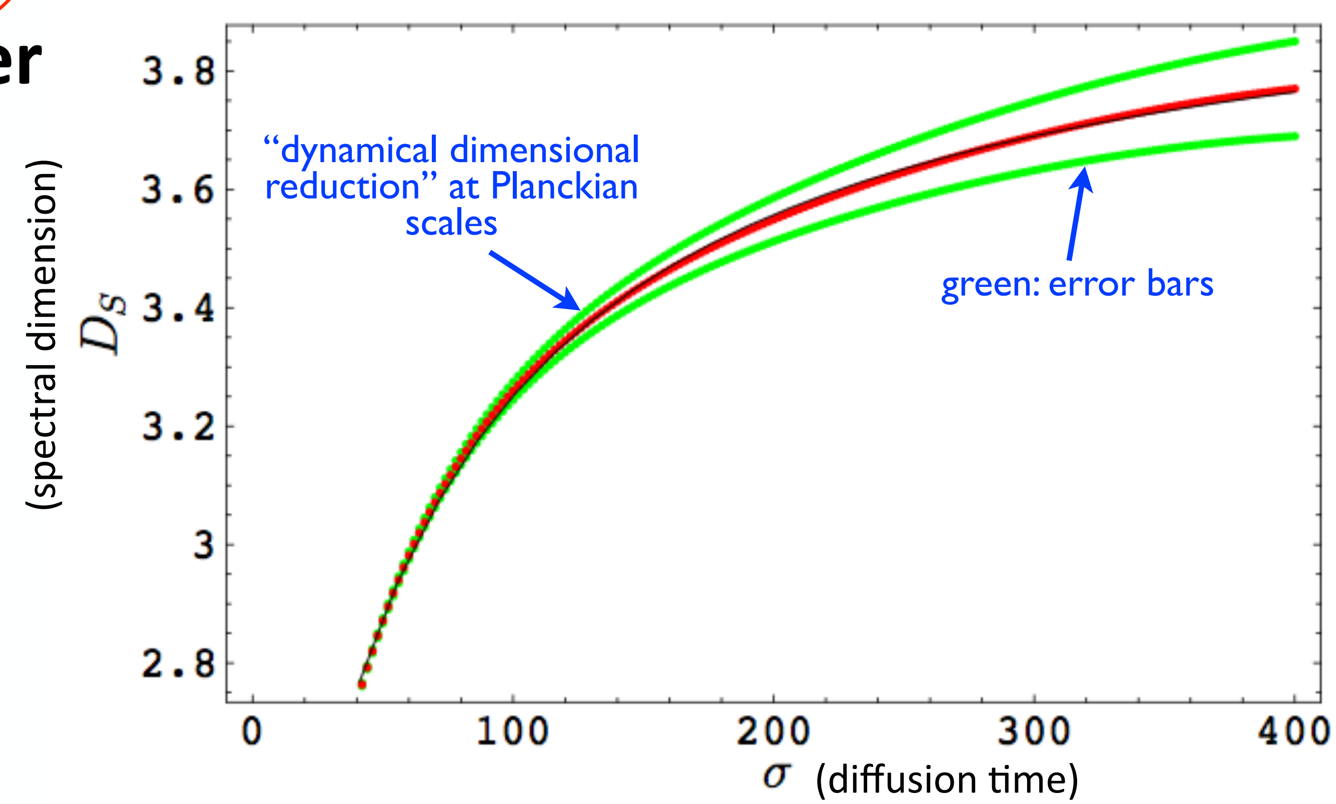
Phase diagram of CDT QG



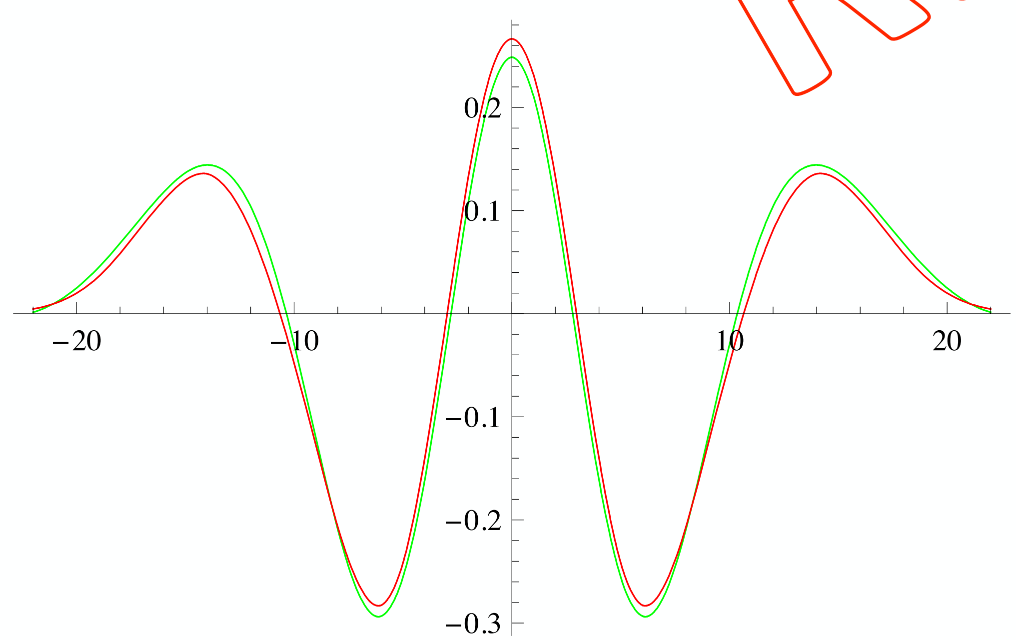
The universe is de Sitter-shaped



Spectral dimension of the universe



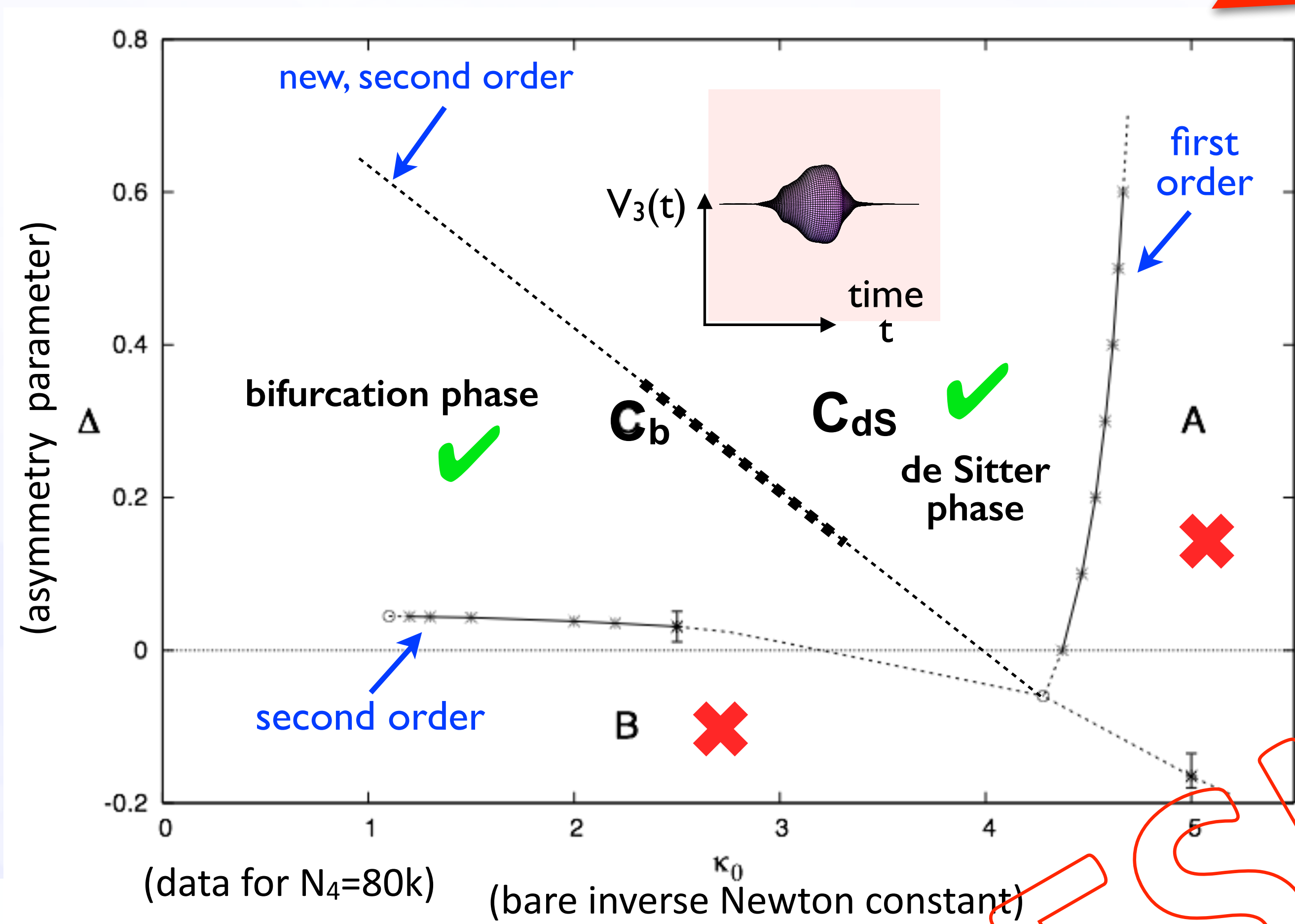
Volume fluctuations around de Sitter



(low-lying eigenmode matches with semiclassical expectation)

RESULTS

Phase diagram of CDT QG



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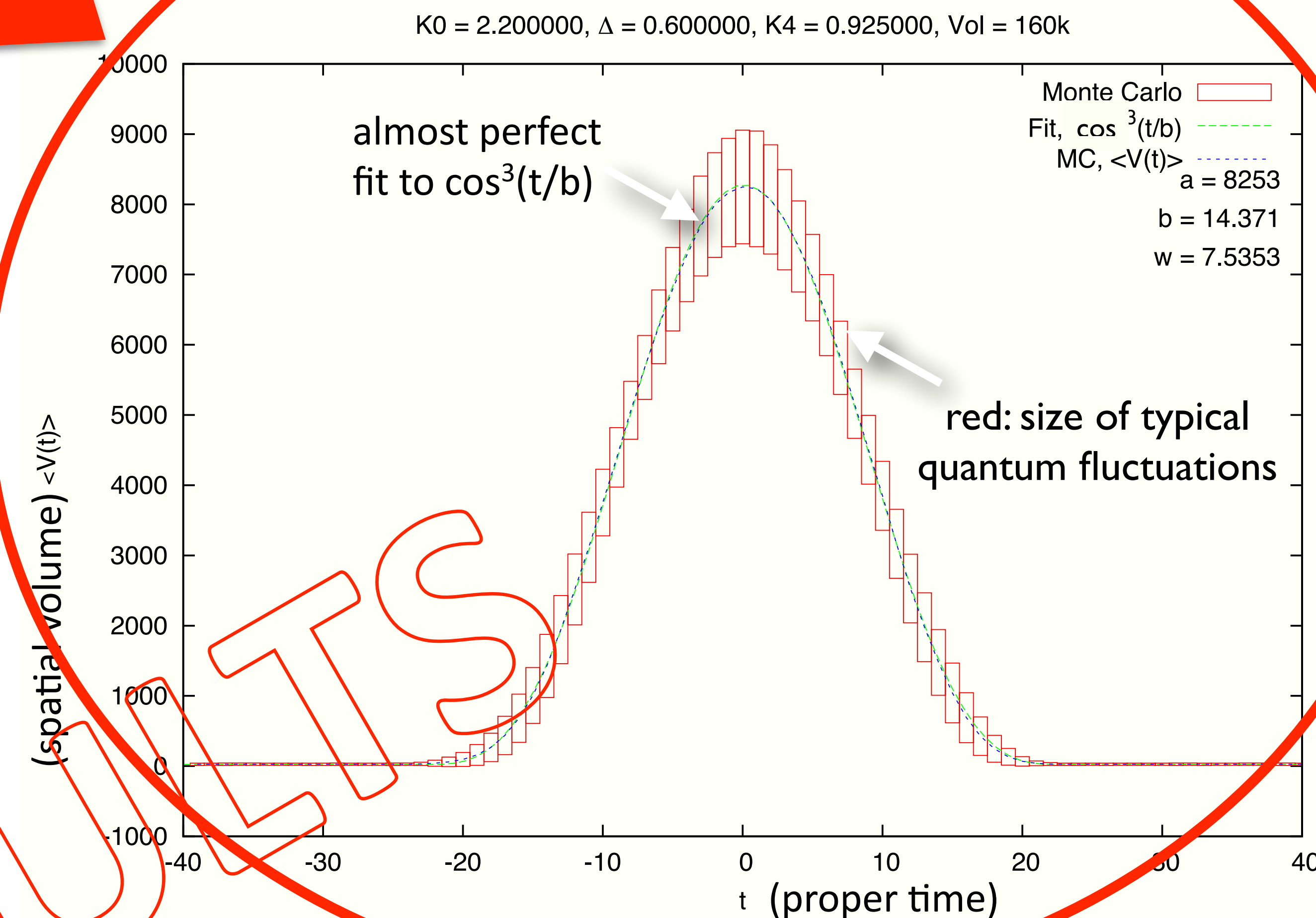

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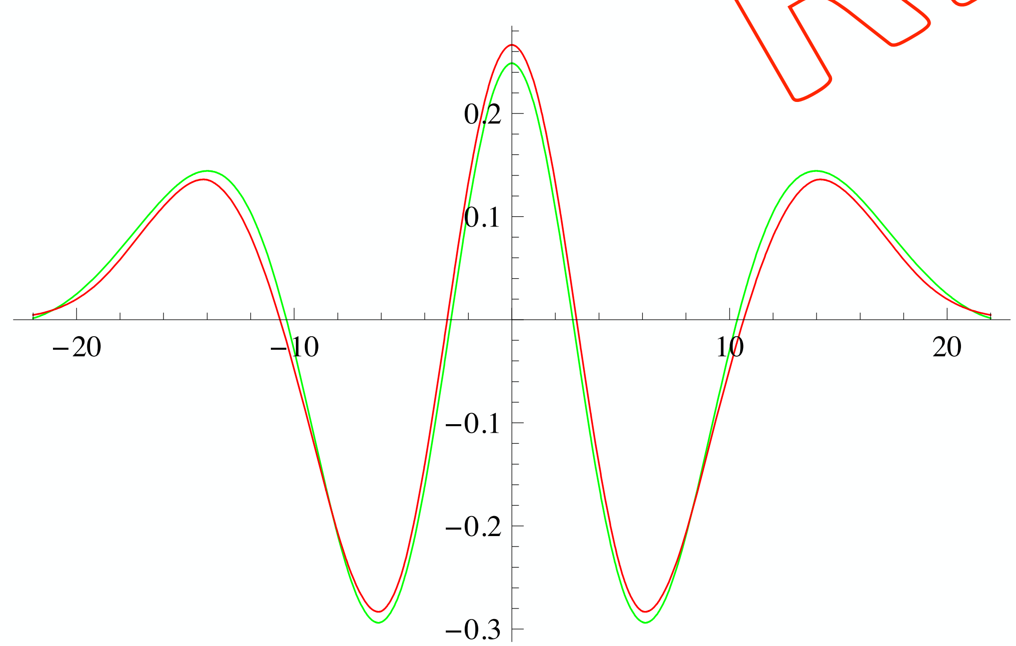

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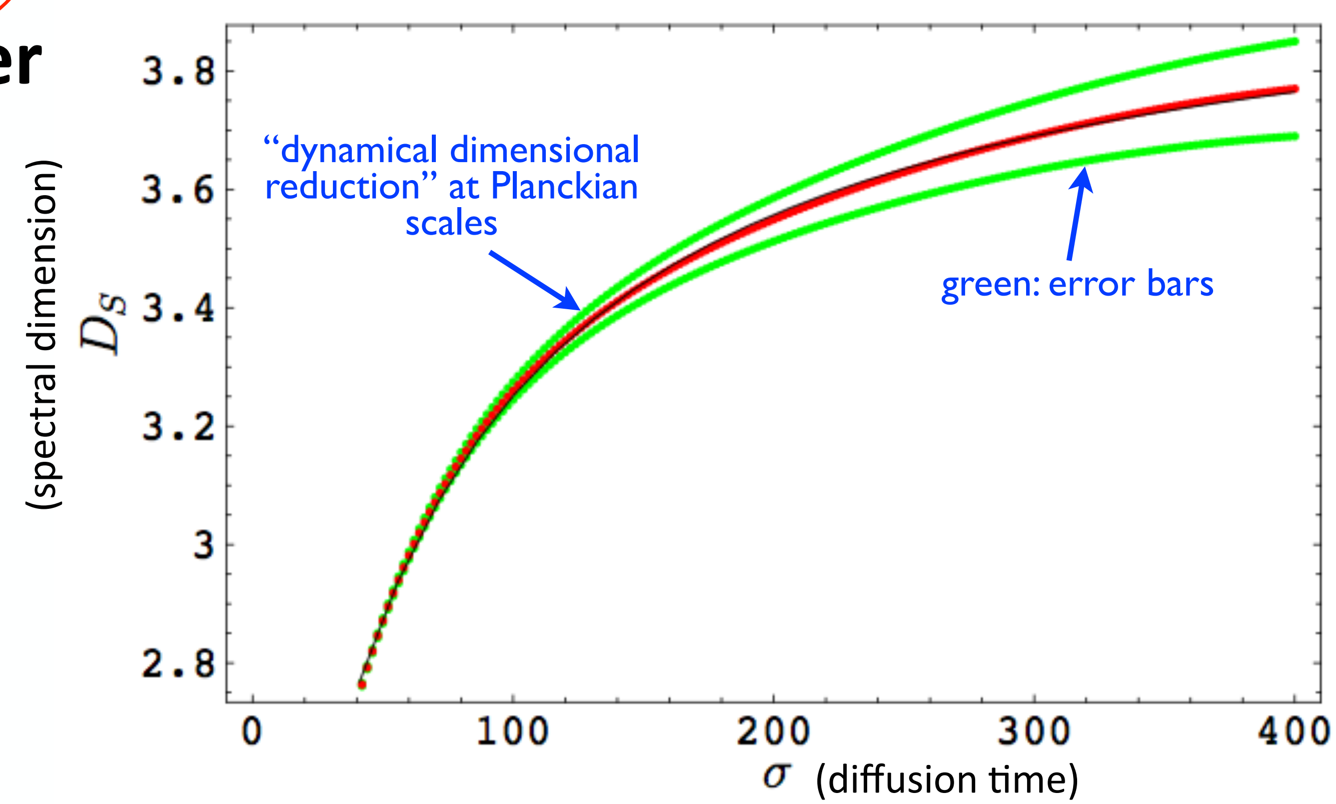


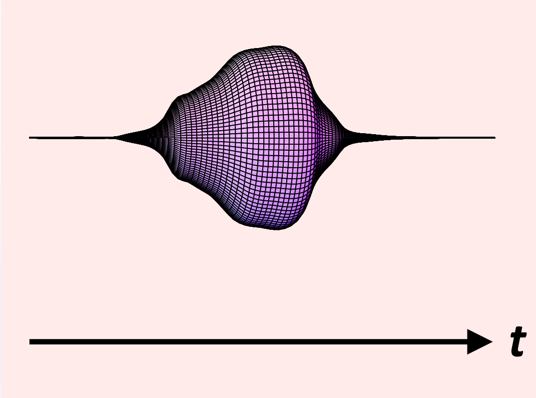
RESULTS

Do we have an emergent de Sitter universe?

Even if the quantum universe is de Sitter-shaped,
it does ***not*** mean it ***is*** a (Euclidean) de Sitter space S^4,

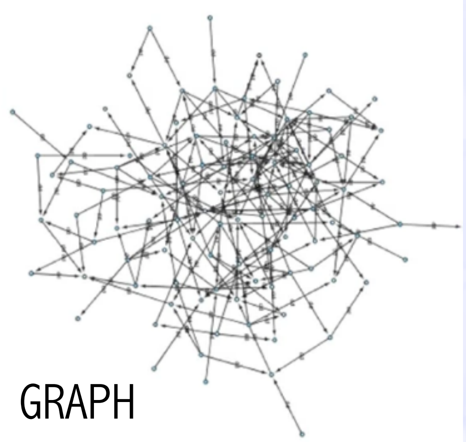
$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2_{(3)}.$$

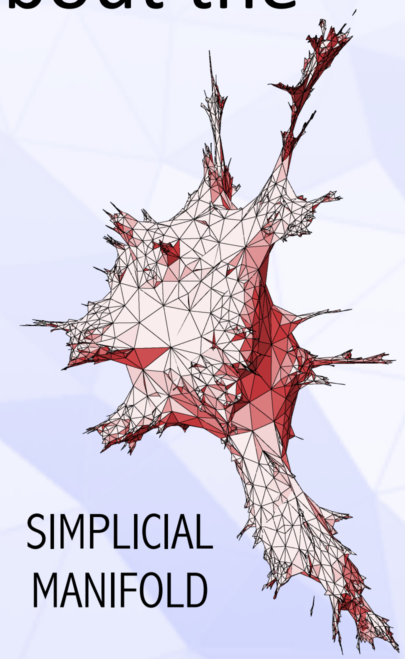
MC snapshot of the shape $\langle V(t) \rangle$ of the universe



The shape of the universe (= the spatial volume $V(t)$ as a function of proper time t) is a single geometric variable, but there are many more. Can we say anything about the **curvature** of this quantum universe?

What is the curvature of a non-smooth metric space? $R^{\kappa}{}_{\lambda\mu\nu}(x) = ?$ Until recently, no well-defined, renormalized notion was known in a Planckian regime.





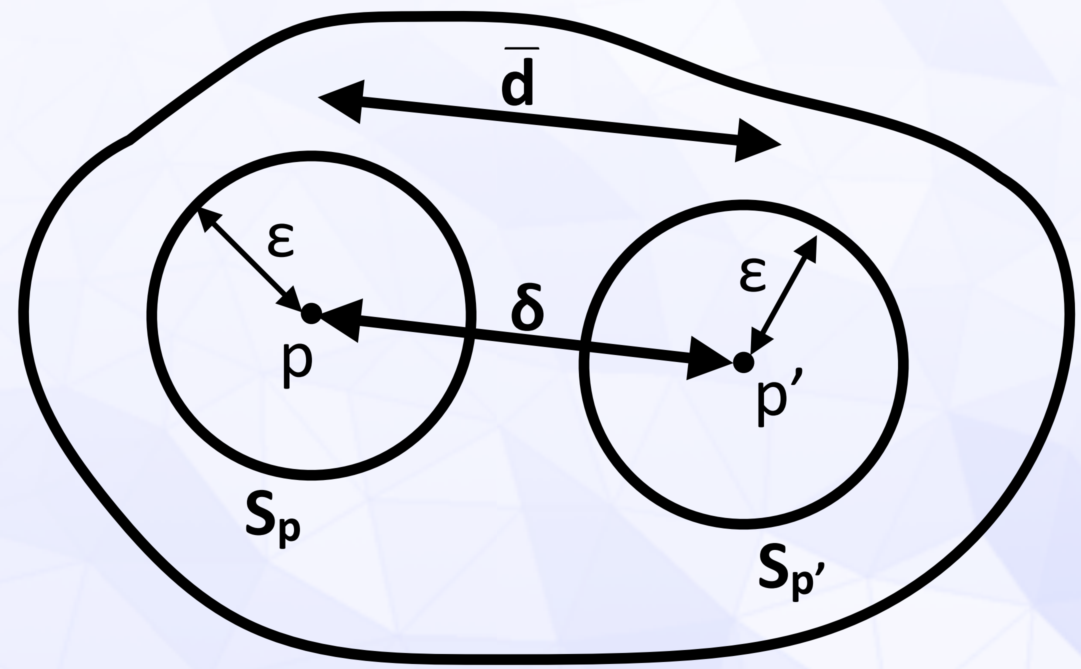
We have successfully defined and tested ***quantum Ricci curvature.***

N. Klitgaard & RL, PRD 97 (2018) 0460008 [arXiv: 1712.08847] and 106017 [arXiv: 1802.10524]

New: Introducing quantum Ricci curvature

In D dimensions, the key idea is to compare the distance $\bar{d}$ between two $(D\text{-}1)$-spheres with the distance δ between their centres.

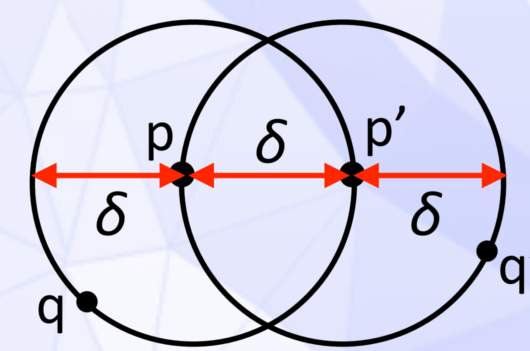
The sphere-distance criterion:
"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres S_p and $S_{p'}$ is smaller (bigger) than the distance δ of their centres."



(inspired by Y. Ollivier, J. Funct. Anal. 256 (2009) 810)

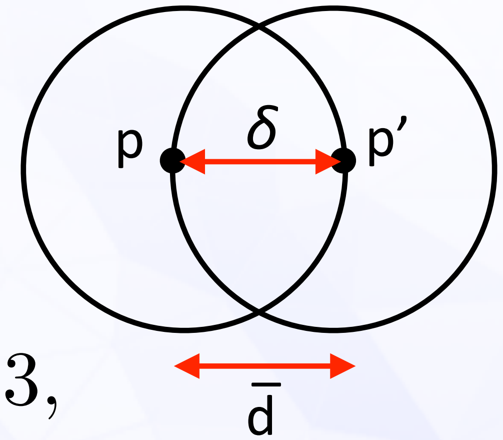
We have adapted this idea to quantum gravity and set $\varepsilon = \delta$ to have only a single scale involved.

⇒ notion of curvature at 'coarse-graining' scale δ



Implementing quantum Ricci curvature

From the quotient of sphere distance and centre distance we extract the **"quantum Ricci curvature K_q at scale δ"**,

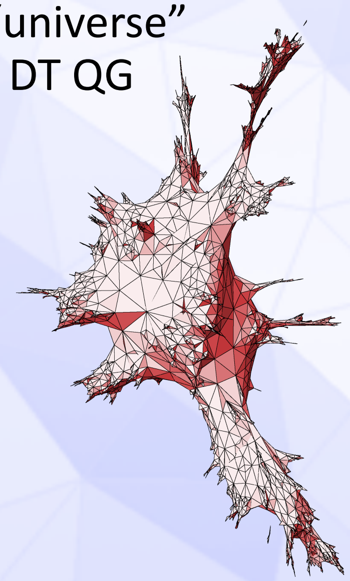


$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad 0 < c_q < 3,$$

where c_q is a non-universal constant and for smooth Riemannian manifolds and $\delta << 1$, $K_q \propto \delta^2\,(Ric(v,v) + c\,R) + O(\delta^3)$.

Beautifully, this involves distance and volume measurements only.

typical "universe" in 2D DT QG



typical "universe" in 2D CDT QG

In-depth investigation of QRC is under way:

- effect of classical inhomogeneity

J.Brunekreef & RL, PRD 103 (2021) 026019 [arXiv: 2011.10168]

- "quantum flatness" in 2D quantum gravity

J.Brunekreef & RL [arXiv: :2110.11100]

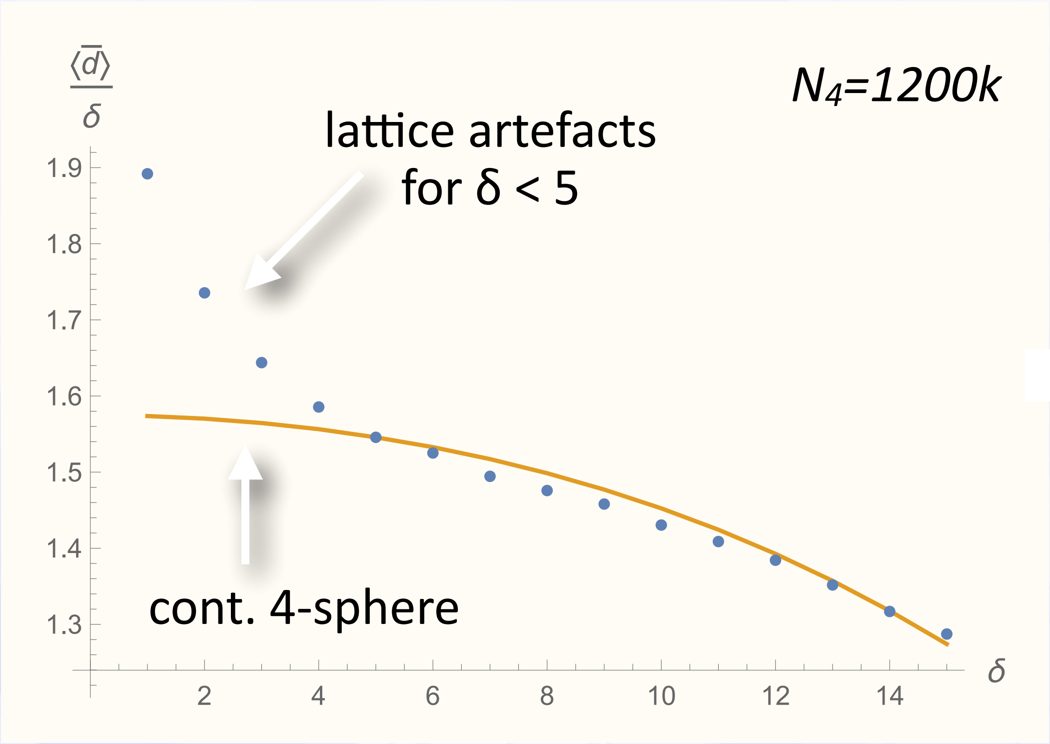
- effect of classical anisotropy

G. Clemente, N. Klitgaard & RL, in preparation

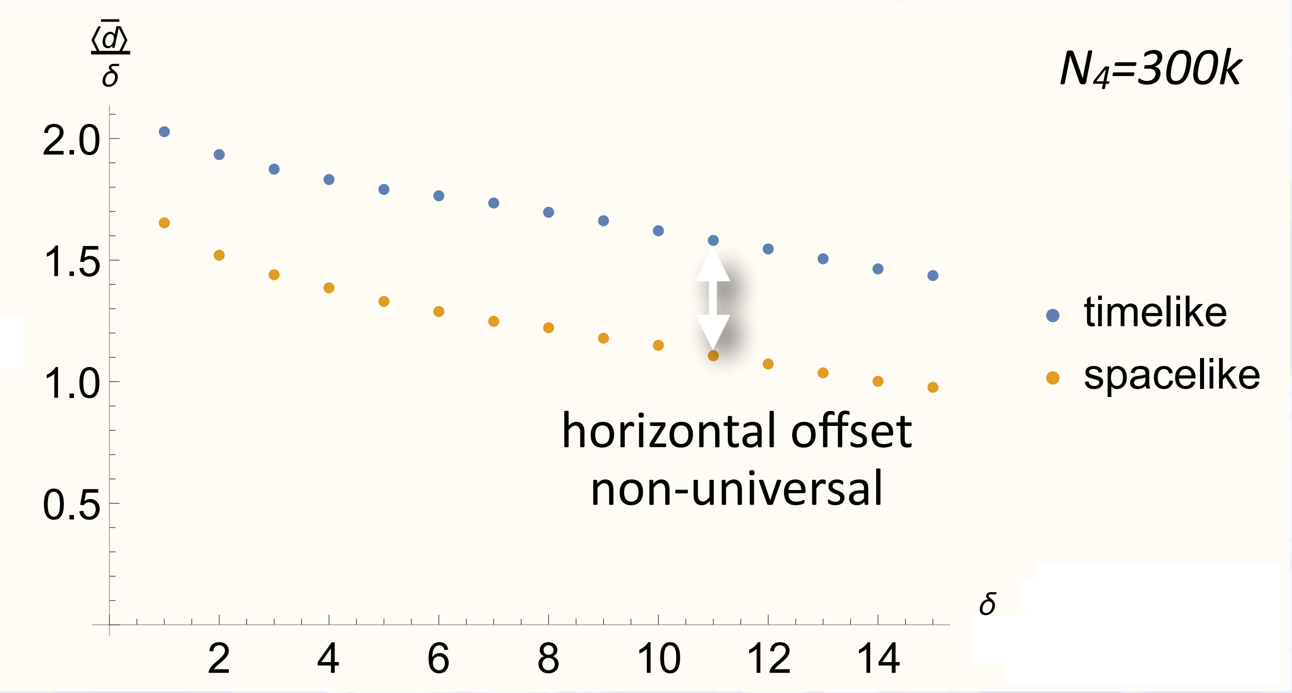
Quantum curvature of the de Sitter universe

Measurements in CDT QG on $S^3 \times S^1$ at volumes $N_4 \leq 1.2 \times 10^6$ show that $\langle K_q \rangle > 0$, with a good fit to S^4! Also evidence that quantum *Ricci* curvature in time- and spacelike directions is the same!

N. Klitgaard & RL, Eur. Phys. J. C 80 (2020) 990 [arXiv: 2006.06263]



$<\bar{d}_{av}/\delta>$ of the dynamically generated dS universe



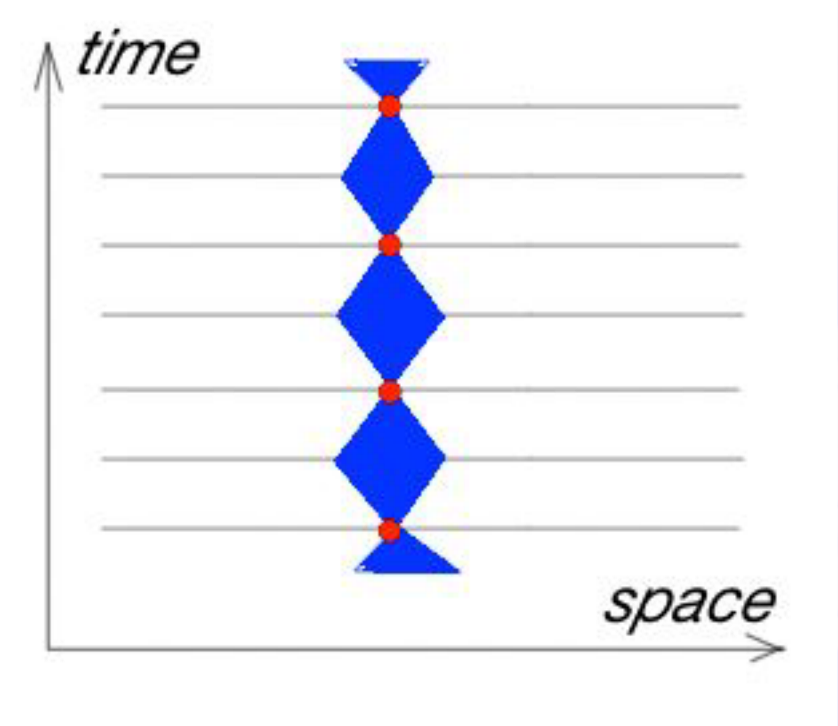
slope of QRC in time & spatial directions

This provides further support for the interpretation of the emergent quantum universe in terms of a de Sitter space, in the sense of EVs.

Quantum universe with de Sitter properties

Health warning: we are in a highly quantum-fluctuating realm; generic observables will *not* behave (semi-)classically!

The ***quantum Ricci curvature*** is a powerful tool to establish properties of quantum spacetime relevant for cosmological applications:



- quantum measures of homogeneity and isotropy
- two-point functions on spatial slices
- investigate the singular, string-like structure appearing in the bifurcation phase C_b of CDT J. Ambjørn, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, JHEP 1406 (2014) 034

⇒ possible candidate for QG structure formation! w.i.p. with G. Clemente

Outlook

- there is genuine progress in full QG; instead of comparing approaches, have started to compare observables and results
- can focus on *physical* questions, like the possible consequences of Planckian dynamics on the behaviour of the early universe
- CDT is a rare example of "spacetime emergence": a background-independent, nonperturbative quantum superposition gives rise to a quantum geometry with large-scale de Sitter features
- we must establish a common language ("observables") between (nonlocal, background-independent) outcomes of QG and (local, background-dependent) descriptions in QFT on curved spacetimes used for the early universe

⇒ lots of interesting, physically relevant and exciting things to do !

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Thank you!