YITP Workshop                                         November 8–10, 2021

*Recent Progress of Quantum Cosmology*

Yukawa Institute for Theoretical Physics, Kyoto University

# Emergent spacetime from the master field of the IIB matrix model

## Frans R. Klinkhamer

Institute for Theoretical Physics,

Karlsruhe Institute of Technology (KIT),

76128 Karlsruhe, Germany

Email: frans.klinkhamer@kit.edu

# 1. Introduction

On the Website of this Workshop is written:

*"Quantum cosmology is an esoteric category of cosmology that pursues the question of how the universe was created, and ..."*

Well, let us try to improve upon the "esoteric." Our approach:

not hand-waving but calculating .

Specifically, we will consider a particular **algebraic equation**, as the solution of <u>that</u> equation may contain information about

the emergence of spacetime and the birth of the Universe .

These results, once established, would "resolve" the Big Bang singularity of the standard Friedmann cosmology.

# 2. Background

We start from the IIB matrix model of Kawai and collaborators [1, 2], which reproduces the structure of the light-cone string field theory of type-IIB superstrings.

The IIB matrix model has a **finite number** of $N \times N$ traceless Hermitian matrices: ten bosonic matrices $A^\mu$ and eight fermionic Majorana–Weyl matrices $\Psi_\alpha$.

The partition function $Z$ of the IIB matrix model is defined by a "path" integral over $A$ and $\Psi$ with a weight factor $\exp[-S_{\text{bos}}(A) - S_{\text{ferm}}(\Psi, A)]$. The fermionic matrices $\Psi$ can be integrated out exactly (Gaussian integrals) and then give the Pfaffian $\mathcal{P}(A)$.
Numerical results have been presented in Ref. [3, 4, 5].

For strings of bosonic observables, the expectation values are defined by the same $A$-integral as $Z$, that is, involving the exponential weight factor with the bosonic action, $\exp[-S_{\text{bos}}(A)]$ and the Pfaffian $\mathcal{P}(A)$.

# 2. Background

As first realized by Witten [6, 7], these expectation values, at large $N$, can also be obtained by inserting the matrices $\widehat{A}^{\mu}$ of the so-called **master field** directly into the observables, without need of integration.

Recently, we have suggested [8] that precisely the master-field matrices $\widehat{A}^{\mu}$ of the IIB matrix model may contain the information of an emergent classical spacetime (the heuristics is clarified in a recent review [9]).

**Assuming** that the matrices $\widehat{A}^{\mu}$ of the IIB-matrix-model master field are known and that they are approximately band-diagonal (as suggested by the numerical results [3, 4, 5]), it is relatively easy [8] to extract a discrete set of spacetime points $\{\widehat{x}^{\mu}_k\}$ and an interpolating metric $g_{\mu\nu}(x)$.

But, instead of <u>assuming</u> the matrices $\widehat{A}^{\mu}$, we want to **calculate** them. And, for that, we need an equation . . .

# 3. Bosonic master-field equation

GOOD NEWS:

the master-field equation has already been established, nearly 40 years ago, by Greensite and Halpern [7], who write in the first line of their abstract:

"We derive an exact algebraic (master) equation for the euclidean master field of any large-$N$ matrix theory, including quantum chromodynamics."

Now, "any" means "any" and we may as well consider the large-$N$ IIB matrix theory [1, 2].

# 3.1 Algebraic equation

Building on the work by Greensite and Halpern [7], we then have the IIB-matrix-model bosonic master field in "quenched" form [8]:

$$\widehat{A}\,^{\mu}_{kl} = e^{i\,\widehat{p}_k\,\tau_{\text{eq}}}\;\widehat{a}\,^{\mu}_{kl}\;e^{-i\,\widehat{p}_l\,\tau_{\text{eq}}}\,. \tag{1a}$$

The $\widehat{p}_k$ are random constants (see below) and the dimensionless time $\tau_{\text{eq}}$ must have a sufficiently large value in order to represent an equilibrium situation ($\tau$ is the fictitious Langevin time of the stochastic-quantization procedure).

The $\tau$-independent matrix $\widehat{a}\,^{\mu}$ on the right-hand side of (1a) solves the following algebraic equation [8]:

$$i\left(\widehat{p}_k - \widehat{p}_l\right)\widehat{a}\,^{\mu}_{kl} \quad = \quad -\frac{\partial\,S_{\text{eff}}}{\partial\,\widehat{a}_{\mu\;lk}} + \widehat{\eta}\,^{\mu}_{kl}\,, \tag{1b}$$

in terms of the master momenta $\widehat{p}_k$ (real uniform random numbers) and the master-noise matrices $\widehat{\eta}\,^{\mu}_{kl}$ (real Gaussian random numbers).

# 3.1 Algebraic equation

Specifically, the IIB-matrix-model algebraic equation (1b) for $D$ traceless Hermitian matrices $\widehat{a}^{\,\mu}$ of dimension $N \times N$ reads:

$$i \left(\widehat{p}_k - \widehat{p}_l\right) \widehat{a}^{\,\mu}_{\,kl} \;\; = \;\; -\left[\widehat{a}^{\,\nu}, \left[\widehat{a}^{\,\nu}, \widehat{a}^{\,\mu}\right]\right]_{kl} + F \, \frac{1}{\mathcal{P}(\widehat{a})} \, \frac{\partial \, \mathcal{P}(\widehat{a})}{\partial \, \widehat{a}^{\,\mu}_{\,lk}} + \widehat{\eta}^{\,\mu}_{\,kl} \,, \quad \text{(2a)}$$

$$\mathcal{P}(\widehat{a}) \;\; = \;\; \text{homogeneous polynomial of degree } K \,, \quad \text{(2b)}$$

$$K \;\; \equiv \;\; (D - 2)\left(N^2 - 1\right), \quad \text{(2c)}$$

$$(D, F) \;\; = \;\; (10, 1), \quad N \gg 1 \,, \quad \text{(2d)}$$

with matrix indices $k$, $l$ running over $\{1, \ldots, N\}$ and directional indices $\mu$, $\nu$ running over $\{1, \ldots, D\}$, while $\nu$ in (2a) is implicitly summed over.

As mentioned before, the $\widehat{p}_k$ are fixed uniform random numbers and the $\widehat{\eta}^{\,\mu}_{\,kl}$ fixed Gaussian random numbers.

# 3.2 Simplified algebraic equation

The algebraic equation (2a) is formidable and it makes sense to first consider the simplified equation obtained by setting $F = 0$:

$$i \left( \widehat{p}_k - \widehat{p}_l \right) \widehat{a}^{\,\mu}_{\;kl} \;\; = \;\; - \left[ \widehat{a}^{\,\nu}, \, \left[ \widehat{a}^{\,\nu}, \, \widehat{a}^{\,\mu} \right] \right]_{kl} + \widehat{\eta}^{\,\mu}_{\;kl} \, . \qquad (3)$$

The matrices $\widehat{a}^{\,\mu}$ are $N \times N$ traceless Hermitian matrices and the number of variables is

$$N_{\mathsf{var}} = D \left( N^2 - 1 \right) . \qquad (4)$$

The simplified equation (3) is essentially a **cubic polynomial**.
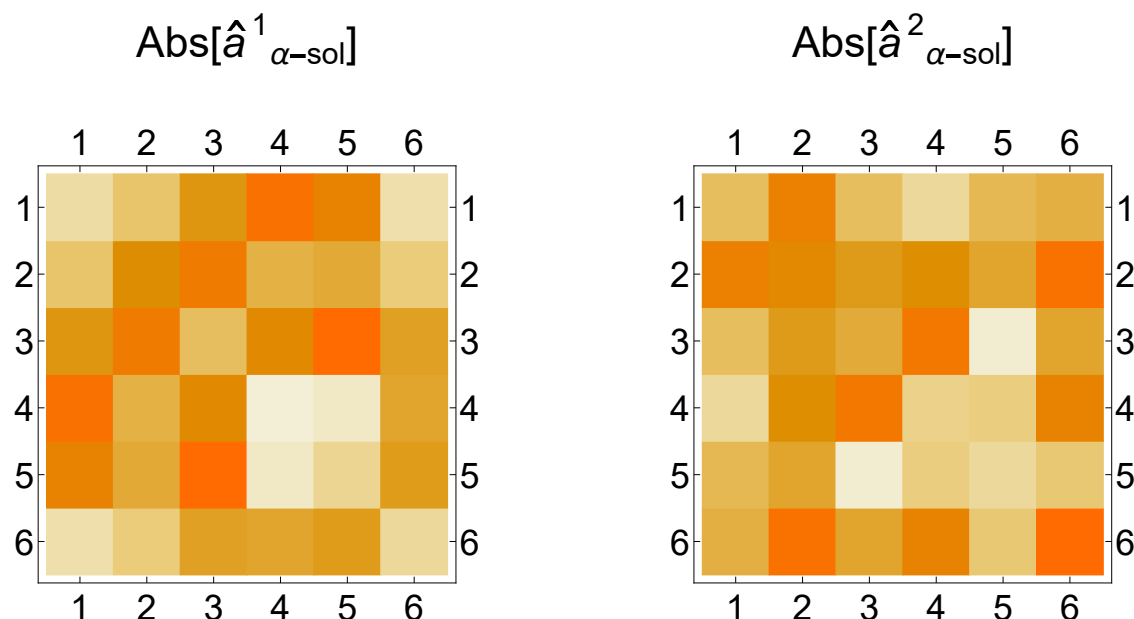
# 3.2 Simplified algebraic equation

It appears impossible to obtain a general analytic solution of (3) in terms of the master constants $\widehat{p}_k$ and $\widehat{\eta}\,^{\mu}_{\,kl}$. Instead, we will try to get solutions for an explicit choice for the random master constants.

For $(D,\, N) = (2,\, 6)$, consider the simplified equation (3) for 70 real variables, with a particular realization, the "alpha-realization," of the pseudorandom numbers entering the equation (details in Ref. [10]).

Other realizations give similar results.

# 3.2 Simplified algebraic equation

A particular solution [10] of the $(D, N) = (2, 6)$ simplified equation (3), with the alpha-realization of pseudorandom constants, is given by $\widehat{a}^1_{\alpha\text{-sol}}$ and $\widehat{a}^2_{\alpha\text{-sol}}$ . Consider the absolute values of the matrix entries and get the following density plots:



$\text{Abs}[\widehat{a}^1_{\alpha-\text{sol}}]$



$\text{Abs}[\widehat{a}^2_{\alpha-\text{sol}}]$

$\Rightarrow$ no obvious band-diagonal structure.

# 3.2 Simplified algebraic equation

Now, change the basis, in order to diagonalize and order the $\mu = 1$ matrix. This gives the matrices $\widehat{a}'^{1}_{\alpha\text{-sol}}$ and $\widehat{a}'^{2}_{\alpha\text{-sol}}$, with density plots:
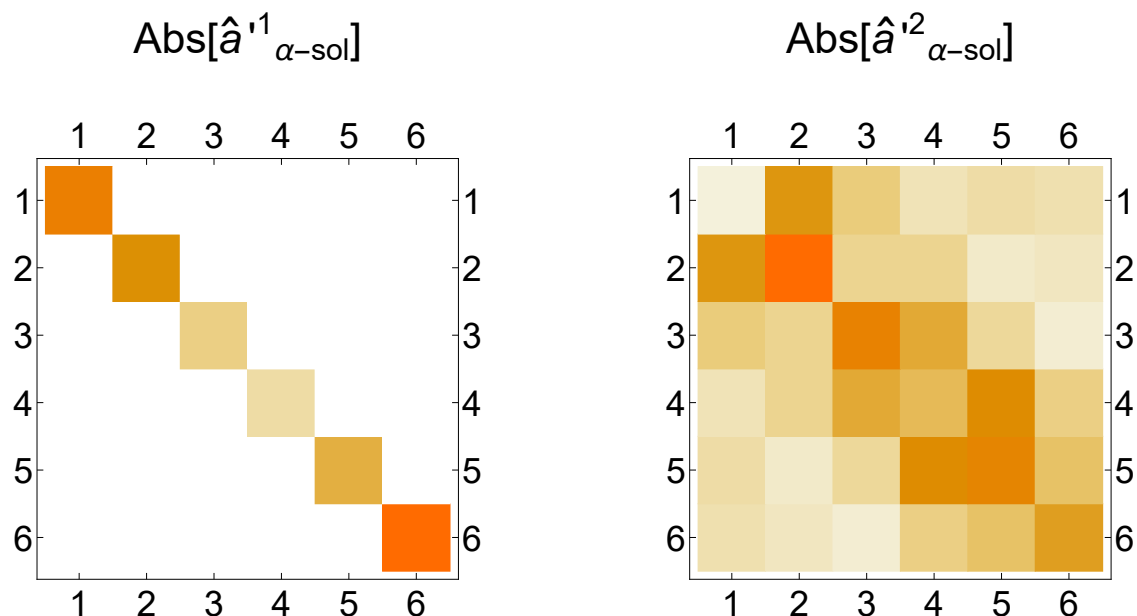


$\Rightarrow$ diagonal/band-diagonal structure, a highly nontrivial result!

$\rightsquigarrow$ But what are the effects of **dynamical fermions** on this structure?

# 3.3 Full algebraic equation

We now look for solutions of the **full** bosonic master-field equation (2a), with $F = 1$ to include the dynamic fermions, but, first, with rather small values of $D$ and $N$.

We have an algebraic solution [11] for the case

$$\{D, \, N, \, F\} = \{3, \, 3, \, 1\} \, . \tag{5}$$

For larger values of $D$ and $N$, we can use a numerical approach [12].

The present status of these calculations is summarized on the next slide.

# 3.3 Full algebraic equation

Table 1: Numerical solutions [12] of the full ($F = 1$) bosonic master-field equation (2a). The number of variables is given by $N_{\text{var}} = D\left(N^2 - 1\right)$ and the order of the Pfaffian by $K = (D - 2)\left(N^2 - 1\right)$.

| | $N_{\text{var}}$ | $K$ | status |
|---|---|---|---|
| $(D, N) = (3, 3)$ | 24 | 8 | done ($\sim 1/2\,$hr) [a] |
| $(D, N) = (10, 3)$ | 80 | 64 | done ($\sim 76\,$hrs) |
| $(D, N) = (10, 4)$ | 150 [b] | 120 | approximate solution ($\gtrsim 90\,$days) |

[a] previous algebraic results reproduced

[b] complex variables in the solution, as the Pfaffian $\mathcal{P}(\widehat{a})$ is complex

# 4. Conclusions

It is conceivable that a **new physics phase** gives rise to classical spacetime, gravity, and matter, as described by our current theories (General Relativity and the Standard Model).

For an <u>explicit</u> calculation, we have considered the **IIB matrix model**, which has been proposed as a nonperturbative formulation of type-IIB superstring theory (M-theory).

The crucial insight is that the emergent classical spacetime may reside in the **large-N master field** $\widehat{A}^{\mu}$ of the IIB matrix model.
The emergence of matter may also be driven by the large-N master field (possibly, its anti-Hermitian part) or by suitable perturbations around the master field. The precise scenario for the matter is not yet clear.

We have now started to **solve** the full bosonic master-field equation of the IIB matrix model: first results are in, but the road ahead is long and arduous . . .

# 5. References

[1]  N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, "A large-$N$ reduced model as superstring," Nucl. Phys. B **498**, 467 (1997), arXiv:hep-th/9612115.

[2]  H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya, and T. Tada, "IIB matrix model," Prog. Theor. Phys. Suppl. **134**, 47 (1999), arXiv:hep-th/9908038.

[3]  S.W. Kim, J. Nishimura, and A. Tsuchiya, "Expanding (3+1)-dimensional universe from a Lorentzian matrix model  for superstring theory in (9+1)-dimensions," Phys. Rev. Lett. **108**, 011601 (2012), arXiv:1108.1540.

[4]  J. Nishimura and A. Tsuchiya, "Complex Langevin analysis of the space-time structure  in the Lorentzian type IIB matrix model," JHEP **1906**, 077 (2019), arXiv:1904.05919.

[5]  K.N. Anagnostopoulos, T. Azuma, Y. Ito, J. Nishimura, T. Okubo, and S. K. Papadoudis, "Complex Langevin analysis of the spontaneous breaking of 10D rotational symmetry in the Euclidean IKKT matrix model," JHEP **06**, 069 (2020), arXiv:2002.07410.

# 5. References

[6] E. Witten, "The $1/N$ expansion in atomic and particle physics," in G. 't Hooft et. al (eds.), *Recent Developments in Gauge Theories*, Cargese 1979 (Plenum Press, New York, 1980).

[7] J. Greensite and M.B. Halpern, "Quenched master fields," Nucl. Phys. B **211**, 343 (1983).

[8] F.R. Klinkhamer, "IIB matrix model: Emergent spacetime from the master field," Prog. Theor. Exp. Phys. **2021**, 013B04 (2021), arXiv:2007.08485.

[9] F.R. Klinkhamer, "M-theory and the birth of the Universe," Acta Phys. Pol. B **52**, 1007 (2021), arXiv:2102.11202.

[10] F.R. Klinkhamer, "A first look at the bosonic master-field equation of the IIB matrix model," Int. J. Mod. Phys. D **30**, 2150105 (2021), arXiv:2105.05831.

[11] F.R. Klinkhamer, "Solutions of the bosonic master-field equation from a supersymmetric matrix model," Acta Phys. Pol. B **52**, 1339 (2021), arXiv:2106.07632.

[12] F.R. Klinkhamer, "Towards a numerical solution of the bosonic master-field equation of the IIB matrix model," arXiv:2110.15309.