Theoretical bound of primordial non-Gaussianity in single field inflation

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Based on arXiv:2104.01953 with Prof. Jun'ichi Yokoyama

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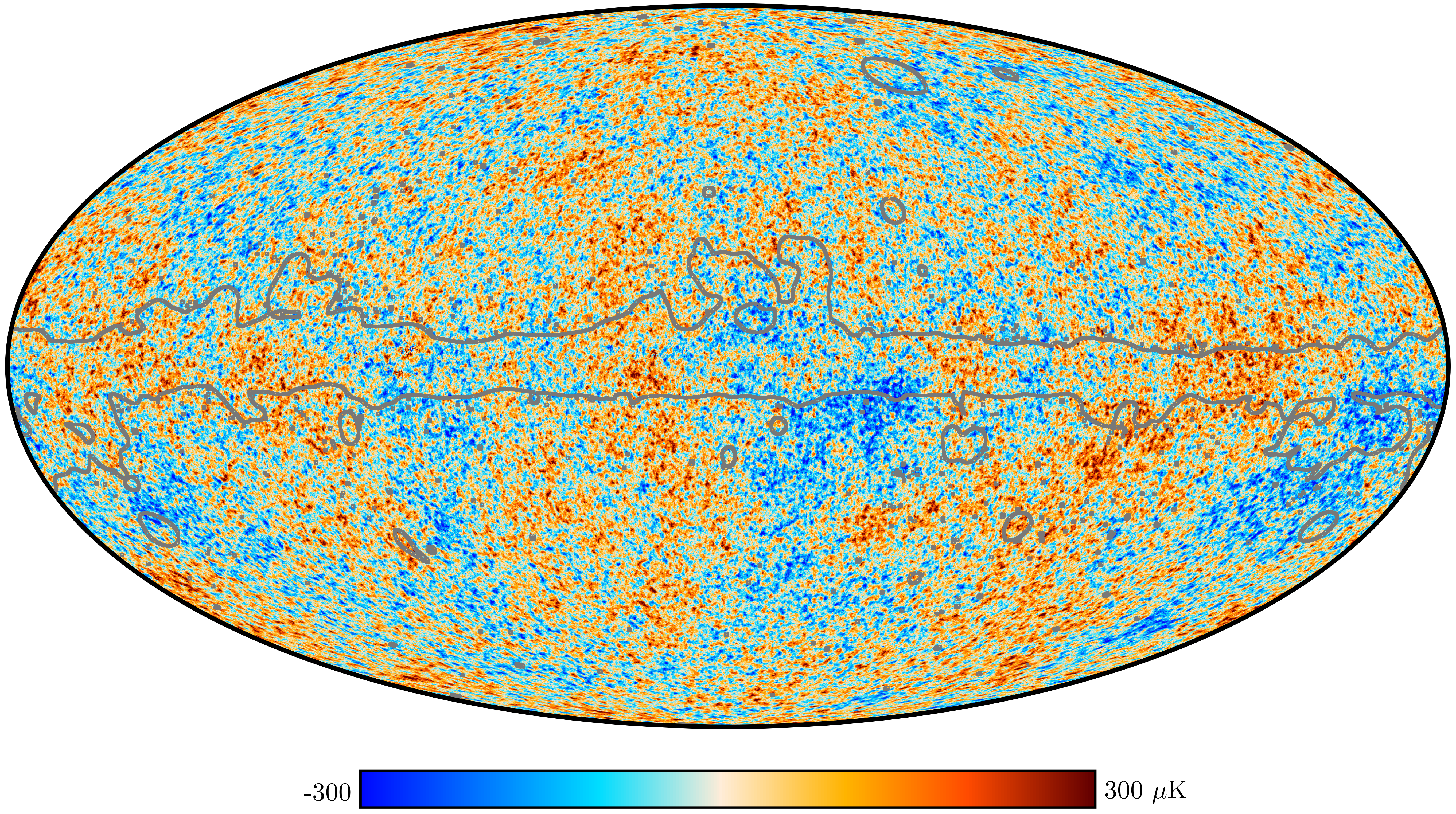
Introduction

Signature of inflation: small quantum fluctuations as the origin of CMB anisotropy and large-scale structures.

Observed by Planck 2018:

- Almost Gaussian.
- Almost scale-invariant.



Planck 2018

Introduction

An inflation model predicts:

- Deviation from scale-invariant (spectral tilt).
- Deviation from Gaussian distribution (non-Gaussianity).

Observation (Planck 2018) still allows quite large non-Gaussianity.

How large non-Gaussianity that still can be explained by cosmological perturbation theory?

k-inflation

Consider k-inflation (Armendariz-Picon et. al., PLB 1998), the simplest model which can generate large spectrum of non-Gaussianity, with action

$$\boxed{S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left[M_{\rm pl}^2 R + 2P(X,\phi)\right],}$$

where $g = \det\ g_{\mu\nu}$, $g_{\mu\nu}$ is spacetime metric, R is Ricci scalar, ϕ is inflaton, and $X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$.

Cosmological perturbations

Small perturbations:

$$\boxed{\begin{gathered}\phi(\mathbf{x},t)=\bar{\phi}(t)+\delta\phi(\mathbf{x},t),\\ \mathrm{d}s^2=g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu=-N^2\mathrm{d}t^2+\gamma_{ij}(\mathrm{d}x^i+N^i\mathrm{d}t)(\mathrm{d}x^j+N^j\mathrm{d}t).\end{gathered}}$$

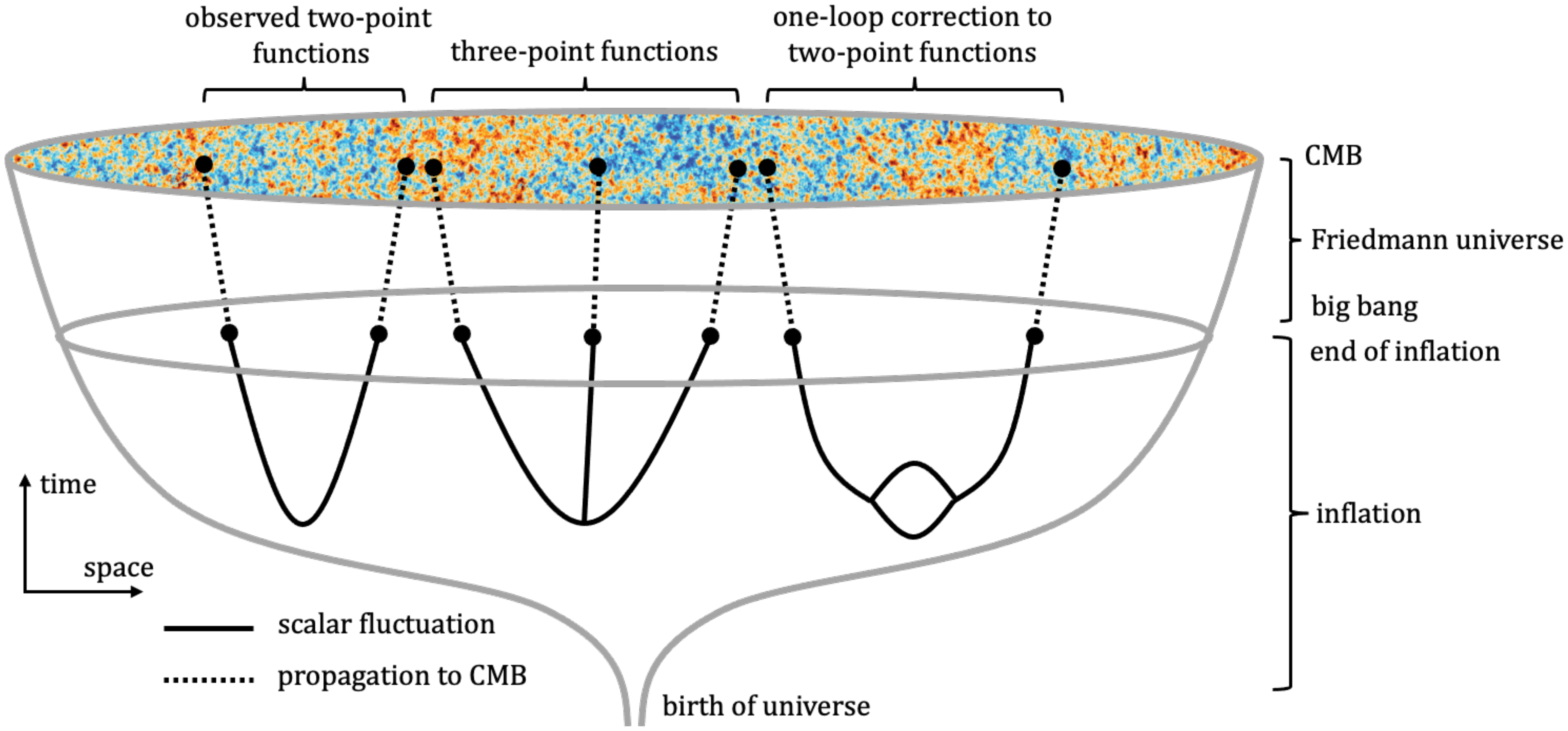
Gauge fixing condition (comoving gauge):

$$\boxed{\delta\phi=0 \text{ and } \gamma_{ij}=a^2(1+2\zeta)\delta_{ij}.}$$

Some parameters ($\epsilon,\eta,s\ll 1$):

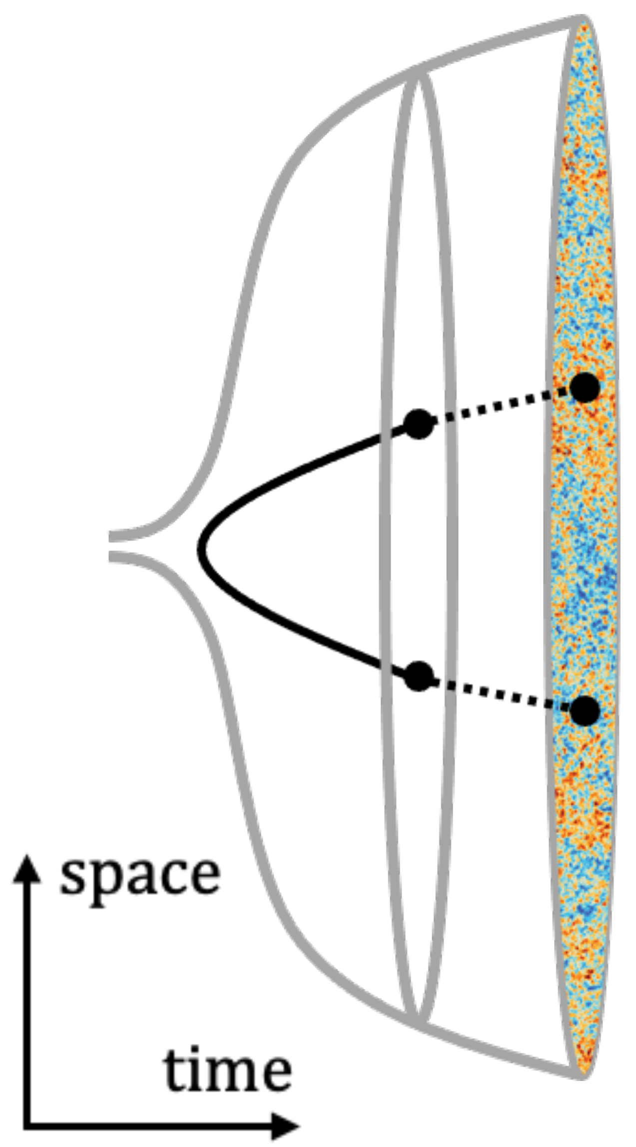
$$\boxed{\epsilon=-\frac{\dot{H}}{H^2},\eta=\frac{\dot{\epsilon}}{\epsilon H},s=\frac{\dot{c}_s}{c_sH},\text{ and } c_s^2=\frac{P_{,X}}{P_{,X}+2XP_{,XX}}.}$$

Feynman-Witten diagram



Cosmological correlators

Two-point functions



Second-order action of ζ:

$$\boxed{S^{(2)} = M_{\rm pl}^2 \int {\rm d}t\ {\rm d}^3x \frac{\epsilon}{c_s^2} a^3 \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial_i \zeta)^2\right]}$$

Inflationary power spectrum:

$$\boxed{\Delta_s^2(k) = \frac{k^3}{2\pi^2}\langle\zeta\zeta\rangle = \left(\frac{H^2}{8\pi^2 M_{\rm pl}^2 c_s \epsilon}\right)_H = \Delta_s^2(k_*)\left(\frac{k}{k_*}\right)^{n_s-1},}$$

where subscript H denotes horizon crossing $c_s k = aH$ and $n_s - 1 = (-2\epsilon - \eta - s)_H$.

Cosmological correlators

Three-point functions

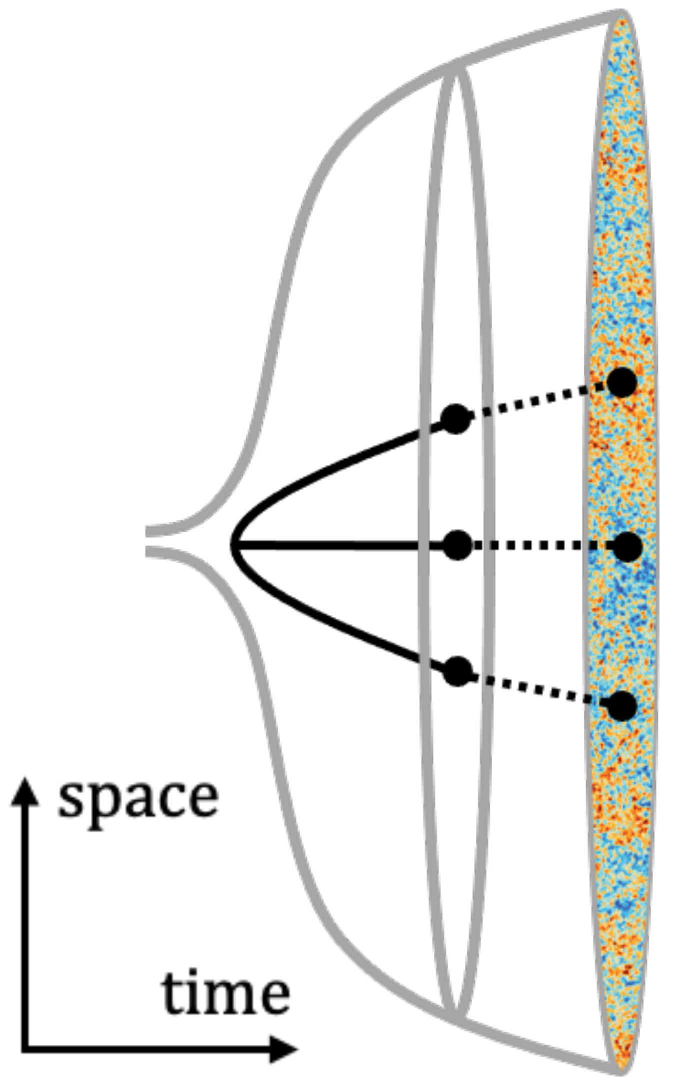
Cubic-order action (Cheung et. al., JHEP 2008) of ζ (for $c_s \ll 1$):

$$\boxed{S_{\rm int} = \int \mathrm{d}t\ \mathrm{d}^3x \left[-\frac{2\lambda}{H^3} a^3 \dot{\zeta}^3 + \frac{\epsilon}{H c_s^2} a \dot{\zeta} (\partial_i \zeta)^2 \right],}$$

where $\lambda = X^2 P_{,XX} + (2/3) X^3 P_{,XXX}$.

Primordial non-Gaussianity:

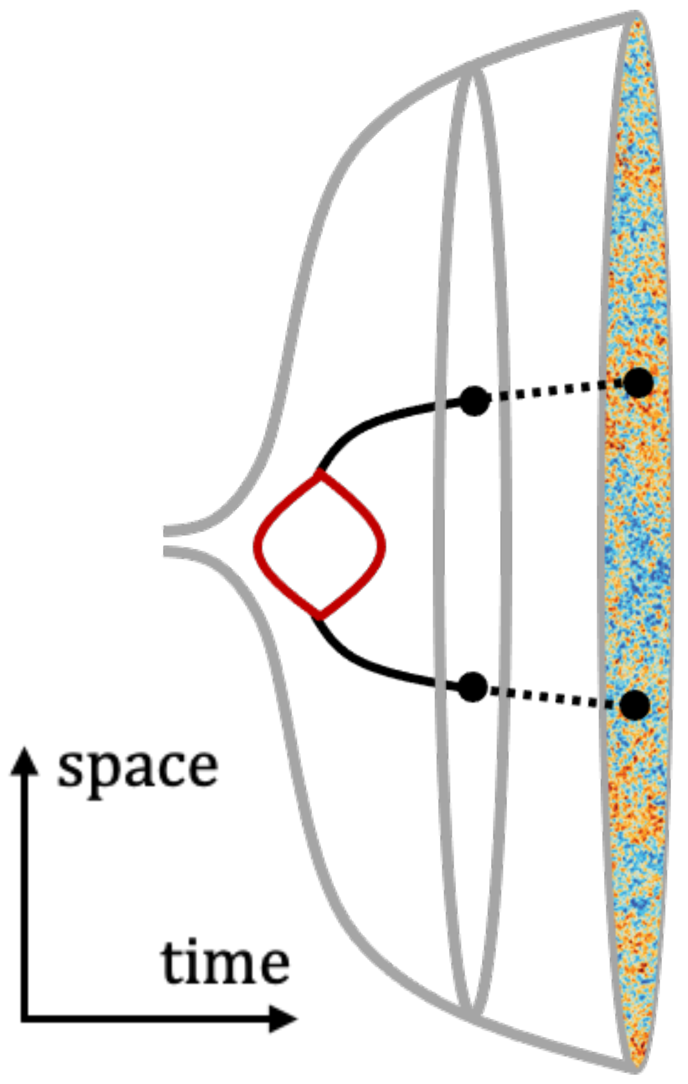
$$\boxed{f_{NL} = \frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^2} \propto \frac{1}{c_s^2}}$$



Cosmological correlators

One-loop correction

One-loop correction generated by cubic-order action is computed using in-in perturbation theory:



$$\langle \zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(1)} = \frac{1}{c_s^4}\langle \zeta(\mathbf{p})\zeta(-\mathbf{p})\rangle_{(0)} \int_p^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3}\langle \zeta(\mathbf{k})\zeta(-\mathbf{k})\rangle_{(0)}$$

$$\Delta^2_{s(1)}(p) = \frac{1}{c_s^4}\Delta^2_{s(0)}(p)\int_p^\infty \frac{\mathrm{d}k}{k}\Delta^2_{s(0)}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$\Delta^2_{s(1)}(p) = \frac{1}{c_s^4}\Delta^2_{s(0)}(p)\frac{\Delta^2_{s(0)}(p)}{n_s - 1}$$

converge for $n_s - 1 < 0$, based on observation $n_s = 0.97$

Renormalization?

Previous research (Senatore and Zaldarriaga, JHEP 2010) did not consider spectral tilt to calculate the loop correction, so they found

$$\boxed{\int_p^\infty \frac{\mathrm{d}^3 k}{(2\pi)^3 k^3} \to \int_p^\infty \frac{\mathrm{d}^{3+\delta} k}{(2\pi)^3 k^3},}$$

and introducing dimensional regularization to make the integral converge.

As a consequence, pole $1/\delta$ appears and it must be renormalized.

Our study shows that considering spectral tilt $n_s - 1$ naturally makes the integral converge and we do not need renormalization.

Conclusion

- Requiring perturbativity of one-loop correction compared to the tree-level power spectrum yields

$$\boxed{\frac{\Delta^2_{s(0)}}{c_s^4(1-n_s)} \ll 1.}$$

- Substituting observation results $\Delta^2_{s(0)} = 2.1 \times 10^{-9}$ and $1 - n_s = 0.03$, theoretical bound of sound speed is $c_s \gg 0.02$.
- Such bound overlaps with observational constraint $c_s > 0.021$.
- Future observation will reveal the validity of cosmological perturbation theory.