2d $\mathcal{N} = (2,2)$ SYM on computer

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- I. Kanamori, H.S., arXiv:0809.2856, Nucl. Phys. B in press
- I. Kanamori, H.S., arXiv:0811.2851

Nonperturbative Formulation of SUSY Theories

- It is widely believed that SUperSYmmetry plays an important role in particle physics beyond SM
  - hierarchy problem
  - superstring theory (gauge/gravity correspondence)
- Nonperturbative phenomena?
  - color confinement, bound states, spontaneous chiral symmetry breaking, quantum tunneling, . . .
  - dynamical spontaneous SUSY breaking
- Nonperturbative formulation? Lattice?

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SUSY on the Lattice?

- Manifest SUSY would be *impossible*, because

$$\{Q^A_\alpha, (Q^B_\beta)^\dagger\} = 2\delta^{AB}\sigma^m_{\alpha\dot\beta}P_m$$

  but *no* infinitesimal translations P_m defined for lattice fields

- However, at least a linear combination Q of Q^A_α and $(Q^B_\beta)^\dagger$ such that

$$\{Q, Q\} = 2Q^2 = 0$$

  could be realized even on the lattice

- Moreover, *if* the continuum action S can be written as

$$S = QX$$

  Q-invariance of S could be promoted to lattice symmetry!

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SUSY on the Lattice? (cont'd)

- (Partial) list of continuum theories with $S = QX$ return
  - 4d $\mathcal{N} = 4$ SYM
  - 3d $\mathcal{N} = 8$ SYM
  - 3d $\mathcal{N} = 4$ SYM
  - 2d $\mathcal{N} = (8, 8)$ SYM
  - 2d $\mathcal{N} = (4, 4)$ SYM
  - 2d $\mathcal{N} = (2, 2)$ SYM (+ matter multiplet)

2d $\mathcal{N} = (2,2)$ Supersymmetric Yang-Mills Theory

- Dimensional reduction of 4d $\mathcal{N} = 1$ SYM from 4 to 2

$$S_{\text{continuum}} = \frac{1}{g^2} \int d^2x \, \text{tr} \left\{ \frac{1}{2} F_{MN} F_{MN} + \Psi^T C \Gamma_M D_M \Psi + \widetilde{H}^2 \right\},$$

where $M, N = 0, 1, 2, 3$, $(\mu, \nu = 0, 1)$ and

$$F_{01} = \partial_0 A_1 - \partial_1 A_0 + i[A_0, A_1] \equiv \Phi/2$$
$$F_{02} = \partial_0 A_2 + i[A_0, A_2] \equiv D_0 A_2, \quad F_{23} = i[A_2, A_3], \quad \text{etc.}$$
$$\phi \equiv A_2 + iA_3, \quad \overline{\phi} = A_2 - iA_3$$
$$\widetilde{H} = H - i\Phi/2$$

- We will use a particular representation

$$\Gamma_0 = \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & -i\sigma_3 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}, \quad \Gamma_3 = C = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$\Psi^T \equiv (\psi_0, \psi_1, \chi, \eta/2)$$

2d $\mathcal{N} = (2,2)$ SYM (cont'd)

- Supersymmetry

$$\delta A_M = i\epsilon^T C\Gamma_M \Psi, \quad \delta\Psi = \frac{i}{2} F_{MN}\Gamma_M\Gamma_N\epsilon + i\widetilde{H}\Gamma_5\epsilon$$

$$\delta\widetilde{H} = -i\epsilon^T C\Gamma_5\Gamma_M D_M\Psi$$

- Setting ◂ return

$$\epsilon^T \equiv -(\varepsilon^{(0)}, \varepsilon^{(1)}, \widetilde{\varepsilon}, \varepsilon), \quad \delta \equiv \varepsilon^{(0)} Q^{(0)} + \varepsilon^{(1)} Q^{(1)} + \widetilde{\varepsilon}\widetilde{Q} + \varepsilon Q$$

we have ◂ return

$$QA_\mu = \psi_\mu \qquad Q\psi_\mu = iD_\mu\phi$$

$$Q\phi = 0$$

$$Q\chi = H \qquad QH = [\phi, \chi]$$

$$Q\overline{\phi} = \eta \qquad Q\eta = [\phi, \overline{\phi}]$$

2d $\mathcal{N} = (2,2)$ SYM (cont'd)

- We see

$$Q^2 = \delta_\phi,$$

where δ_ϕ is an infinitesimal gauge transformation by the parameter ϕ, and thus

$$Q^2 = 0 \text{ on gauge invariant combinations}$$

- The action is moreover Q-exact

$$S_{\text{continuum}} = Q\frac{1}{g^2}\int d^2x \, \text{tr}\left\{\frac{1}{4}\eta[\phi,\overline{\phi}] - i\chi\Phi + \chi H - i\psi_\mu D_\mu\overline{\phi}\right\}$$

return

2d $\mathcal{N} = (2,2)$ SYM (cont'd)

- Global symmetries
  - $U(1)_A$ symmetry ($\Leftarrow$ 2-3 plane rotation in 4d)

$$\Psi \to \exp\{\alpha\Gamma_2\Gamma_3\}\Psi, \quad \phi \to \exp\{2i\alpha\}\phi, \quad \overline{\phi} \to \exp\{-2i\alpha\}\overline{\phi}$$

  - $U(1)_V$ symmetry ($\Leftarrow$ $U(1)_R$ symmetry in 4d SYM)

$$\Psi \to \exp\{i\alpha\Gamma_5\}\Psi$$

  - a Z_2 symmetry ($\Leftarrow$ reflection in 2-direction in 4d)

$$\Psi \to i\Gamma_2\Psi, \quad \phi \to -\overline{\phi}, \quad \overline{\phi} \to -\phi$$

2d $\mathcal{N} = (2,2)$ SYM (cont'd)

- This is a “toy” field theory, but no obvious low-energy description
- In 2d, no SSB of bosonic global symmetries (no chiral lagrangian)
- no controllable parameter except N_c (large N_c limit is non-trivial) gauge coupling g simply provides a mass scale, like Λ_{QCD}
- flat directions $[\phi, \overline{\phi}] = 0$, but (probably) no vacuum modulus in 2d, Witten index is unknown (SSUSYB?, Hori-Tong)

LATTICE FORMULATION

Recent Developments in Lattice Formulation

- lattice formulations with exact fermionic symmetr(ies) Q of various gauge theories see
  - Cohen, Kaplan, Katz, Ünsal, Endres
  - Sugino
  - Catterall
  - D'Adda, Kanamori, Kawamoto, Nagata
  - Damgaard, Matsuura
  - Kikukawa, Sugino
- cf. 2d $\mathcal{N} = (2, 2)$ WZ model, Sakai-Sakamoto ('83 !)

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Sugino's Lattice Formulation of 2d $\mathcal{N} = (2,2)$ SYM

- 2d Lattice

$$\Lambda = \left\{ x \in a\mathbb{Z}^2 \mid 0 \leq x_0 < \beta,\ 0 \leq x_1 < L \right\}$$

- Lattice Q-transformation see

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu) \quad \text{Link variables}$$

$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i\Big(\phi(x) - U(x,\mu)\phi(x+a\hat{\mu})U(x,\mu)^{-1}\Big)$$

$$Q\phi(x) = 0$$

$$Q\chi(x) = H(x) \quad QH(x) = [\phi(x), \chi(x)]$$

$$Q\bar{\phi}(x) = \eta(x) \quad Q\eta(x) = [\phi(x), \bar{\phi}(x)]$$

is nilpotent on the lattice

$$Q^2 = \delta_\phi \simeq 0$$

Lattice Action

- Imitating the continuum action see we adopt

$$S = Q\frac{1}{a^2 g^2}\sum_{x\in\Lambda}\mathrm{tr}\left\{\frac{1}{4}\eta(x)[\phi(x),\bar{\phi}(x)] - i\chi(x)\hat{\Phi}(x)\right.$$

$$\left. + \chi(x)H(x) - i\sum_{\mu=0}^{1}\psi_\mu(x)\Big(U(x,\mu)\bar{\phi}(x+a\hat{\mu})U(x,\mu)^{-1} - \bar{\phi}(x)\Big)\right\},$$

where the lattice field strength $\hat{\Phi}$ is

$$\hat{\Phi}(x) \simeq -iU(x,0)U(x+a\hat{0},1)U(x+a\hat{1},0)^{-1}U(x,1)^{-1} + \text{h.c.}$$

with some *important* modification

Restoration of Full SUSY?

- The above lattice formulation possesses a manifest lattice symmetry Q (and $U(1)_A$)
- But how about other $Q^{(0)}$, $Q^{(1)}$, $\widetilde{Q}$? (and $U(1)_V$, Z_2)? see
- The best thing we can hope is that these are restored in the continuum limit $a \to 0$
- Is this really the case?

This is our main objective here!

In what follows, the gauge group is $SU(N_c)$: Our numerical results are for $SU(2)$ only

RESTORATION OF SUSY?

How SUSY (Other than Q) Is Restored?

- Perturbative argument (Kaplan et at.):
  - SUSY breaking (owing to the lattice regularization) can be removed by *local* counterterms in the continuum limit
  - Possible local term in the effective action in the ℓ-loop

$$a^{p+2\ell-4}(g^2)^{\ell-1}\int d^2x\,\varphi^a\partial^b\psi^{2c},\qquad p\equiv a+b+3c\geq 0$$

(up to some powers of $\ln a$)

  - Operators with $p+2\ell-4\leq 0$ survive in the continuum limit $a\to 0$. It is enough to consider $\ell=0,1,2$
- For $\ell=0$, the continuum limit coincides with the target theory

How SUSY (Other than Q) Is Restored? (cont'd)

- For $\ell = 1$, only $p = 0, 1, 2$ are dangerous

  $p = 0 \Rightarrow$ identity operator, no dynamical effect

  $p = 1 \Rightarrow \varphi$, but $\mathrm{tr}\{\varphi\} \equiv 0$

  $p = 2 \Rightarrow \varphi\varphi \leftarrow$ prohibited by gauge, $U(1)_A$, Q symmetries

One-loop scalar self-energy

- Each of these is logarithmically divergent
- If SUSY, the sum vanishes at zero external momentum

- For $\ell = 2$, only $p = 0$ is marginal (i.e., the identity)

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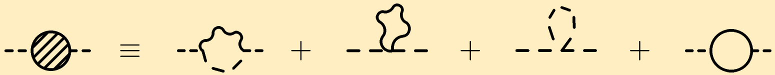

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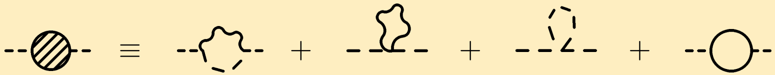

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Motivation for Direct Confirmation

- Although the above argument is highly plausible, it is not completely free from question (at least for me)
- There is a “hidden” dimensionful parameter L, the physical size of the system. If this is relevant to the argument,

$$L^{p+2\ell-4}(g^2)^{(\ell-1)}\int d^2x\,\varphi^a\partial^b\psi^{2c},\qquad p\equiv a+b+3c,$$

  for example, does survive in all loops
- Another concern: Is the non-linear symmetry Q realized as it stands in the 1PI effective action? (Probably OK in 1 loop level)
- In any case, direct (nonperturbative) confirmation of SUSY restoration by numerical means is certainly desirable

But How?

- It is not so straightforward
  - We cannot directly measure $\langle\phi(x)\overline{\phi}(0)\rangle$, because it is not gauge invariant
  - We must consider something gauge invariant (that is necessarily *composite field*)
  - The above argument however refers to the effective action of *elementary fields*
- (After many trial and fails) we finally decided to observe the conservation law of the supercurrent

$$s_\mu \equiv -\frac{1}{g^2} C\Gamma_M\Gamma_N\Gamma_\mu \,\mathrm{tr}\left\{F_{MN}\Psi\right\}$$

- The 4 spinor components of s_μ correspond to

$$(s_\mu)_1 \to Q^{(0)}, \quad (s_\mu)_2 \to Q^{(1)}, \quad (s_\mu)_3 \to \widetilde{Q}, \quad (s_\mu)_4 \to Q$$

SUSY Ward-Takahashi (WT) identities

- More definitely, we take the fermionic operator

$$f_\mu \equiv \frac{1}{g^2}\Gamma_\mu \left(\Gamma_2 \operatorname{tr}\{A_2\Psi\} + \Gamma_3 \operatorname{tr}\{A_3\Psi\}\right)$$

and examine SUSY Ward-Takahashi identities

$$\partial_\mu \langle (s_\mu)_1(x)(f_\nu)_1(0)\rangle = -i\delta^2(x)\left\langle Q^{(0)}(f_\nu)_1(0)\right\rangle$$

$$\partial_\mu \langle (s_\mu)_2(x)(f_\nu)_2(0)\rangle = -i\delta^2(x)\left\langle Q^{(1)}(f_\nu)_2(0)\right\rangle$$

$$\partial_\mu \langle (s_\mu)_3(x)(f_\nu)_3(0)\rangle = -i\delta^2(x)\left\langle \widetilde{Q}(f_\nu)_3(0)\right\rangle$$

$$\partial_\mu \langle (s_\mu)_4(x)(f_\nu)_4(0)\rangle = -i\delta^2(x)\langle Q(f_\nu)_4(0)\rangle$$

- NB: These should hold irrespective of boundary conditions

Lattice Artifacts in WT Identities

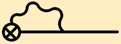
- Composite operator $s_\mu(x)$, for example, has $O(a)$ discretization ambiguity
- We must be sure that this ambiguity, when combined with UV divergence arising from the composite operator, does not modify the WT identities

General rule

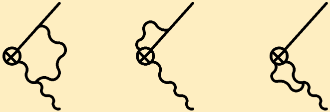
UV finite functions are safe

Possible UV Divergences in WT Identities

Supercurrent itself is UV finite in 2d



$$\propto \quad C\Gamma_{\mu}\,\mathrm{tr}\{\Psi\} = 0$$



$=$ finite! and no mixing

The one-loop scalar self-energy in sub-diagrams

$=$ finite for SUSY field content

Possible UV Divergences in WT Identities (cont'd)

Lowest 1-loop diagram

$=$ UV diverging if $x = 0$

- In fact, the divergence at $x = 0$ may modify the WT identities, as

$$\partial_\mu \langle (s_\mu)_1(x)(f_\nu)_1(0)\rangle$$
$$= -i\delta^2(x)\left\langle Q^{(0)}(f_\nu)_1(0)\right\rangle + \frac{1}{4\pi}(N_c^2 - 1)(c-1)\partial_\nu\delta^2(x)$$

- Conclusion:
  - $\langle s_\mu(x) f_\nu(0)\rangle$ is UV convergent for $x \neq 0$
  - We should examine the WT identities for $x \neq 0$!

One More (Final and Crucial) Element

- We need to introduce a scalar mass term

$$S_{\rm mass} = \frac{\mu^2}{g^2}\int d^2x\, {\rm tr}\left\{\overline{\phi}\phi\right\} \Longrightarrow \frac{\mu^2}{g^2}\sum_{x\in\Lambda}{\rm tr}\left\{\overline{\phi}(x)\phi(x)\right\}$$

- This (softly) breaks SUSY and the WT identifies become

"PCSC" relation (for $x \neq 0$) return

$$\partial_\mu \left\langle (s_\mu)_i(x)(f_\nu)_i(0)\right\rangle - \frac{\mu^2}{g^2}\left\langle (f)_i(x)(f_\nu)_i(0)\right\rangle = 0 \quad \text{no sum over } i,$$

where

$$f \equiv -2C\left(\Gamma_2\,{\rm tr}\{A_2\Psi\} + \Gamma_3\,{\rm tr}\{A_3\Psi\}\right)$$

- The reason for $S_{\rm mass}$ will be elucidated later

MONTE CARLO RESULTS

Before Going to That...

- Simulation with dynamical fermions (tough task...)
  - Partition function

$$\mathcal{Z} = \mathcal{N} \int d\mu \, e^{-S} = \mathcal{N}' \int d\mu_{\mathrm{B}} \, e^{-S_B} \, \mathrm{Pf}\{D\}$$

  - Pseudo-fermion

$$\begin{aligned} \mathrm{Pf}\{D\} &= e^{i \,\mathrm{Arg}\,\mathrm{Pf}\{D\}} (\det D^\dagger D)^{1/4} \\ &= e^{i \,\mathrm{Arg}\,\mathrm{Pf}\{D\}} \int d\varphi \, d\overline{\varphi} \, e^{-\overline{\varphi}(D^\dagger D)^{-1/4} \varphi} \end{aligned}$$

  - Rational approximation (RHMC '04)

$$x^{-1/4} \simeq \alpha_0 + \sum_{i=1}^{N} \frac{\alpha_i}{x + \beta_i}$$

Remez algorithm, multi-shift solver, ...

Simulation Parameters ($\sim 20,000\,\mathrm{CPU}\cdot\mathrm{hour}$)

- 2d rectangular lattice

$$\Lambda \equiv \left\{ x \in a\mathbb{Z}^2 \mid 0 \leq x_0 < 2L,\ 0 \leq x_1 < L \right\}, \qquad Lg = 1.414$$

- Lattice sizes

$$12 \times 6, \quad 16 \times 8, \quad 20 \times 10$$

- Lattice spacings

$$ag = 0.2357, \quad 0.1768, \quad 0.1414$$

- Scalar masses

$$\mu^2/g^2 = 0.04, \quad 0.25, \quad 0.49, \quad 1.0, \quad 1.69$$

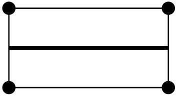
- Number of uncorrelated configurations

800–1800

Correlation Functions with antiPeriodic BC (aPBC)

- Following 4 ($i = 1, 2, 3, 4$) coincide in the continuum theory

$$\langle (s_0)_i(x)(f_0)_i(0)\rangle / g^2 \quad i = 1, 2, 3, 4$$



owing to the $U(1)_V$ and the Z_2 symmetries

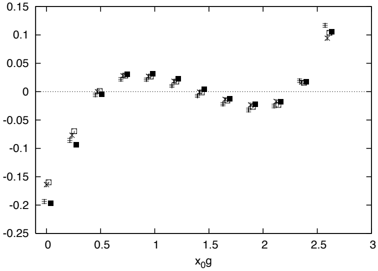


Figure: 12×6, $ag = 0.2357$, $\mu^2/g^2 = 1.0$. Along the line $x_1 = L/2$. $i = 1$ (+), $i = 2$ ($\times$), $i = 3$ ($\square$), $i = 4$ ($\blacksquare$)

SUSY WT identity (aPBC)

- The left-hand side of the WT identity see

$$\partial_\mu \langle (s_\mu)_1(x)(f_0)_1(0)\rangle /g^3 - \frac{\mu^2}{g^2}\langle (f)_1(x)(f_0)_1(0)\rangle /g^3$$

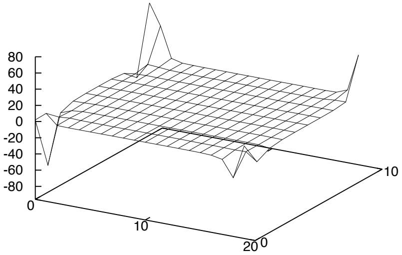
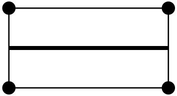


Figure: 20×10, $ag = 0.1414$, $\mu^2/g^2 = 1.0$

SUSY WT identities (PCSC relation) (aPBC)

- The ratio

$$\frac{\partial_\mu \langle (s_\mu)_1(x)(f_0)_1(0)\rangle}{\langle (f)_1(x)(f_0)_1(0)\rangle} \left(\Rightarrow \frac{\mu^2}{g^2}\right)$$



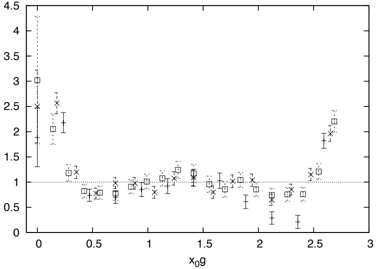


Figure: $\mu^2/g^2 = 1.0$. Along the line $x_1 = L/2$. $ag = 0.2357$ (+), $ag = 0.1768$ ($\times$), $ag = 0.1414$ ($\Box$)

χ^2-fit for the Plateau Region (aPBC)

- The ratio

$$\frac{\partial_\mu \langle (s_\mu)_1(x)(f_0)_1(0)\rangle}{\langle (f)_1(x)(f_0)_1(0)\rangle}$$

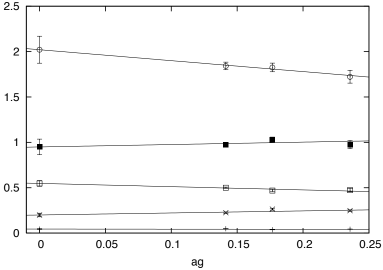


Figure: $\mu^2/g^2 = 0.04$ $(+)$, $\mu^2/g^2 = 0.25$ $(\times)$, $\mu^2/g^2 = 0.49$ $(\square)$, $\mu^2/g^2 = 1.0$ $(\blacksquare)$, $\mu^2/g^2 = 1.69$ $(\bigcirc)$

We Observe PCSC! (aPBC)

- The continuum limit of the ratio

$$\frac{\partial_\mu \langle (s_\mu)_i(x)(f_0)_i(0)\rangle}{\langle (f)_i(x)(f_0)_i(0)\rangle} \left(\Rightarrow \frac{\mu^2}{g^2}\right)$$

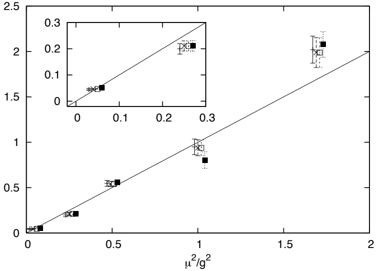


Figure: $i = 1$ (+), $i = 2$ (×), $i = 3$ (□), $i = 4$ (■)

Summary at This Stage

- For $\mu^2/g^2 > 0$, with aPBC, PCSC is observed in the continuum limit
  - Breaking of SUSY (and other symmetries) owing to lattice regularization in fact disappears
  - The target (2d $\mathcal{N} = (2, 2)$ SYM with SUSY breaking scalar mass) seems to be obtained in the continuum limit
- This is the first example in lattice gauge theory in which the restoration of SUSY was clearly confirmed!

How about the Periodic BC (PBC) Case?

- Following 4 ($i = 1, 2, 3, 4$) coincide in the continuum theory

$$\langle (s_0)_i(x)(f_0)_i(0)\rangle / g^2 \quad i = 1, 2, 3, 4$$

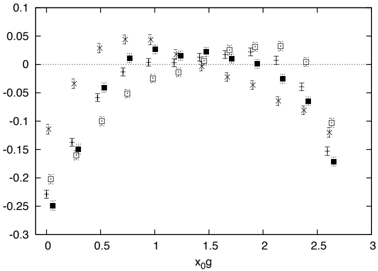


Figure: PBC. 12×6, $ag = 0.2357$, $\mu^2/g^2 = 1.0$. Along the line $x_1 = L/2$. $i = 1$ ($+$), $i = 2$ ($\times$), $i = 3$ ($\square$), $i = 4$ ($\blacksquare$)

How about the Periodic BC (PBC) Case? (cont'd)

- For $\mu^2/g^2 > 0$, PBC case is the subject of further study

Without the Scalar Mass? $\mu^2/g^2 = 0$

- We could not achieve the thermalization…

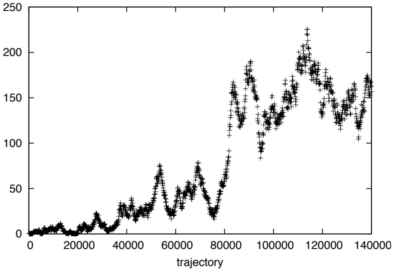


Figure: Monte Carlo evolution of $a^2\,\mathrm{tr}\{\bar{\phi}\phi\}$ with aPBC. 12×12, $ag = 0.1179$

Without the Scalar Mass? $\mu^2/g^2 = 0$ (cont'd)

- and, generally, scalar fields acquire very large value along the flat directions

$$\phi, \overline{\phi} \gtrsim \frac{1}{a}$$

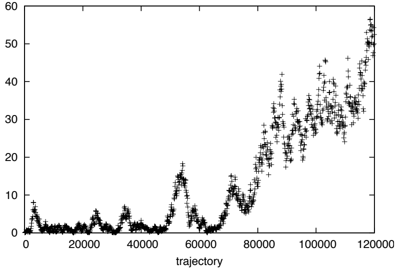


Figure: Monte Carlo evolution of $a^2\,\mathrm{tr}\{\overline{\phi}\phi\}$ with aPBC. 18×12, $ag = 0.1179$

Very Large Value of Scalars may Cause Trouble

- Such very large value could amplify $O(a)$ quantities to $O(1)$,
$$a\phi \sim O(1), \text{ instead of } O(a)$$
and could ruin the power counting. For example, the combination
$$Q(a\,\mathrm{tr}\{\bar{\phi}\psi_\mu\}) = a\,\mathrm{tr}\{\eta\psi_\mu\} + a\,\mathrm{tr}\{\bar{\phi} i D_\mu \phi\},$$
might be $O(1)$. This is invariant under gauge, $U(1)_A$, Q transformations, but is not invariant under $Q^{(0)}$, $Q^{(1)}$, $\widetilde{Q}^{(0)}$

SOME PHYSICS

2d $\mathcal{N} = (2, 2)$ SYM with (small) SUSY breaking scalar mass

Correlation Functions with Power-like Behavior

- This system has no mass gap (Witten) $\Leftarrow$ 't Hooft anomaly matching condition
- More definitely, on $\mathbb{R}^2$ (Fukaya, Kanamori, H.S., Hayakawa, Takimi)

$$
\begin{aligned}
&-\frac{i}{2}\langle j_\mu(x)\epsilon_{\nu\rho}j_{5\rho}(0)\rangle \\
&=\frac{1}{4\pi}(N_c^2-1)\int\frac{d^2p}{(2\pi)^2}\,e^{ipx}\left\{-\frac{1}{p^2}(p_\mu p_\nu-\epsilon_{\mu\rho}\epsilon_{\nu\sigma}p_\rho p_\sigma)+\widetilde{c}\delta_{\mu\nu}\right\} \\
&=\frac{1}{4\pi}(N_c^2-1)\left\{\frac{1}{\pi}\frac{1}{(x^2)^2}(x_\mu x_\nu-\epsilon_{\mu\rho}\epsilon_{\nu\sigma}x_\rho x_\sigma)+\widetilde{c}\delta_{\mu\nu}\delta^2(x)\right\},
\end{aligned}
$$

where j_μ and $j_{5\rho}$ are $U(1)_V$ and $U(1)_A$ currents, respectively ($\widetilde{c}$ is ambiguity in operator definition)

Can We See This Massless Bosonic State?

- Power-like behavior on $\mathbb{R}^2$

$$-\frac{i}{2}\langle j_0(x)\epsilon_{0\rho}j_{5\rho}(0)\rangle = \frac{3}{4\pi^2}\frac{1}{(x_0)^2}, \qquad \text{for } N_c = 2 \text{ along } x_1 = 0$$

- If so, the $U(1)_V$ symmetry is restored

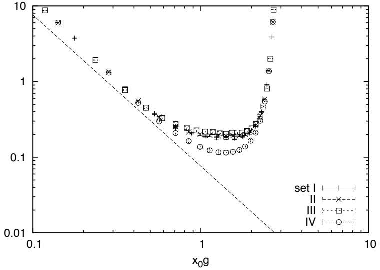


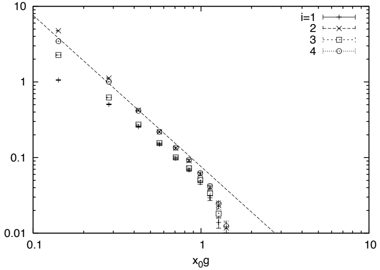
Figure: IV: $\mu^2/g^2 = 0.25$. 20×16, $ag = 0.1414$. aPBC

Almost Degenerated Fermionic State

- SUSY WT identity

$$\langle (s_0)_i(x)(f_0)_i(0)\rangle = -\frac{i}{2}\langle j_0(x)\epsilon_{0\rho}j_{5\rho}(0)\rangle \overbrace{-\left\langle j_0(x)\epsilon_{0\rho}\frac{1}{g^2}\,\mathrm{tr}\left\{A_3(0)F_{\rho 2}(0)-A_2(0)F_{\rho 3}(0)\right\}\right\rangle}^{O(g^2);\ \text{no massless singularity}}$$

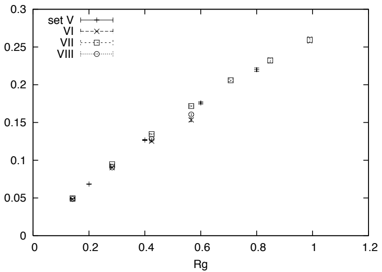
(This follows from $\delta\langle j_\mu(x)f_\nu^T(0)\rangle = 0$, neglecting μ^2 and aPBC)



Static Potential between Charges in Fund. Reps.

- Static potential between charges in the fundamental representation $V(R)/g$

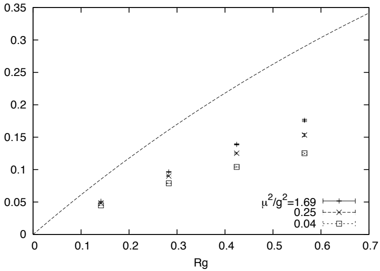
$$-\ln\{W(T,R)\} = V(R)T + c(R)$$



- This confining behavior appears distinct with a conjecture in the '90s by Armoni, Frishman and Sonnenschein

Static Potential (cont'd)

- Static potential between charges in the fundamental representation $V(R)/g$ for various scalar masses



- The broken line: Gross, Klebanov, Matytsin, Smilga for $\mu^2/g^2 \to \infty$

SUMMARY

Summary

- SUSY breaking owing to lattice regularization certainly disappears in the continuum limit (this is the first firm demonstration!)
- It appears that 2d $\mathcal{N} = (2, 2)$ SYM with a (small) SUSY breaking scalar mass is realized in the machine
- We illustrated some physical application
- Outlook
  - Physical questions: Further study of the static potential, spectrum of excited states, etc. . .
  - SUSY theory by $\mu^2/g^2 \to 0$ limit
  - Spontaneous SUSY breaking in this limit (Kanamori, Sugino, H.S.)
  - Issue of the vacuum modulus
  - Other theories, other formulation on the basis of similar idea. . .