GEOMETRIC ENTROPY AND HAGEDORN/DECONFINEMENT TRANSITION

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Introduction

- The Entanglement Entropy is a measure for counting the number of the quantum mechanically entangled states.

- Application
- Condensed matter physics
  → An order parameter of the phase transition
- The AdS/CFT correspondence
  → The dual gravity interpretation
- Topological field theory

- We want to find the similar measure which is an order parameter of the phase transition.
- Especially, for the YM theory on the compactified space.

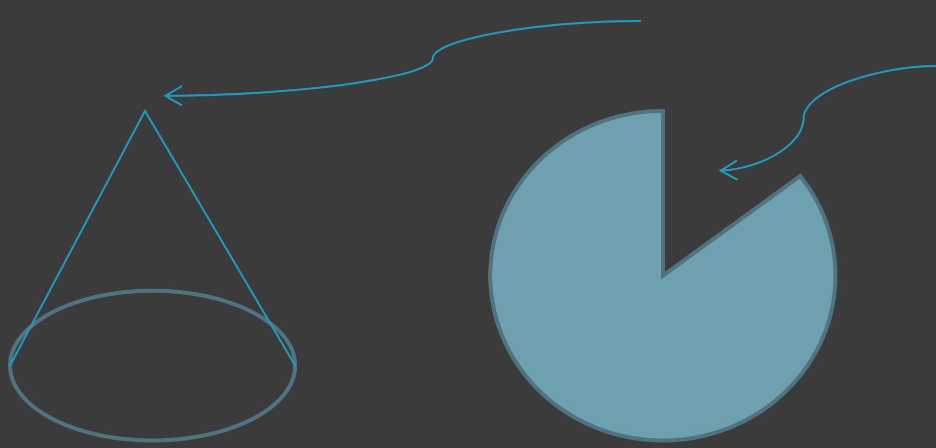
N=4 SYM on the orbifold S^3/Z_n

- $S^1 \times S^3$ compactification of N=4 SYM

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta\,(d\psi^2 + \sin^2\psi\, d\phi^2)$$

$$0 \le \phi \le 2\pi \;\rightarrow\; 0 \le \phi \le \frac{2\pi}{n} \quad (n \ne 1)$$

- Note that n is not always integer.
- The conical singularity at $\psi = 0,\ \pi$



The deficit angle $2\pi(1\text{-}1/n)$

The Geometric entropy

- The partition function of the N=4 SYM on the orbifold S^3/Z_n is Z(n).
- Z(1): the partition function on S^3.

- The identification $$\frac{Z(n)}{\left(Z(1)\right)^n} = Tr\rho^n, \qquad \rho = e^{-2\pi H}$$

- ρ: the density matrix, H: φ's Hamiltonian
- We define the geometric entropy by,

$$S_G = -Tr\rho\log\rho = -\frac{\partial}{\partial n}\log\left[\frac{Z(n)}{Z(1)^n}\right]\Bigg|_{n=1}$$

The Geometric entropy and Hagedorn/Deconfinement Transition

- **Free N=4 SYM** on S^3 can capture the confinement/deconfinement phase transition.

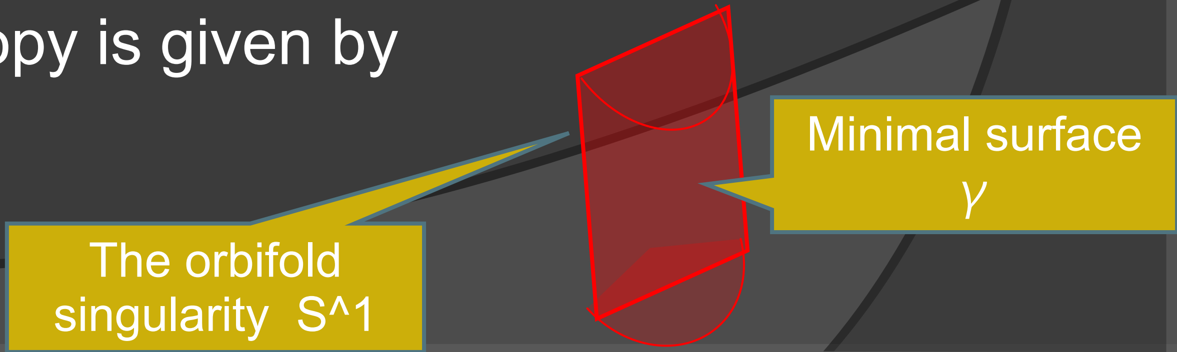
  - The partition function of the free N=4 SYM on the orbifold

$$Z(n)=\int[dU]e^{\sum_{m=1}^{\infty}\frac{1}{m}\left(z_s(x^m)+z_v(x^m)+(-1)^{m+1}z_f(x^m)\right)Tr(U^m)Tr(U^{\dagger m})},\quad x=e^{-1/T}$$

$$z_s(x)=6\frac{x(1+x^n)}{(1-x)^2(1-x^n)},\quad z_v(x)=\frac{2x^2(1+2x^{n-1}-x^n)}{(1-x)^2(1-x^n)},\quad z_f=\frac{16x^{\frac{n}{2}+1}}{(1-x)^2(1-x^n)},$$

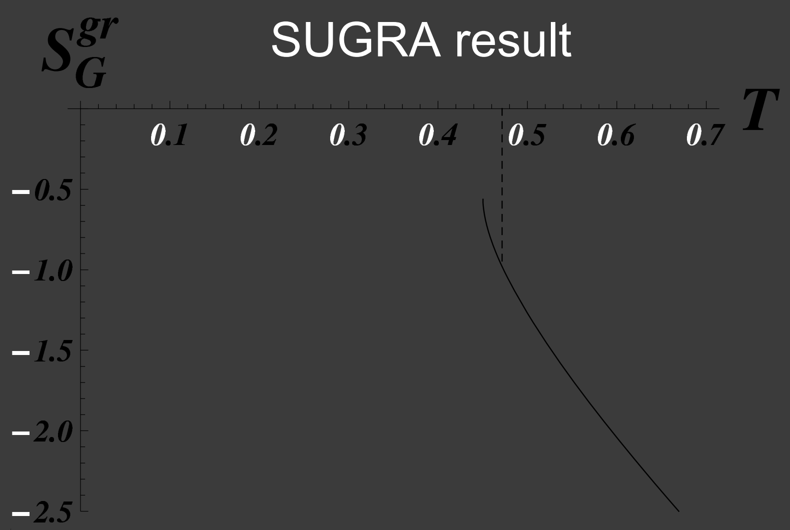
- **SUGRA analysis** (strongly coupled SYM)
  - The geometric entropy is given by

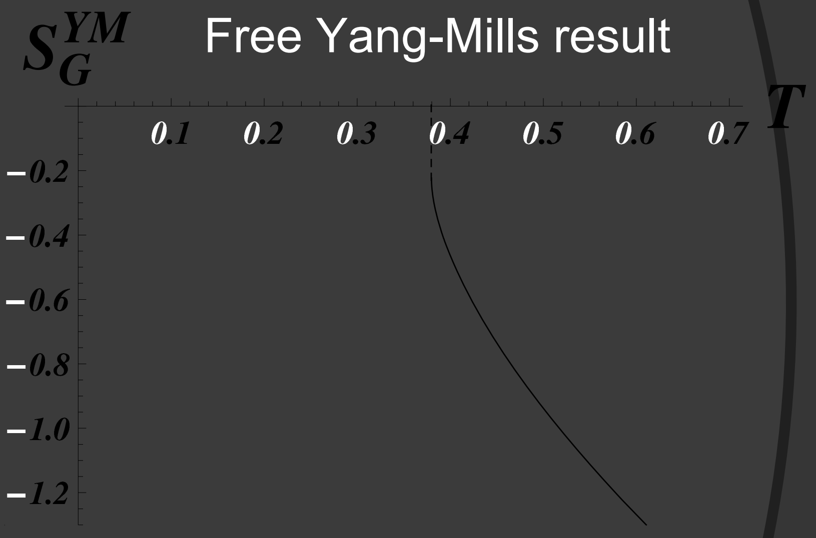
$$S_G=\frac{Area(\gamma)}{4G_N^{(5)}}$$



The Geometric entropy and Hagedorn/Deconfinement Transition

- Below, we compare the geometric entropy from gravity with that of the free Yang-Mills





⇒ The geometric entropy as an order parameter of these phase transitions.

Geometric Entropy in 2-dim Yang-Mills (Large N)

- The partition function of the U(N) 2-dim Yang-Mills theory:

$$Z_g(A) = \sum_R \dim R^{2-2g} e^{-\frac{1}{2N} C_2(R) A}$$

A: the area of the Rieman surface in units of the 't Hooft coupling

- We set the genus g=0.
- The geometric entropy:

Area of n-sheeted surface =nA

$$S_G(g) = -\frac{\partial}{\partial n} \log \left[\frac{Z(nA)}{(Z(A))^n} \right] \Bigg|_{n=1} = N^2 (AF'(A) - F(A))$$

- The strong-weak coupled phase transition at A=Ac

$$\Delta S_G(A) \approx N^2 (A - A_c)^2$$

Conclusion

- In the N=4 SYM, the geometric entropy can be used as the order parameter of confinement/deconfinement transition.
- In the dual gravity description, the geometric entropy is also the order parameter of the Hagedorn transition.
- The geometric entropy is well defined in 2-dim Yang-Mills theory.
- Application for the other CFT with the holographic dual and the other topological field theory.