Super Schrödinger algebra
&
non-relativistic limits of supersymmetric Chern-Simons-matter theories

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@ 理 研シンポジウム「場と弦の理論の新展開に向けて」

with Kentaroh Yoshida (KITP, UCSB)

- "Super Schrödinger algebra in AdS/CFT,"
  J. Math. Phys. 49 (2008) 102302 [arXiv:0805.2661]
- “More super Schrödinger algebras from psu(2,2|4),"
  JHEP 0808 (2008) 049 [arXiv:0806.3612]

with Yu Nakayama, Shinsei Ryu (UC Berkeley) and Kentaroh Yoshida

- "A family of super Schrödinger invariant Chern-Simons matter systems,“
  JHEP 0901 (2009) 006 [arXiv:0811.2461]
- "Interacting SUSY-singlet matter in non-relativistic Chern-Simons theory"
  arXiv:0812.1564

and in preparation

Motivation

AdS/CFT対応

$psu(2,2|4) \supset so(2,4) \times su(4)$

IIB string in $AdS_5 \times S^5$ ⟺ N=4 super YM in 4-dim

弱結合 *強結合*

応用　AdS/CMP

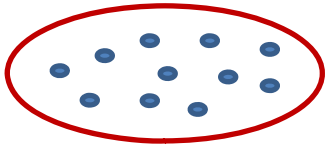
superconductor [Gubser, Hartnoll-Herzog-Horowitz],…

quantum Hall effect [Davis-Kraus-Shar],…

Fermions at unitarity

蒸発冷却、レーザー冷却 ⇒ (neutral) 原子気体

Cold atoms are trapped



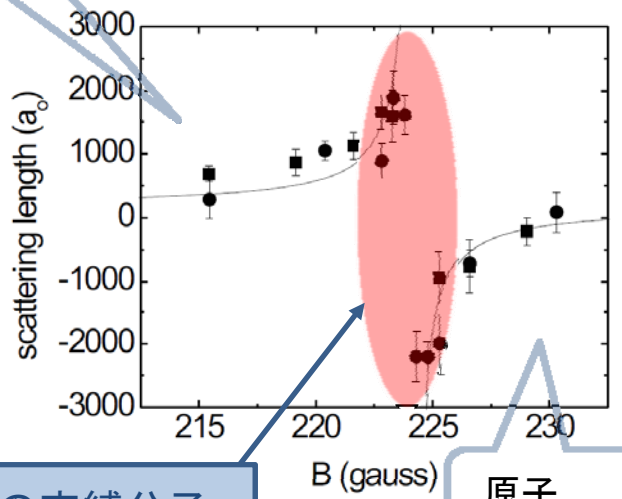
Superfluidity of the atomic gas of ^6Li and ^{40}K (fermions)　[2004]

Feshbach 共鳴

[Regal-Jin,PRL90(2003)230404]

原子間の引力の強さを外磁場によって
自由に制御できる　$a(B) \sim 1 - \dfrac{w}{B - B_0}$

a_0：ボーア半径



相互作用の強さを特徴付ける散乱長 a：

弱い引力　$-\infty < a^{-1} < +\infty$　強い引力

原子間力の到達距離：　r_0~$60a_0$ (^{40}K), $30a_0$ (^{6}Li)

$r_0/a \to 0$ と近似　相互作用の詳細によらなくなる（universal）

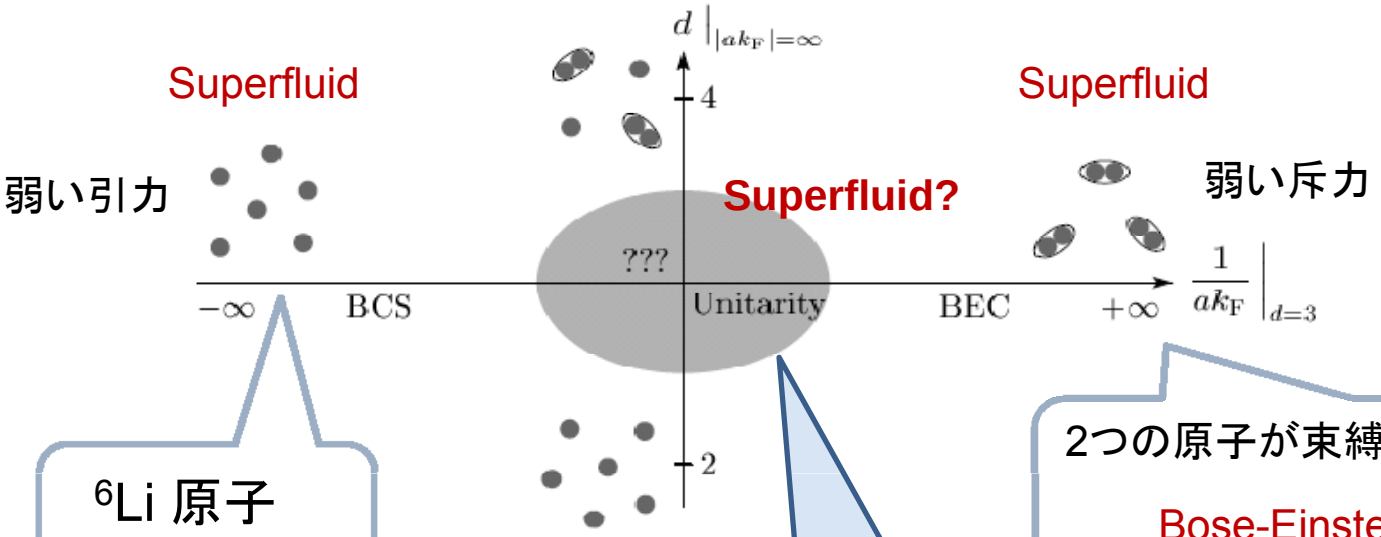
短距離力で相互作用する spin 1/2のfermion

$$\mathcal{L} = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi_\sigma - c_0 \psi_\downarrow^\dagger \psi_\uparrow^\dagger \psi_\uparrow \psi_\downarrow$$

Schrödinger invariant

BCS-BEC crossover

[Nishida's figure]



^{6}Li 原子
BCS 機構

強結合
Scale invariant

2つの原子が束縛分子(boson)を構成
Bose-Einstein 凝縮 (BEC)

両端で摂動論 $|ak_F|<<1$

a: scattering length
k_F : Fermi momentum
d: spatial dimensions

Unitarity limit ⇒ **Schrödinger symmetry**

AdS/NRCFT

[Son,Balasubramanian-McGreevy,…]
[Goldberger, Bgarbon-Fuertes]

(D+3)-dim. AdS geometry ⟺ (D+2)-dim. CFT

⇓ Light-cone reduction

Deformed AdS geometry ⟺ **(D+1)-dim. Schrödinger-invariant FT**

Schrödinger-invariant

- LC reduction of AdS_{D+3}
- (D+3)-dim. Schrödinger inv. Geometry (deformation of AdS_{D+3})
- (D+2)-dim. Schrödinger-inv. geometry

Klein-Gordon in (D+2)-dim $\partial^2\phi = 0$ [Gomis-Pons]

$$t = \frac{1}{\sqrt{2}}\left(x^0 - x^{D+1}\right) \ , \quad \xi = \frac{1}{\sqrt{2}}\left(x^0 + x^{D+1}\right)$$

⇓ $\phi = \mathrm{e}^{-im\xi}\psi(t, x^i)$

Schrödinger in (D+1)-dim.

$$\left(i\partial_t + \frac{\nabla^2}{2m}\right)\psi = 0$$

A mode with a definite momentum along ξ

Contents

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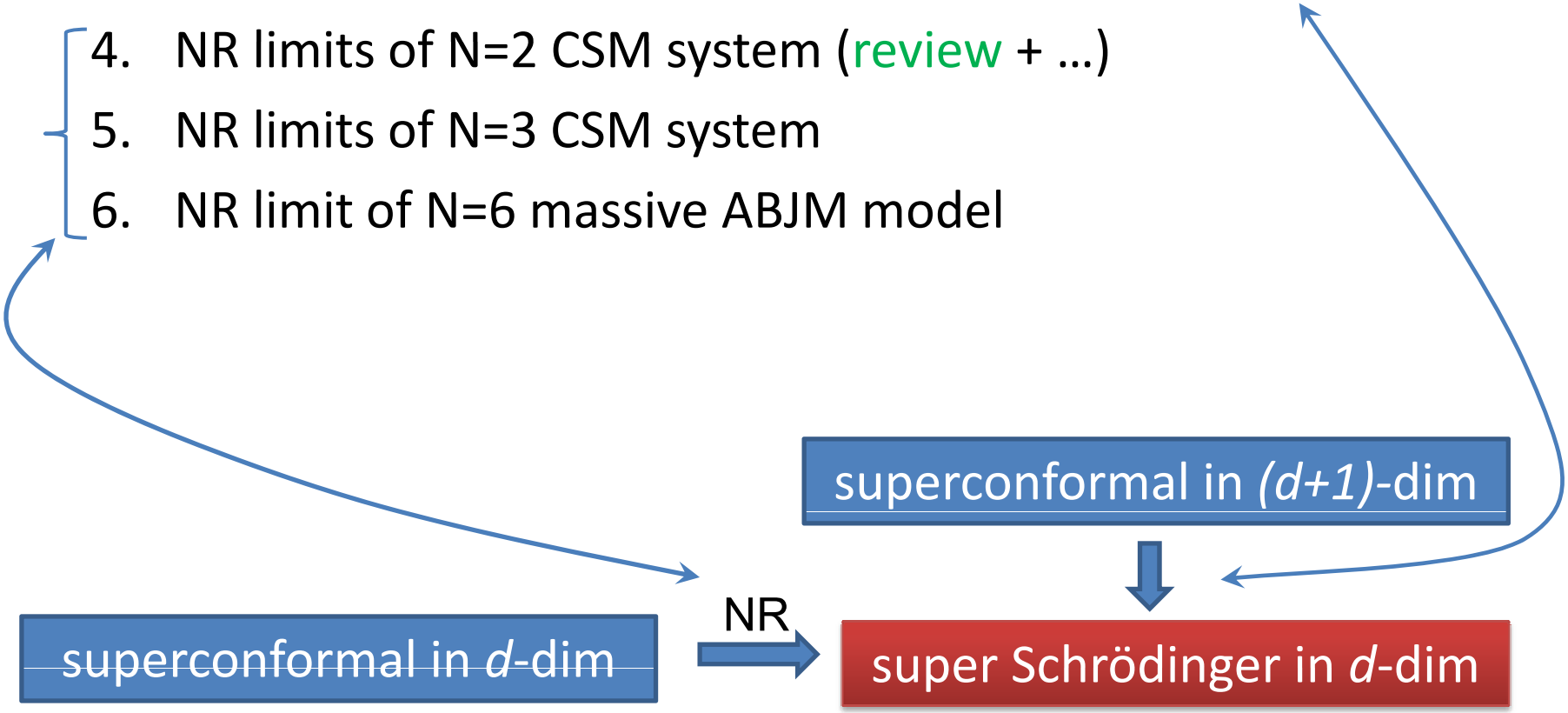
2. AdS/NRCFT (review)
3. Super Schrodinger algebra from superconformal algebra
4. NR limits of N=2 CSM system (review + …)
5. NR limits of N=3 CSM system
6. NR limit of N=6 massive ABJM model



Results

sch(d-1) from so(2,d+1) (Sch(d-1) $\subset$ conf$_{d+1}$ = AdS$_{d+2}$)

[MS-Yoshida]

super Schrödinger algebra from psu(2,2|4), osp(8|4) and osp(8*|4)

	# susy	R-symmetry	superconformal	
sch(2)	24	su(4)	N=4, (1+3)-dim	psu(2,2|4)
sch(2)	12	$su(2)^2 \times u(1)$	N=2*, (1+3)-dim	$\subset$ psu(2,2|4)
sch(2)	12	$su(2) \times u(1)$	N=2, (1+3)-dim	su(2,2|2)
sch(2)	6	$u(1)^3$	N=1*, (1+3)-dim	$\subset$ psu(2,2|4)
sch(2)	6	u(1)	N=1, (1+3)-dim	su(2,2|1)
sch(1)	24	so(8)	N=8, (1+2)-dim	osp(8|4)
sch(4)	24	sp(4)	N=2, (1+5)-dim	osp(8*|4)

Field theories which are invariant under these symmetry ?

symmetry of NR limit of N=2 Chern-Simons-matter system

NR limit of supersymmetric field theories

[Nakayama-Ryu-MS-Yoshida]

N=2 CSM system [Leblanc-Lozano-Min'92]

$$\{H,\ P_i,\ J,\ G_i,\ D,\ K,\ N_{\rm B},\ N_{\rm F},\ Q_1,\ Q_2,\ S\}$$

super Schrödinger algebra with 6 susy and u(1) R-symmetry

N=3 CSM system (maximal susy with one A)

$$\{H,\ P_i,\ J,\ G_i,\ D,\ K,\ M,\ N_B,\ N_F,\ Q_1^{(1)},\ Q_1^{(2)},\ Q_2,\ S\}$$

super Schrödinger algebra with 8 susy and $u(1)^2$ R-symmetry

N=6 massive ABJM model

$$\{H,\ P_i,\ J,\ G_i,\ D,\ K,\ M,\ R^{mn},\ R,\ Q_1^0,\ Q_1^m,\ Q_2,\ S\}$$

super Schrödinger algebra with 14 susy and $su(2)^2$ x u(1) R-symmetry

New super Schrödinger algebras