Finite Gluon Fusion Amplitude in the Gauge-Higgs Unification

Nobuhito Maru
(Chuo University)

Based on
N.M., MPL A23 2737 (2008)

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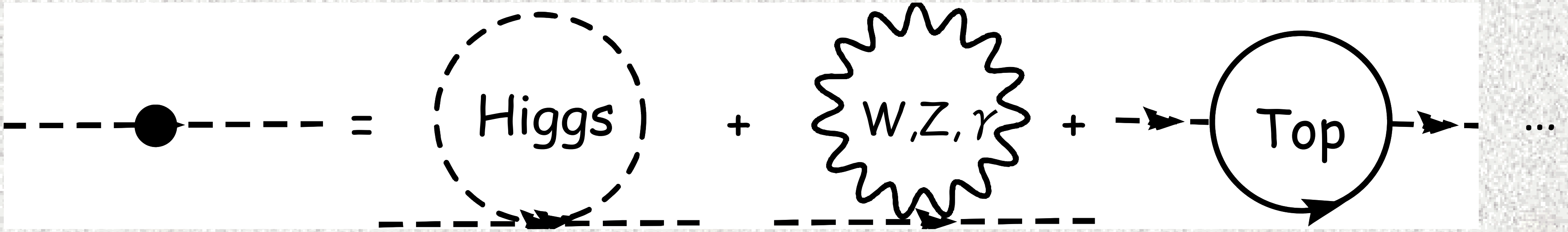
"Towards New Developments in Field & String theories"@RIKEN

Introduction

One of the problems in the Standard Model:
Gauge Hierarchy Problem

Quantum corrections to Higgs mass
is sensitive to the cutoff scale of the theory



$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!!
(Natural cutoff scale is
Planck/GUT scale)

In GHU, extra component of higher dim gauge field is identified with Higgs

where the (local) mass term A_y^2 is forbidden by its gauge transformation

$$\because A_y \to A_y + \partial_y \varepsilon(x,y) + i\left[\varepsilon(x,y), A_y\right]$$

In other words, no local counter term is allowed
⇒ No quadratic divergence, finite regardless of the nonrenormalizability

$$m_{A_y}^2 \simeq \frac{1}{16\pi^2}\frac{1}{R^2}$$

Explicit calculations of Higgs mass

- D-dim QED on S^1@1-loop Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2@1-loop
  Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2@1-loop
  Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2@1-loop Lim, N.M. & Hasegawa (2006)
- 5D QED on S^1@2-loop
  N.M. & Yamashita (2006), Hosotani, N.M., Takenaga & Yamashita (2007)

...

GHU has opened up a new avenue
to solve the gauge hierarchy problem

Question:

Is there any other finite physical observable?

Natural to guess in the gauge-Higgs sector of the SM

One of the candidates: **Gluon Fusion amplitude gg -> H**

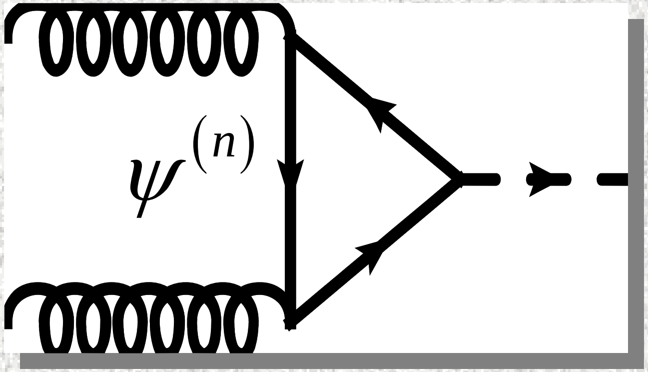
Coefficient of Dim 5 operator

$$\langle H^\dagger \rangle H G^a_{\mu\nu} G^{a\mu\nu} \Rightarrow \langle A_y \rangle A_y G^a_{\mu\nu} G^{a\mu\nu}$$

We can expect them to be finite due to the higher dim. gauge invariance

$$A_y \to A_y + \partial_y \varepsilon(x,y) + i\left[\varepsilon(x,y), A_y\right]$$

This process are very important because the dominant Higgs production process @LHC



Explicit calculation in (D+1)-dim SU(3) gauge theory with a triplet fermion on S^1/Z2

$$= -\frac{m}{v} g_s^2 \frac{2^{[D/2]} R^2}{(4\pi)^{(D-3)/2}} \int_0^1 dx \int_0^{1-x} dy \left(1 - 4xy\right) \left(xy m_H^2\right)^{(D-3)/4}$$

$$\times \sum_{n=1}^{\infty} \frac{n \sin\left(2\pi nmR\right)}{\left(\pi Rn\right)^{(D-3)/2}} K_{(D-3)/2}\left(2\pi Rnm_H \sqrt{xy}\right)$$

$$\simeq -\frac{m g_s^2}{3 \bullet 2^{3(D-1)/2-[D/2]} \pi^{3(D-3)/2} v R^{D-5}} \Gamma\left(\frac{D-3}{2}\right) \sum_{n=1}^{\infty} \frac{\sin\left(2\pi nmR\right)}{n^{D-4}}$$

$$\left(\sum_{n=1}^{\infty} \sin\left(2\pi nmR\right) = \frac{1}{2} \cot\left(\pi mR\right) : D = 4, \sum_{n=1}^{\infty} \frac{\sin\left(2\pi nmR\right)}{n} = \frac{\pi}{2}\left(1 - mR\right) : D = 5 \right)$$

Comments

- Even in realistic compactifications, the finiteness remains unchanged since the information of compactification is an IR nature

- At higher order loops, divergences from subdiagrams have to be renormalized by the ordinary procedures

- Two photon decay amplitude H -> $\gamma\gamma$ is also finite (Main discovery mode in case m_H < 150GeV)

$$\langle H^\dagger \rangle H F_{\mu\nu} F^{\mu\nu} = 0 \quad \because \left[\langle A_y^{(0)} \rangle, \gamma_\mu \right] = 0$$

Summary

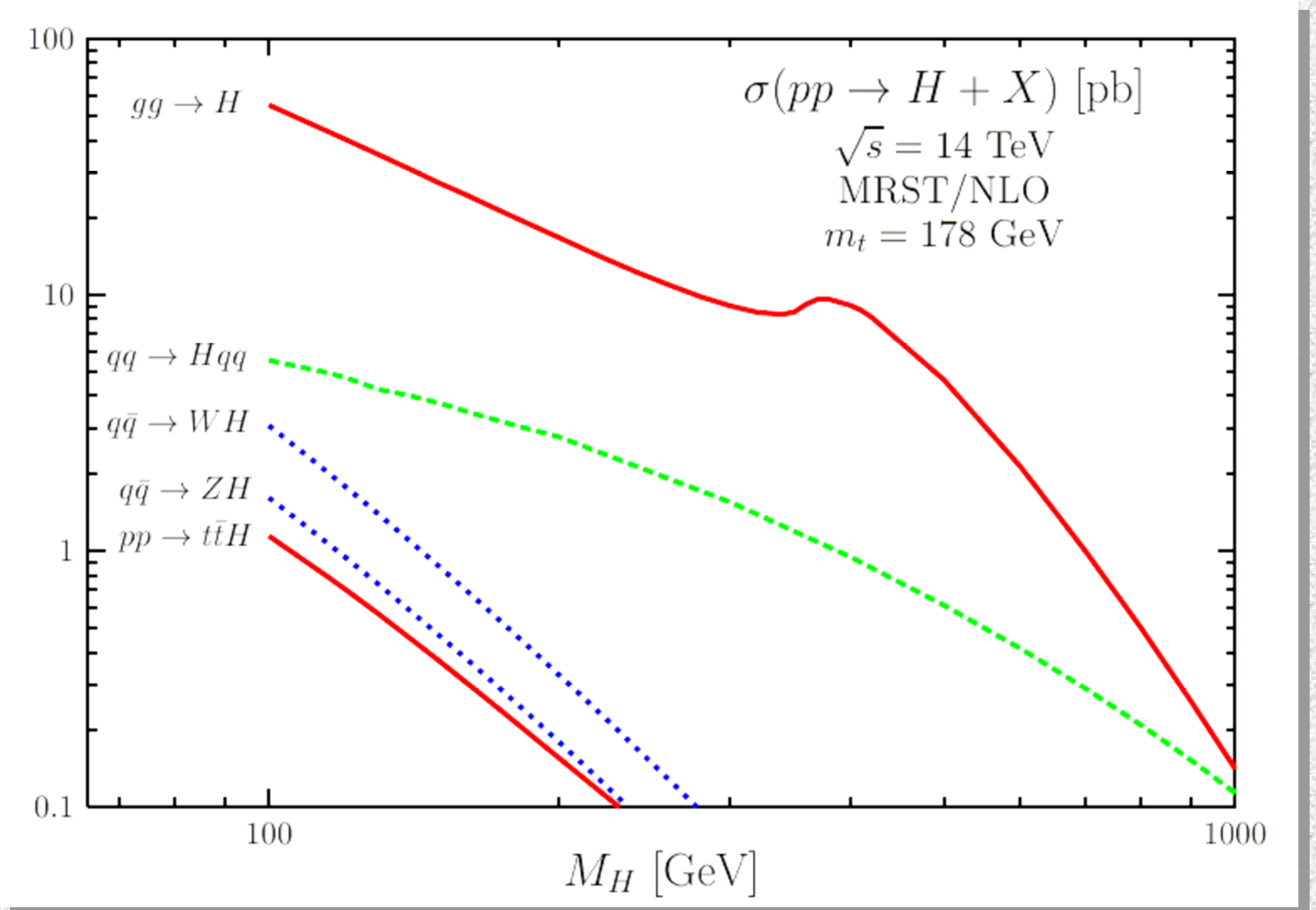
- We have shown that the gluon fusion amplitude
  gg -> H
  is finite in gauge-Higgs unification in any dimension
  regardless of its nonrenormalizablity
  (striking impact on collider physics!!)

- Gauge invariant operator describing the amplitude
  is forbidden
  by higher dimensional gauge symmetry

- This is a new finite physical observable
  other than Higgs mass

Backup Slides

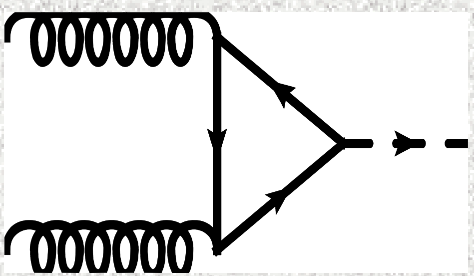
Gluon fusion is the dominant Higgs production process



SM contributions

gg -> H

$$\mathcal{L}_{eff} = C_{glue}^{SM} h G^{a\mu\nu} G_{\mu\nu}^{a}$$

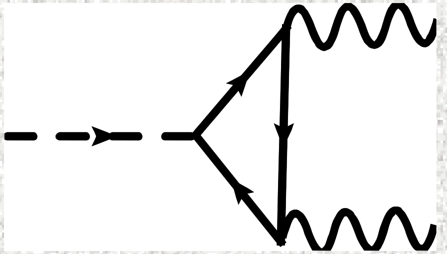


$$C_{glue}^{SM} = -\frac{m_t}{v} \times \frac{\alpha_S F_{1/2}\left(4m_t^2/m_h^2\right)}{8\pi m_t} \times \frac{1}{2}$$

$$F_{1/2}(x) = -2x\left[1+(1-x)\left(\sin^{-1}\left(1/\sqrt{x}\right)\right)^2\right] \to -\frac{4}{3}(x \gg 1)$$

H -> $\gamma\ \gamma$

$$\mathcal{L}_{eff} = C_{\gamma}^{SM} h F^{\mu\nu} F_{\mu\nu}$$



$$C_{\gamma}^{SM} = C_{\gamma}^{t} + C_{\gamma}^{W}$$

$$C_{\gamma}^{t} = -\frac{m_t}{v} \times \frac{\alpha_{em} F_{1/2}\left(4m_t^2/m_h^2\right)}{8\pi m_t} \times \frac{4}{3},\ C_{\gamma}^{W} = -\frac{m_W^2}{v} \times \frac{\alpha_{em} F_1\left(4m_W^2/m_h^2\right)}{8\pi m_W^2}$$

$$F_1(x) = 2+3x+3x(2-x)\left(\sin^{-1}\left(1/\sqrt{x}\right)\right)^2 \to 7\,(x \gg 1)$$

Consider (D+1)-dim SU(3) gauge theory on S^1/Z2

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + i\bar{\Psi}\Gamma^M D_M \Psi \qquad \Gamma^M = \left(\gamma^\mu, i\gamma^5\right)$$

$$F_{MN} = \partial_M F_N - \partial_N F_M - ig_5\left[A_M, A_N\right]\left(M, N = 0,1,2,3,D\right)$$

$$D_M = \partial_M - ig_5 A_M \left(A_M = A_M^a \lambda^a/2 : \lambda^a : \text{Gell-Mann matrices}\right)$$

$$\Psi = \left(\psi_1, \psi_2, \psi_3\right)^T$$

Boundary conditions: (+,+) only has massless mode

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \Psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}$$

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SU(3) → SU(2) x U(1)

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Higgs is identified with 0 mode of A5

(KK modes of A5 are absorbed into KK gauge bosons)

Consider (D+1)-dim SU(3) gauge theory on S^1/Z2

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Chiral fermions are easily obtained

we obtain D-dim effective Lagrangian of fermion

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\Bigg[i\left(\bar\psi_1^{(n)},\bar\psi_2^{(n)},\bar\psi_3^{(n)}\right)\gamma^\mu\partial_\mu \begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}$$

$$+\frac{g}{2}\left(\bar\psi_1^{(n)},\bar\psi_2^{(n)},\bar\psi_3^{(n)}\right)\begin{pmatrix} W_\mu^3+\frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0\\ \sqrt{2}W_\mu^- & -W_\mu^3+\frac{B_\mu}{\sqrt{3}} & 0\\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}$$

$$-\left(\bar\psi_1^{(n)},\bar\psi_2^{(n)},\bar\psi_3^{(n)}\right)\begin{pmatrix} m_n & 0 & 0\\ 0 & m_n & -m\\ 0 & -m & m_n\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}\Bigg]$$

$$+i\bar t_L\gamma^\mu\partial_\mu t_L+\bar b\left(i\gamma^\mu\partial_\mu-m\right)b+\frac{g}{\sqrt{2}}\left(\bar t\gamma^\mu LbW_\mu^+ +\bar b\gamma^\mu LtW_\mu^-\right)+\frac{g}{2}\left(\bar t\gamma^\mu Lt+\bar b\gamma^\mu Lb\right)W_\mu^3$$

$$+\frac{\sqrt{3}g}{6}\left(\bar t\gamma^\mu Lt+\bar b\gamma^\mu Lb-2\bar b\gamma^\mu Rb\right)B_\mu$$

$$L\equiv\frac{1}{2}(1-\gamma_5),\ R\equiv\frac{1}{2}(1+\gamma_5), m_n=\frac{n}{R},\ g=\frac{g_5}{\sqrt{2\pi R}},\ m=\frac{1}{2}gv(=M_W)$$

D-dim effective Lagrangian in terms of mass eigenbasis

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty} \Bigg[i\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\gamma^\mu \partial_\mu \begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}$$

$$\begin{pmatrix} \tilde{\psi}_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}$$

$$+\frac{g}{2}\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right) \begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & W_\mu^+ & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} \\ W_\mu^- & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}} \end{pmatrix} \gamma^\mu \begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}$$

$$-\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right) \begin{pmatrix} m_n & 0 & 0 \\ 0 & m_n + m & 0 \\ 0 & 0 & m_n - m \end{pmatrix} \begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix} \Bigg]$$

$$+ i\bar{t}_L \gamma^\mu \partial_\mu t_L + \bar{b}\left(i\gamma^\mu\partial_\mu - m\right)b + \frac{g}{\sqrt{2}}\left(\bar{t}\gamma^\mu L b W_\mu^+ + \bar{b}\gamma^\mu L t W_\mu^-\right) + \frac{g}{2}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b\right)W_\mu^3$$

$$+\frac{\sqrt{3}g}{6}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b - 2\bar{b}\gamma^\mu R b\right)B_\mu$$

$$L \equiv \frac{1}{2}(1-\gamma_5),\ R \equiv \frac{1}{2}(1+\gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\left(= M_W\right)$$

D-dim effective Lagrangian in terms of mass eigenbasis

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\Bigg[i\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\gamma^\mu\partial_\mu \begin{pmatrix}\psi_1^{(n)}\\ \tilde{\psi}_2^{(n)}\\ \tilde{\psi}_3^{(n)}\end{pmatrix}$$

$$\begin{pmatrix}\tilde{\psi}_1^{(n)}\\ \tilde{\psi}_2^{(n)}\\ \tilde{\psi}_3^{(n)}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{2} & 0 & 0\\ 0 & 1 & -1\\ 0 & 1 & 1\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}$$

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$$-\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} m_n & 0 & 0\\ 0 & m_n + m & 0\\ 0 & 0 & m_n - m\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \tilde{\psi}_2^{(n)}\\ \tilde{\psi}_3^{(n)}\end{pmatrix}\Bigg]$$

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$$L \equiv \frac{1}{2}(1-\gamma_5),\ R \equiv \frac{1}{2}(1+\gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\left(= M_W\right)$$

Fermion contributions to gluon fusion amplitude

(D+1)-dim SU(3) gauge theory
with a triplet fermion on S^1/Z2

Essential Point for calculation

KK spectrum: Mass splitting

$$\begin{cases} m^{+(n)} = \dfrac{n}{R} + m & - \dfrac{m}{v} \\ m^{-(n)} = \dfrac{n}{R} - m & + \dfrac{m}{v} \end{cases}$$

Top Yukawa: Sign flipping

Crucial for cancellation of divergences