Domain Walls with Non-Abelian Clouds

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1 Non-Abelian orientational moduli

BPS soliton has similarity to **D-branes** in string theory

Parameters of the Solution = **Moduli**

Moduli dynamics = **Effective field theory** of massless fields

Non-Abelian orientational moduli

Single instanton in $\boldsymbol{SU(N)}$ gauge theory

$$\boldsymbol{A_\mu = U \begin{pmatrix} A_\mu^{\mathrm{BPST}}(x_0, \rho) & 0 \\ 0 & 0_{N-2} \end{pmatrix} U^\dagger, \quad U \in \frac{SU(N)}{SU(N-2) \times U(1)}}$$

Non-Abelian clouds: non-Abelian orientational moduli (E.Weinberg)

Our purpose:

Study **Domain walls** with **non-Abelian orientational moduli**

Non-Abelian moduli occur when Higgs masses are (partially) **degenerate**

2 Models, BPS equations and the moduli matrix

SUSY $U(N_{\rm C})$ Gauge Theory with $N_{\rm F}$ Higgs fields

Higgs fields H as an $N_{\rm C} \times N_{\rm F}$ matrix, adjoint scalar Σ

$$\mathcal{L} = \mathcal{L}_{\rm kin} - V$$

$$\mathcal{L}_{\rm kin} = {\rm Tr}\left(-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{g^2}\mathcal{D}_\mu\Sigma\,\mathcal{D}^\mu\Sigma + \mathcal{D}^\mu H\,(\mathcal{D}_\mu H)^\dagger\right)$$

$$V = {\rm Tr}\Big[\frac{g^2}{4}\left(c\mathbf{1} - HH^\dagger\right)^2 + (\Sigma H - HM)(\Sigma H - HM)^\dagger\Big]$$

Gauge coupling g, Fayet-Iliopoulos parameter c, diagonal mass matrix M

Bogomol'nyi bound (dependence on y only)

$$E = \int_{-\infty}^{\infty} dy\,{\rm Tr}\Big[(\mathcal{D}_y H - HM + \Sigma H)^2 + \frac{1}{g^2}\Big(\mathcal{D}_y\Sigma - \frac{g^2}{2}\left(c\mathbf{1} - HH^\dagger\right)\Big)^2 + c\,\mathcal{D}_y\Sigma\Big] \geq c\Big[{\rm Tr}\,\Sigma(\infty) - {\rm Tr}\,\Sigma(-\infty)\Big]$$

Lower bound is saturated if the **BPS equations** are satisfied

$$\mathcal{D}_y H = HM - \Sigma H, \qquad \mathcal{D}_y \Sigma = g^2 \left(c\mathbf{1} - HH^\dagger \right) / 2$$

Solution of BPS equations

$$H = S^{-1}(y) H_0 e^{My}, \quad \Sigma + i W_y = S^{-1}(y) \partial_y S(y)$$

Moduli matrix H_0: $N_{\rm C} \times N_{\rm F}$ constant complex matrix of rank $N_{\rm C}$,
contains all the moduli parameters ϕ^i

Remaining BPS eq.(**Master eq.**): $\Omega \equiv SS^\dagger$, $\Omega_0 \equiv \frac{1}{c} H_0 H_0^\dagger$

$$\partial_y (\Omega^{-1} \partial_y \Omega) = c g^2 \left(\mathbf{1}_{N_{\rm C}} - \Omega^{-1} \Omega_0 \right)$$

3 Non-Abelian Clouds in Abelian Gauge Theories

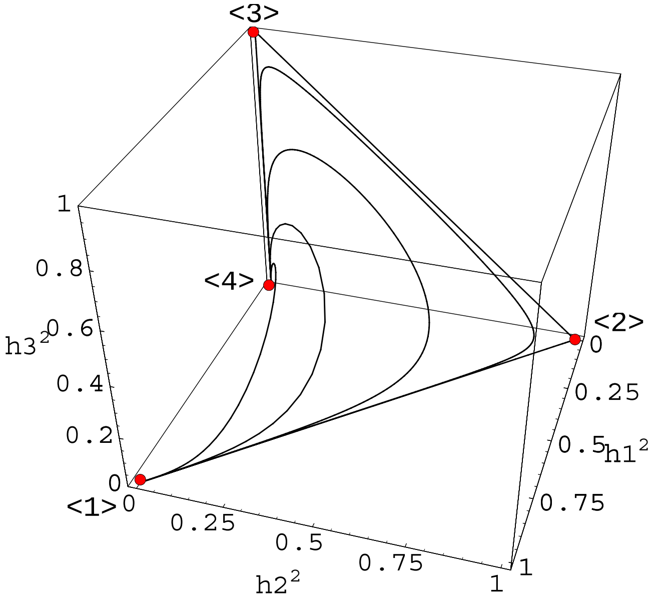
A simple example: $N_{\rm F} = 4$ with $M = \mathrm{diag}\,(m,\ m\epsilon/2,\ -m\epsilon/2,\ -m)$

4 isolated vacua $\langle 1 \rangle, \cdots, \langle 4 \rangle$, flavor symmetry $U(1)^3$ ($\epsilon \neq 0$)

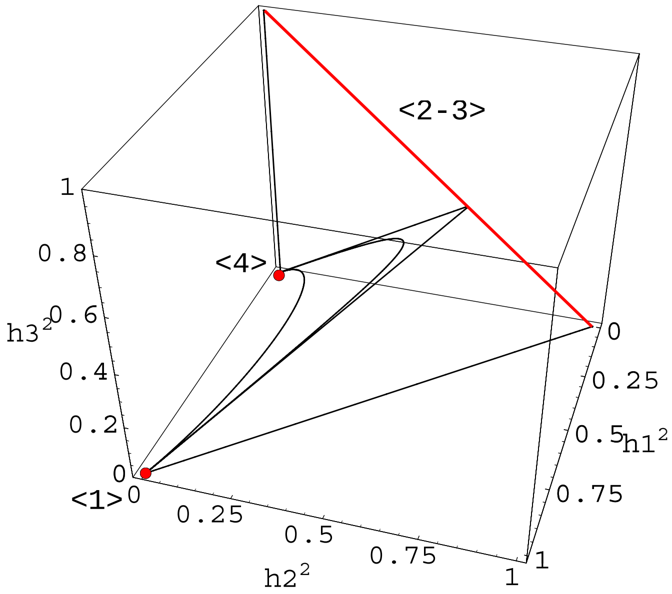
Explicit solution at $g^2 \to \infty$ limit: $H = \frac{1}{\sqrt{\Omega_0}} H_0 e^{My}$

Moduli parameters are $\varphi_1, \varphi_2, \varphi_3$

$$H_0 = \left(1,\ e^{\varphi_1},\ e^{\varphi_1 + \varphi_2},\ e^{\varphi_1 + \varphi_2 + \varphi_3} \right) = (1, \phi_2, \phi_3, \phi_4)$$



(a) non-degenerate mass



(b) degenerate mass

Domain wall trajectories in terms of $|\boldsymbol{h_i}|^2$ of $\boldsymbol{H} = \sqrt{\boldsymbol{c}}(\boldsymbol{h_1}, \boldsymbol{h_2}, \boldsymbol{h_3}, \boldsymbol{h_4})$ in the target space $\mathbf{C}\boldsymbol{P}^{\mathbf{3}}$ (as Toric diagram)

Wall configuration in terms of $\boldsymbol{\Sigma}$

Well-separated walls: $\boldsymbol{\varphi_i}$ is ***i*-th wall position** and **phase difference** wave function is localized around the $\boldsymbol{i}$-th wall

As $\boldsymbol{\epsilon}$ decreases, $\boldsymbol{\varphi_2}$ wave function spreads between **2** walls

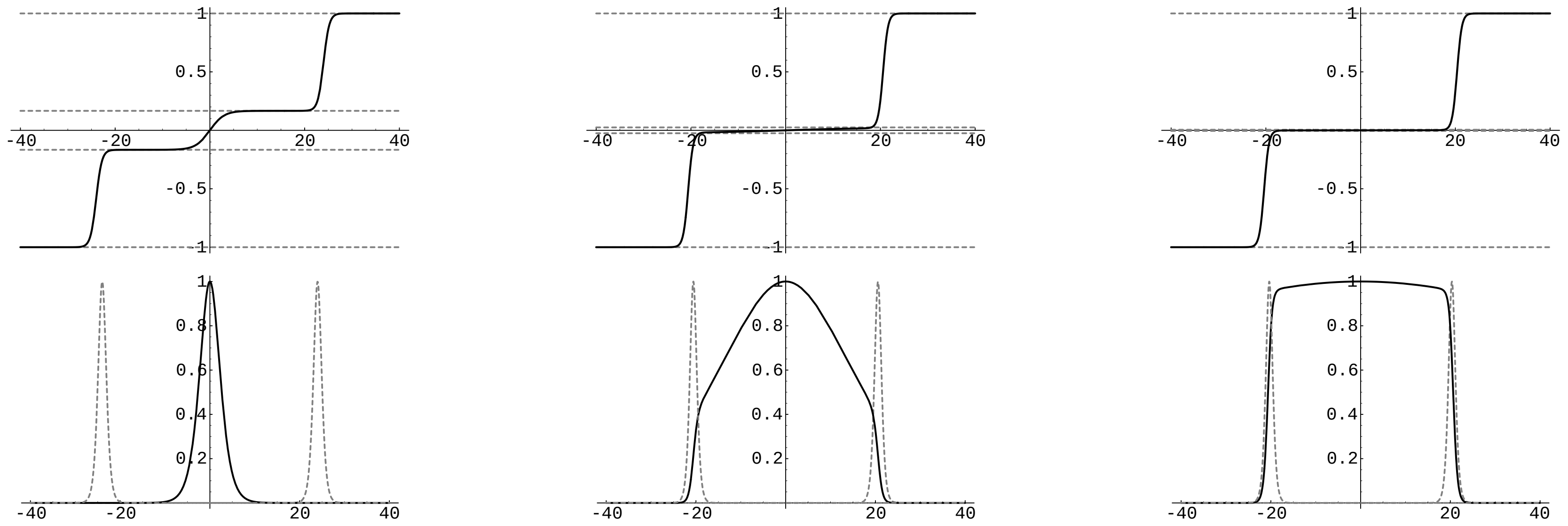


Figure 1: Configuration of $\boldsymbol{\Sigma}$ (first row) and density of the Kähler metric of $\boldsymbol{\varphi_1}$, $\boldsymbol{\varphi_2}$ and $\boldsymbol{\varphi_3}$ (second row). Moduli parameters are $(\boldsymbol{\varphi_1}, \boldsymbol{\varphi_2}, \boldsymbol{\varphi_3}) = (\mathbf{20}, \mathbf{0}, -\mathbf{20})$ and $\boldsymbol{m} = \mathbf{1}$.

$\longrightarrow$ **Non-Abelian clouds**

$\boldsymbol{\epsilon} \longrightarrow \mathbf{0}$ (degenerate mass) limit: enhanced flavor symmetry $\boldsymbol{U}(\mathbf{1})^{\mathbf{2}} \times \boldsymbol{SU}(\mathbf{2})$

$\mathbf{2}$ isolated vacua $\langle \mathbf{1} \rangle$, $\langle \mathbf{4} \rangle$ and a degenerate vacuum $\langle \mathbf{2} - \mathbf{3} \rangle$

(with vacuum moduli $\mathbf{C}\boldsymbol{P}^{\mathbf{1}} \simeq \boldsymbol{SU}(\mathbf{2})/\boldsymbol{U}(\mathbf{1})$)

When domain walls are **coincident**,

all **6** massless modes are localized at the wall with identical wave functions

When domain walls are **separated**, **6** massless modes consist of

positions of two walls: $\{|\phi_2|^2 + |\phi_3|^2, |\phi_4|^2\}$ (**1** NG, **1** qNG)

Localized at each wall

N-G bosons for $U_1(1) \times U_2(1) \times SU(2)/U(1)$ (**4** NG)

$U_1(1), U_2(1)$: **localized at each wall**

$SU(2)/U(1)$: **spread between 2 walls**

4 Conclusion

1. Domain walls in models with (partially) degenerate masses for Higgs scalars have normalizable **non-Abelian Nambu-Goldstone** (NG) **modes**, which are called **Non-Abelian clouds**.

2. When walls **coincide**, all the massless modes are **localized** at the walls with identical wave functions.

3. When walls **separate**, we find **non-Abelian clouds** which **spread between two domain walls**.

4. **Effective Lagrangians** are explicitly obtained.

5. When all the walls **coincide** in the $U(N)$ gauge model, symmetry breaking $SU(N)_{\mathrm{L}} \times SU(N)_{\mathrm{R}} \times U(1) \to SU(N)_{\mathrm{V}}$ gives $U(N)_{\mathrm{A}}$ **NG modes**. In addition, there are $N^2 - 1$ **quasi-NG modes** besides **1** NG mode for broken translation. All these modes have identical wave function and are localized at the wall.

6. When n walls **separate** in the $U(N)$ gauge model, off-diagonal elements of $U(n)$ NG modes have wave function **spreading between two separated walls** (**non-Abelian clouds**). Some **quasi-NG modes turn to NG modes** because of further symmetry breaking $U(n)_{\mathrm{V}} \to U(1)^n_{\mathrm{V}}$.

7. The **number of massless modes remain unchanged** as wall positions change.

8. In $4 + 1$-dimensions, we **dualize** the effective theory on the $3 + 1$ dimensional world-volume to the supersymmetric Freedman-Townsend model of **2-form fields** valued in $\mathcal{U}(N)$.

9. **Moduli matrix** approach is extremely useful to describe non-Abelian clouds of domain walls.