Casimir Energy of Higher Dimensional theories and New Regularization

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(*ArXiv:0801.3064 flat*; *ArXiv:0812.1263v2 warped*)

Introduction

Energy Spectrum of Radiation Filed

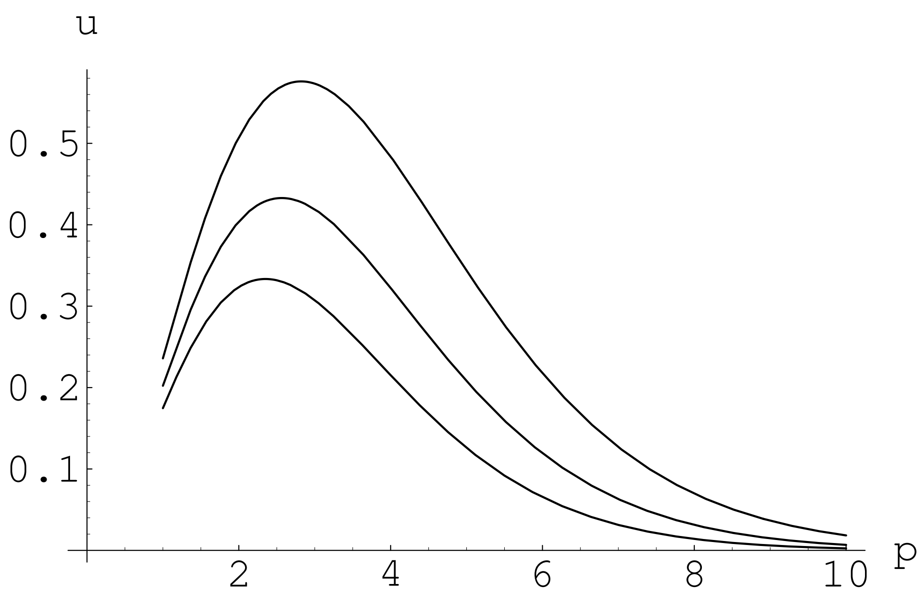

Fig.1 Planck Distribution

Rayley-Jeans formula $\longrightarrow$ Divergences of Specific Heat

Planck's formula $$\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} = h\nu \left(e^{-\frac{h\nu}{kT}} + e^{-\frac{2h\nu}{kT}} + e^{-\frac{3h\nu}{kT}} + \cdots \right)$$

—— sum over all ν $\longrightarrow$ Stephan's formula $U = 3\,\zeta(4) \times T^4$

Energy is discretized : $0,\ h\nu,\ 2h\nu, 3h\nu, \cdots$ $\qquad [x,p] = i\hbar$

$\frac{h\nu}{2}$ Zero-Point Energy – sum over all ν $\longrightarrow$ Casimir Energy

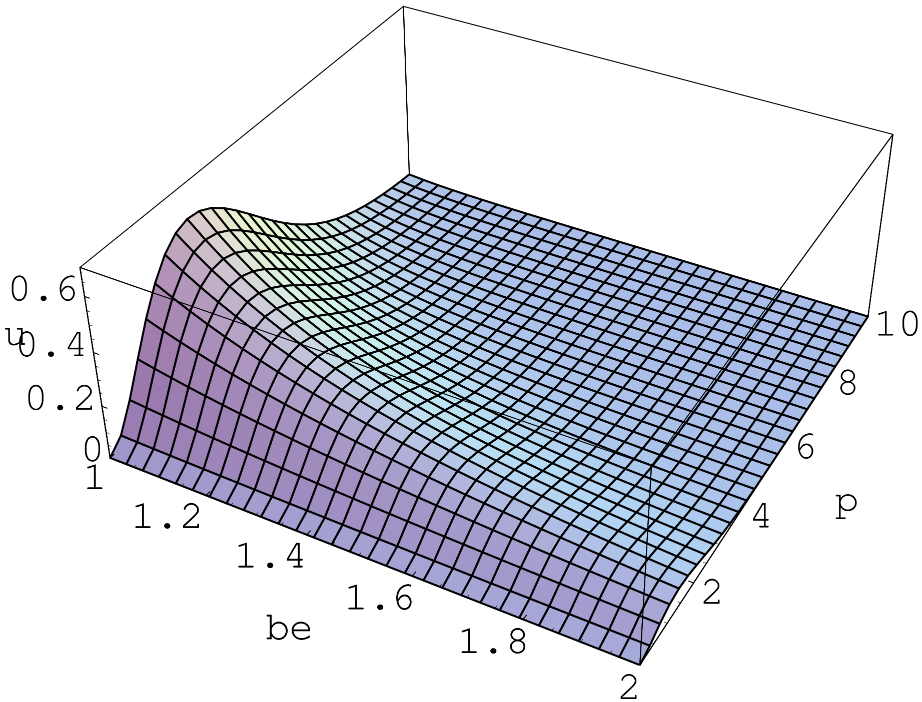

Fig.1 Planck Distribution, "extra" axis: 1/T

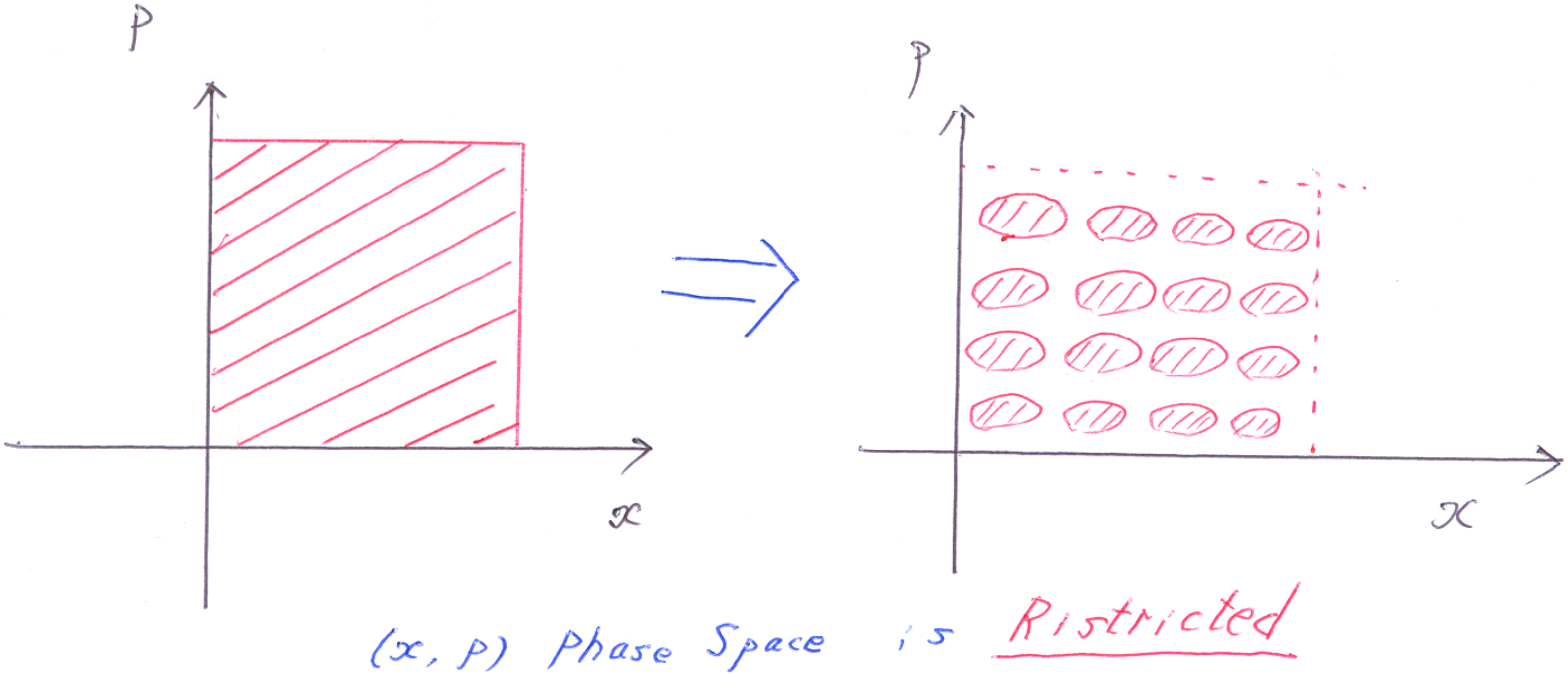

Fig.1'

Quantization: Phase Space (x,p) Restricted

Figure 4: Space of $(z,\tilde{p})$ for the integration. The hyperbolic curve will be used in Sec.6.

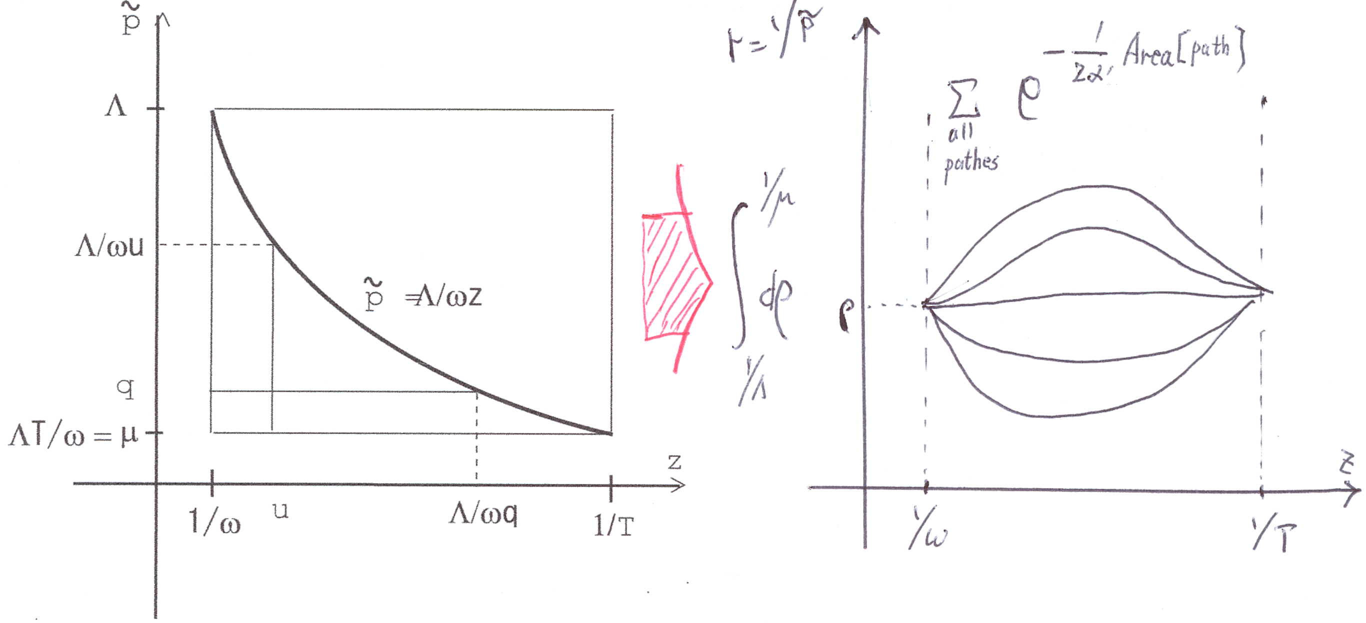

Fig.1'b Naive Integral over Rectangle Region is replaced by Path Integral over Extra Axis with Area Hamiltonian

An Effective Approach: Weight Function

We introduce a *weight function* $W(\tilde{p},y)$ (to suppress UV and IR divergences).

$$E^W_{Cas}(l) \equiv \int \frac{d^4p}{(2\pi)^4}\int_0^l dy\ W(\tilde{p},y)F(\tilde{p},y) \quad ,$$

Trial Examples of $W(\tilde{p},y)$:

$$\left\{ \begin{array}{c} e^{-\frac{1}{2}l^2\tilde{p}^2-\frac{1}{2}(y^2/l^2)} \equiv W_1(\tilde{p},y),\ \text{elliptic} \\ e^{-\frac{1}{2}\tilde{p}^2y^2} \equiv W_3(\tilde{p},y),\ \text{hyperbolic, R-S type} \\ e^{-\frac{1}{2}l^2(\tilde{p}^2+1/y^2)} \equiv W_8(\tilde{p},y),\ \text{reciprocal} \end{array} \right. \qquad (1)$$

Numerical result 1. Flat Case:

$$E^W_{Cas}\times(\frac{1}{\Lambda l})\times 8\pi^2 =$$

$$\left\{\begin{array}{lll} \frac{-21.4}{l^4}\left[1-(0.258,0.130,0.0650)\cdot 10^{-3}\ln\Lambda\right] & \text{for} & W_1(\tilde{p},y) \\ -0.270\frac{\Lambda^3}{l}\left[1-(21.9,10.9,5.44)\cdot 10^{-5}\ln\Lambda\right] & \text{for} & W_3(\tilde{p},y) \\ \frac{-1.00}{l^4}\left[1-(4.04,2.02,1.01)\cdot 10^{-4}\ln\Lambda\right] & \text{for} & W_8(\tilde{p},y) \end{array}\right. \tag{2}$$

Numerical result 2. Warped Case:

$$E^W_{Cas}\times(\frac{T}{\Lambda})\times 4\pi^2 =$$

$$\left\{\begin{array}{lll} -0.336\omega^4\left[1+3.15\cdot 10^{-2}\ln\Lambda\right] & \text{for} & W_1(\tilde{p},z) \\ -2.62\cdot 10^{-2}\omega\Lambda^3\left[1-4.85\cdot 10^{-5}\ln\Lambda\right] & \text{for} & W_3(\tilde{p},z) \\ -0.104\omega^4\left[1+2.56\cdot 10^{-2}\ln\Lambda\right] & \text{for} & W_8(\tilde{p},z) \end{array}\right. \tag{3}$$

The (UV) divergences much reduces compared with the un-weighted case $W(\tilde{p}, y) = 1$ of Λ^5. Λl and Λ/T are dimensionless and are big normalization constants.
W_3: Randall-Schwartz's proposal.

Renormalization of ω

$$
\begin{aligned}
E^W_{Cas}/\Lambda T^{-1} = -\alpha\omega^4\left(1 - 4c\ln(\Lambda/\omega)\right) = -\alpha\omega'^4 \quad , \\
\omega' = \omega\sqrt[4]{1 - 4c\ln(\Lambda/\omega)} \quad .
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
|c| \ll 1 \quad , \quad \omega' = \omega(1 - c\ln(\Lambda/\omega)) \quad , \\
\beta_\omega = \frac{\partial}{\partial(\ln\Lambda)}\ln\frac{\omega'}{\omega} = -c \quad .
\end{aligned} \tag{5}
$$

Present New Definition

We newly define the Casimir energy in the higher-dimensional theory as follows.

$$\mathcal{E}_{Cas}(\omega,T) \equiv \int_{1/\Lambda}^{1/\mu} d\rho \int_{r(1/\omega)=r(1/T)=\rho} \prod_{a,z} \mathcal{D}x^a(z) F(\frac{1}{r},z)$$
$$\times \exp\left[-\frac{1}{2\alpha'}\int_{1/\omega}^{1/T} \frac{1}{\omega^4 z^4}\sqrt{r'^2+1}\; r^3 dz\right] \quad , \tag{6}$$

where $\mu = \Lambda T/\omega$ and the limit $\Lambda \to \infty$ is taken. The string (surface) tension $T = 1/2\alpha'$ is introduced. The above expression manifestly shows the 4D coordinates x^a are the quantum statistical operators and the extra coordinate

z is the inverse temperature. The Hamiltonian is given by the area of the "closed-string" surface.

Other Results

We have analyzed 5D quantum electro-magnetism in the recent standpoint. To make the theory finite, we have proposed a new regularization procedure based on the minimal area principle. Casimir energy is finitely obtained.

- formulation in terms of the heat-kernel
- Casimir energy is expressed in a closed form
- UV and IR regularization surfaces and minimal area principle.
- Numerical evaluation of Casimir energy and the minimal surface curve.

- Sphere lattice structure and renormalization flow, the β function

- A dynamical reason of the smallness of the cosmological constant is presented: The cosmological term (dark energy) is given by $\sim -\omega^4$ and the warp parameter ω does renormalization flow to the small number as Λ decreases.

We hope the present analysis advances further development of the higher dimensional QFT.