CFT Duals for Extreme Black Holes

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Introduction

It is known that the black hole has an entropy given by the Bekenstein-Hawking area law (Hawking '74, Bekenstein '73)

$$S_{BH} = \frac{\text{Area(Horizon)}}{4}$$

within the general relativity, but

- it is mysterious that the entropy is proportional to the area of the black hole, not its volume
- we cannot see the inside of the black hole
- its (microscopic) origin remains to be fully understood

Various approach

For specific case, there are several explanations

- Counting BPS states (SUSY BH) (Strominger-Vafa '96)
- Attractor mechanism (Extremal) (Ferrrara-Kallosh-Strominger '95, Sen '05 Goldstein-Iizuka-Jena-Trivedi '05)
- AdS_3/CFT_2 (BTZ) (Strominger '97)
- Near horizon symmetry (Carlip '98 '99)
- Entanglement entropy (Extremal) (Azeyanagi-TN-Takayanagi '07)

⋮

Remarkably, the extremality plays an important role even though the approached are quite different

The Kerr/CFT correspondence

Recently, a new duality called the Kerr/CFT correspondence was proposed between the extreme Kerr black hole in four-dimension and a two-dimensional CFT

(Guica-Hartman-Song-Strominger '08)

The prescription to obtain the dual CFT is

- take the near horizon limit of the extremal Kerr black hole
- determine the asymptotic "boundary condition" in order that the "SL(2,R) Virasoro" algebra appears
- evaluate the central charge c of this Virasoro algebra
- define the dual temperature T_L analogous to the Hartle-Hawking vacuum

Purpose

We will see that the statistical entropy computed by using the Cardy formula agrees with the black hole entropy

$$S_{CFT} = \frac{\pi^2}{3} c T_L = S_{BH}$$

We can obtain the (in a sence) microscopic interpretation of the black hole entropy

The natural question is

Why Kerr ? Can we apply this strategy to more general black holes ?

The answer is yes, and we can construct the dual CFT thanks

to the extremality

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Generalization to Kerr-Newman-(A)dS black hole

To illustrate the construction of the dual CFT to the extremal black hole, we consider the Kerr-Newman-(A)dS black hole

This is the most general solution in the four-dimensional Einstein-Maxwell theory

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R + \frac{6}{\ell^2} - \frac{1}{4} F^2 \right)$$

Notice that

- Once we set the electric and magnetic charges to zero, we obtain the Kerr black hole
- Also we obtain the Reissner-Nordstrom black hole in the limit of zero angular momentum

Kerr-Newman-(A)dS black hole

The metric is given by (Caldarelli-Cognola-Klemm '99)

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(d\hat{t} - \frac{a}{\Xi}\sin^2\theta d\hat{\phi}\right)^2 + \frac{\rho^2}{\Delta_r}d\hat{r}^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\left(a d\hat{t} - \frac{\hat{r}^2 + a^2}{\Xi}d\hat{\phi}\right)^2$$

with

$$\Delta_r = (\hat{r}^2 + a^2)\left(1 + \frac{\hat{r}^2}{\ell^2}\right) - 2M\hat{r} + q^2 , \qquad \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta ,$$

$$\rho^2 = \hat{r}^2 + a^2\cos^2\theta , \qquad \Xi = 1 - \frac{a^2}{\ell^2} , \qquad q^2 = q_e^2 + q_m^2$$

Thermodynamic quantities

The angular velocity of the horizon and the entropy are

$$\Omega_H = \frac{\Xi a}{(r_+^2 + a^2)}, \qquad S = \pi \frac{r_+^2 + a^2}{\Xi},$$

$$\Omega_H^\infty = \Omega_H + \frac{a}{\ell^2} = \frac{a(1 + r_+^2/\ell^2)}{r_+^2 + a^2}$$

The Hawking temperature is

$$T_H = \frac{r_+(1 + a^2/\ell^2 + 3r_+^2/\ell^2 - (a^2 + q^2)/r_+^2)}{4\pi(r_+^2 + a^2)}$$

The physical mass, angular momentum, and electric and magnetic charges are

$$M_{\mathrm{ADM}} = \frac{M}{\Xi^2}, \quad J = \frac{aM}{\Xi^2}, \quad Q_e = \frac{q_e}{\Xi}, \quad Q_m = \frac{q_m}{\Xi}$$

Gauge field

Now we consider the Einstein-Maxwell theory, there is a gauge field

The gauge field and field strength are

$$A = -\frac{q_e \hat{r}}{\rho^2}\left(d\hat{t} - \frac{a\sin^2\theta}{\Xi}d\hat{\phi}\right) - \frac{q_m\cos\theta}{\rho^2}\left(a d\hat{t} - \frac{\hat{r}^2 + a^2}{\Xi}d\hat{\phi}\right),$$

$$F = -\frac{q_e(\hat{r}^2 - a^2\cos^2\theta) + 2q_m\hat{r}a\cos\theta}{\rho^4}\left(d\hat{t} - \frac{a\sin^2\theta}{\Xi}d\hat{\phi}\right)\wedge d\hat{r}$$
$$+ \frac{q_m(\hat{r}^2 - a^2\cos^2\theta) - 2q_e\hat{r}a\cos\theta}{\rho^4}\sin\theta d\theta \wedge \left(a d\hat{t} - \frac{\hat{r}^2 + a^2}{\Xi}d\hat{\phi}\right)$$

Extreme limit

In the extreme limit ($T_H \to 0$), the inner and outer horizons degenerate to a single horizon at r_+

The extremality condition is

$$a^2 = \frac{r_+^2(1 + 3r_+^2/\ell^2) - q^2}{1 - r_+^2/\ell^2}$$

$$M = \frac{r_+[(1 + r_+^2/\ell^2)^2 - q^2/\ell^2]}{1 - r_+^2/\ell^2}$$

and the entropy at extremality is

$$S(T_H = 0) = \frac{\pi(2r_+^4/\ell^2 + 2r_+^2 - q^2)}{1 - 2r_+^2/\ell^2 - 3r_+^4/\ell^4 + q^2/\ell^2}$$

Near horizon limit

To take the near horizon limit, we introduce new coordinates

(Bardeen-Horowitz '99)

$$\hat{r} = r_+ + \epsilon r_0 r \,, \qquad \hat{t} = t r_0/\epsilon \,, \qquad \hat{\phi} = \phi + \Omega_H \frac{t r_0}{\epsilon}$$

In the limit of $\epsilon \to 0$, the metric becomes

$$\boxed{ds^2 = \Gamma(\theta)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + \alpha(\theta) d\theta^2\right] + \gamma(\theta)(d\phi + k r dt)^2}$$

where

$$\Gamma(\theta) = \frac{\rho_+^2 r_0^2}{r_+^2 + a^2} \,, \quad \alpha(\theta) = \frac{r_+^2 + a^2}{\Delta_\theta r_0^2} \,, \quad \gamma(\theta) = \frac{\Delta_\theta (r_+^2 + a^2)^2 \sin^2\theta}{\rho_+^2 \Xi^2}$$

and we have defined

$$\rho_+^2 = r_+^2 + a^2 \cos^2\theta \,, \quad r_0^2 = \frac{(r_+^2 + a^2)(1 - r_+^2/\ell^2)}{1 + 6 r_+^2/\ell^2 - 3 r_+^4/\ell^4 - q^2/\ell^2} \,, \quad k = \frac{2 a r_+ \Xi r_0^2}{(r_+^2 + a^2)^2}$$

Near horizon limit: gauge field

The field strength becomes

$$F = f(\theta)kdr \wedge dt + f'(\theta)(d\theta \wedge d\phi + krd\theta \wedge dt)$$

and the near horizon gauge field is

$$\boxed{A = f(\theta)(d\phi + krdt)}$$

with

$$f(\theta) = \frac{(r_+^2 + a^2)[q_e(r_+^2 - a^2\cos^2\theta) + 2q_m ar_+ \cos\theta]}{2\rho_+^2 \Xi ar_+}$$

Isometry

- The Kerr-Newman-(A)dS black hole has the complicated metric
- But it becomes fairly simple form once we take the near horizon limit of the extremal black hole
- The isometry is $U(1) \times SL(2, R)$ ($U(1) : \phi$, $SL(2, R)$: AdS$_2$ part)

We will calculate the central charge for this general form for simplicity

Surprisingly, this simple form appears as the near horizon limit of the extreme black hole in the fairly general gravity theory as we will see later

Asymptotic Symmetry Group and Boundary Conditions

The asymptotic symmetry group (ASG) of a spacetime is

- A symmetry which obeys the boundary conditions in the diffeomorphism

Example: AdS_3

$$ds^2 = -(1 + \frac{r^2}{l^2})dt^2 + \frac{dr^2}{1 + \frac{r^2}{l^2}} + \frac{r^2}{l^2}d\phi^2$$

Choose the boundary condition as (Brown-Henneaux '86)

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} 1 & 1/r^2 & 1 \\ & 1/r^2 & 1/r^2 \\ & & 1 \end{pmatrix}$$

- This allows the BTZ black hole
- The ASG is $SL(2,R)_L \times SL(2,R)_R$ Virasoro algebras
- The central charges are $c_L = c_R = 3l/2$

How to choose boundary conditions?

For the general form

$$
\begin{aligned}
ds^2 &= \Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right] + \gamma(\theta)(d\phi + krdt)^2 \\
A &= f(\theta)(d\phi + krdt)
\end{aligned}
$$

we choose the boundary condition by demanding that

- the ASG includes the $SL(2,R)$ Virasoro algebra
- the charges is finite

like the Brown-Henneaux's case

The appropriate boundary conditions determine the family of the geometries in which the charges are finite

Boundary conditions

Such a boundary condition is (in the basis (t, ϕ, θ, r))

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} r^2 & 1 & 1/r & 1/r^2 \\ & 1 & 1/r & 1/r \\ & & 1/r & 1/r^2 \\ & & & 1/r^3 \end{pmatrix}$$

For the gauge field we impose the boundary condition

$$a_\mu \sim \mathcal{O}(r, 1/r, 1, 1/r^2) \ .$$

(This condition is important to obtain the $SL(2,R)$ Virasoro symmetry uniquely)

ASG

The most general diffeomorphisms which preserve the boundary conditions are

$$\boxed{\zeta_\epsilon = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r}$$

The gauge field transforms under ζ_ϵ as

$$\delta_\epsilon A = f\epsilon'(d\phi - krdt)$$

This does not satisfy the boundary condition, so we must add a compensating $U(1)$ gauge transformation to restore $\delta A_\phi = O(1/r)$

$$\boxed{\Lambda = -f(\theta)\epsilon(\phi)}$$

Under the combined gauge + diffeomorphism transformation,

$$\delta_\epsilon A = -krf(\theta)\epsilon'(\phi)dt - f'(\theta)\epsilon(\phi)d\theta$$

Conserved charge

We focus on the Einstein-Maxwell theory. There are two symmetries under which the action is invariant

- Diffeomorphism: $\delta_\zeta g_{\mu\nu} = \mathcal{L}_\zeta g_{\mu\nu}, \quad \delta_\zeta A_\mu = \mathcal{L}_\zeta A_\mu$
- $U(1)$ gauge symmetry: $\delta_\Lambda A_\mu = \partial_\mu \Lambda$

The associated charge $Q_{\zeta,\Lambda}$ is defined by

$$\delta Q_{\zeta,\Lambda} = \frac{1}{8\pi} \int_\infty \left(k_\zeta^{grav}[h; g] + k_{\zeta,\Lambda}^{gauge}[h, a; g, A] \right)$$

where we denote the infinitesimal field variations by $a_\mu = \delta A_\mu$ and $h_{\mu\nu} = \delta g_{\mu\nu}$

Conserved charge

The contribution from the Einstein action is (Barnich-Brandt '01)

$$k_\zeta^{grav}[h,g] = \frac{1}{4}\epsilon_{\alpha\beta\mu\nu}[\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2}hD^\nu\zeta^\mu - h^{\nu\sigma}D_\sigma\zeta^\mu + \frac{1}{2}h^{\sigma\nu}(D^\mu\zeta_\sigma + D_\sigma\zeta^\mu)]dx^\alpha \wedge dx^\beta$$

The Maxwell contribution is (Barnich-Compere '05)

$$k_{\zeta,\Lambda}^{gauge}[\delta\phi,\phi] = \frac{1}{8}\epsilon_{\alpha\beta\mu\nu}[(-\frac{1}{2}hF^{\mu\nu} + 2F^{\mu\gamma}h_\gamma^{\ \nu} - \delta F^{\mu\nu})(\zeta^\rho A_\rho + \Lambda) - F^{\mu\nu}\zeta^\rho a_\rho - 2F^{\alpha\mu}\zeta^\nu a_\alpha - a^\mu(\mathcal{L}_\zeta A^\nu + \partial^\nu\Lambda)]dx^\alpha \wedge dx^\beta$$

where $\delta F^{\mu\nu} \equiv g^{\mu\alpha}g^{\nu\beta}(\partial_\alpha a_\beta - \partial_\beta a_\alpha)$ $\quad (\delta\phi \equiv \delta(g_{\mu\nu}, A_\mu) = (h_{\mu\nu}, a_\mu))$

Conserved charge

The algebra of the ASG is the Dirac bracket algebra of the charges themselves

$$\begin{aligned}\{Q_{\zeta,\Lambda}, Q_{\tilde{\zeta},\tilde{\Lambda}}\}_{DB} &= (\delta_{\tilde{\zeta}} + \delta_{\tilde{\Lambda}})Q_{\zeta,\Lambda} \\ &= Q_{[(\zeta,\Lambda),(\tilde{\zeta},\tilde{\Lambda})]} + (\text{central term})\end{aligned}$$

$$(\text{central term}) = \frac{1}{8\pi}\int \Big(k^{grav}_{\zeta}[\mathcal{L}_{\tilde{\zeta}}\bar{g};\bar{g}] + k^{gauge}_{\zeta,\Lambda}[\mathcal{L}_{\tilde{\zeta}}\bar{g}, \mathcal{L}_{\tilde{\zeta}}\bar{A} + d\tilde{\Lambda}; \bar{g}, \bar{A}]\Big)$$

where $(\bar{g}, \bar{A})$ denote the background solution

The central term give us the central charge for dual CFT as we will see in the following

Virasoro algebra

We expand the arbitrary function $\epsilon(\phi)$ by the fourier mode $\epsilon_n = -e^{-in\phi}$, and define

$$\zeta_n \equiv \zeta_{\epsilon_n}, \qquad \Lambda_n \equiv \Lambda(\epsilon = \epsilon_n)$$

Combining the diffeomorphism and gauge transformation as (ζ_n, Λ_n), this becomes the $SL(2,R)$ Virasoro algebra

$$i[(\zeta_n, \Lambda_n), (\zeta_m, \Lambda_m)] = (n-m)(\zeta_{n+m}, \Lambda_{n+m})$$

without the central charge

But when we calculate the Dirac bracket between the symmetry generators $Q_{\zeta,\Lambda}$, we will obtain the central charge from the central term

Central charge

The Dirac brackets between symmetry generators are

$$i\{Q_{\zeta_\epsilon,\Lambda}, Q_{\zeta_{\tilde{\epsilon}},\tilde{\Lambda}}\}_{DB} = iQ_{[(\zeta_\epsilon,\Lambda),(\zeta_{\tilde{\epsilon}},\tilde{\Lambda})]}$$
$$- \frac{ik}{16\pi}\int d\theta d\phi \sqrt{\frac{\alpha(\theta)\gamma(\theta)}{\Gamma(\theta)}}\Big(f(\theta)\Lambda\tilde{\epsilon}' + \Gamma(\theta)\epsilon'\tilde{\epsilon}'' + [f(\theta)^2 + \gamma(\theta)]\epsilon\tilde{\epsilon}'$$
$$- (\epsilon, \Lambda \leftrightarrow \tilde{\epsilon}, \tilde{\Lambda})\Big)$$

The algebra of the ASG is the Virasoro algebra generated by (ζ_n, Λ_n) with charges Q_n

$$i\{Q_m, Q_n\}_{DB} = (m-n)Q_{m+n} + \frac{c}{12}(m^3 - Bm)\delta_{m+n,0}$$

where B is a constant that can be absorbed by a shift in Q_0

Central charge

The central charge has contributions from k^{grav} and k^{gauge}

$$c = c_{grav} + c_{gauge}$$

We find

$$c_{grav} = 3k \int_0^{\pi} d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$
$$c_{gauge} = 0$$

For the Kerr-Newman-(A)dS black hole

$$c = \frac{12 r_+ \sqrt{(3r_+^4/\ell^2 + r_+^2 - q^2)(1 - r_+^2/\ell^2)}}{1 + 6r_+^2/\ell^2 - 3r_+^4/\ell^4 - q^2/\ell^2}$$

Temperature

The extremality constraint requires that any fluctuations satisfy

$$0 = T_H dS = dM_{ADM} - (\Omega_H dJ + \Phi_e dQ_e + \Phi_m dQ_m)$$

For such constrained variations we may write

$$-dI_{gr} = dS = \frac{dJ}{T_L} + \frac{dQ_e}{T_e} + \frac{dQ_m}{T_m}$$

Like GKP-W relation, we identify the density matrix of the bulk with that of the boundary

$$\rho_{gravity} \equiv \rho_{CFT}$$

$$\rho_{gravity} = e^{-I_{gr}}, \qquad \rho_{CFT} = e^{-\frac{L_0}{T_L} - \frac{\hat{q}_e}{T_e} - \frac{\hat{q}_m}{T_m}}$$

Then we obtain the temperature T_L of dual CFT

Entropy

For Kerr-Newman-(A)dS case

$$T_L = \frac{(1 + 6r_+^2/\ell^2 - 3r_+^4/\ell^4 - q^2/\ell^2)[2r_+^2(1 + r_+^2/\ell^2) - q^2]}{4\pi r_+[(1 + r_+^2/\ell^2)(1 - 3r_+^2/\ell^2) + q^2/\ell^2]\sqrt{(1 - r_+^2/\ell^2)(3r_+^4/\ell^2 + r_+^2 - q^2)}}$$

Assuming the Cardy formula, we obtain the statistical entropy of the CFT

$$S_{CFT} = \frac{\pi^2}{3}cT_L = \frac{\pi(2r_+^4/\ell^2 + 2r_+^2 - q^2)}{1 - 2r_+^2/\ell^2 - 3r_+^4/\ell^4 + q^2/\ell^2}$$

This agrees in precise with the Bekenstein-Hawking entropy of the Kerr-Newman-(A)dS black hole!

Notice that the temperature is rewritten as the surprisingly simple form

$$\boxed{T_L = \frac{1}{2\pi k}}$$

The Extreme Black Hole/CFT correspondence

We treated the KNAdS black hole as the following general form

$$ds^2 = \Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + \alpha(\theta)d\theta^2\right] + \gamma(\theta)(d\phi + krdt)^2$$

It was shown that the above form is obtained as the near horizon geometry of the extremal black hole constructed in the following action (Kunduri-Lucietti-Reall '07)

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big(R - \frac{1}{2}f_{AB}(\chi)\partial_\mu\chi^A\partial^\mu\chi^B - V(\chi) - \frac{1}{4}g_{IJ}(\chi)F^I_{\mu\nu}F^{J\mu\nu}\Big) + \frac{1}{2}\int h_{IJ}(\chi)F^I\wedge F^J$$

The near horizon scalar fields and gauge fields have the form

$$\chi^A = \chi^A(\theta)\ , \quad A^I = f^I(\theta)(d\phi + krdt)$$

Construct dual CFT

The Bekenstein-Hawking entropy of such a black hole is

$$S_{grav} = \frac{\pi}{2}\int_0^{\pi} d\theta\,\sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

We would like to explain the black hole entropy as the statistical entropy of dual CFT

- the calculation of the central charge is the same as before
- but we must take the contribution of the non-gravitational part such as the scalar fields into account

We derive the expression for the conserved charges of the general action following the covariant phase method (Wald '93, Iyer-Wald '94)

Conserved charge

The final resuts are

$$\boldsymbol{k}_{\xi}^{grav} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}dx^{\alpha} \wedge dx^{\beta}\Big\{\xi^{\nu}\nabla^{\mu}h - \xi^{\nu}\nabla_{\sigma}h^{\mu\sigma} + \xi_{\sigma}\nabla^{\nu}h^{\mu\sigma} + \frac{1}{2}h\nabla^{\nu}\xi^{\mu} - h^{\rho\nu}\nabla_{\rho}\xi^{\mu}\Big\}$$

$$\boldsymbol{k}_{\xi,\Lambda}^{F} = \frac{1}{8}\epsilon_{\mu\nu\alpha\beta}dx^{\alpha} \wedge dx^{\beta}\Big[\Big\{ - k_{IJ,A}(\chi)F^{J\mu\nu}\delta\chi^{A} + 2k_{IJ}(\chi)h^{\mu\lambda}F^{J}{}_{\lambda}{}^{\nu}$$
$$- k_{IJ}(\chi)\delta F^{J\mu\nu} - \frac{1}{2}hk_{IJ}(\chi)F^{J\mu\nu}\Big\}(A^{I}_{\rho}\xi^{\rho} + \Lambda^{I}) - k_{IJ}(\chi)F^{J\mu\nu}a^{I}_{\rho}\xi^{\rho} + 2\xi^{\nu}k_{IJ}(\chi)F^{J\mu\lambda}a^{I}_{\lambda}\Big]$$

$$\boldsymbol{k}_{\xi,\Lambda}^{CS} = \frac{1}{8}\epsilon_{\mu\nu\alpha\beta}dx^{\alpha} \wedge dx^{\beta}\Big[\epsilon^{\mu\nu\lambda\sigma}\{h_{IJ,A}(\chi)F^{J}_{\lambda\sigma}\delta\chi^{A} + h_{IJ}(\chi)\delta F^{J}_{\lambda\sigma}\}(A^{I}_{\rho}\xi^{\rho} + \Lambda^{I})$$
$$+ \epsilon^{\mu\nu\lambda\sigma}h_{IJ}(\chi)F^{J}_{\lambda\sigma}a^{I}_{\rho}\xi^{\rho} - 2\xi^{\nu}h_{IJ}(\chi)\epsilon^{\mu\lambda\rho\sigma}F^{J}_{\rho\sigma}a^{I}_{\lambda}\Big]$$

$$\boldsymbol{k}_{\xi}^{\chi} = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}dx^{\alpha} \wedge dx^{\beta}\xi^{\nu}f_{AB}(\chi)\nabla^{\mu}\chi^{B}\delta\chi^{A}$$

Central charge

We can calculate the central charge as before

- In KNAdS case (or the Einstein-Maxwell theory), we checked that the gauge field does not contribute to the central charge
- Remarkably, even in the presence of the non-gravitational fields, the central charge is always given by (to appear)

$$c = c_{grav} = 3k \int_0^\pi d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

as before

Entropy

We saw that the temperature of the KNAdS is given by

$$T_L = \frac{1}{2\pi k}$$

This does not depend on the specific form of the metric, then we naively apply this formula to the general cases[1]

Using the Cardy formula

$$\begin{aligned} S_{CFT} &= \frac{\pi^2}{3} c_{grav} T_L \\ &= \frac{\pi}{2} \int_0^\pi d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)} = \frac{\text{Area(horizon)}}{4} \end{aligned}$$

in agreement with the Bekenstein-Hawking entropy!

[1] Recently, this conjecture has been checked in five-dimensinal case (Chow-Cvetic-Lu-Pope)

Relation to higher dimension

- The four-dimensional action we study is very general in its own right (but we exclude the non-abelian gauge field)
- Once we reduce the higher dimensional action by torus compactification, it always takes that form
- The interesting example is that we can explain indirectly the entropy of the nontrivial solutions such as the black rings and saturns in five-dimension

Reissner-Nordstrom-AdS black hole: limit of KNAdS

In the limit of $J \to 0$ of the Kerr-Newman-AdS black hole, we obtain the Reissner-Nordstrom-AdS black hole, and reproduce the Bekenstein-Hawking entropy

$$S_{RN} = \pi r_+^2$$

This is a satisfactory result, but we should notice the subtleties

- the central charge approaches zero
- the temperature goes to infinity
- these singular behaviors cancel against each other to reproduce the finite entropy

Below we propose a dual description which does not require a singular temperature and central charge

Another description: embedding into 5D space

We assume that we can embed the Kerr-Newman-AdS black hole into 5D space by combining the $U(1)$ gauge bundle with the geometry as

$$ds^2 = ds^2_{BH} + (dy + A)^2$$

We shift a gauge field A as follows in order to choose it non-singular in the $a \to 0$ limit

$$A \to A - \frac{q_e r_+}{2a} d\phi$$

Setting $a = 0$

$$A = q_e r \frac{\bar{r}_0^2}{r_+^2} dt + q_m \cos\theta d\phi$$

Another description: entropy

Once we embed the RN black hole into 5D dimension

- choose boundary conditions appropriately
- the $SL(2,R)$ Virasoro algebra from the gauge fiber direction

$$\zeta^{(y)} = \epsilon(y)\partial_y - r\epsilon'(y)\partial_r \ .$$

- we can calculate its central charge similar to 4D case

$$c_{(y)} = 6q_e\bar{r}_0^2 \qquad (\bar{r}_0^2 \equiv r_0^2|_{a\to 0})$$

- the temperature conjugate to the electric charge is defined by $T_e dS = dQ_e$ and we find

$$T_e = \frac{r_+^2}{2\pi q_e \bar{r}_0^2}$$

- Using Cardy formula

$$S_{CFT} = \frac{\pi^2}{3}c_{(y)}T_e = \pi r_+^2 = S_{BH}$$

Summary

- The entropy of the Kerr-Newman-(A)dS black hole is reproduced as the statistical entropy of dual CFT
- If we assume the formula for the temperature of CFT, we can apply this idea to the fairly general four-dimensional extremal black holes
- The Reissner-Nordstrome black hole also can be treated, but there is a dual description by embedding it into 5D space

Futher application

- We can generalize the Kerr/CFT correspondence to the higher dimension (Lu-Mei-Pope, Azeyanagi-Terashima-Ogawa, Nakayama, Chow-Cvetic-Lu-Pope '08)
- There are several cycles along which we can construct the Virasoro algebra
- The central charge is given by

$$c_i = \frac{3k_i}{2\pi G_N} Area(horizon)$$

(we are now checking this formula for five-dimension including non-gravitational fields)

- The temperature associated with i-th cycle is

$$T_i = \frac{1}{2\pi k_i}$$

Open problem

- We can explain the entropy of the extremal black holes by the Cardy formula in dual CFTs
- but we don't know the complete spectrum of dual CFT which accounts the statistical entropy
- It is important to embed the extremal black holes into the string theory in order to understand the CFTs

Appendix: derivation of conserved charges

We consider the n-form Lagrangian $\boldsymbol{L}(\Phi)$

$$\delta \boldsymbol{L}(\Phi) = \boldsymbol{E}(\Phi)\delta\Phi + d\boldsymbol{\Theta}(\Phi, \delta\Phi)$$

Then, equations of motion are given by $\boldsymbol{E}(\Phi) = 0$

On the other hand, from the definition of the Lie derivative, we can obtain

$$\mathcal{L}_\xi \boldsymbol{L}(\Phi) = \xi \cdot d\boldsymbol{L}(\Phi) + d(\xi \cdot \boldsymbol{L}(\Phi)) = d(\xi \cdot \boldsymbol{L}(\Phi))$$

The conserved charge whose exterior derivative vanishes on-shell is defined as

$$\boldsymbol{J}_\xi(\Phi) = \boldsymbol{\Theta}(\Phi, \mathcal{L}_\xi\Phi) - \xi \cdot \boldsymbol{L}(\Phi)$$

Appendix:

We can define $(n-2)$-form $\boldsymbol{Q}_\xi(\Phi)$ by

$$\boldsymbol{J}_\xi(\Phi) = d\boldsymbol{Q}_\xi(\Phi)$$

From the current, we can construct the generator of the diffeomorphism as

$$H_\xi[\Phi] = \int_C \boldsymbol{J}_\xi(\Phi) + \int_{\partial C} \boldsymbol{B}_\xi(\Phi)$$

We can determine the boundary term $\boldsymbol{B}_\xi(\Phi)$ to require the variation of H_ξ

$$\delta \boldsymbol{B}_\xi(\Phi) = -\xi \cdot \boldsymbol{\Theta}(\Phi, \delta\Phi) \ .$$

Then we obtain

$$\delta H_\xi \equiv \frac{1}{16\pi} \int \boldsymbol{k}_{\xi,\Lambda}(\Phi, \delta\Phi)$$

$$\boldsymbol{k}_\xi(\Phi, \delta\Phi) = \delta\boldsymbol{Q}_\xi(\Phi) - \xi \cdot \boldsymbol{\Theta}(\Phi, \delta\Phi)$$