On $SU(2) \times U(1) \times SO(4) \times R$ Symmetric 1/8 BPS Geometries of IIB Supergravity

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Introduction: Backreacted region in AdS/CFT

AdS/CFT at finite N

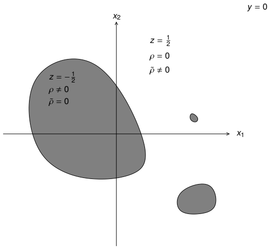
string theory on $AdS_{d+1} \times M$	$\longleftrightarrow$	CFT in d-dimensions
fluctuations around $AdS_{d+1} \times M$	$\longleftrightarrow$	operators with small Δ
deformations of $AdS_{d+1} \times M$	$\longleftrightarrow$	operators with large Δ

$AdS_5 \times S^5$ case

Lin-Lunin-Maldacena (LLM) investigated $SO(4) \times SO(4) \times R$ symmetric geometries in IIB SUGRA dual to the $\frac{1}{2}$ BPS primary operators like

$$Z_1^p .$$

(hep-th/0409174)



Generalization of LLM

Gava-Milanesi-Narain-O'Loughlin (GMNO) investigated $SU(2) \times U(1) \times SO(4) \times R$ symmetric geometries in IIB SUGRA dual to the $1/8$ BPS primary operators like

$$Z_1^p Z_2^q Z_3^q + \cdots .$$

(hep-th/0611065)

They replaced one of the two S^3s in LLM by a *squashed S^3*.

idea

LLM		GMNO
$SO(4) = SU(2)_L \times SU(2)_R$	$\longrightarrow$	$SU(2)_L \times U(1)_R$
z_1^p	$\longrightarrow$	$z_1^p z_2^q z_3^q + \cdots$
$d\Omega_{S^3}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2$	$\longrightarrow$	$\sigma_1^2 + \sigma_2^2 + a\sigma_3^2$

σ_i : left-invariant 1-forms

The geometries are written by four functions on (x_1, x_2, y) space

$$m(x_1,x_2,y) \quad , \quad n(x_1,x_2,y) \quad , \quad p(x_1,x_2,y) \quad , \quad T(x_1,x_2,y)$$

Result

$$ds^2 = -h^{-2}(dt + V_i dx_i) + h^2 \frac{\rho_1^2}{\rho_3^2}\left(T^2\left(dx_1^2 + dx_2^2\right) + dy^2\right) + \rho_1^2\left[\sigma_{\hat{1}}^2 + \sigma_{\hat{2}}^2\right] + \rho_3^2\left(\sigma_{\hat{3}} - A_\mu dx^\mu\right)^2 + \tilde{\rho}^2 d\tilde{\Omega}_3^2$$

$$F_5 = -\left(\rho_1^2\rho_3 G_2 \wedge \sigma_{\hat{1}} \wedge \sigma_{\hat{2}} \wedge \sigma_{\hat{3}} + \rho_1^2 *_4 \tilde{V} \wedge \sigma_{\hat{1}} \wedge \sigma_{\hat{2}} + \rho_3 *_4 \tilde{g} \wedge \sigma_{\hat{3}}\right)$$
$$+ \left(\tilde{G}_2 + \rho_3 \tilde{V}_{\bar{\mu}} e^{\bar{\mu}} \wedge \sigma_{\hat{3}} + \rho_1^2 \tilde{g} \sigma_{\hat{1}} \wedge \sigma_{\hat{2}}\right) \wedge \tilde{\rho}^3 d\tilde{\Omega}_3$$

where $\quad h^{-2} = \tilde{\rho}^2 + \rho_3^2(1 + A_t)^2, \quad \tilde{g} = \frac{1}{2\tilde{\rho}}\left(1 - \frac{\rho_3^2}{\rho_1^2}(1 + A_t)\right), \quad \tilde{V} = \frac{1}{2\rho_3\tilde{\rho}^3} d\left(\tilde{g}\rho_1^2\tilde{\rho}^3\right)$

$$\rho_1^2\rho_3 G_2 = d(B_t(dt + V) + \hat{B}), \quad \tilde{\rho}^3\tilde{G}_2 = -\frac{1}{2}\tilde{g}\rho_1^2\tilde{\rho}^3 dA + d(\tilde{B}_t(dt + V) + \hat{\tilde{B}})$$

$$A_i = A_t V_i + \frac{1}{2}\epsilon_{ij}\partial_j \ln T$$

where $\quad \rho_1^4 = \frac{mp + n^2}{m} y^4, \quad \rho_3^4 = \frac{p^2}{m(mp + n^2)}, \quad \tilde{\rho}^4 = \frac{m}{mp + n^2}, \quad A_t = \frac{n - p}{p}$

$$dV = y *_3 \left[dn + \left(nD + 2ym(n - p) + \frac{2n}{y}\right)dy\right]$$

$$B_t = \frac{y^2}{4}\frac{n}{m} \quad , \quad d\hat{B} = \frac{y^3}{4} *_3 [dp + 4yn(p - n)dy] \quad , \quad \tilde{B}_t = \frac{y^2}{4}\frac{n - \frac{1}{y^2}}{p} \quad , \quad d\hat{\tilde{B}} = \frac{y^3}{4} *_3 [dm + 2mDdy],$$

where $D = 2y(m + n - 1/y^2)$.

Constraints for m, n, p, T

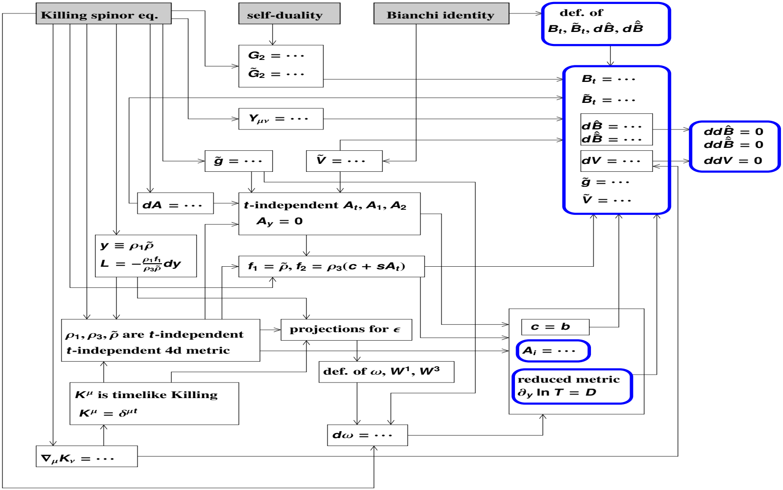
- m, n, p, T satisfy 4 differential equations

$$y^3\left(\partial_1^2+\partial_2^2\right)n+\partial_y\left(y^3T^2\partial_y n\right)+y^2\partial_y\left[T^2\left(yDn+2y^2m(n-p)\right)\right]+4y^2DT^2n=0$$
$$y^3\left(\partial_1^2+\partial_2^2\right)m+\partial_y\left(y^3T^2\partial_y m\right)+\partial_y\left(2y^3T^2mD\right)=0$$
$$y^3\left(\partial_1^2+\partial_2^2\right)p+\partial_y\left(y^3T^2\partial_y p\right)+\partial_y\left[4y^3T^2ny(n-p)\right]=0$$
$$\partial_y \ln T=D.$$

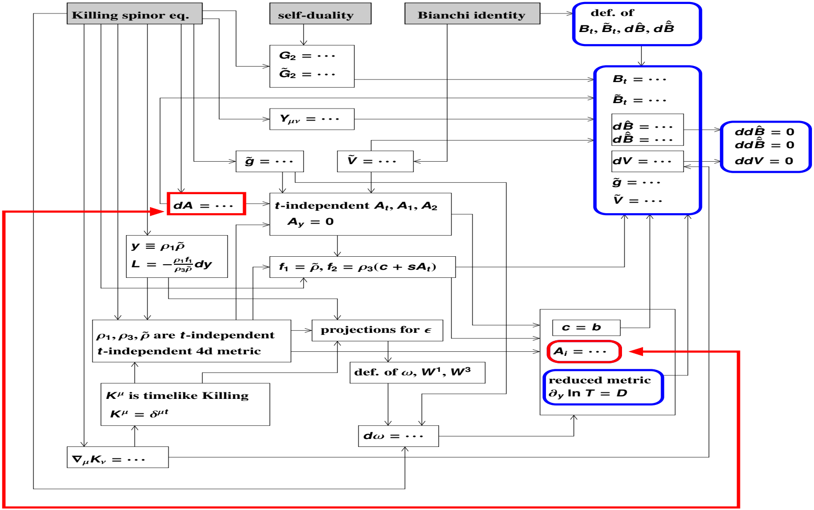
- "A solution" $n=0,\ m=1/y^2,\ p:\text{const.},\ T:\text{const.}$ does not give a solution of the supergravity.

m, n, p, T must be constrained more.

Closer Look: New Constraint



Closer Look: New Constraint



Another constraint

We obtain a new constraint

$$\frac{1}{2}\left(\partial_1^2 + \partial_2^2\right)\ln T = -T^2 y\partial_y n - T^2 y\partial_y m + 2T^2\left(m - 2m^2y^2 - 4mny^2 - n^2y^2 + mpy^2\right).$$

♣ Independent from the other four differential equations.

Any other constraints?

We will exhaust the constraints only for a restricted case

$n = 0,\ m = 1/y^2.$

Limit to a Liouville Equation

We take $n = 0,\ m = \frac{1}{y^2}$. Then p and T is y-independent.

- Killing spinor eq., self-duality, Bianchi identity
  ⇒No other constraint on the metrics and fluxes
- tt-component of the e.o.m ⇒ p:const..

Result

$$ds^2 = \overbrace{-\frac{1}{\sqrt{p}}(dt+V)^2 + \sqrt{p}T^2\left(dx_1^2 + dx_2^2\right)}^{AdS_3} + \overbrace{\sqrt{p}\left(dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2\right)}^{R^4} + \overbrace{\frac{1}{\sqrt{p}}d\tilde{\Omega}_3^2}^{S^3}$$

$$F_5 = \frac{p}{2}T^2 dt \wedge dx_1 \wedge dx_2 \wedge (dy_1 \wedge dy_2 + dy_3 \wedge dy_4) + \text{dual}$$

$$V = \frac{1}{4}\epsilon_{ij}\partial_j \ln T^2 dx_i.$$

A Liouville eq. $\left(\partial_1^2 + \partial_2^2\right)\ln\left(T(x)^2\right) = 8pT(x)^2.$

This $AdS_3 \times S^3 \times R^4$ is the near horizon geometry of a intersecting D3-branes.

Concluding Remarks

We have found a new constraint imposed on GMNO geometries for general cases and have picked up all the constrains for the case of $n = 0,\ m = 1/y^2$.

Future

- exhausting all the constraints for more general cases
- relating different CFTs