# A lattice study of $\mathcal{N}$=2 Landau-Ginzburg model using a Nicolai map

based on arXiv:1005.4671

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# 1 Purpose

## 2d $\mathcal{N}$=2 Landau-Ginzburg model (LG model)

$$S = \int d^2x d^4\theta\, K(\Phi, \bar{\Phi}) + \left( \int d^2x d^2\theta\, W(\Phi) + c.c. \right)$$

$\Phi \cdots$ chiral superfield

At the IR fixed point, $W(\Phi) = \lambda \Phi^k$ is believed to describe...

$\lambda_{\text{eff}} \to \infty$, lattice !

- $\mathcal{N} = 2$ minimal model $\leftarrow$ check for $K(\Phi, \bar{\Phi}) = \bar{\Phi}\Phi$ (WZ model)
- $\hookrightarrow$ Gepner model (compactified string), ...

## Why it is believed that LG models describe CFTs ?

- 2d bosonic case '86 A.B.Zamolodchikov

In the $c = 1 - \frac{6}{p(p+1)}$ minimal model, the fusion rule implies $\cdots\ \phi_{(2,2)}^{2p-3} \propto \partial^2 \phi_{(2,2)}$

In the 2d bosonic LG model $\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial_\mu \phi + g\phi^{2p-2}$, EOM is $\cdots\ \phi^{2p-3} \propto \partial^2 \phi$

$\overset{\text{conjecture}}{\Rightarrow}$ $\phi = \phi_{(2,2)}$ at the IR fixed point.

$\Rightarrow$ Extending this idea, ...

## How to check the conjecture

- early studies

| | |
|---|---|
| RG flow of $c$-functions | → '89 Kastor, Martinec and Shenker |
| catastrophe theory | → '89 Vafa and Warner |
| $\epsilon$-expansion | → '89 Howe and West |
| elliptic genus, SCA | → '93 Witten |
| ... | |

→ For $W(\Phi) = \lambda\Phi^k$,

$$\begin{cases} c = 3(1 - \frac{2}{k}) \\ \Phi : (h, \bar{h}) = (\frac{1}{2k}, \frac{1}{2k}) \\ \Phi^2 : (h, \bar{h}) = (\frac{2}{2k}, \frac{2}{2k}) \\ \vdots \\ \Phi^{k-2} : (h, \bar{h}) = (\frac{k-2}{2k}, \frac{k-2}{2k}) \end{cases}$$

- We computed **correlation functions** non-perturbatively for $W(\Phi) \propto \Phi^3$.

susceptibility of CFT:

$$\chi \equiv \int d^2x \langle \phi(x)\phi^*(0)\rangle \xrightarrow{\text{finite volume}} \int_V d^2x \frac{1}{|x|^{2h+2\bar{h}}} \propto V^{1-h-\bar{h}}$$

$$\Rightarrow \log \chi = \underline{(1 - h - \bar{h})} \log V + \text{const.}$$

For the present $W(\Phi) \propto \Phi^3$, $1 - h - \bar{h} = 1 - \frac{1}{6} - \frac{1}{6} = 0.666\ldots$

## 2 Lattice Formulation of WZ model

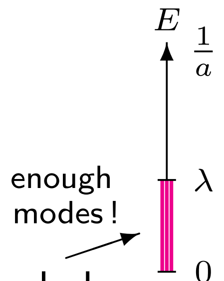
lattice action:

'83 Sakai and Sakamoto
'02 Catterall and Karamov
'02 Kikukawa and Nakayama
'09,'10 Kadoh and Suzuki,..

$$\mathcal{S} = \sum \left\{ \phi^* T \phi + W^*(1 - \frac{a^2}{4} T) W + \left( W'(-S_1 + iS_2)\phi + c.c. \right) \right.$$
$$\left. + \bar{\psi} \left( D + \frac{1+\gamma_3}{2} W'' \frac{1+\hat{\gamma}_3}{2} + \frac{1-\gamma_3}{2} W''^* \frac{1-\hat{\gamma}_3}{2} \right) \psi \right\}$$

where $D = \frac{1}{2}\left[ 1 + \frac{X}{\sqrt{X^\dagger X}} \right] = T + \gamma_1 S_1 + \gamma_2 S_2, \quad W = \frac{\lambda}{3}\Phi^3$

- $\lambda$ is the unique mass parameter (besides $a$) $\Rightarrow$ $\begin{cases} \text{continuum limit : } a\lambda \to 0 \\ \text{To see CFT, } L \gg (a\lambda)^{-1} \text{ is needed.} \end{cases}$




- no extra fine-tunings $\Leftarrow$ $\begin{cases} ◎ \text{ one SUSY } Q \\ ◎\ Z_3 \text{ R-symmetry } \leftarrow \text{overlap fermion} \end{cases}$

- This lattice model faces the sign problem

  $|D+F|$ is real, but can be negative. $\Leftarrow \gamma_1(D+F)\gamma_1 = (D+F)^*$

## 3 Simulation

We utilized the Nicolai map : $\eta = W' + (\phi - \frac{a}{2}W')T + (\phi^* - \frac{a}{2}W^{*\prime})(S_1 + iS_2)$.

$$\langle \mathcal{O} \rangle = \frac{\langle \sum_{i=1}^{N(\eta)} \mathcal{O}(\phi_i)\text{sgn}|D + F(\phi_i)|\rangle_\eta}{\langle \sum_{i=1}^{N(\eta)} \text{sgn}|D + F(\phi_i)|\rangle_\eta} \stackrel{a\to 0}{\longrightarrow} \textbf{Witten index } \Delta = 2 \text{ (cubic potential)}$$

where

$$\langle X \rangle_\eta \equiv \frac{\int \mathcal{D}\eta\mathcal{D}\bar{\eta}\ X\ e^{-\sum_x |\eta|^2}}{\int \mathcal{D}\eta\mathcal{D}\bar{\eta}\ e^{-\sum_x |\eta|^2}}$$

$N(\eta)$ counts the solutions of the Nicolai map $\phi_1, .., \phi_{N(\eta)}$

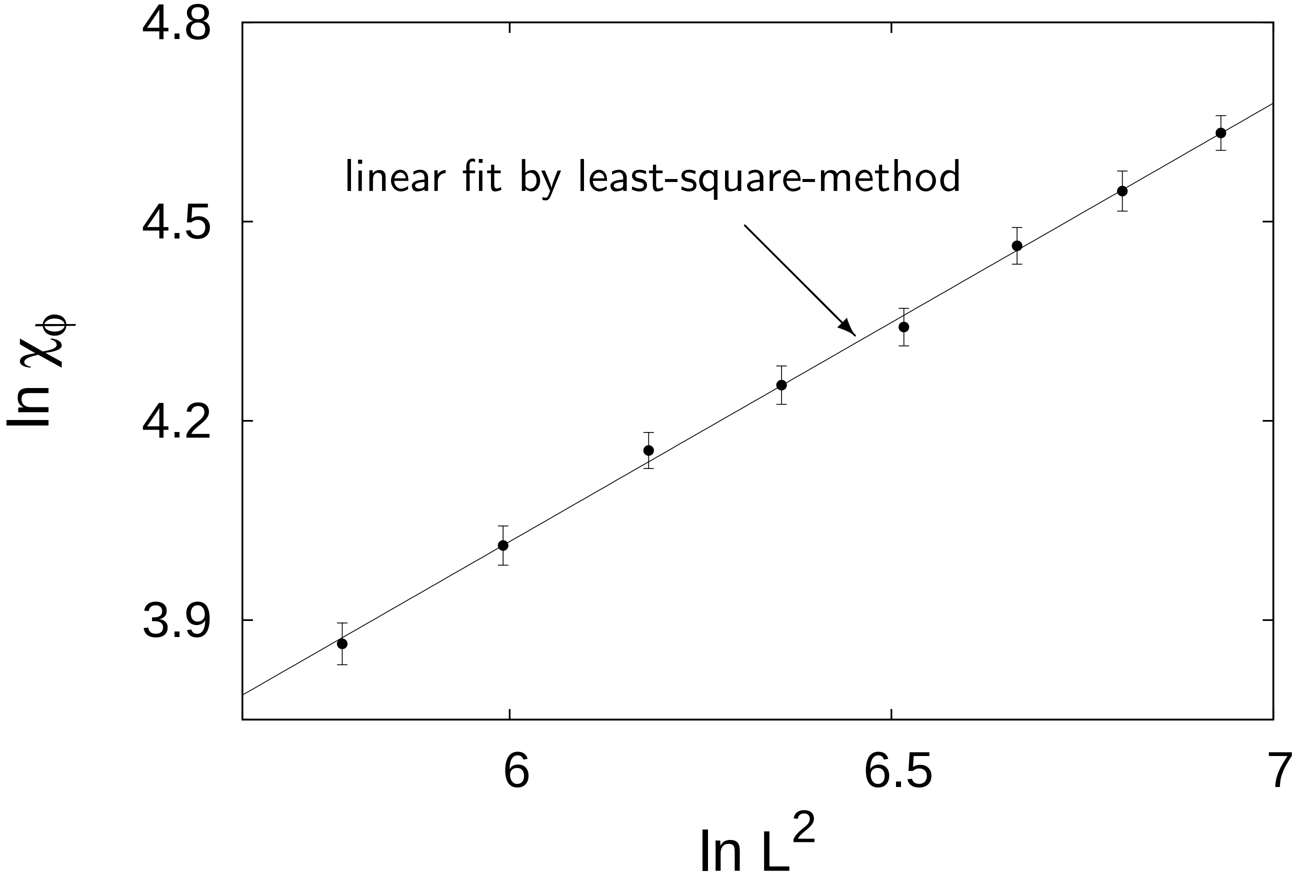
1. Assigning $\{\eta,\ \eta^*\}$ as the standard normal distribution,
2. Solving the Nicolai map by the Newton-Raphson algorithm,
3. Sample the configurations of $\{\phi,\ \phi^*\}$.

- advantage $\cdots$ no autocorrelation
- difficulty $\cdots$ $N(\eta)$

**Susceptibility:** $\chi_\phi \equiv \sum_{x\geq 3}\langle\phi(x)\phi(0)\rangle$

$W(\Phi)=\frac{\lambda}{3}\Phi^3, \quad a\lambda=0.3, \quad L=18,\ 20, .., \ 32$

(Newton iter. from 100 initial config. for each noise) × 320 noises




⇒ $\chi_\phi \propto V^{0.660\pm 0.011}$

⇒ consistent with the conjecture $\chi_\phi \propto V^{0.666\ldots}$ ◎

## 4 Summary and future plan

### Summary

- We observed $\chi = \int_V dx^2 \langle \phi(x)\phi^*(0)\rangle$ in the cubic potential case, and got the consistent result with the conjecture $\chi \sim V^{0.660\pm0.011}$.

- We also extracted the effective coupling constant $K$ of the Gaussian model, and obtained $K = 0.242 \pm 0.010$ which is consistent with the $\mathcal{N} = 2$ SUSY point $K = \frac{3}{4\pi} = 0.238...$
This implies the restoration of all supersymmetries in the IR. (see more detail in arXiv:1005.4671)

### Future Plan

- further check of the A-D-E classification:

$$\begin{aligned} W = \quad & \Phi^4 && \longrightarrow A_3 \text{ model ?} \\ & \Phi^3 + \Phi'^4 && \longrightarrow E_6 = A_2 \otimes A_3 \text{ model ?} \\ & \Phi^3 + \Phi\Phi'^2 && \longrightarrow D_4 \text{ model ?} \end{aligned}$$

- c-function $\longrightarrow$ central charge, c-theorem

- 2d $\mathcal{N} = 1$ LG model with $W \propto \Phi^3$ ( $\overset{\text{infrared}}{\longrightarrow}$ tricritical ising model)
$\Rightarrow$ dynamical SUSY breaking