Index Theorem and Overlap Formalism with Naive and Minimally Doubled Fermions

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December 17-19, 2010
Towards New Developments in Field and String Theories

based on a work with
M. Creutz (BNL) and T. Misumi (YITP, BNL)

JHEP **1012** (2010) 041 [arXiv:1011.0761]

Introduction

- Topological aspects of QFT
  - non-perturbative analysis

Index theorem

a link between gauge field topology and fermionic zero modes:

$$\mathrm{Ind}(D) = (-1)^{d/2} Q$$

It gives a theoretical foundation of topological study.

Introduction

- Topological aspects of QFT **on lattice**
  - non-perturbative analysis

Index theorem on lattice?

a link between gauge field topology and fermionic zero modes:

$$\mathrm{Ind}(D) = (-1)^{d/2} Q$$

It gives a theoretical foundation of topological study.

- Lattice regularization $\longrightarrow$ non-perturbative calculation
- Difficulty on lattice fermions
  - doublers, chiral symmetry, fine-tuning, etc...

Introduction

- Index theorem on lattice
  - doubler-free fermions: Wilson, Overlap, Domain-wall
  - doubling fermions: naive, staggered, minimal-doubling
    $\longrightarrow$ Index cancels out between doublers.

    cf. Nielsen-Ninomiya theorem (Poincaré-Hopf theorem)

Index theorem is hidden?

Our results

[Creutz-TK-Misumi '10 JHEP]

Index theorem with naive and minimally doubled fermions

We identify the would-be zero modes of naive and minimally doubled fermions even away from the continuum limit.

- How to extract the index?
  $\longrightarrow$ spectral flow of the hermitian Dirac operator

$$H(m) = \gamma_5(D - m)$$

- Key point

We have to assign **flavor sensitive mass** to hidden multiple degrees.

$\downarrow$

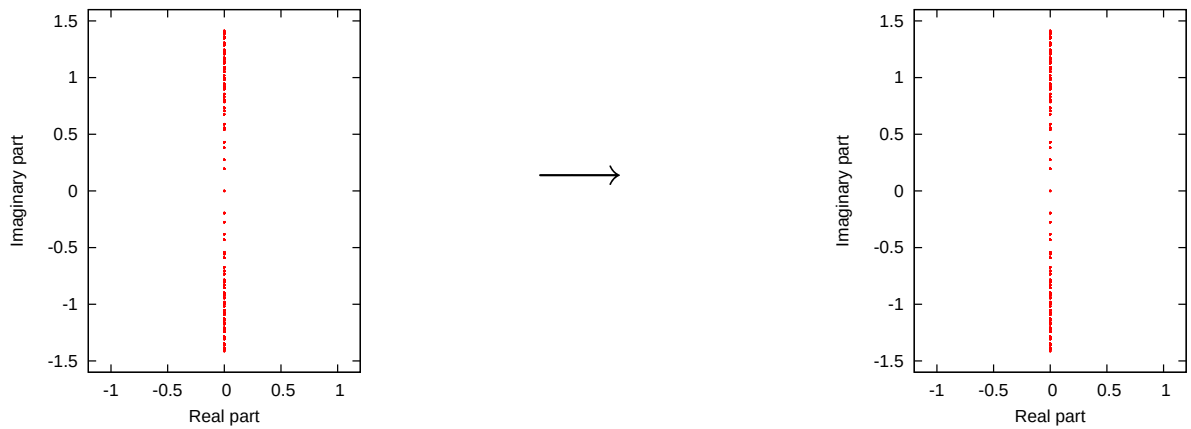
How to do that for species doublers?

- cf. staggered fermions [Adams '10 PRL] [Hoelbling '10]
  $\longrightarrow$ flavor(taste) structure is well known.

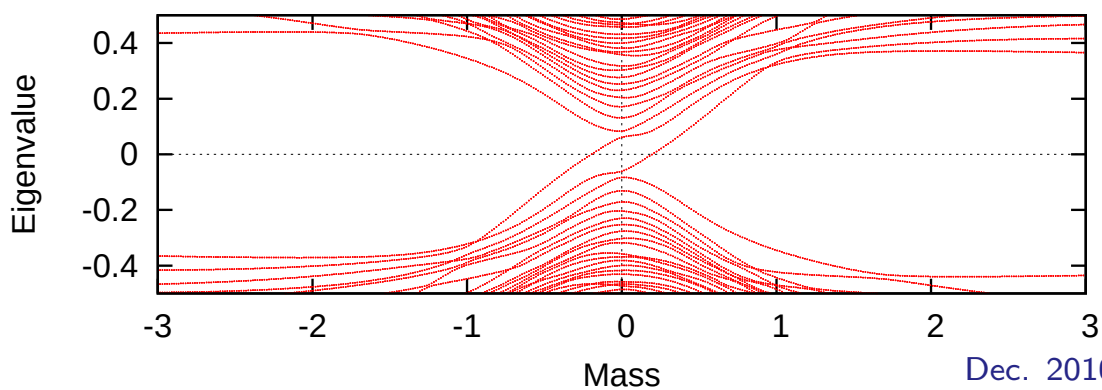
Our results

- Flavored mass term
  $\longrightarrow$ momentum depend, including hopping terms

  - Dirac spectrum: $D - M_f$, $\quad M_f = \mathrm{diag}(+1, -1, -1, +1)$



  - Spectral flow: $H(m) = \gamma_5(D - M_f)$



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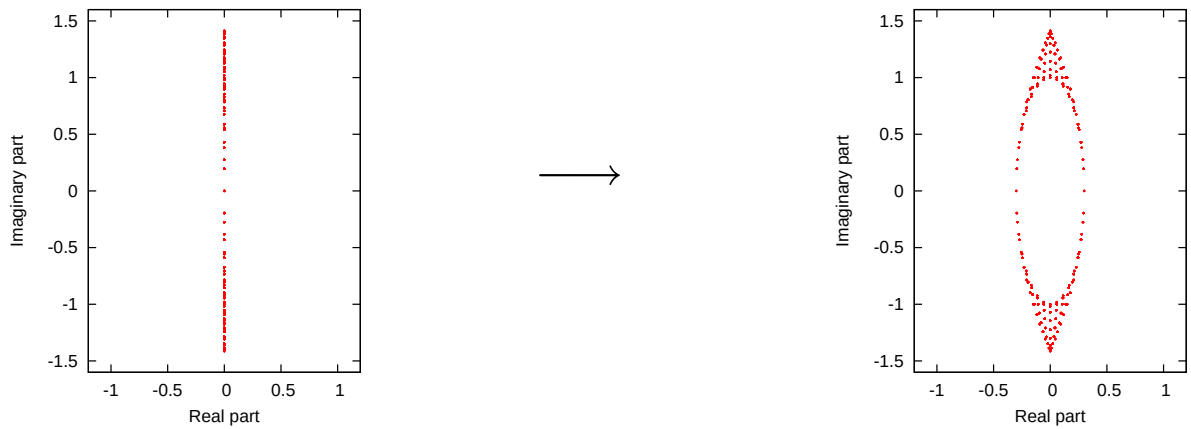

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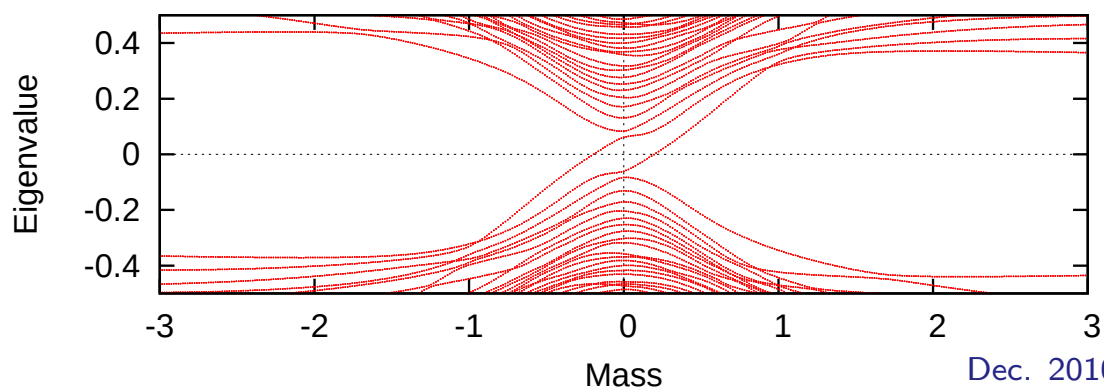

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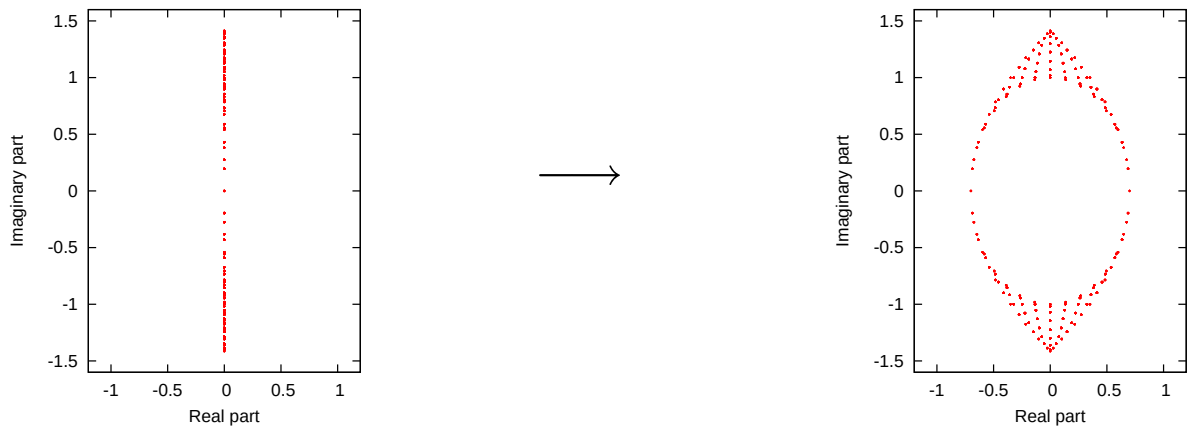

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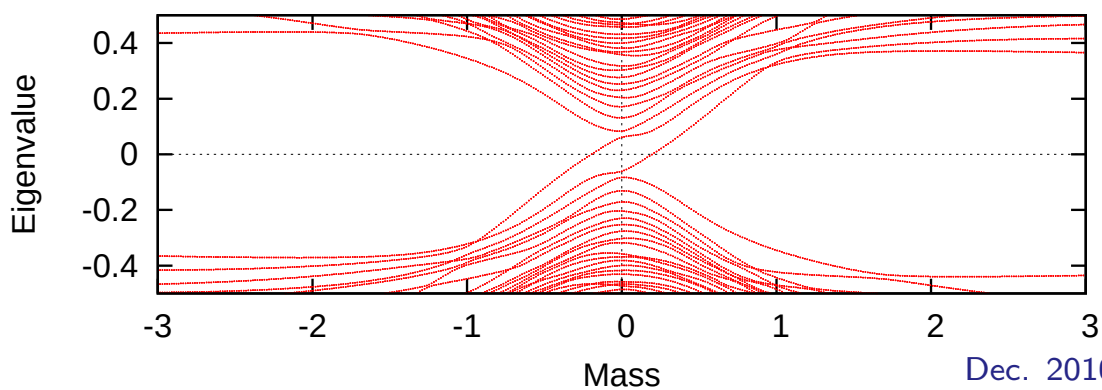

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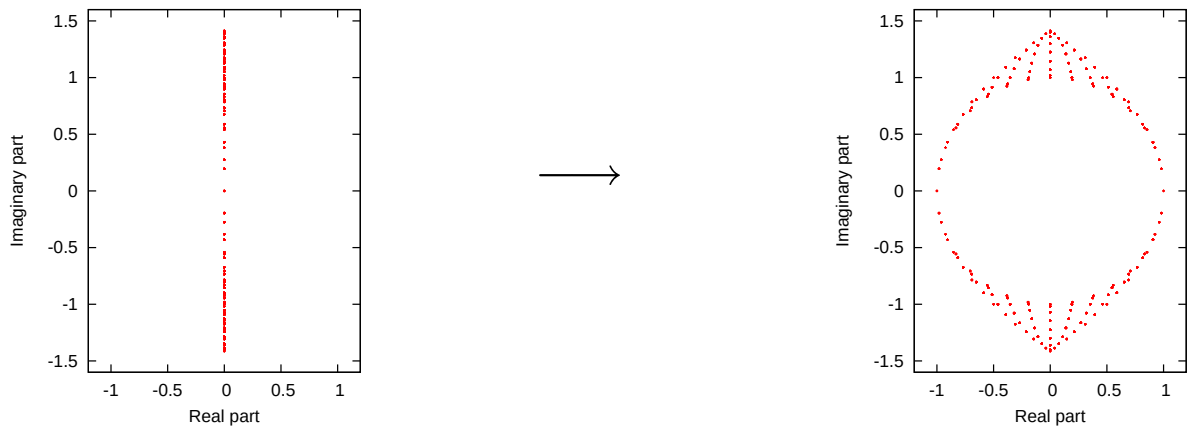

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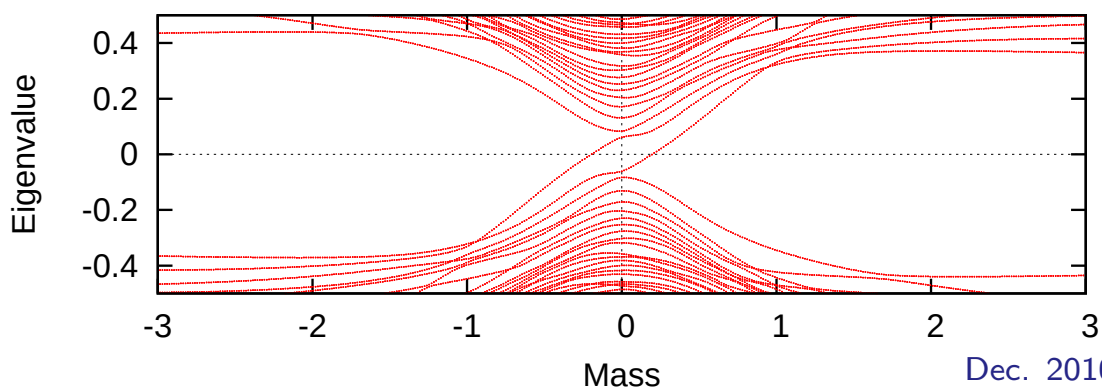

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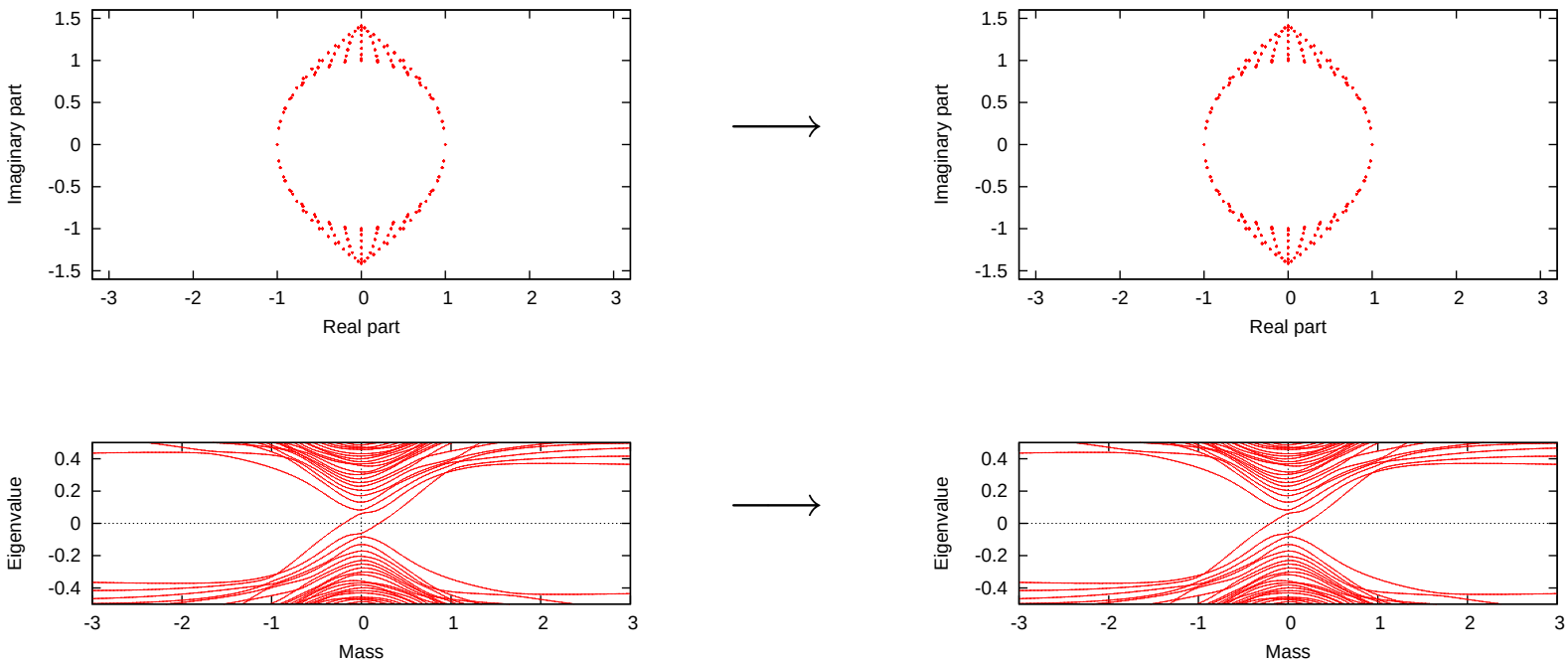
Overlap formalism with naive fermion kernel

We can formulate a new version of overlap fermion built with naive fermion kernel, especially yielding a **single-flavor** naive overlap fermion by choosing a certain flavored mass.

- Lifting the degeneracy



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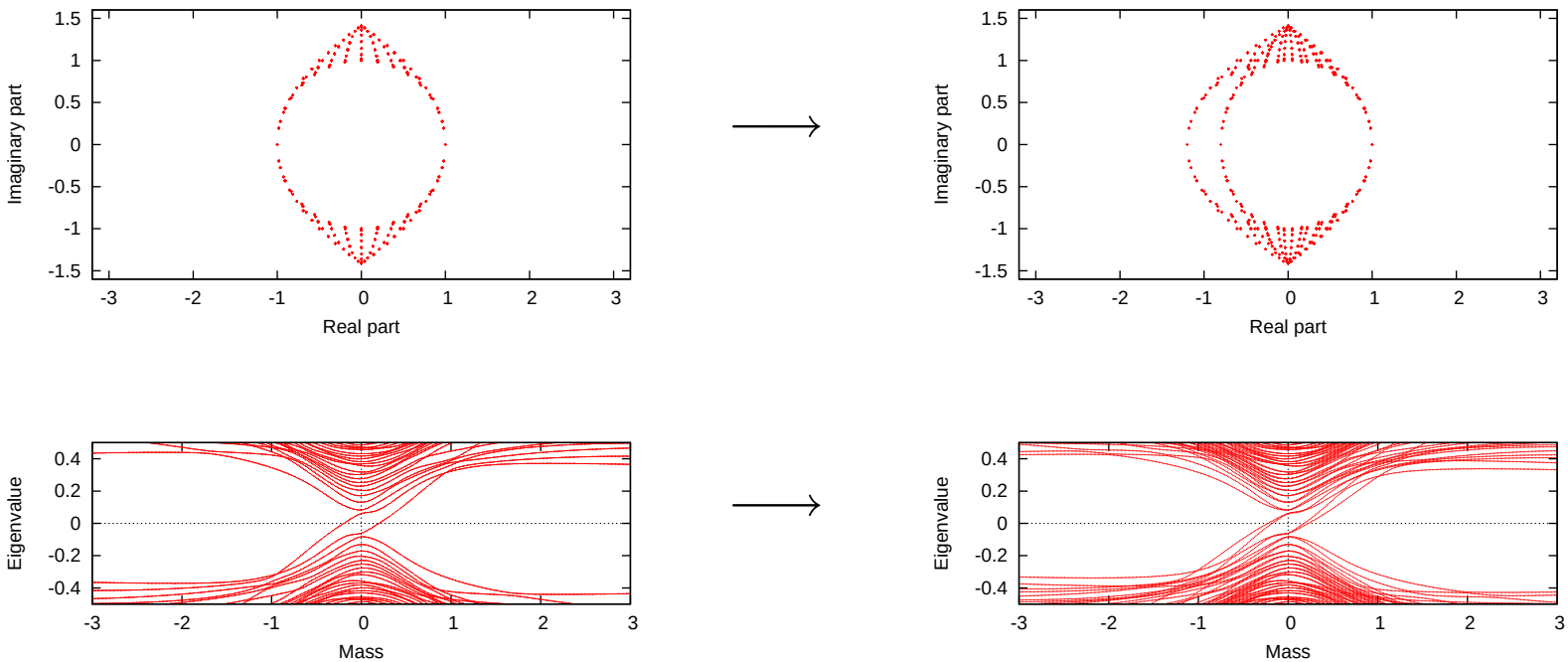

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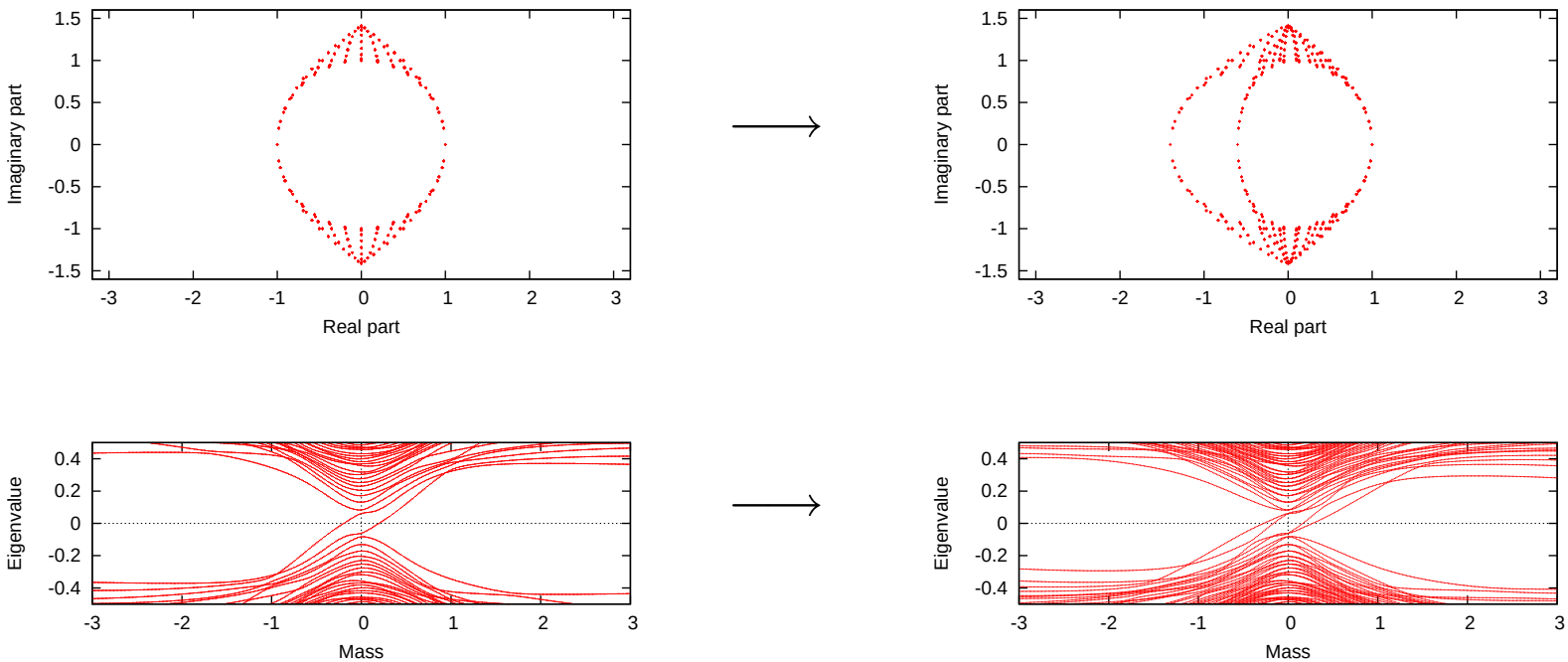

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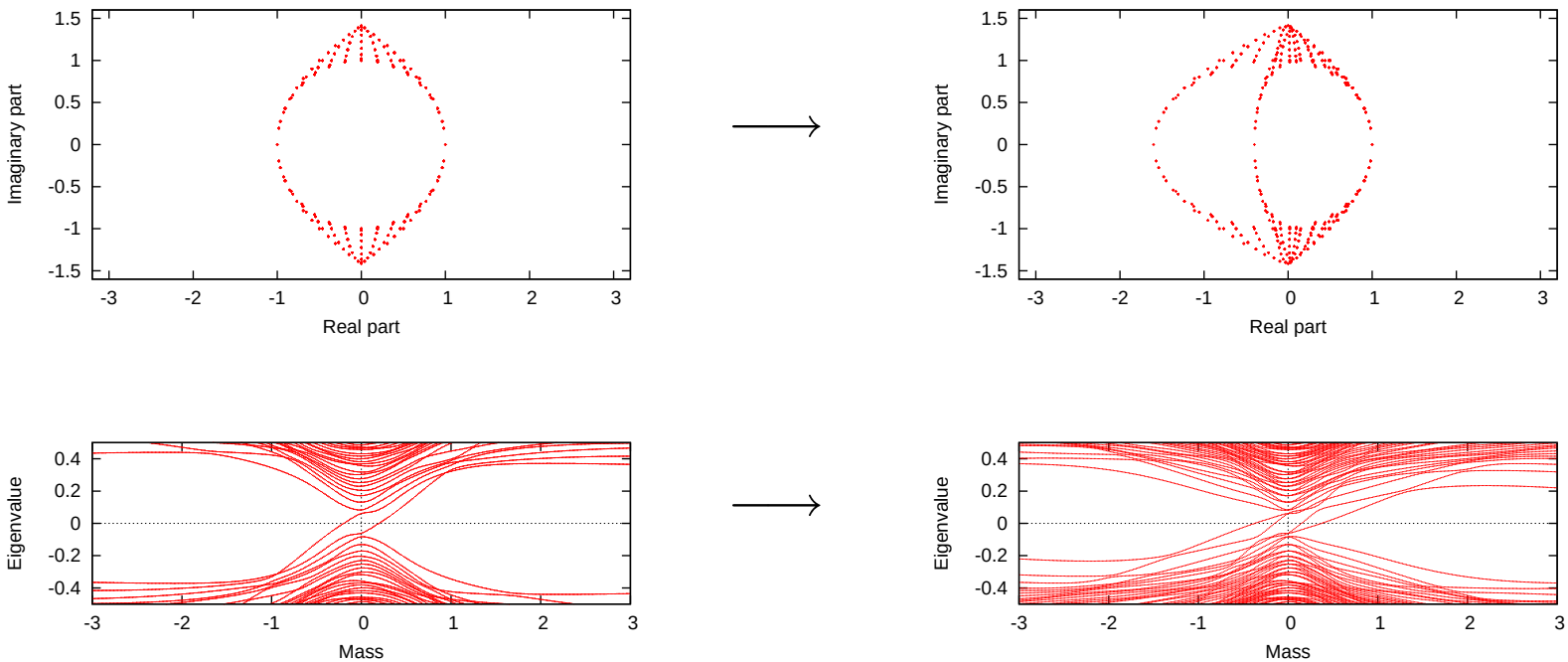

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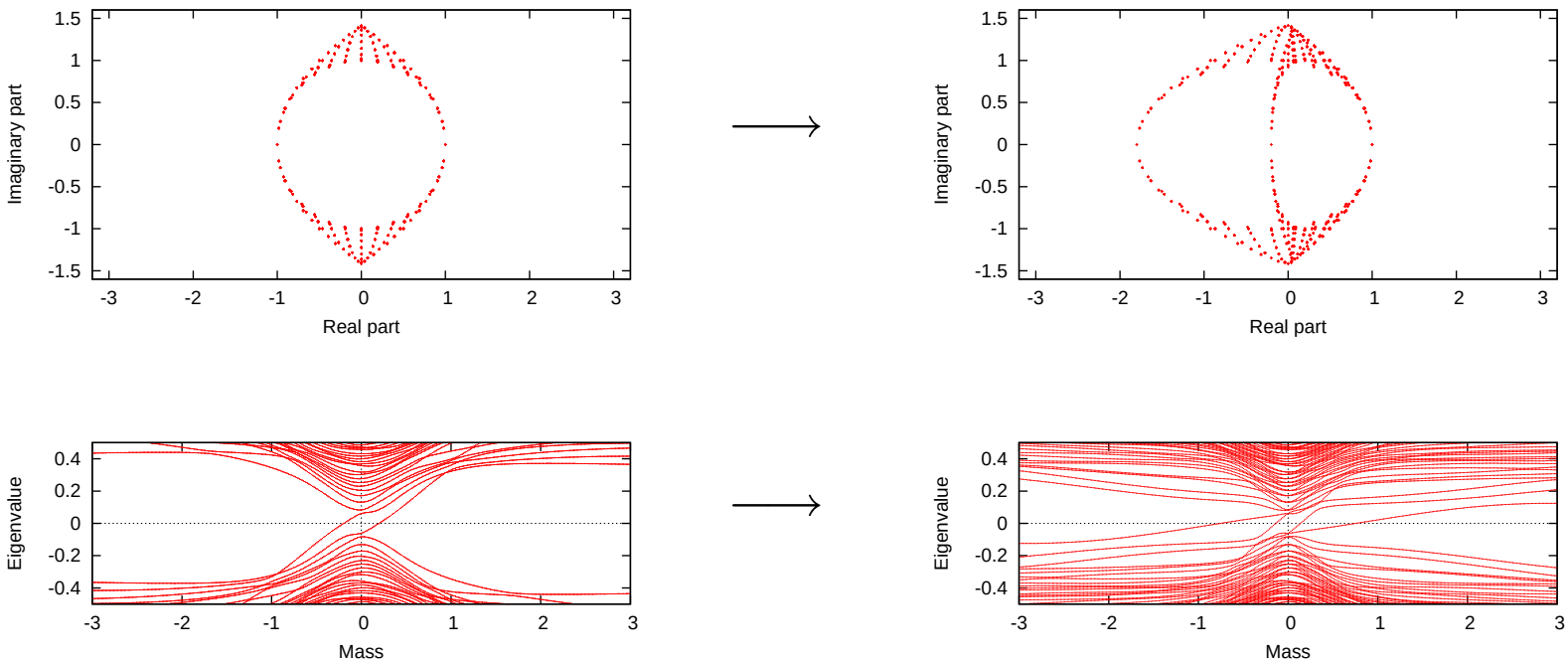

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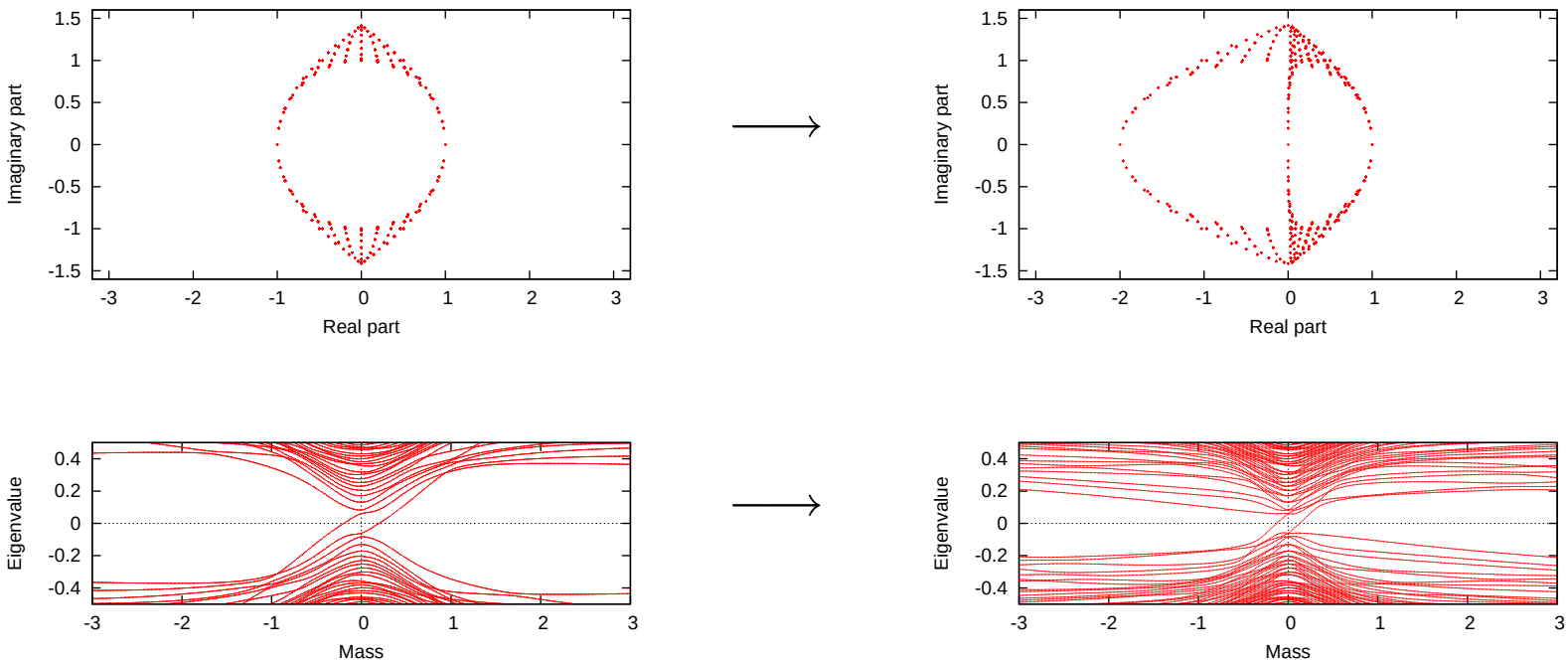

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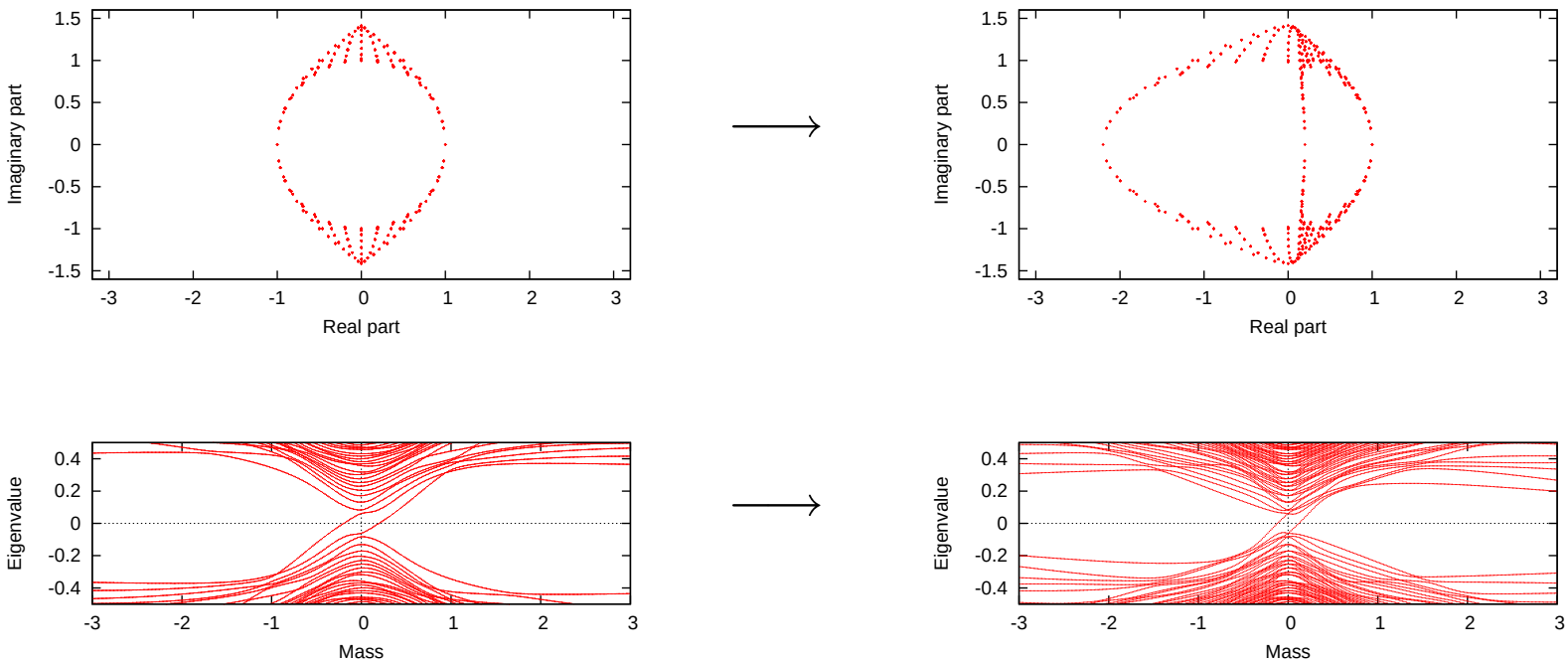

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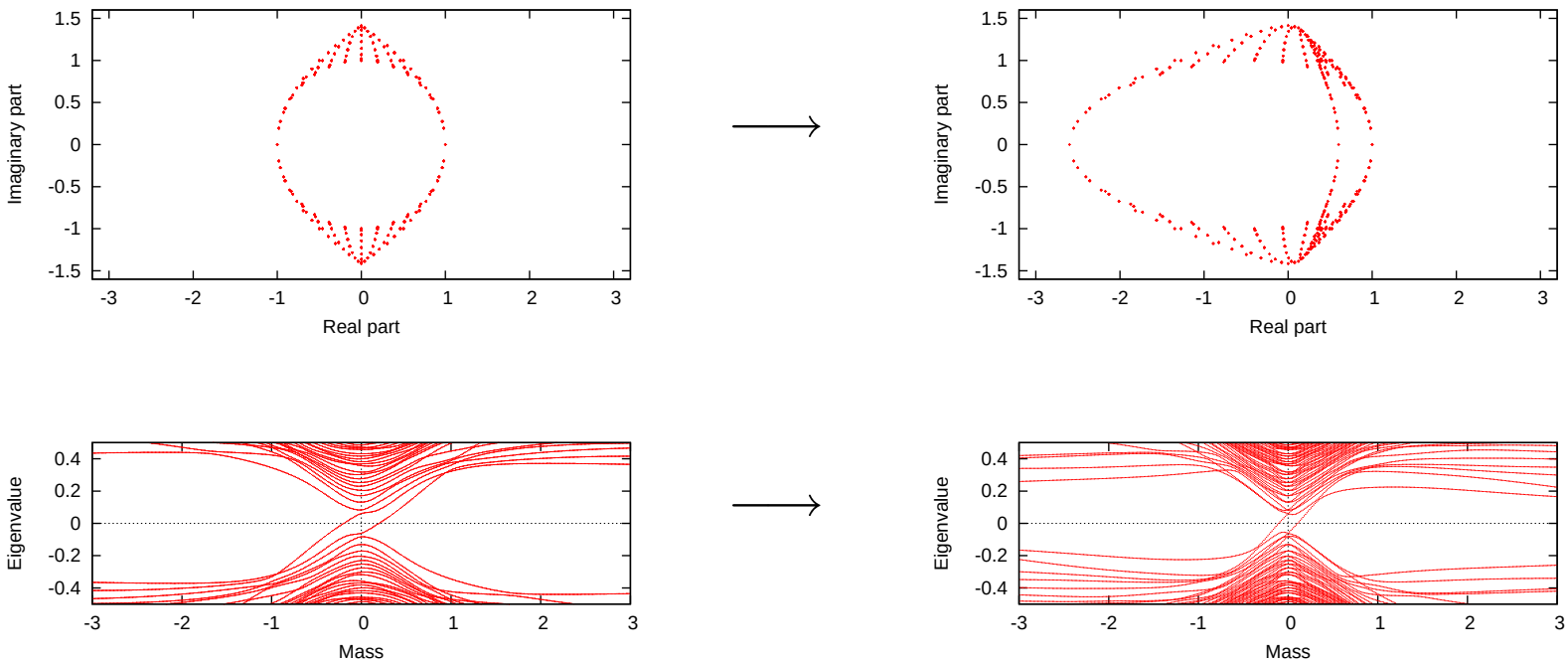

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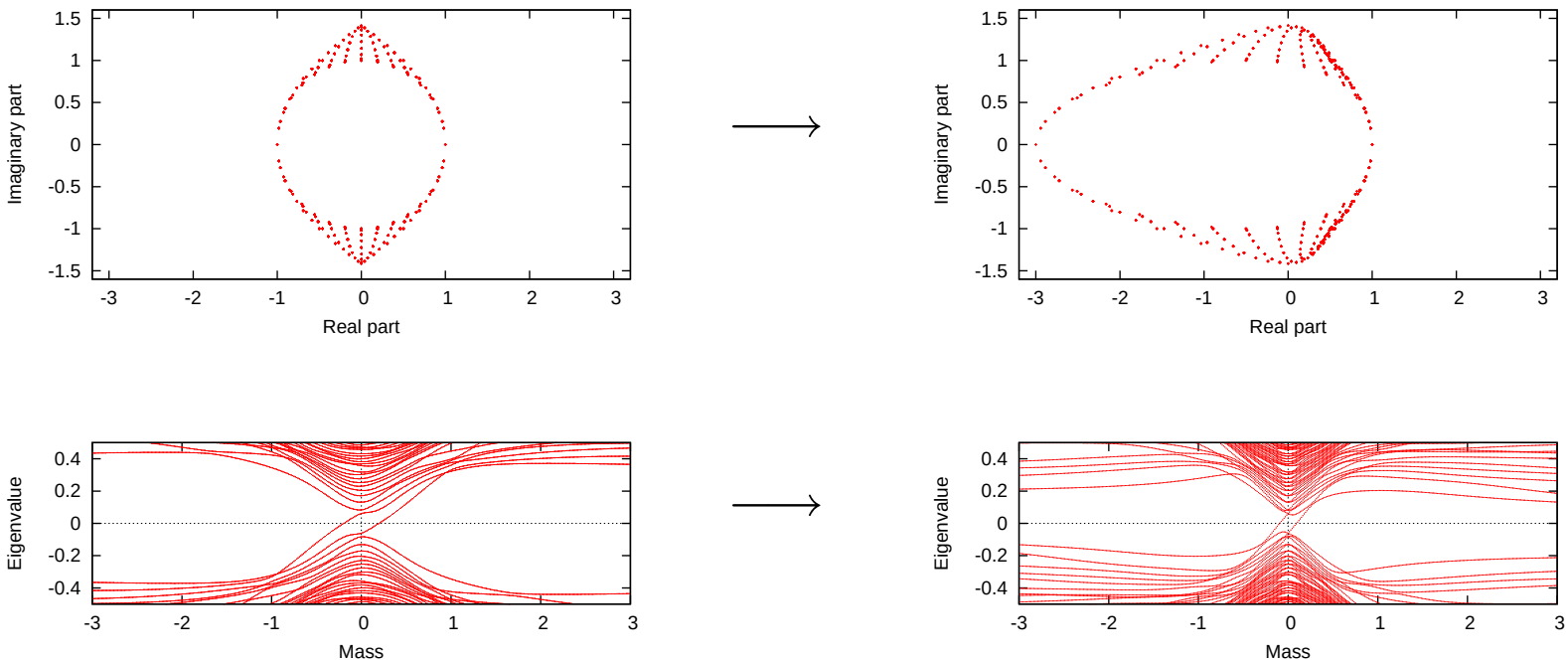

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