Multiple M5-branes' theory with Lie 3-algebra

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Construction of BLG model

[Bagger-Lambert '07][Gustavsson '07]

1. Conjecture the supersymmetry transformation for multiple M2-branes' system.

   ✓ The clues are provided by that of a single M2-brane's and multiple D2-branes' (3-dim super Yang-Mills) system.

   ✓ To do this, Lie 3-algebra is naturally introduced as the gauge symmetry algebra.

2. Obtain the equations of motion, by checking the closure of this transformation.

3. Write down the action which reproduces these equations of motion.

Action of BLG model

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}h^{ab}D_\mu X^I_a D_\mu X^I_b + \frac{i}{2}h^{ab}\bar{\Psi}_a\Gamma^\mu D_\mu \Psi_b + \frac{i}{4}h^{ae}f^{bcd}{}_e\bar{\Psi}_a\Gamma_{IJ}X^I_b X^J_c \Psi_d \\
& -\frac{1}{12}h^{gh}f^{abc}{}_g f^{def}{}_h X^I_a X^J_b X^K_c X^I_d X^J_e X^K_f \\
& +\frac{1}{2}\epsilon^{\mu\nu\lambda}\left(h^{de}f^{abc}{}_e A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}h^{bh}f^{cda}{}_g f^{efg}{}_h A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right)
\end{aligned}
$$

The fields on M2-branes' worldvolume are…

- ✓ scalars (transverse directions) – 8 d.o.f. / mass dim. = 1/2
- ✓ spinors – 8 d.o.f. / mass dim. = 1
- ✓ Chern-Simons gauge field – 0 d.o.f.

The Lie 3-algebra is denoted as…

metric

$$
\begin{aligned}
\langle T^a, T^b \rangle &= h^{ab} \\
[T^a, T^b, T^c] &= f^{abc}{}_d T^d
\end{aligned}
$$

structure constants

Lambert-Papageorgakis model

[Lamber-Papageorgakis '10]

By similar procedure to BLG model, a model of multiple M5-branes' system can be constructed:

$$
\begin{aligned}
D^2 X^I_a + \frac{i}{2}[\bar{\Psi}, C^\mu, \Gamma_\mu \Gamma^I \Psi]_a - [C^\mu, X^J, [C_\mu, X^J, X^I]]_a &= 0 \\
\Gamma^\mu D_\mu \Psi_a + \Gamma_\mu \Gamma^I [C^\mu, X^I, \Psi]_a &= 0 \\
D_{[\mu} H_{\nu\rho\sigma]a} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[C^\lambda, X^I, D^\tau X^I]_a - \frac{i}{8}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[\bar{\Psi}, C^\lambda, \Gamma^\tau \Psi]_a &= 0 \\
\tilde{F}_{\mu\nu}{}^b{}_a - C^\rho_c H_{\mu\nu\rho,d} f^{cdb}{}_a &= 0 \\
D_\mu C^\nu_a &= 0
\end{aligned}
$$

$$
C^\mu_c D_\mu X^I_d f^{cdb}{}_a = C^\mu_c D_\mu \Psi_d f^{cdb}{}_a = C^\mu_c D_\mu H_{\mu\rho\sigma,d} f^{cdb}{}_a = C^\mu_c C^\nu_d f^{cdb}{}_a = 0
$$

✓ The action cannot be written down, unfortunately.

Lambert-Papageorgakis model

$$
\begin{aligned}
D^2 X_a^I + \frac{i}{2}[\bar{\Psi}, C^\mu, \Gamma_\mu \Gamma^I \Psi]_a - [C^\mu, X^J, [C_\mu, X^J, X^I]]_a &= 0 \\
\Gamma^\mu D_\mu \Psi_a + \Gamma_\mu \Gamma^I [C^\mu, X^I, \Psi]_a &= 0 \\
D_{[\mu} H_{\nu\rho\sigma]a} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[C^\lambda, X^I, D^\tau X^I]_a - \frac{i}{8}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[\bar{\Psi}, C^\lambda, \Gamma^\tau \Psi]_a &= 0 \\
\tilde{F}_{\mu\nu}{}^b{}_a - C_c^\rho H_{\mu\nu\rho,d} f^{cdb}{}_a &= 0 \\
D_\mu C_a^\nu &= 0
\end{aligned}
$$

The fields on M5-branes' worldvolume are…

- ✓ scalars – 5 d.o.f. / mass dim.= 2
- ✓ spinors – 8 d.o.f. / mass dim. = 5/2
- ✓ 2-form field B – 3 d.o.f. / mass dim.= 2 (only H=dB appears above.)
- ✓ gauge field – 0 d.o.f.? / mass dim.= 1 (closely related to 2-form field.)
- ✓ new field C – 0 d.o.f.? / mass dim.= -1 (needed for comformality.)

M5 to D4 / meaning of field C ?

- Lie 3-algebra for reproduction of D4-branes $\{T^i, u, v\}$

$$[u, T^i, T^j] = f^{ij}{}_k T^k, \quad [T^i, T^j, T^k] = -f^{ijk} v, \quad [v, *, *] = 0.$$

  - ✓ This reproduces D2-branes in BLG model. [Ho-Imamura-Matsuo '08]
  - ✓ This is related to the compactification of M-direction.

- VEV's for u-component fields can be set, without breaking supersymmetry and gauge symmetry.

$$C_u^\mu = \lambda \delta_5^\mu, \quad \text{otherwise} = 0\,. \quad \text{and} \quad C_i^\mu = C_v^\mu = 0$$

- The new field C seems to relate to the gauge fixing of worldvolume coordinates: [Honma-Ogawa-SS, to appear]

$$X^\mu(\sigma) = \sigma^\mu \mathbf{1} + C_a^\mu T^a \quad \text{instead of} \quad X^\mu(\sigma) = \sigma^\mu$$

M5 to Dp / U-duality ?

[Honma-Ogawa-SS, to appear]

- Lie 3-algebra for reproduction of Dp-branes on T^{p-4}

  (a kind of central extension of Kac-Moody algebra) $\{T^i_{\vec{m}}, u, v, u_a, v_a\}$

$$f^{u_a (i\vec{m})(j\vec{n})} = m_a \delta^{ij} \delta_{\vec{m}+\vec{n}}\,, \quad f^{(i\vec{m})(j\vec{n})(k\vec{l})} = f^{ijk} \delta_{\vec{m}+\vec{n}+\vec{l}}\,; \quad \langle u_a, v_b \rangle = \delta_{ab}\,.$$

  - ✓ This reproduces Dp-branes on T^{p-2} in BLG model.
  - ✓ This is related to the compactification of M-direction and T-duality. [Ho-Matsuo-SS '08][Kobo-Matsuo-SS '08]

- VEV's can be set as $C^\mu_u = \lambda \delta^\mu_5\,, \quad X^I_{u_a} = \lambda^I_a\,, \quad \text{otherwise} = 0\,.$

  Field redefinition is needed like $\Phi_i(x, y) = \sum_{\vec{m}} \Phi_{i\vec{m}}(x) e^{i\vec{m}\vec{y}}$

- U-duality ⊃ relation among M5-branes and Dp-branes on T^{p-4}

  T-duality, T-transformation, S-duality can be discussed…