Topological Phases of Eternal Inflation

Yasuhiro Sekino
（Okayama Institute for Quantum Physics)

w/ Stephen Shenker (Stanford), Leonard Susskind (Stanford),

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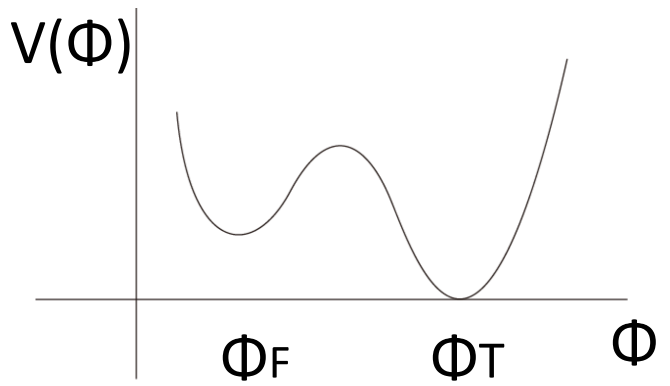
The problem addressed in this work:

What happens when gravity is coupled to a theory with metastable vacuum?

- e.g. scalar field which has a false vacuum and a true vacuum

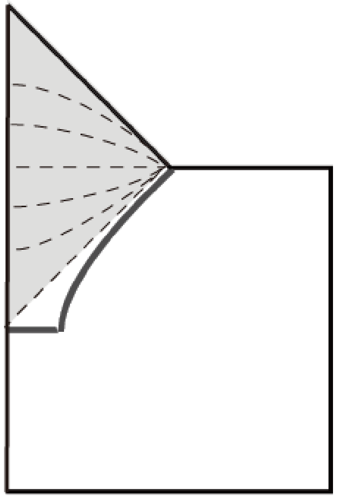
  $V(\Phi_F)>0, \ V(\Phi_T)=0$



- If we ignore gravity, first order phase transition:
  - Nucleation of bubbles of true vacuum (Callan, Coleman, …)
  - The whole space eventually turns into true vacuum.

Bubble of true vacuum

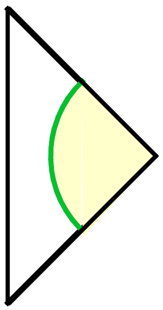
- Described by Coleman-De Luccia instanton (Euclidean "bounce" solution). Nucleation rate: $\Gamma \sim e^{-(S_{\rm cl} - S_{\rm deSitter})}$

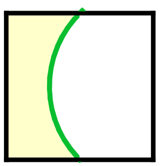


- Open FRW universe inside a bubble: Spatial slice: 3D hyperboloid

$$ds^2 \sim -dt^2 + t^2(dR^2 + \sinh^2 R d\Omega^2)$$

- If $\Gamma \ll H^4$, bubble nucleation cannot catch up the expansion of space, and false vacuum exists forever ("Eternal Inflation")

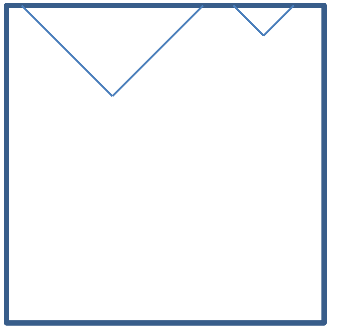




View from the future infinity

- Consider conformal future infinity of de Sitter.

$$ds^2 \sim \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2} \qquad (-\infty < \eta < 0)$$



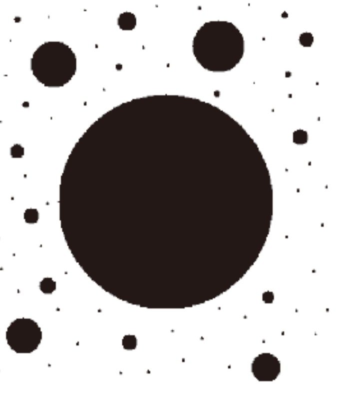
- A bubble: represented as a sphere cut out from de Sitter.

- "Scale invariant" distribution of bubbles

  Bubbles nucleated earlier:

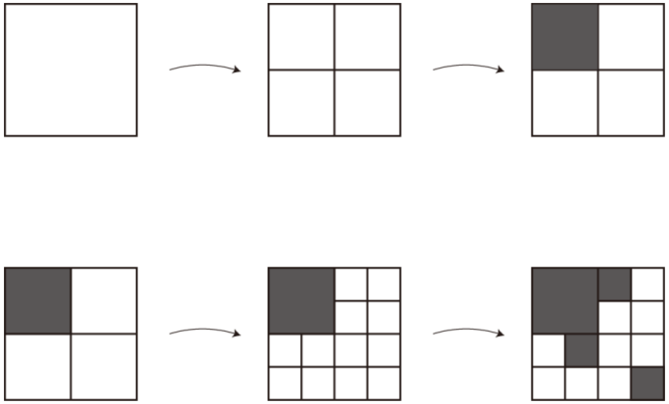
  appear larger: radius $\sim H^{-3}|\eta|^3$

  rarer: volume of nucleation sites $\sim |\eta|^{-3}$



Model for eternal inflation

- Mandelbrot model (Fractal percolation)
  - Start from a white cell.
    (One horizon volume of inflating region)
  - Divide the cell into cells with half its linear size.
    (The space grows by a factor of 2.)



Picture of the 2D version

  - Paint each cell in black with probability P.
    (Bubble is nucleated and takes up a horizon volume. P ~ Γ)
  - Subdivide the surviving (white) cells, and paint cells in black w/ probability P. Repeat this infinite times.

Three phases of eternal inflation

From the result on the 3D Mandelbrot model

[Chayes et al, Probability Theory and Related Fields 90 (1991) 291]

In order of increasing P (or Γ), there are

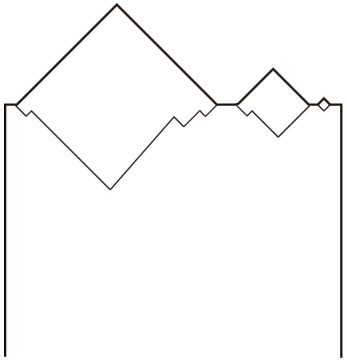
(white = inflating, black = non-inflating)

- Black island phase: Black regions form isolated clusters; ∃ percolating white sheets.
- Tubular phase: Both regions form tubular network; ∃ percolating black and white lines.
- White island phase: White regions are isolated; ∃ percolating black sheets.

Spacetime inside the (cluster of) bubbles

Black island phase (isolated cluster of bubbles)

Small deformations of open FRW universe.



- Basic fact: A collision of two bubbles (of the same vacuum) does not destroy the bubble
  [c.f. Bousso, Freivogel, Yang, '07]
  - Spatial geometry approaches smooth H^3 at late time.
  - Residual symmetry SO(2,1): spatial slice has H^2 factor
  - Negative curvature makes the space expand.

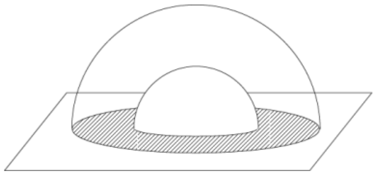
- Local geometry near collision will be similar to the two bubble case even when many bubbles collide.

Tubular phase (tube-like structure of bubbles)

In the late time limit: spatial slice is a negatively curved space whose boundary has infinite genus.

Late time geometry: $ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma}$

- Spatial geometry: H^3 modded out by discrete elements of isometry
- Boundary genus = # of elements
- The whole space is accessible to a single observer.

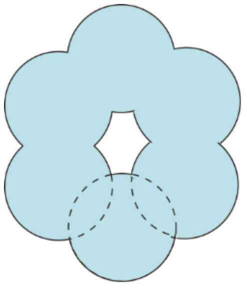


Genus 1 case

- Simpler example: true vacuum with toroidal boundary

[Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, '08]

Consider ring-like initial configuration of bubbles



Late time geometry: negatively curved space with toroidal boundary.

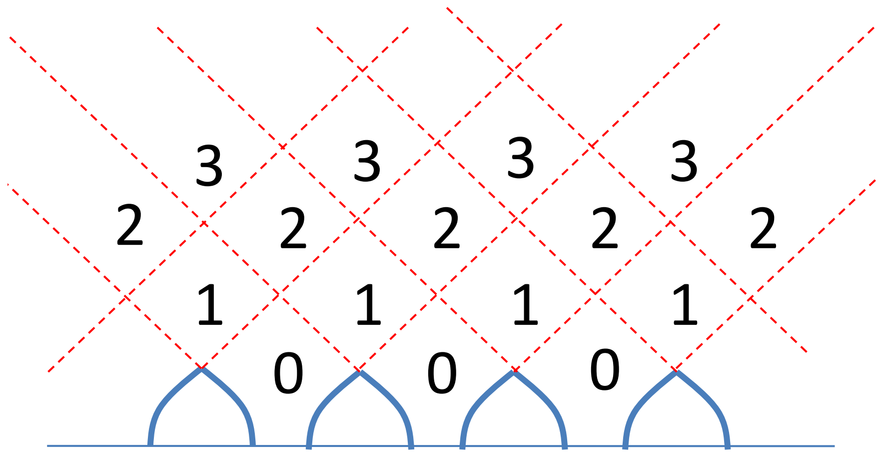
$$ds^2 = -f(t)dt^2 + f^{-1}(t)dz^2 + t^2 dH_2^2$$

$$f(t) = 1 + t^2/\ell^2 \quad \text{(de Sitter)}$$

$$f(t) = 1 - t_n/t \quad \text{(in region } n\text{; } t_n\text{: const.)}$$

$$f_{n+2}(t_{*,n+2}) f_n(t_{*,n+2}) = \left(f_{n+1}(t_{*,n+2})\right)^2$$

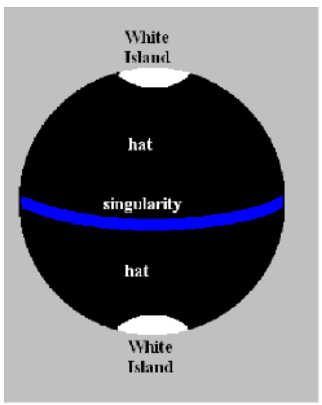
$$t_{*,n} : \text{ time of the } n\text{-th collision}$$



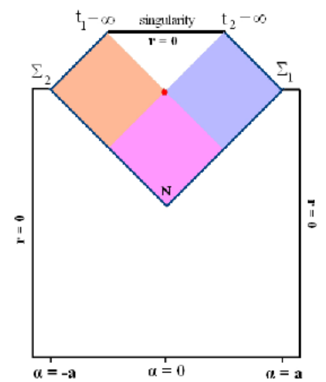
White island phase (isolated inflating region)

<u>An observer in the black region is "surrounded" by the white region (contrary to the intuition from Mandelbrot model).</u>

- Simple case: two white islands (with S^2 symmetry)
  [Kodama et al '82, BFSSSYY '08]



Global slicing (S^3) of de Sitter



Penrose diagram

  - An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. "non-traversability of a wormhole", "topological censorship"]

Summary

Three phases of eternal inflation and their cosmology:

- Black island phase:
  Small deformation of an open FRW
- Tubular phase:
  Negatively curved space with an infinite genus boundary
- White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).