Numerical approaches to string theory

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Gauge/gravity duality

SYM difficult		STRING
large-N, strong coupling	$\longleftarrow$	SUGRA easier
large-N, finite coupling	$\longrightarrow$	tree-level string (SUGRA+α') more difficult
finite-N, finite coupling	$\longrightarrow$	Quantum string (g_{string}>0) very difficult

use Monte Carlo to study string theory!

- Form the string theory point of view, *SYM theories in less than four spacetime dimensions are as interesting as four dimensional theories!*

(0+1)-d SYM $\Leftrightarrow$ Black hole

(1+1)-d SYM $\Leftrightarrow$ Black 1-brane, black string

(3+1)-d SYM $\Leftrightarrow$ Black 3-brane ($AdS_5 \times S^5$)

Plan

(1) What is Monte Carlo?

(2) Why lattice SUSY is hard（fine tuning, sign problem）

(3) Simulation of (0+1)-d SYM (D0-brane quantum mechanics)

(4) Simulation of (1+1)-d SYM (D0-brane quantum mechanics)

What is Monte Carlo?

The principle of Monte-Carlo

- Consider field theory on Euclidean spacetime with the action $S[\phi]$.
- Generate field configurations with probability $e^{-S[\phi]}$. Then,

$$\langle \mathcal{O} \rangle = \frac{\int [d\phi] \mathcal{O}[\phi] e^{-S[\phi]}}{\int [d\phi] e^{-S[\phi]}} \simeq \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}[\phi_i]$$

- Such a set of configurations can be generated as long as $e^{-S[\phi]} > 0$
(not 'probability' otherwise...)

Algorithm

- generate a chain of field configurations with the transition probability $P[C \to C']$

$$C_0 \to C_1 \to C_2 \to \cdots$$

- 'Markov chain' : transition probability from C_k to C_{k+1} does not depend on $C_0,\ldots,C_{k-1}$

$w_k[C]$: probability of obtaining C at k-th step

Choose $P[C \to C']$ so that

$$\lim_{k\to\infty} w_k[C] \propto e^{-S[C]}$$

Algorithm (cont'd)

'algorithm' = choice of $P[C \to C']$

- Metropolis
  simplest
  useful for fermions
- Hybrid Monte Carlo (HMC)
- Rational Hybrid Monte Carlo (RHMC)

.....etc etc...

Nicholas Constantine Metropolis
(1915 – 1999)

The simplest example (Gaussian integral)

Metropolis algorithm

(Metropolis-Rosenbluth-et al, 1953)

- Consider the Gaussian integral,

$$S[x] = \frac{x^2}{2}, \qquad Z = \int_{-\infty}^{\infty} dx e^{-S[x]}.$$

(1) vary the 'field' x randomly:

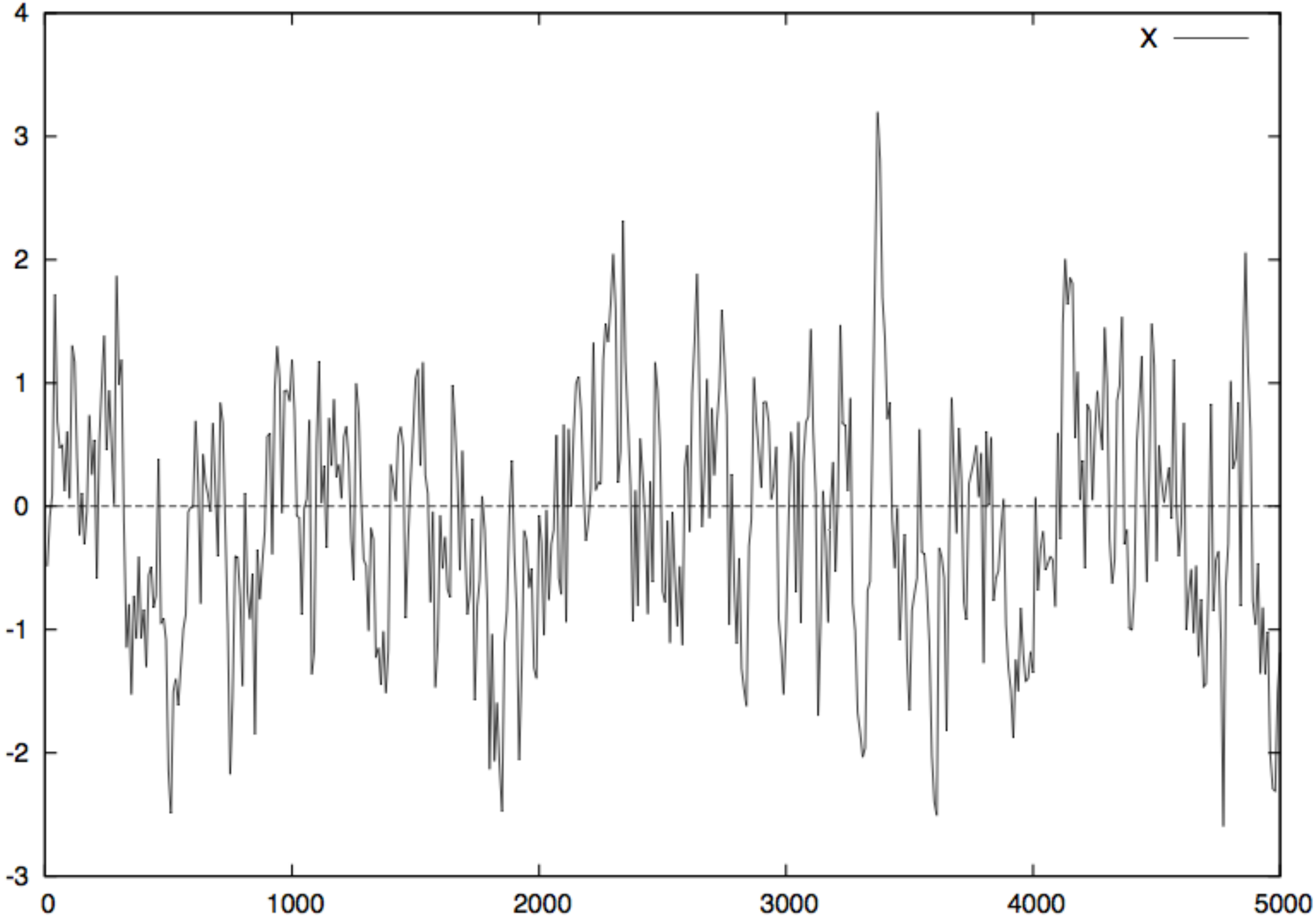
$$x \to x + \Delta x, \qquad -0.5 < \Delta x < 0.5$$

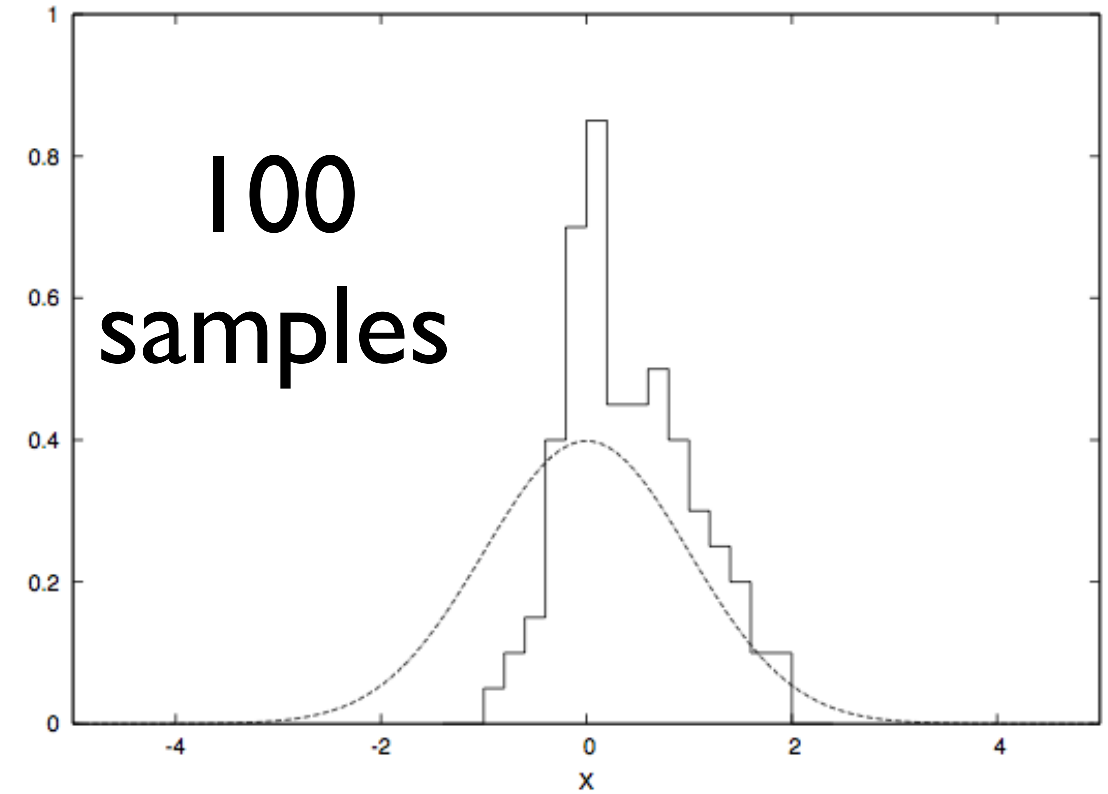
(2) accept the new 'configuration' with a probability

$\min\{1, e^{-\Delta S}\}$ where $\Delta S = S[x + \Delta x] - S[x]$

'Metropolis test'

Initial condition : x=0

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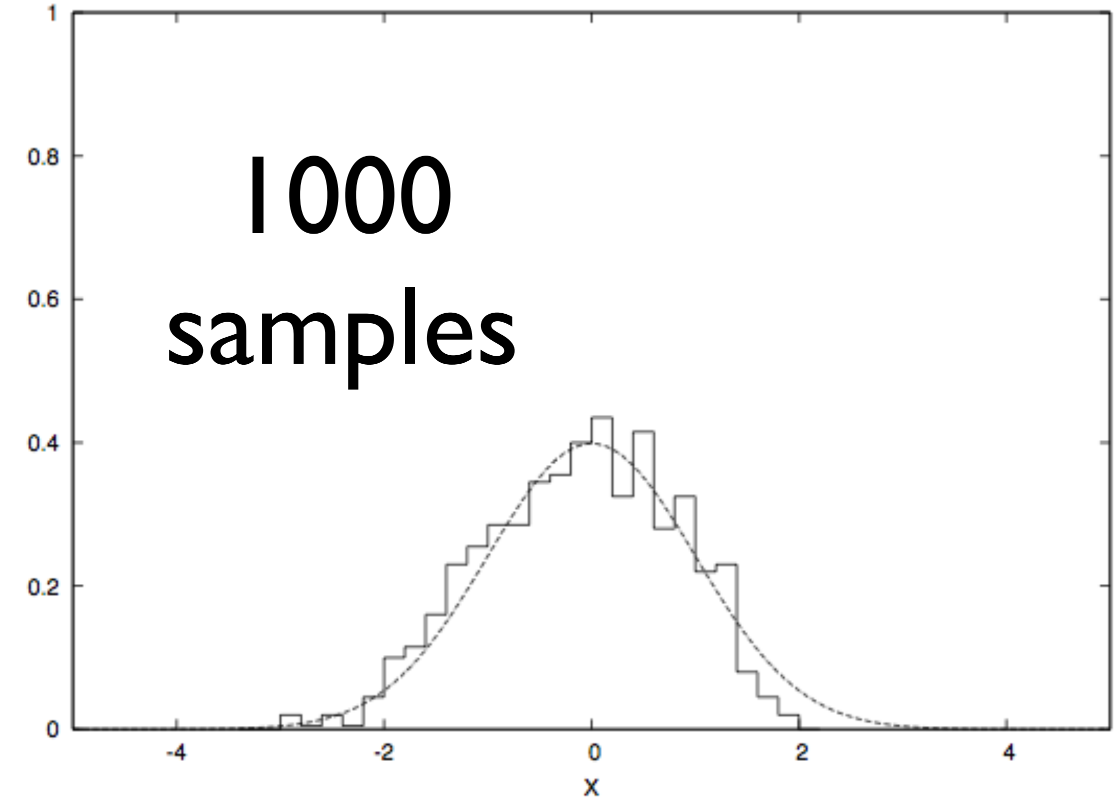
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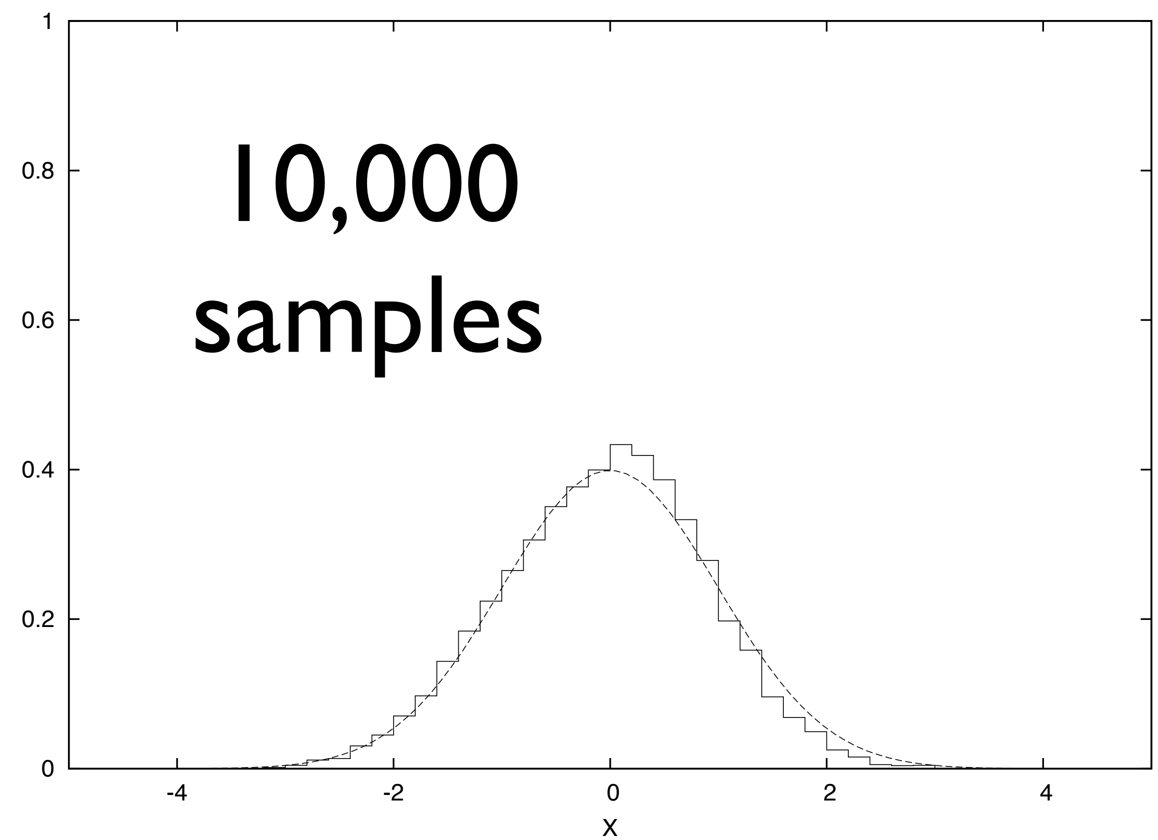
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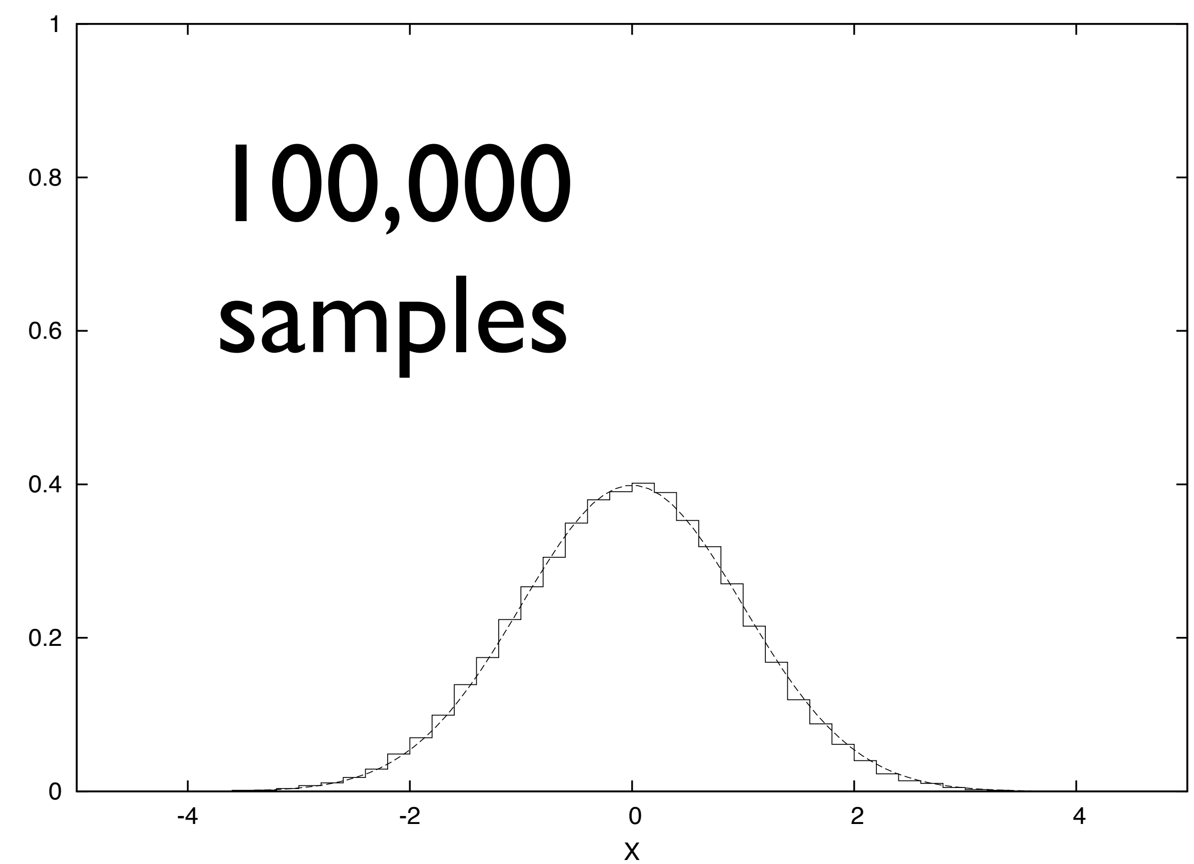
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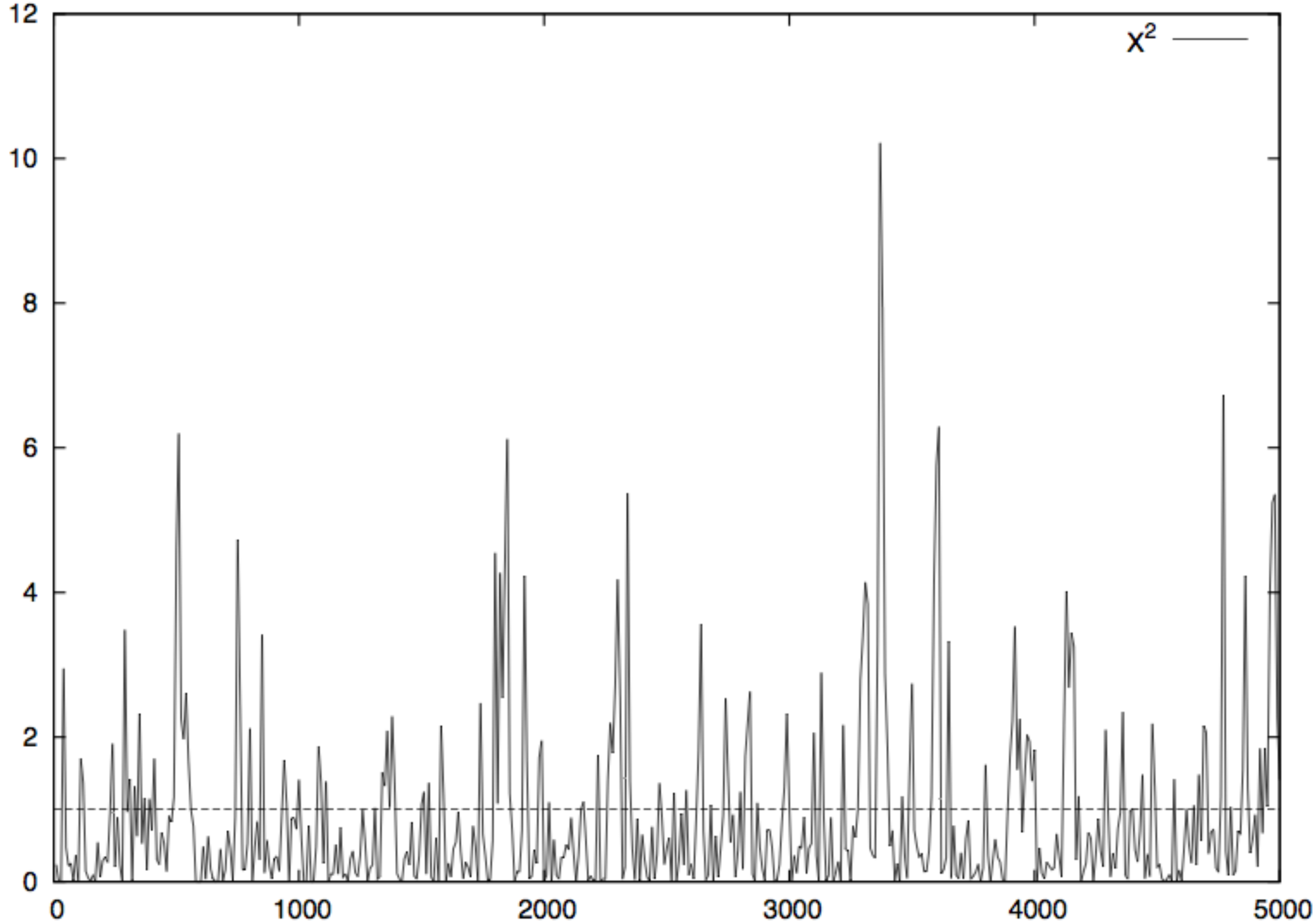
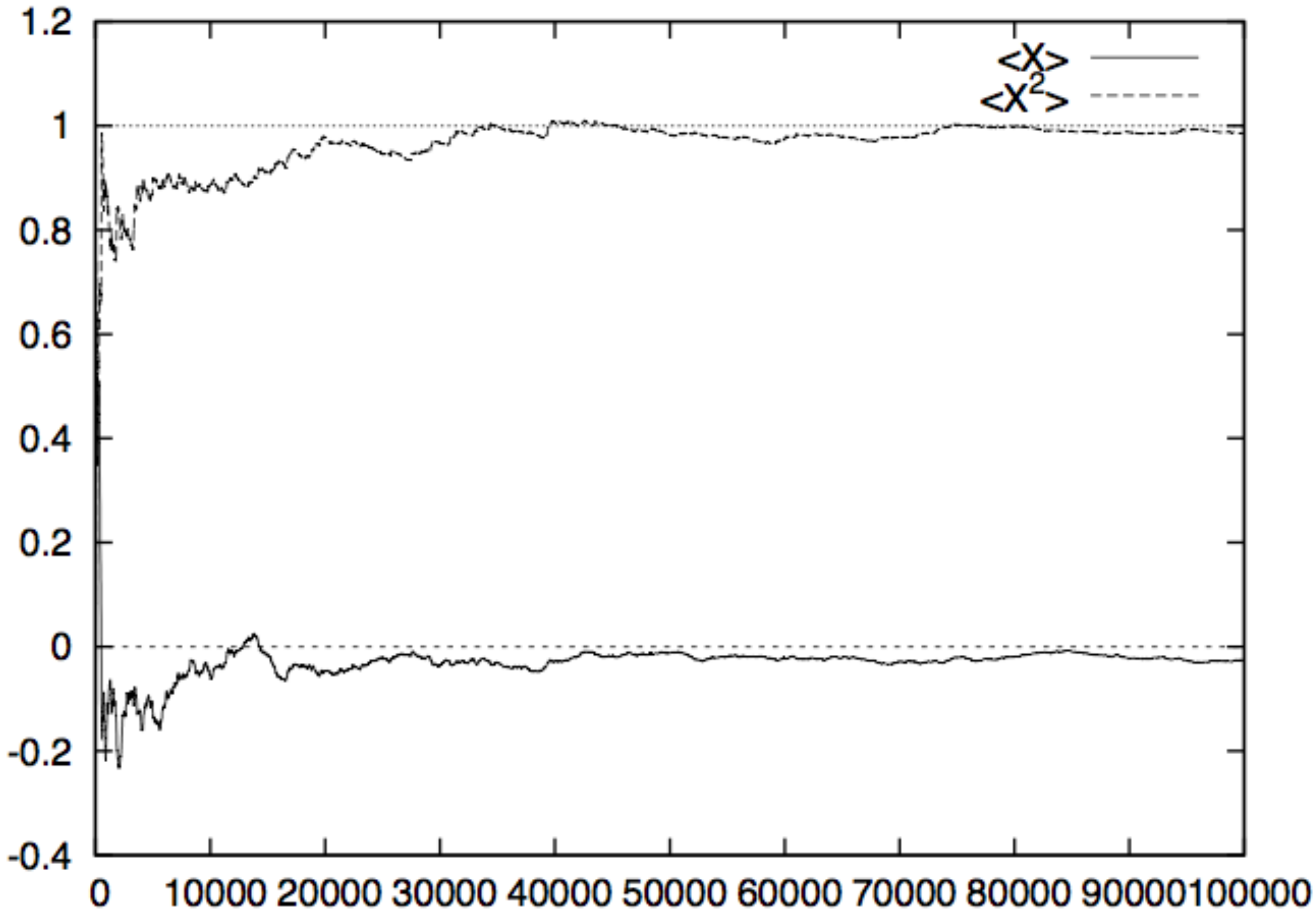
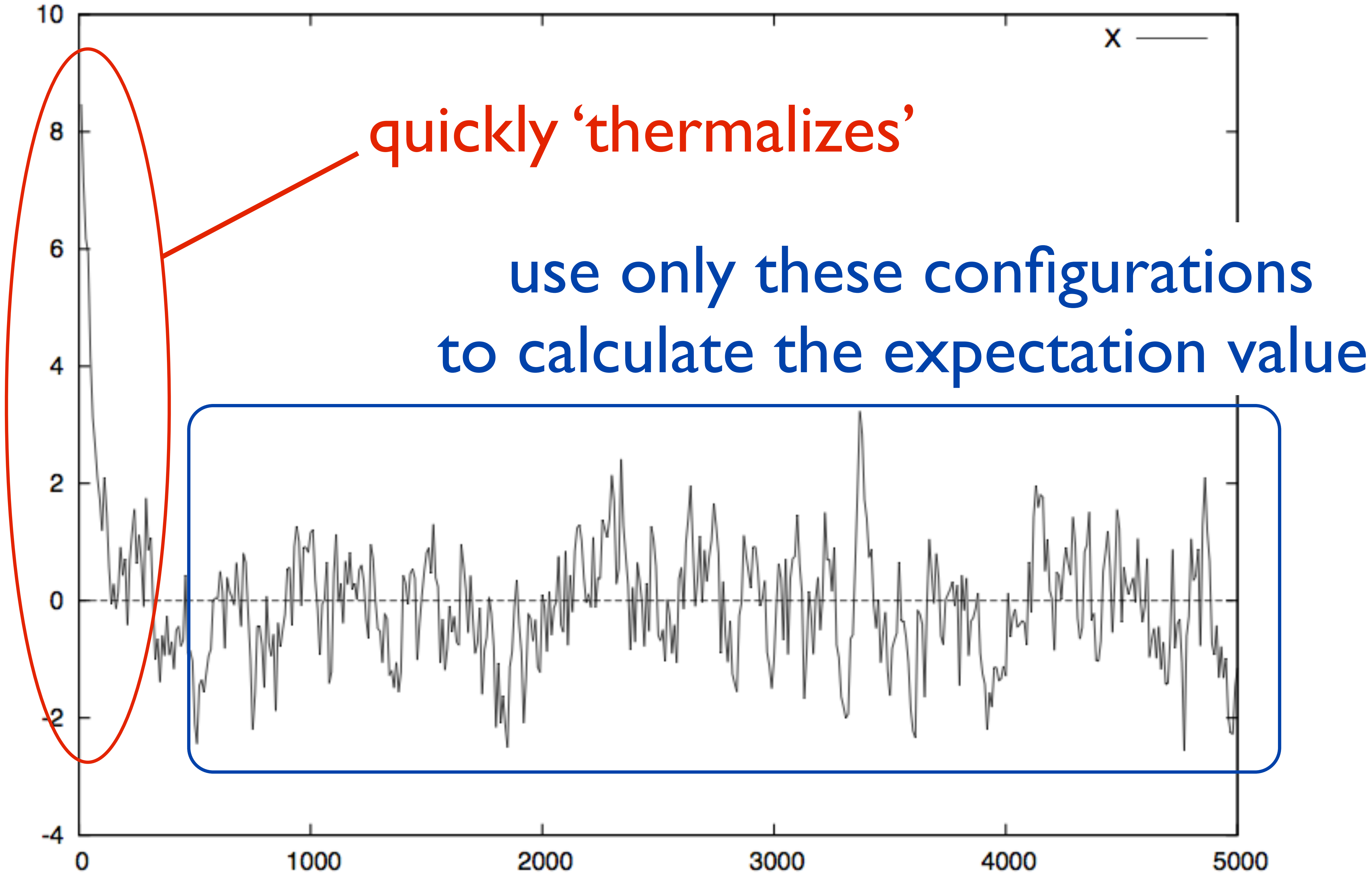


history of x^2



<u>Note</u> In typical YM simulations, with better algorithm, reasonable results can be obtained from 100 - 1000 configurations, if the theory does not suffer from the 'sign problem'.

Initial condition : x=10



after the thermalization, configuration
with small weight never appears in practice
→ "*importance sampling*"

Fermion

$$S = S_B + S_F, \qquad S_F = \int d^4x \bar{\psi} D \psi$$

$$D = \gamma^\mu(\partial_\mu - iA_\mu)$$

Fermions appear in a bilinear form.
(if not.. make them bilinear by introducing auxiliary fields!)

→ can be integrated out *by hand*.

$$\int [dA][d\psi] e^{-S_B[A] - S_F[A,\psi]} = \int [dA] \det D[A] \cdot e^{-S_B[A]}$$

So, simply use the 'effective action',

$$S_{eff}[A] = S_B[A] - \log\det D[A]$$

(crucial assumption : det D > 0)

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warm-up example :

Pure Yang-Mills (bosonic)

Wilson's lattice gauge theory

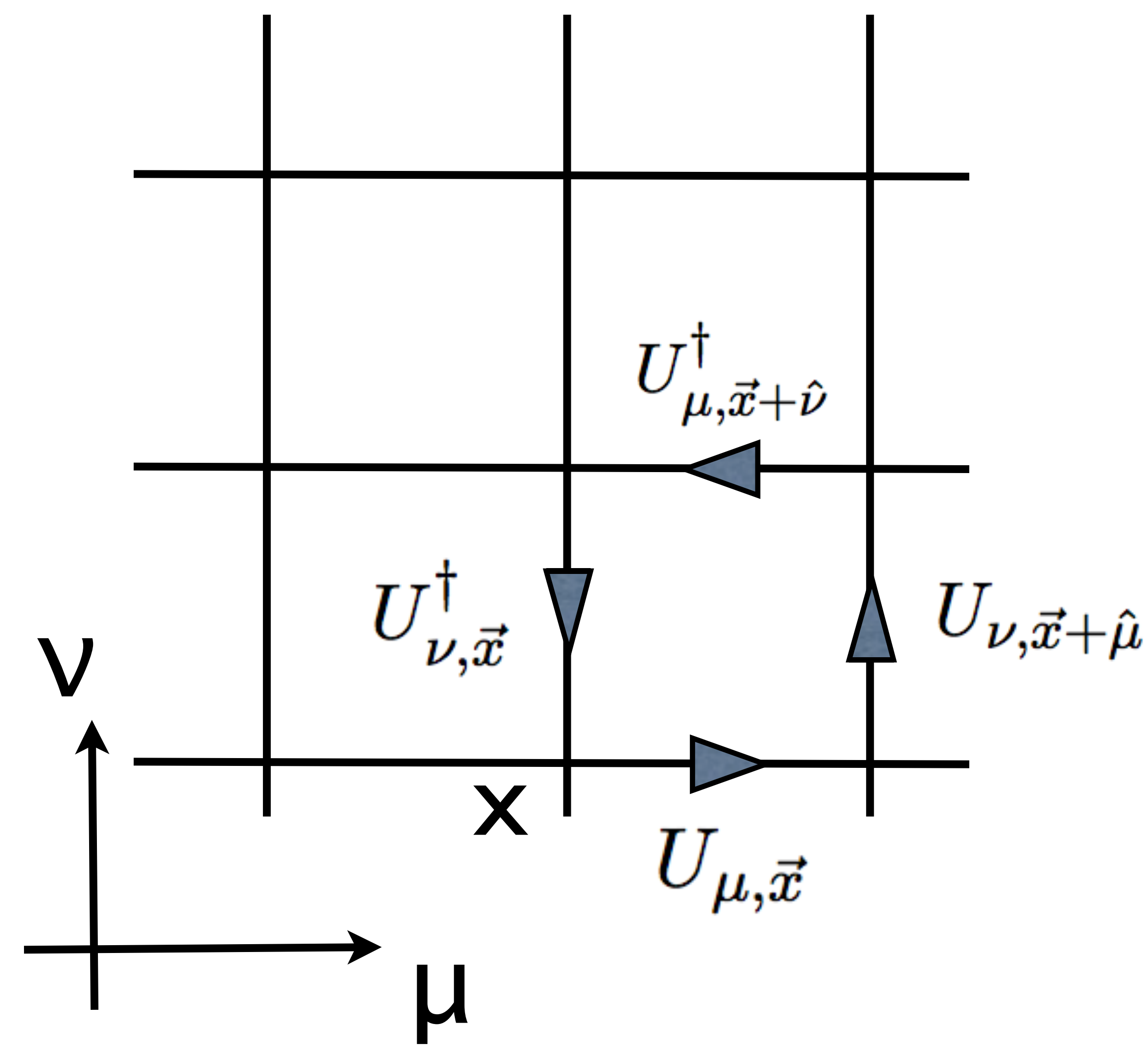
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_{\mu}(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degrees rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

Continuum limit $a \to 0$ respects exact symmetries at discretized level.

Exact symmetries at discretized level

$\downarrow$

gauge invariance, translational invariance, rotationally invariant,... in the continuum limit.

What happens if the gauge symmetry is explicitly (not spontaneously) broken, (e.g. the sharp momentum cutoff prescription)?

- We are interested in low-energy, long-distance physics (compared to the lattice spacing a).
- So let us integrate out high frequency modes.

Then...

gauge symmetry breaking radiative corrections can appear.

To kill them, one has to add counterterms to lattice action, whose coefficients must be fine-tuned!

'fine tuning problem'

This is the reason why we *must* preserve symmetries exactly.

Super Yang-Mills

‘No-Go’ for lattice SYM

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.
- So it is impossible to keep all supercharges exactly on lattice.
- Still it is possible to preserve a part of supercharges. (subalgebra which does not contain ∂)

Strategy

Use other exact symmetries and/or a few exact SUSY to forbid SUSY breaking radiative correction.

- 1d : no problem thanks to UV finiteness. Lattice is not needed; momentum cutoff method is much more powerful.
  (M.H.-Nishimura-Takeuchi 2007)

- 2d : lattice with a few exact SUSY+R-symmetry
  - no fine tuning at perturbative level (Cohen-Kaplan-Katz-Unsal 2003, Sugino 2003, Catterall 2003, D'Adda et al 2005, ...)
  - works even nonperturbatively (←simulation)
  (Kanamori-Suzuki 2008, M.H.-Kanamori 2009, 2010)

- 3d N=8 : "Hybrid" formulation:
  BMN matrix model + fuzzy sphere
  (Maldacena-Seikh Jabbari-Van Raamsdonk 2002)

- 4d N=1 *pure* SYM : lattice chiral fermion assures SUSY
  (Kaplan 1984, Curci-Veneziano 1986)

- 4d N=4 :
  - again "Hybrid" formulation:Lattice + fuzzy sphere
    (M.H.-Matsuura-Sugino 2010, M.H. 2010)
  - Large-N Eguchi-Kawai reduction(Ishii-Ishiki-Shimasaki-Tsuchiya, 2008)
  - Another Matrix model approach(Heckmann-Verlinde, 2011)
  - recent analysis of 4d lattice:
    Fine tuning is needed, but only for 3 bare lattice couplings.
    (Catterall-Dzienkowski-Giedt-Joseph-Wells, 2011)

SIGN PROBLEM

$$S = S_B + S_F, \qquad S_F = \int d^4x \bar{\psi} D \psi$$

$$D = \gamma^\mu(\partial_\mu - iA_\mu)$$

Fermions appear in a bilinear form.

(if not.. make them bilinear by introducing auxiliary fields!)

$\longrightarrow$ can be integrated out *by hand*.

$$\int [dA][d\psi] e^{-S_B[A]-S_F[A,\psi]} = \int [dA] \, \underline{\det D[A]} \cdot e^{-S_B[A]}$$

Monte Carlo cannot be used if it is not real positive

‘reweighting method’

- Use the ‘phase-quenched’ effective action

$$S_{eff}[A] = S_B[A] - \log|\det D[A]|$$

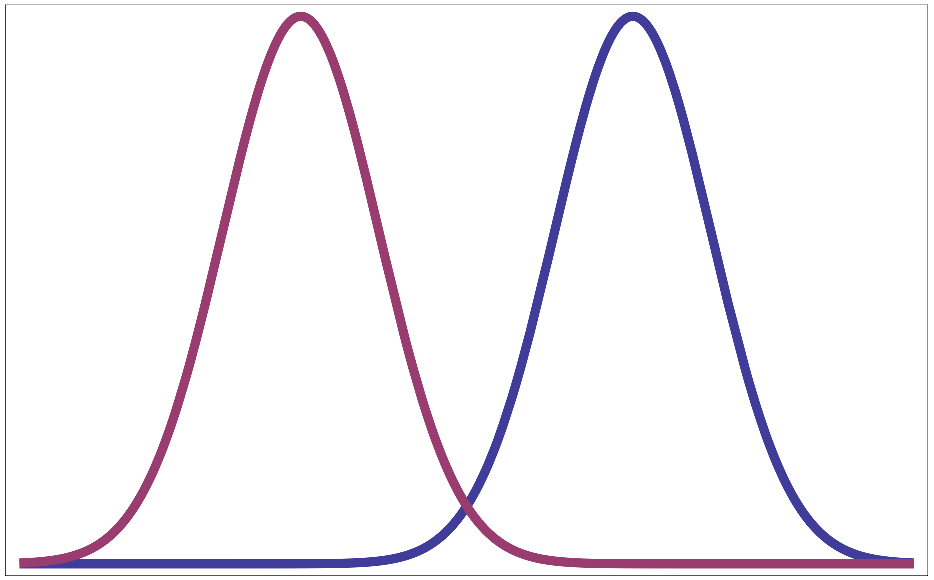
- Phase can be taken into account by the ‘phase reweighting’ :

$$\begin{aligned}
\langle \mathcal{O} \rangle &= \frac{\int [dA] \det D \cdot e^{-S_B} \cdot \mathcal{O}}{\int [dA] \det D \cdot e^{-S_B}} \\
&= \frac{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} \cdot \mathcal{O} / \int [dA] |\det D| \cdot e^{-S_B}}{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} / \int [dA] |\det D| \cdot e^{-S_B}} \\
&= \frac{\langle (phase) \cdot \mathcal{O} \rangle_{phase\ quench}}{\langle (phase) \rangle_{phase\ quench}}
\end{aligned}$$

usually the reweighting does not work in practice...

- violent phase fluctuation
  → both numerator and denominator becomes almost zero. 0/0 = ??
- vacua of full and phase-quenched model can disagree.

  'overlapping problem'



Miracles happen in SYM!

- Almost no phase except for very low temperature and/or SU(2).

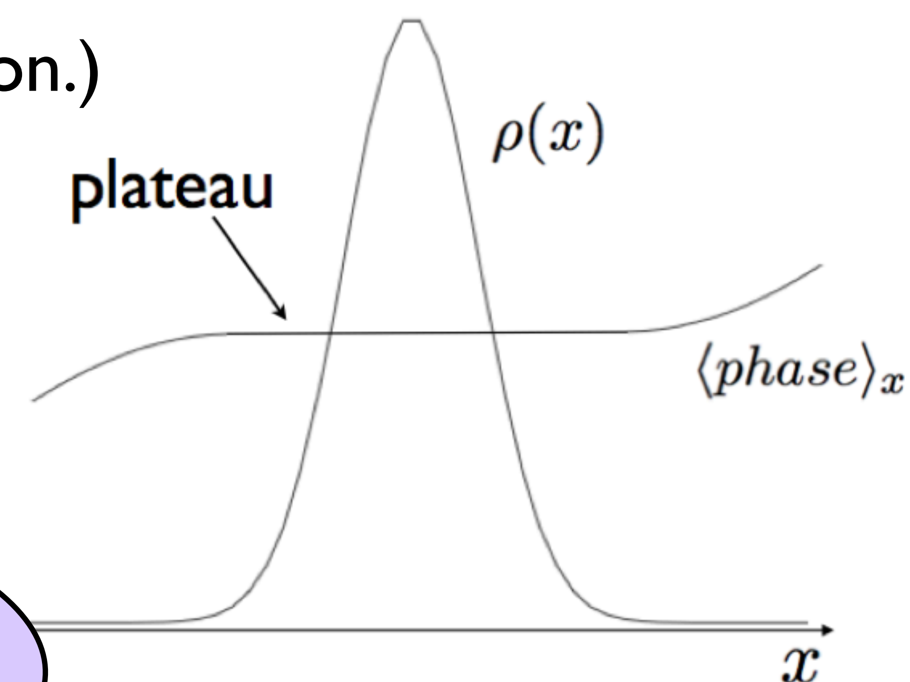
  (Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007, Catterall-Wiseman 2008, Catterall et al 2011, Buchoff-M.H.-Matsuura, in progress.)

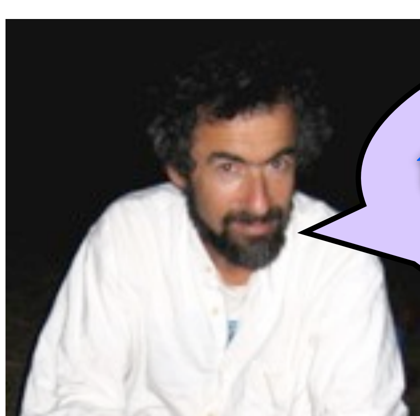
- Even when the phase fluctuates, phase quench gives right answer.

  ('right' in the sense it reproduces gravity prediction.)

- Can be justified numerically.

  (M.H.-Nishimura-Sekino-Yoneya 2011)





This is the only theory in which we can believe in any miracle.

(D.B.Kaplan 2010, private communication.)

Plan

(1) What is Monte Carlo?

(2) Why lattice SUSY is hard（fine tuning, sign problem）

(3) Simulation of (0+1)-d SYM (D0-brane quantum mechanics)

(4) Simulation of (1+1)-d SYM (D0-brane quantum mechanics)

$$S = \frac{N}{\lambda}\int dt\ Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi}D_t\psi - \frac{1}{2}\bar{\psi}\gamma^i[X_i, \psi]\Big\}$$

- Dimensional reduction of 4d N=4 (or 10d N=1)
- D0-brane effective action
- Matrix model of M-theory
- gauge/gravity duality →dual to black 0-brane

Simple but can be more interesting than AdS_5/CFT_4 from string theory point of view!

- Matrix quantum mechanics is UV finite.

No fine tuning!

(4d N=4 is also UV finite, but that relies on cancellations of the divergences...)

- We don’t have to use lattice. Just fix the gauge & introduce momentum cutoff!
(M.H.-Nishimura-Takeuchi, 2007)

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

"(p+1)-d maximally supersymmetric U(N) YM and type II superstring on black p-brane background are equivalent"

p=3 : AdS_5/CFT_4

p<3 : nonAdS/nonCFT

large-N, strong coupling = SUGRA

finite coupling = α' correction

finite N = g_s correction

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N} U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$\gg 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$\ll 1$

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature
$$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature
$$\lambda^{-1/3} T \ll 1.$$

The dictionary

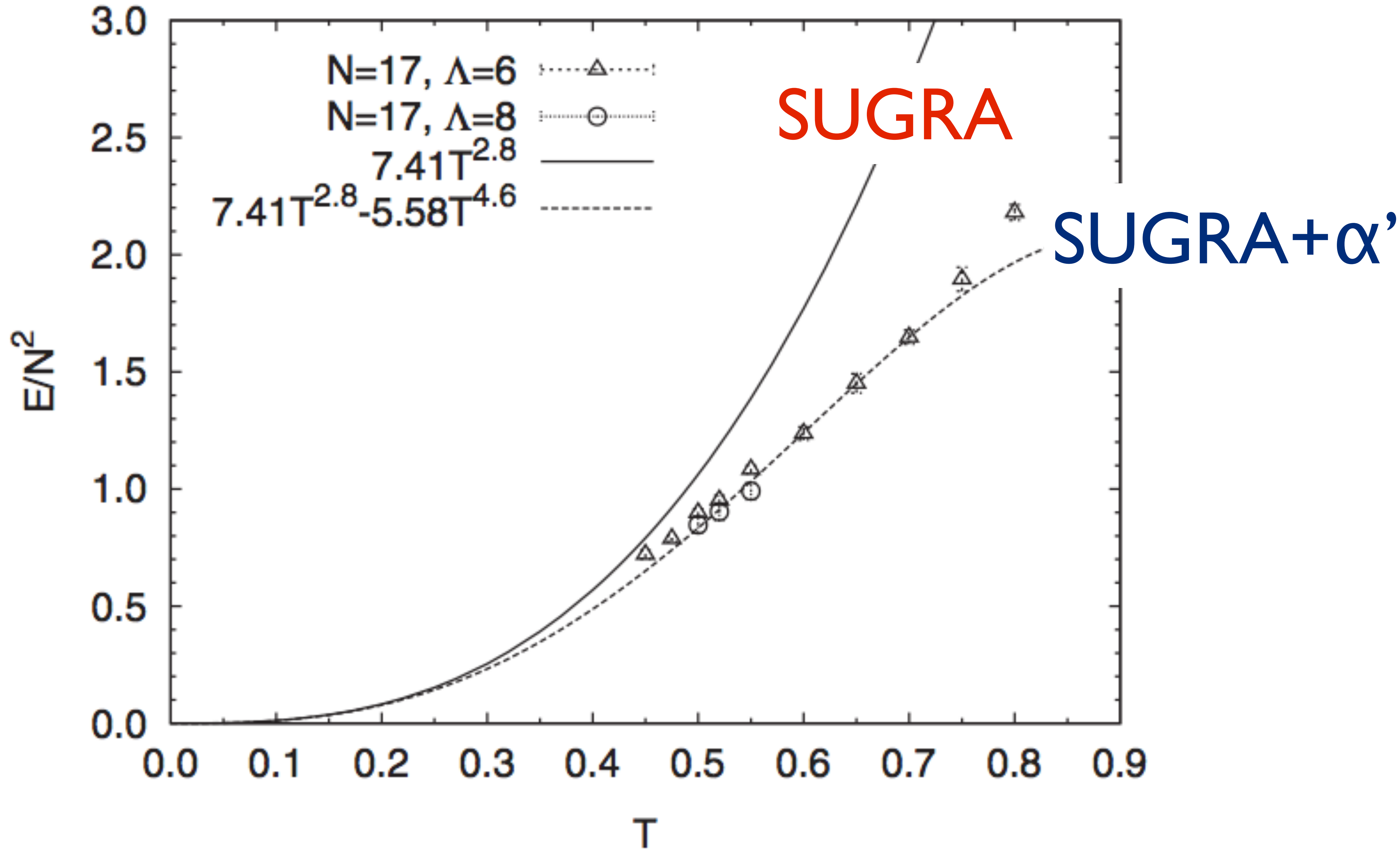
Gravity	SYM
ADM mass	Energy density
minimal surface	Wilson/Polyakov loop
mass of field excitation	scaling dimension

ADM mass vs energy density

$$E_{D0} = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

$$\boxed{\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)}$$

at large-N & low temperature (strong coupling)

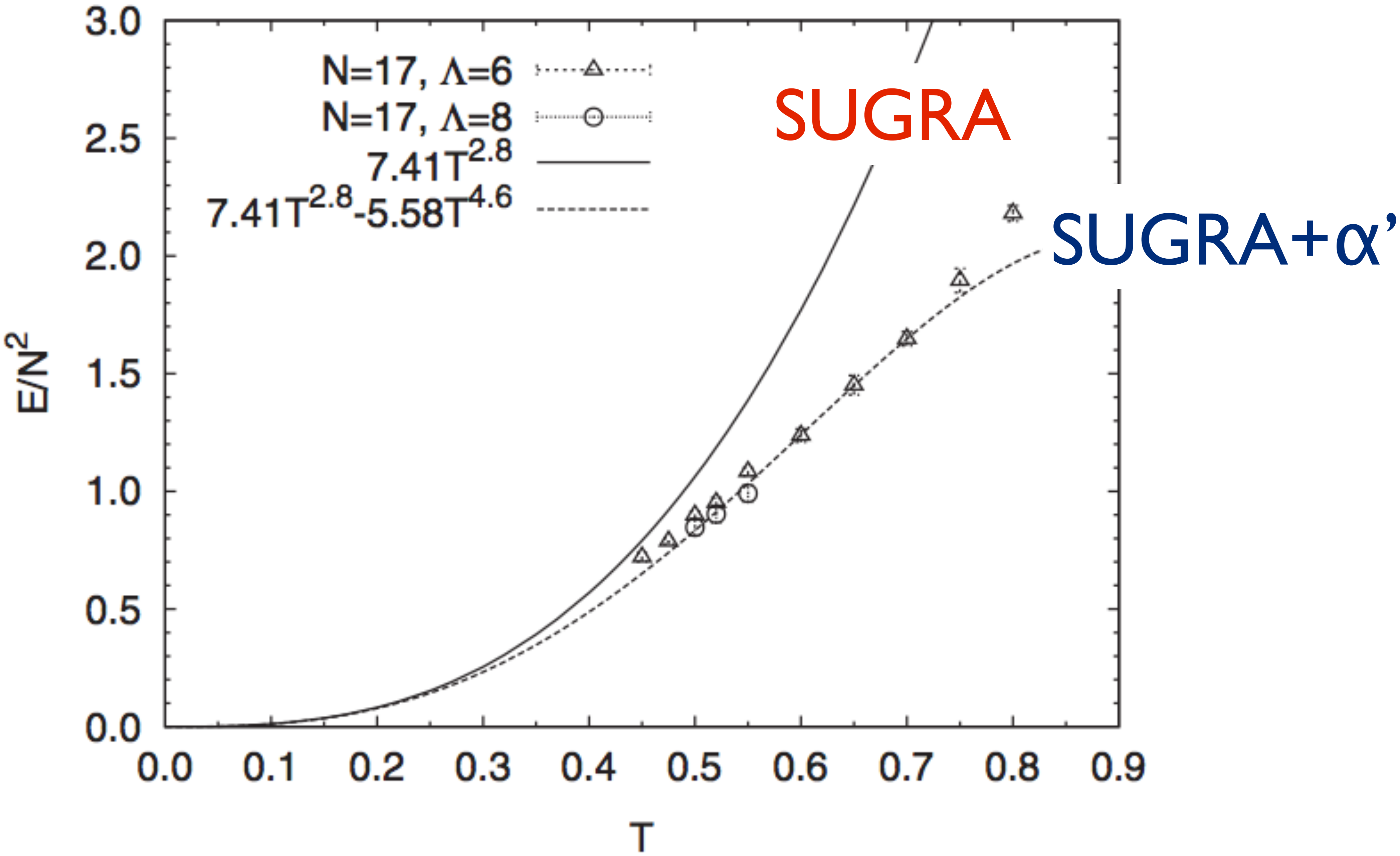


Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,
M.H.-Hyakutake-Nishimura-Takeuchi 2008

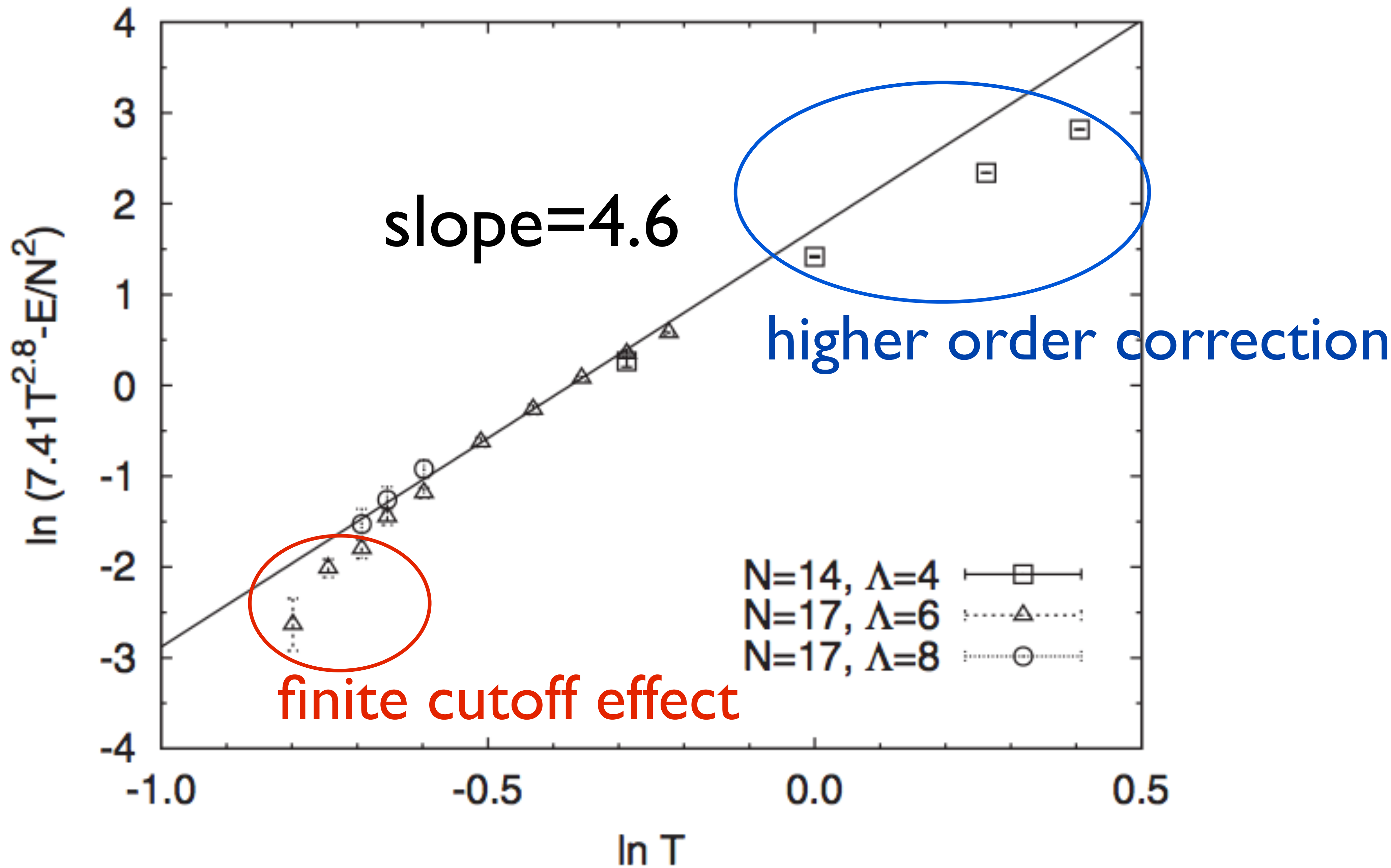
α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

'prediction' by SYM simulation



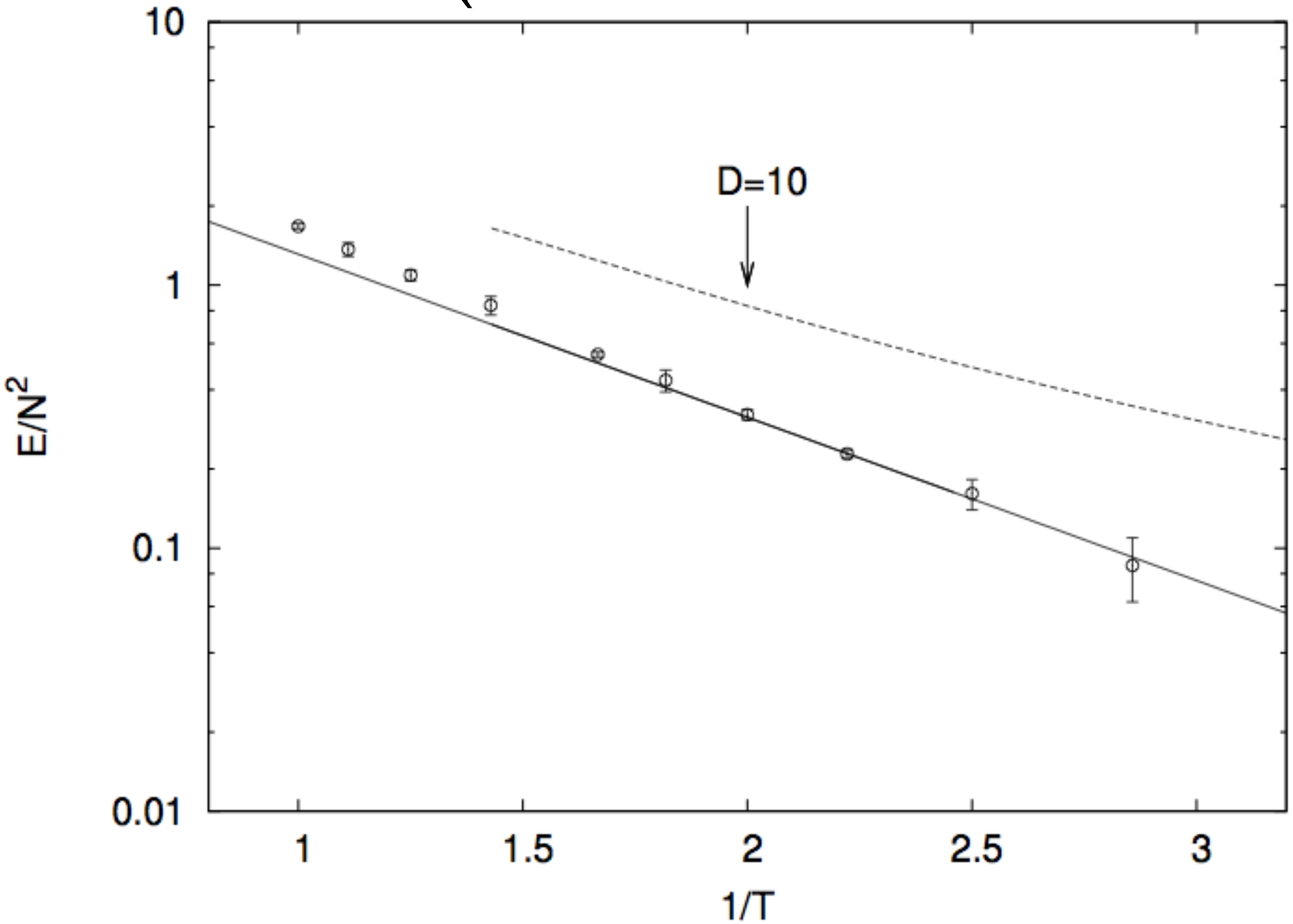
M.H.-Hyakutake-Nishimura-Takeuchi 2008



M.H.-Hyakutake-Nishimura-Takeuchi 2008

4-SUSY MQM

(M.H.-Matsuura-Nishimura-Robles 2010)



Exponential $E/N^2 \sim \exp(-a/T)$
rather than power
→ consistent with the absence of
the zero-energy normalizable state

Correlation functions (GKPW relation)

- AdS/CFT (D3-brane) $\rightarrow$ GKPW relation

  (Gubser-Klebanov-Polyakov 1998, Witten 1998)

- Similar relation in D0-brane theory :

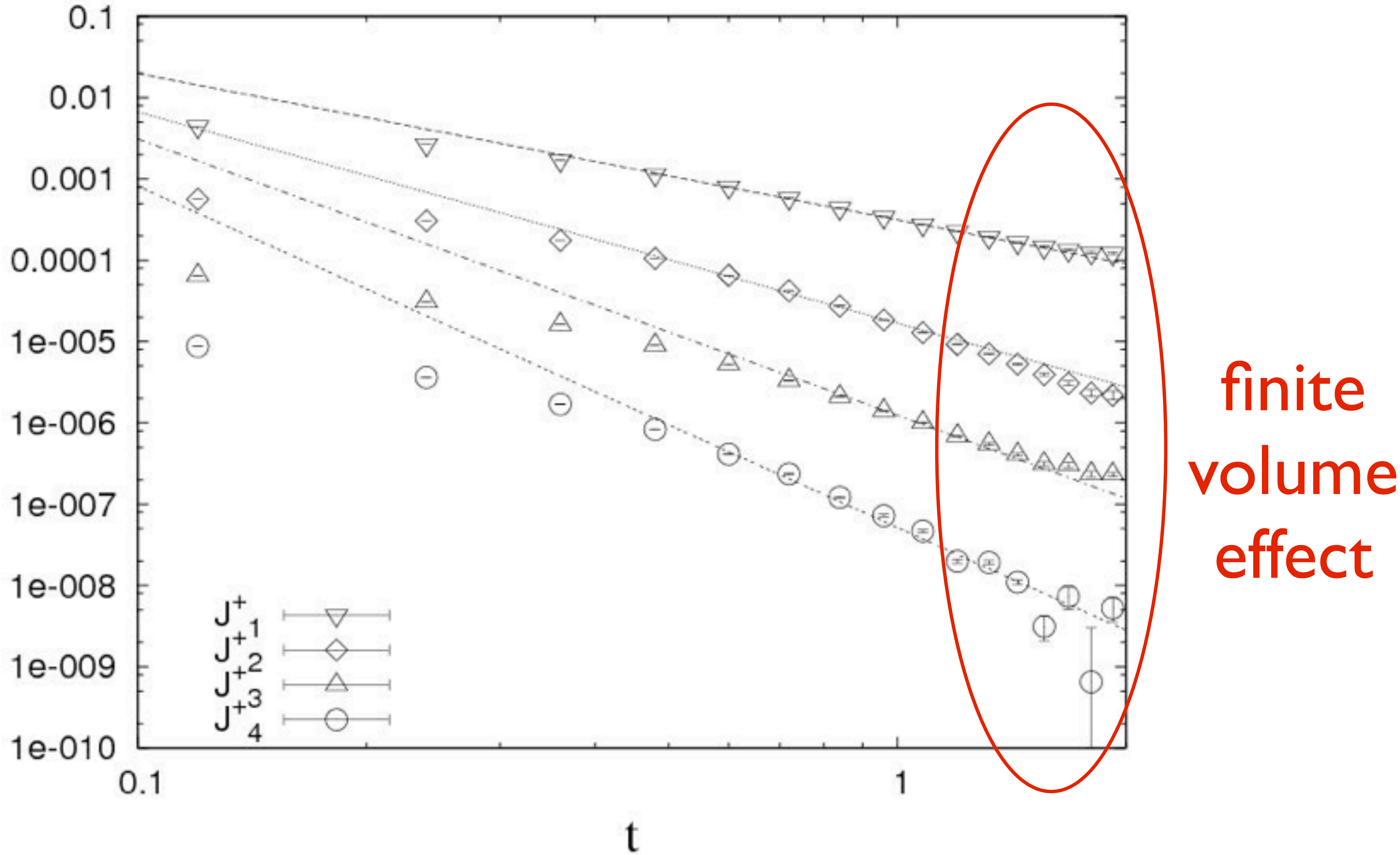
  "generalized" conformal dimension

  $\Leftrightarrow$ mass of field excitations

  (Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

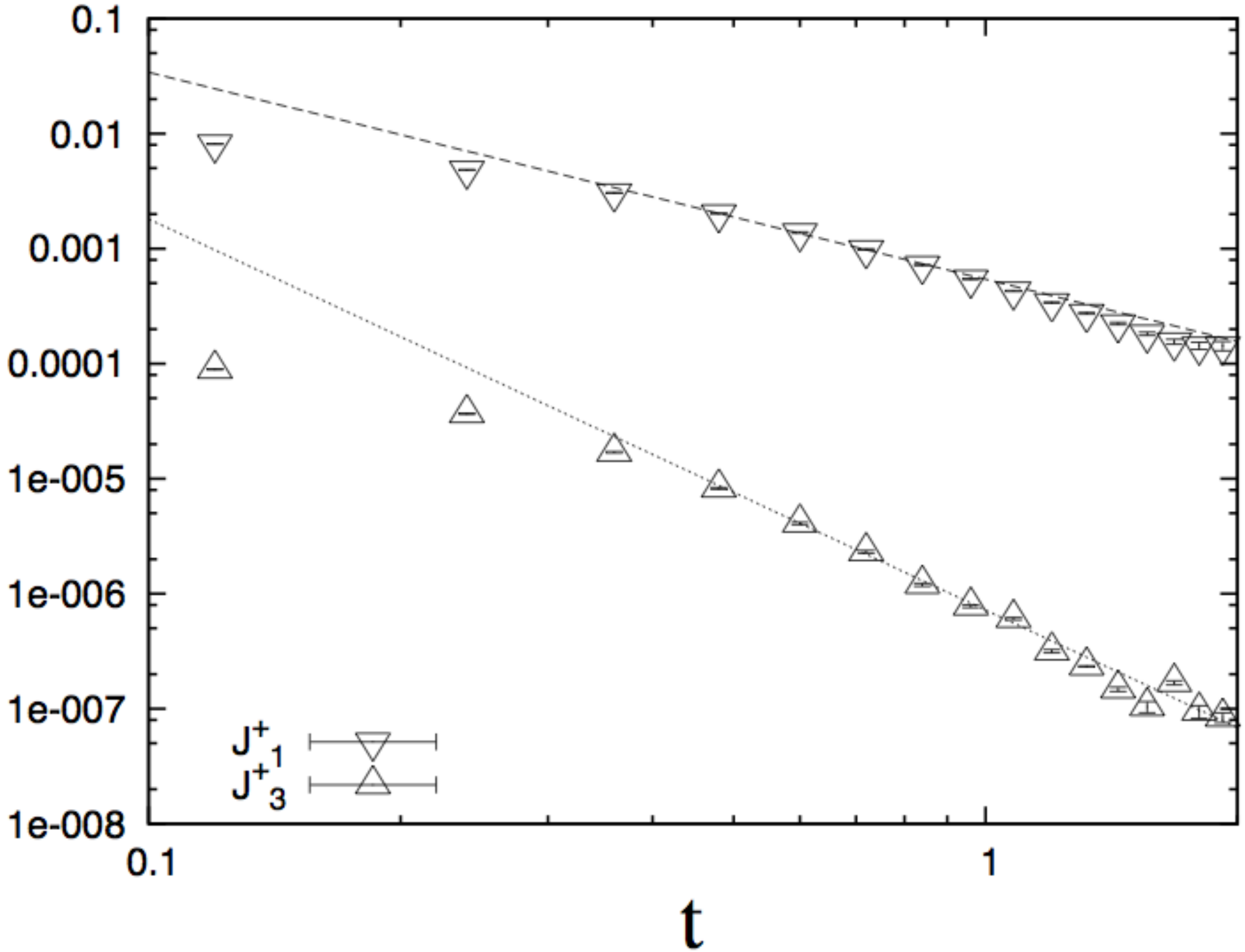
calculable via SUGRA



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\,(F_{ij} X_{i_1} \cdots X_{i_l}) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya 2009)



two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya 2011)

Next targets:

- 1/N correction to BH mass
  (M.H.-Hyakutake-Ishiki-Nishimura, in progress)
- Correlators of massive stringy modes
  Sekino-Yoneya's prediction vs Yin's prediction
  (Azeyanagi-M.H.-Nishimura-..., in progress)

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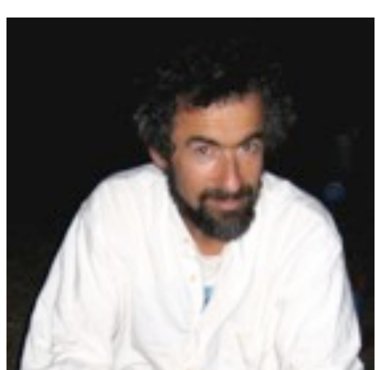
Basic ideas

- Keep a few supercharges exact on lattice.
- Use it (and other discrete symmetries) to forbid SUSY breaking radiative corrections.
  (Kaplan-Katz-Unsal 2002)
- Only "extended" SUSY can be realized for a technical reason. (4, 8 and 16 SUSY)
- Below we consider 16 SUSY theory.



Several lattice theories exists (from around 2002-2005)

- Cohen, Kaplan, Katz, Unsal
- Sugino
- Catterall
- Suzuki, Taniguchi
- D'Adda, Kanamori, Kawamoto, Nagata

Explained below
(conceptually the simplest,
according to my taste)

$$S_0 = \frac{1}{g_{2d}^2}\int d^2x\, \mathrm{Tr}\Big\{F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2$$
$$+\Psi^T\left(D_1 + \gamma_2 D_2\right)\Psi + i\Psi^T\gamma_I\left[X^I, \Psi\right]\Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2}\int d^2x\, \mathrm{Tr}\Big\{-iB_A\Phi_A - \frac{1}{3}\epsilon_{ABC}B_A[B_B, B_C]$$
$$-\psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_-\Big\},$$

$$\Phi_1 = 2(-D_1X_3 - D_2X_4),\ \Phi_2 = 2(-D_1X_4 + D_2X_3),$$
$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

$$
\begin{aligned}
&Q_{\pm}^{(0)} A_{\mu} = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_{\mu}\phi_{\pm}, \\
&Q_{\mp}^{(0)} \psi_{\pm\mu} = \frac{i}{2} D_{\mu} C \mp \tilde{H}_{\mu}, \\
&Q_{\pm}^{(0)} \tilde{H}_{\mu} = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}\left[C, \psi_{\pm\mu}\right] \mp \frac{i}{2} D_{\mu}\eta_{\pm}, \\
&Q_{\pm}^{(0)} X_{i} = \rho_{\pm i}, \quad Q_{\pm}^{(0)} \rho_{\pm i} = \mp \left[X_{i}, \phi_{\pm}\right], \\
&Q_{\mp}^{(0)} \rho_{\pm i} = -\frac{1}{2}[X_{i}, C] \mp \tilde{h}_{i}, \\
&Q_{\pm}^{(0)} \tilde{h}_{i} = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}\left[C, \rho_{\pm i}\right] \pm \frac{1}{2}\left[X_{i}, \eta_{\pm}\right], \\
&Q_{\pm}^{(0)} B_{A} = \chi_{\pm A}, \quad Q_{\pm}^{(0)} \chi_{\pm A} = \pm[\phi_{\pm}, B_{A}], \\
&Q_{\mp}^{(0)} \chi_{\pm A} = -\frac{1}{2}[B_{A}, C] \mp H_{A}, \\
&Q_{\pm}^{(0)} H_{A} = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}\left[B_{A}, \eta_{\pm}\right] \mp \frac{1}{2}\left[C, \chi_{\pm A}\right], \\
&Q_{\pm}^{(0)} C = \eta_{\pm}, \quad Q_{\pm}^{(0)} \eta_{\pm} = \pm\left[\phi_{\pm}, C\right], \\
&Q_{\mp}^{(0)} \eta_{\pm} = \mp\left[\phi_{+}, \phi_{-}\right], \\
&Q_{\pm}^{(0)} \phi_{\pm} = 0, \quad Q_{\mp}^{(0)} \phi_{\pm} = \mp \eta_{\pm}.
\end{aligned}
$$

Nilpotency

$$\left(Q_+^{(0)}\right)^2 = \left(Q_-^{(0)}\right)^2 = \{Q_+^{(0)}, Q_-^{(0)}\} = 0$$

$\longrightarrow$ $Q_\pm^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$
\begin{aligned}
Q_\pm U_\mu(x) &= i\psi_{\pm\mu}(x)U_\mu(x), \\
Q_\pm \psi_{\pm\mu}(x) &= i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x), \\
Q_\mp \psi_{\pm\mu}(x) &= \frac{i}{2}\left\{\psi_{+\mu}(x), \psi_{-\mu}(x)\right\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x), \\
Q_\pm \tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\quad \pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\quad \mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right] \\
&\quad +\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]
\end{aligned}
$$

$$
D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)
$$

Sugino, 2003

Absence of fine tuning (to all order in perturbation)

(Cohen-)Kaplan-Katz-Unsal, 2002&2003

- Possible correction from UV is

$$\left(\underbrace{\frac{1}{g_{2d}^2}c_0 a^{p-4}}_{\text{tree}}+c_1 a^{p-2}+g_{2d}^2 c_2 a^p+\cdots\right)\int d^2x\,\mathcal{O}_p(x)$$

up to log(a), where

$$\mathcal{O}_p(x)=\varphi(x)^\alpha\partial^\beta\psi(x)^{2\gamma},\qquad p=\alpha+\beta+3\gamma$$

- Only p=1,2 are dangerous.

$$\underline{\varphi},\ \varphi^2 \qquad (\partial\varphi \text{ is a total derivative})$$

SU(2)$_R$ allows only TrB_A and TrX_i .

Exact SUSY kills them.

φ^2 term is forbidden in a similar manner.

Does it work at nonperturbative level?

4 SUSY model (dimensional redcution of 4d N=1; sign-free) has been studied extensively.

(16 SUSY: in progress by Buchoff, M.H. and Matsuura)

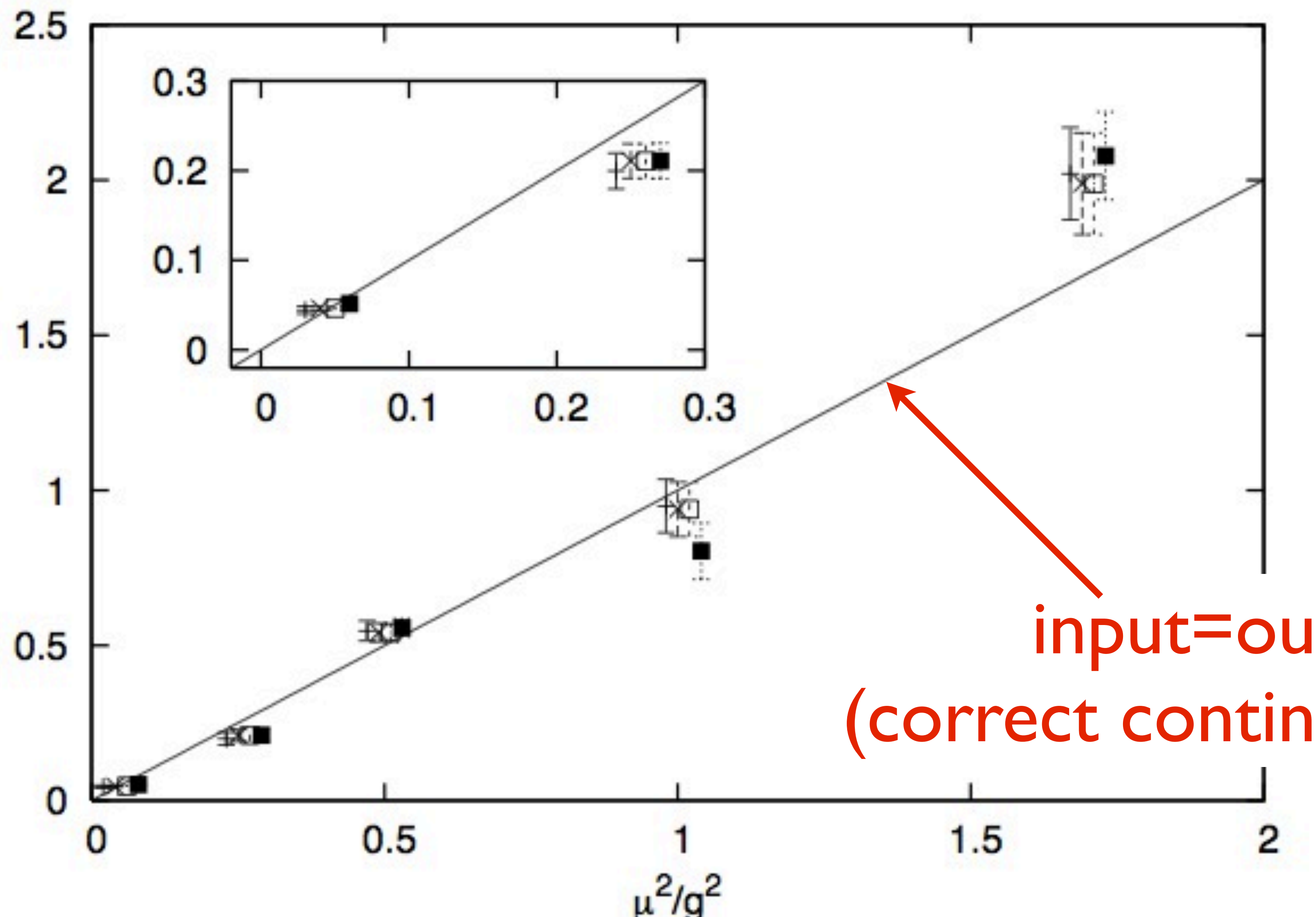
- Conservation of supercurrents. (Suzuki 2007, Kanamori-Suzuki 2008)
- Comparison with analytic results at small volume & large-N behaviors. (M.H.-Kanamori 2009)
- Comparison to Cohen-Kaplan-Katz-Unsal model. (M.H.-Kanamori 2010)

All results supports the emergence of the correct continuum limit without fine tuning.

Supercurrent conservation in the SU(2) Sugino model

soft SUSY breaking mass (output)

$\sim \partial_\mu J^\mu$

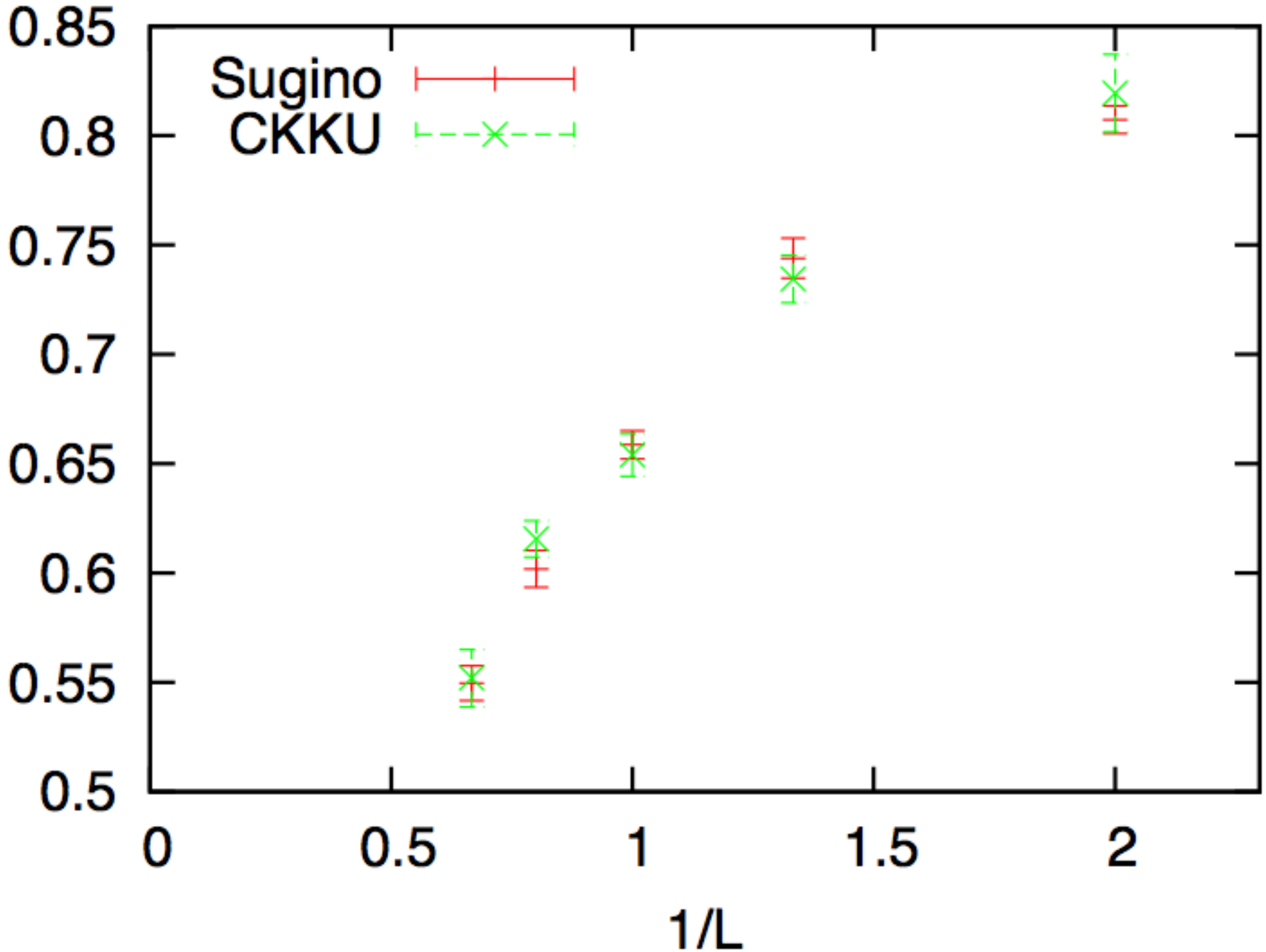


input=output
(correct continuum limit)

soft SUSY breaking mass (input)

(Kanamori-Suzuki 2008)

(see also Kadoh-Suzuki 2009)



Polyakov loop vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)

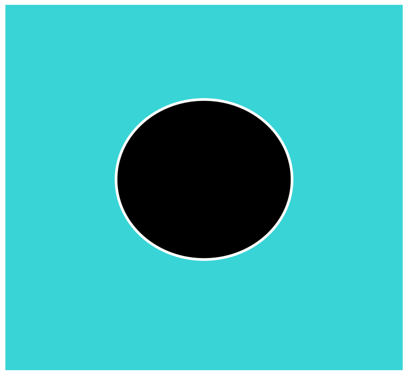
Application :
black hole/black string transition

Susskind, Barbon-Kogan-Rabinovici,
Li-Martinec-Sahakian,
Aharony-Marsano-Minwalla-Wiseman,…

SYM simulation : Catterall-Wiseman, 2010

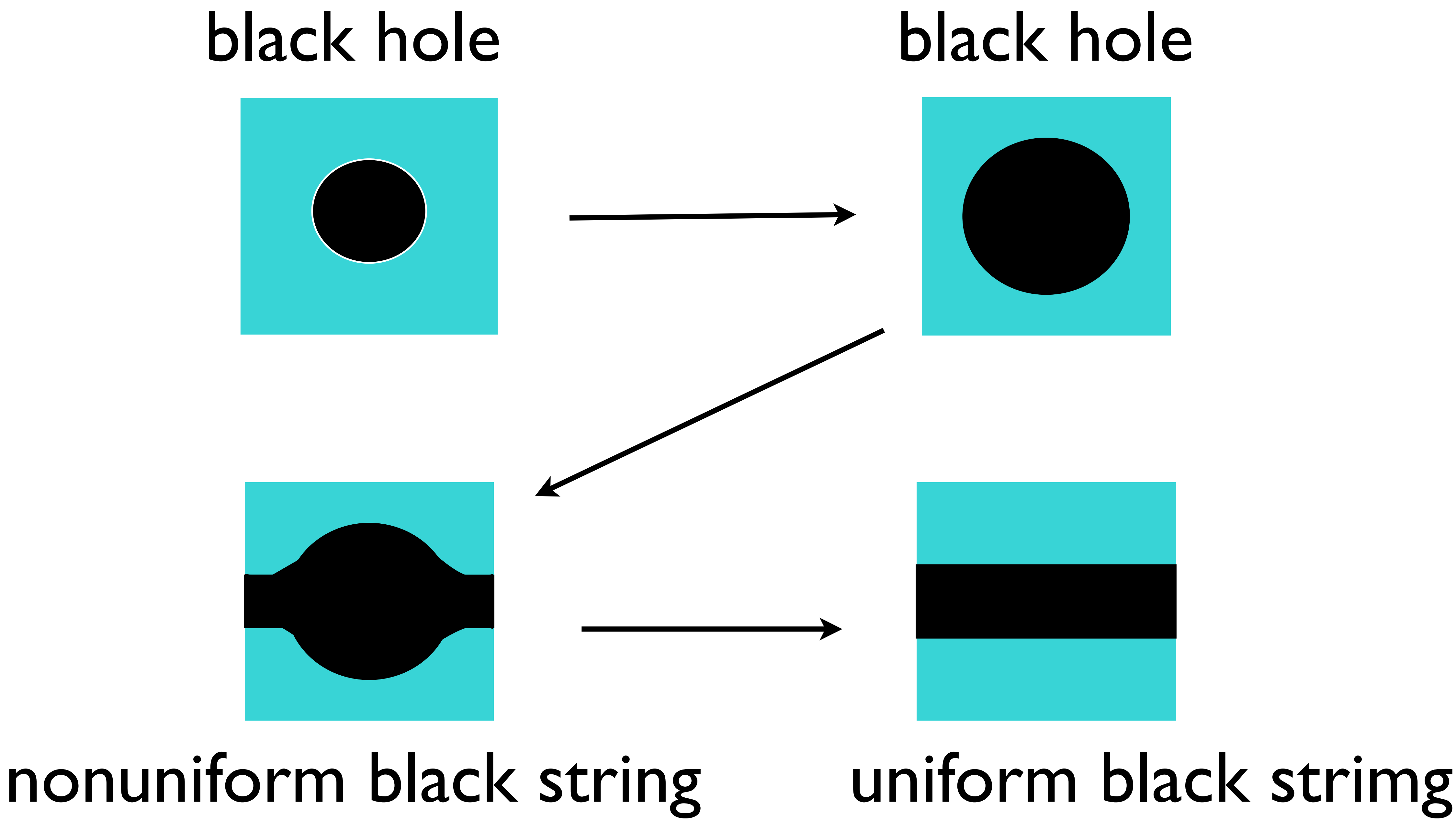

- Consider 2d U(N) SYM on a spatial circle. It describes N D1-branes in $R^{1,8} \times S^1$, winding on S^1.
- T-dual picture : N D0-branes in $R^{1,8} \times S^1$.
- Wilson line phase = position of D0

uniform distribution
= 'black string'



localized distribution
= 'black hole'

Fix the mass (or temparature) and shrink the compactification radius. Then...



Counterpart in SYM
= center symmetry breakdown

- Wilson line phase = position of D0

$$W = diag(e^{i\theta_1}, \cdots, e^{i\theta_N})$$

- Center symmetry

$$\theta_i \to \theta_i + const.$$

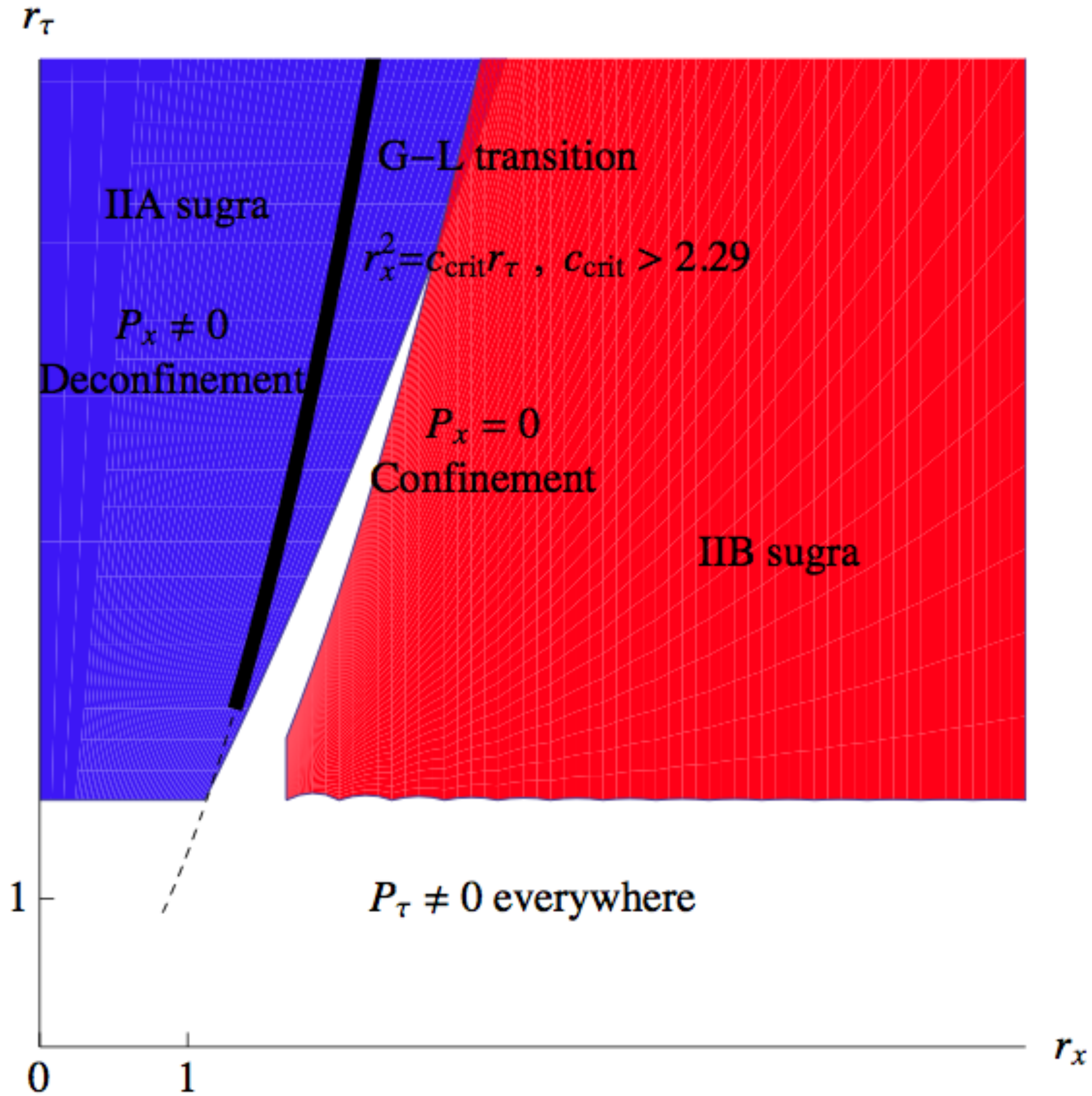
Uniform = center unbroken $\left\langle \frac{1}{N} TrW \right\rangle = 0$

Non-uniform = center broken $\left\langle \frac{1}{N} TrW \right\rangle \neq 0$

Phase diagram

(Theoretical prediction)

(Temperature)$^{-1}$



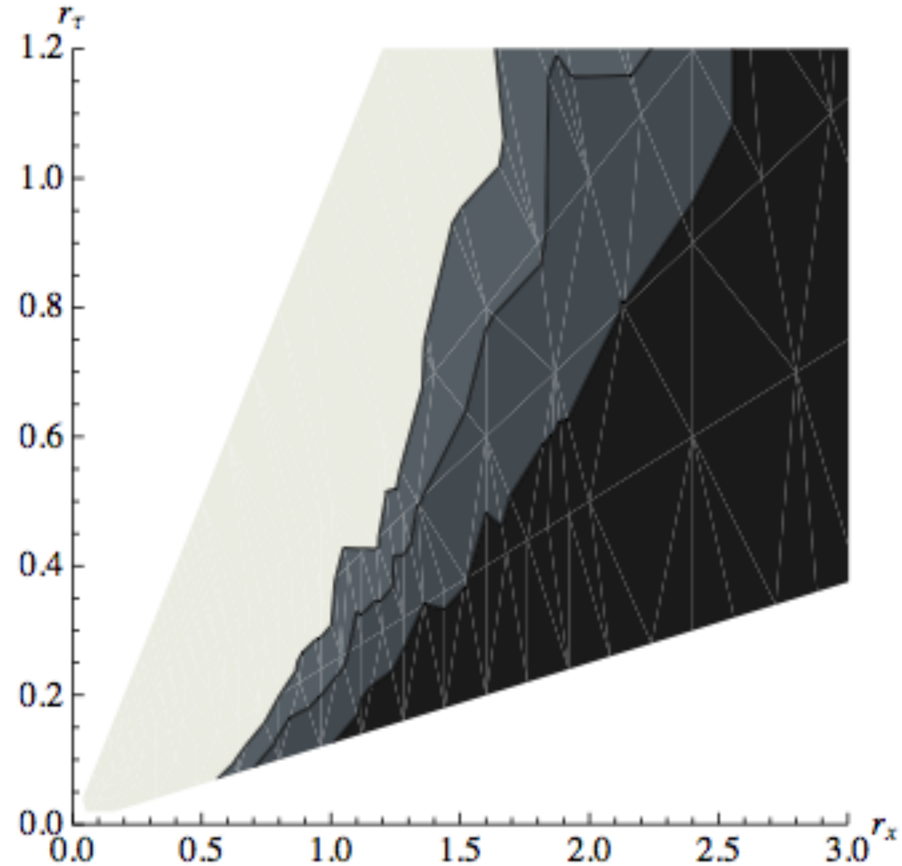
radius of spatial circle

Low temperature:
1st order
BH→uniform BS
(Aharony et al, 2004)

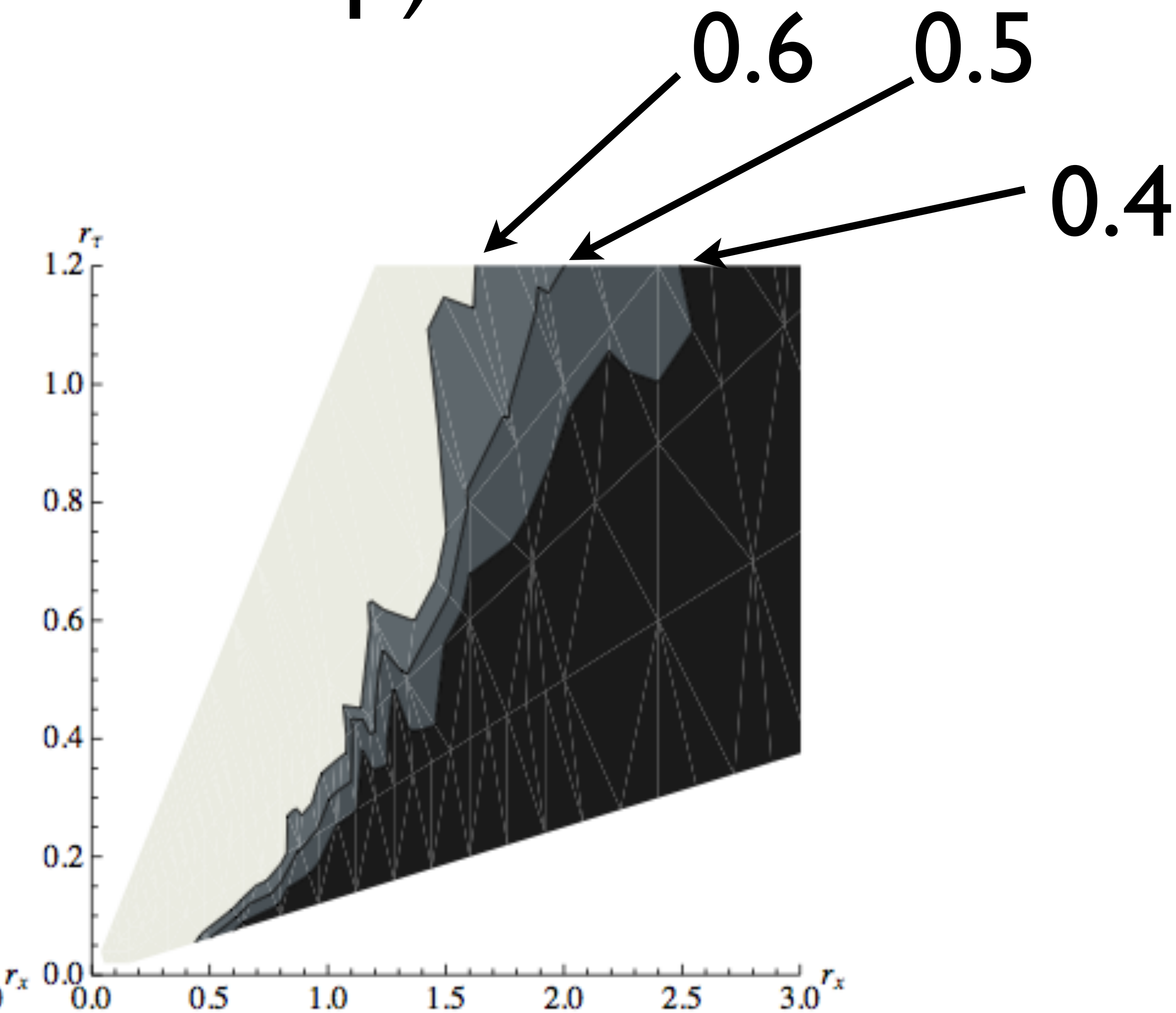
High temperature:
2nd + 3rd
BH→nonuniform BS
→uniform BS
(Kawahara et al, 2007)

Figure from Catterall-Wiseman, 2010

Value of spatial Wilson loop
(‘t Hooft loop)

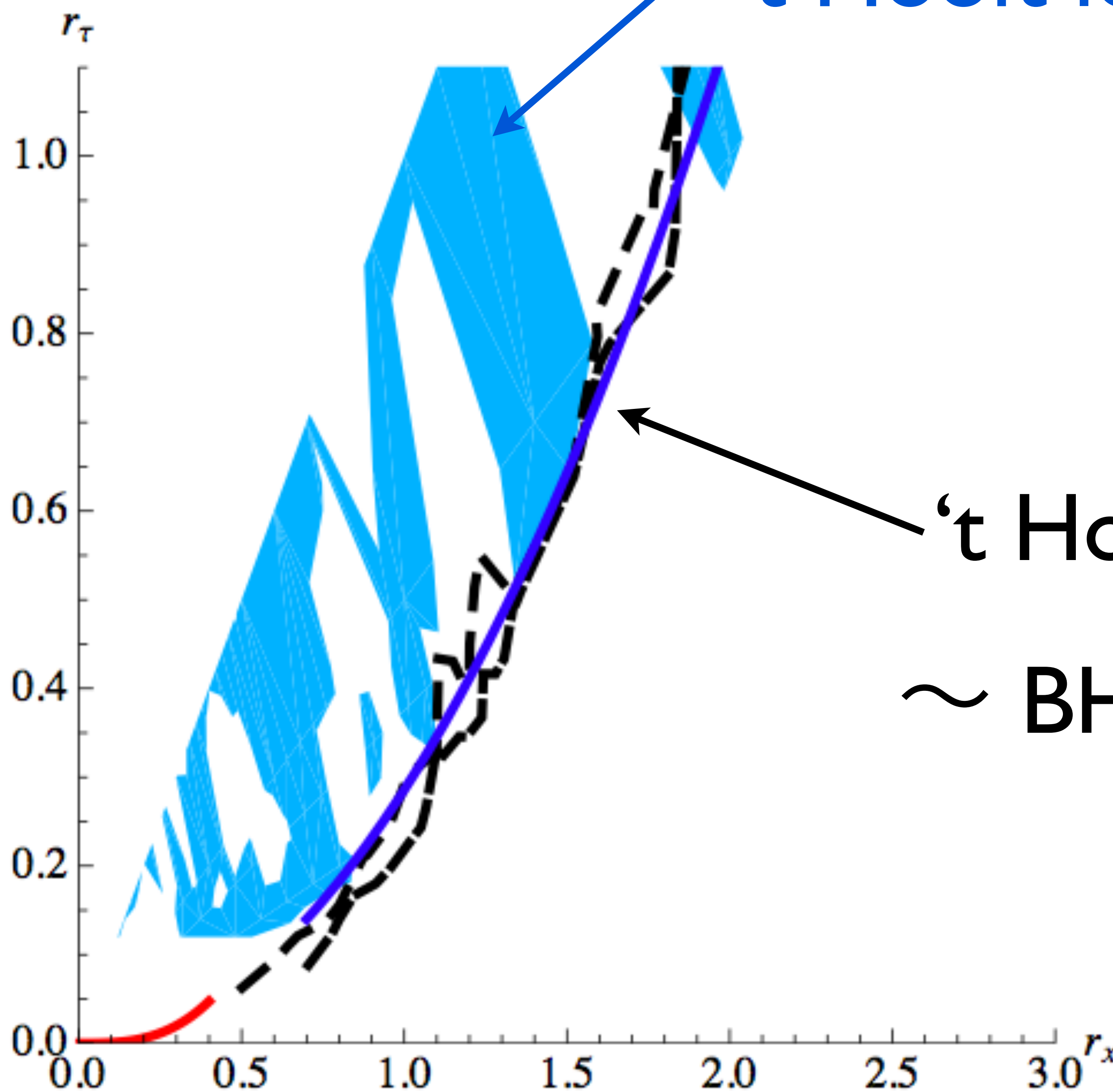


SU(3)



SU(4)

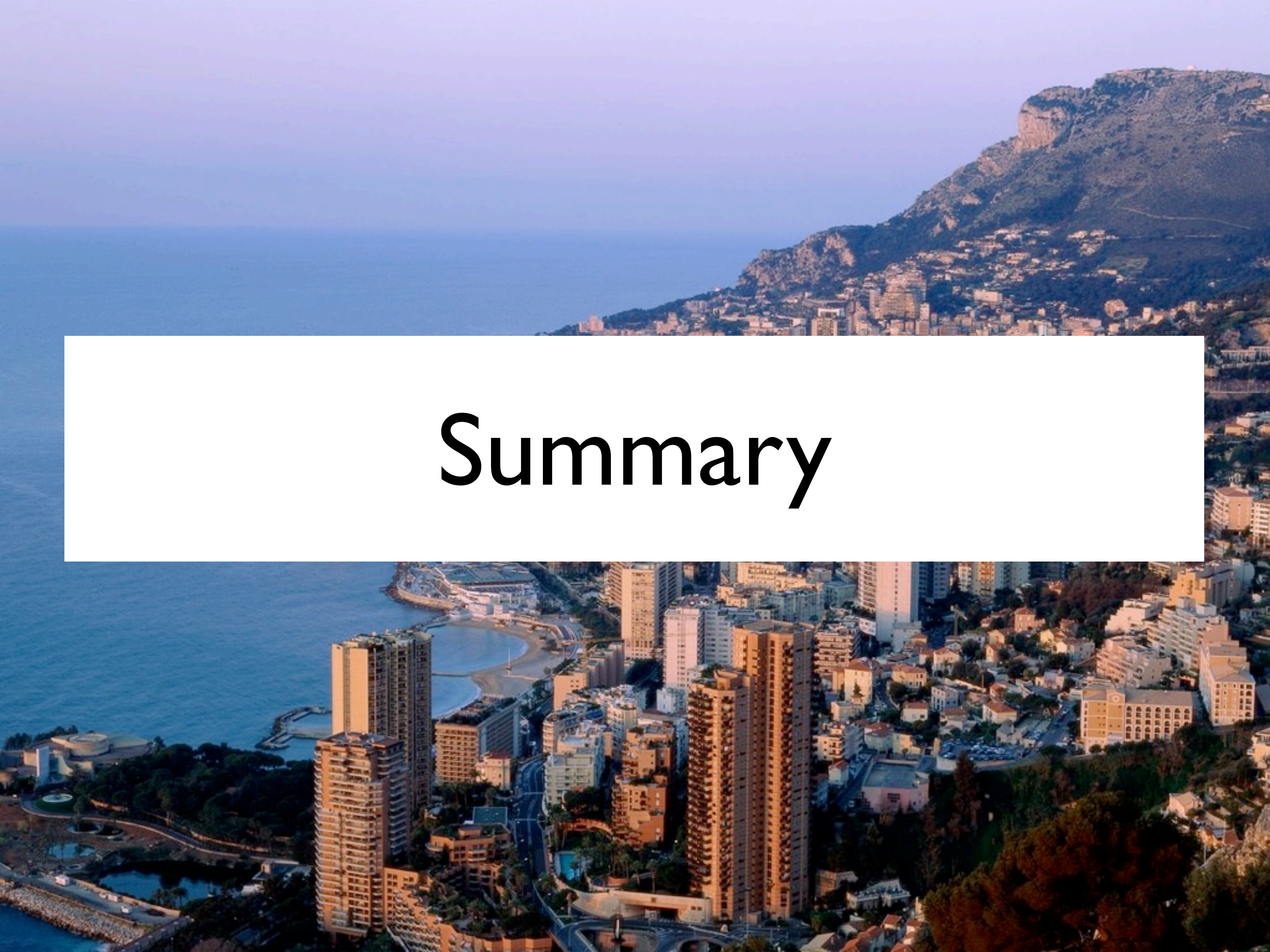
SU(4) gives bigger value of 't Hooft loop than SU(3)



't Hooft loop = 0.5

$\sim$ BH/BS transition

Summary



- Monte Carlo is a useful tool to study SYM.

  ‘simulation of superstring’

- Sign problem? No problem!
- 1d (non-lattice) : nice & precise results.
- 2d (lattice) : ongoing.
- 3d, 4d (fuzzy sphere, lattice) : coming soon.
- For other theories (e.g. SUSY QCD) new ideas are needed.

THE END