古典**Yang-Mills**系における
カラーグラス凝縮状態からの熱化過程

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Thermal Quantum Field Theory and Their Applications 2012
@YITP, 22 Aug., 2012

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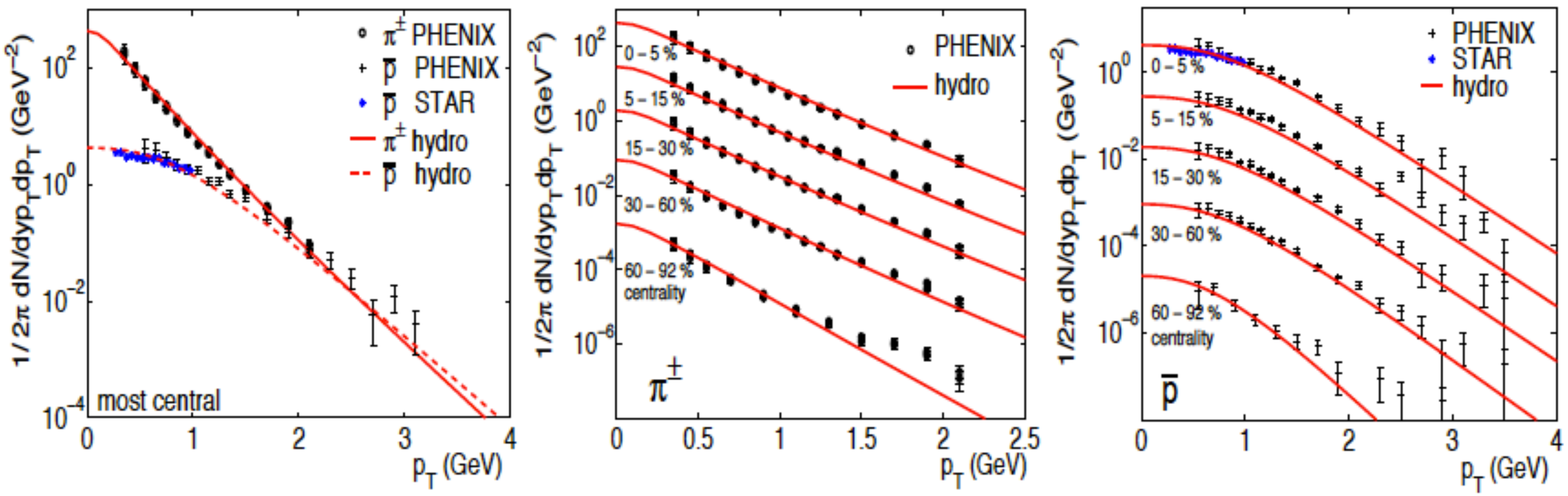
Introduction

- Early thermalization: mystery of Heavy Ion Collision



P_T and centrality dependence of π, p in Au+Au collision at \root s=130 A GeV, taken by the PHENIX and STAR collab.

(RHIC white paper & U.Heinz and P.Kolb, arXiv:hep-ph/0111075.)

τ_{eq}=0.6fm/c … early thermalization

cf) pQCD: τ_{eq}>2fm/c (K.Geiger 1992)

- **Thermalization process in heavy ion collision is important.**

Introduction

[Question]

How is entropy created in Heavy Ion Collision?

How to define entropy in non-equilibrium in isolated system?

(cf: entropy of a subsystem due to entanglement of its environment…entanglement entropy)

- Entropy production in isolated system in
  - Classical dynamics … Kolmogorov-Sinai entropy (entropy production rate)

    $$S_{\mathrm{KS}} = \sum_k \lambda_k \theta(\lambda_k) \qquad \lambda_k \text{ : Lyapunov exponent } \quad |\delta X_i(t)| \propto e^{\lambda_i t}$$

    … related to mixing property in chaos theory
    (complexity of orbits in phase space in certain time interval)
  - Quantum mechanics … Husimi-Wehrl entropy

    $$S_{H,\Delta}(t) = -\int \frac{dpdx}{2\pi h} H_\Delta(p,x;t) \ln H_\Delta(p,x;t)$$
-

Husimi-Wehrl entropy

Can we construct a quantity like H function in quantum mechanics? $H(t) = \int dp \int dx f(x,p;t) \ln f(x,p;t)$

▪ **Distribution function in q.m. …Wigner function??:**

$$W(p,x;t) = \int du\, e^{\frac{i}{\hbar}pu} \left\langle x - \frac{u}{2} \right| \hat{\rho}(t) \left| x + \frac{u}{2} \right\rangle$$ **… not always positive**

▪ **Husimi function:**

$$H_\Delta(p,x;t) \equiv \int \frac{dp'\,dx'}{\pi\hbar} \exp\left(-\frac{1}{\hbar\Delta}(p-p')^2 - \frac{\Delta}{\hbar}(x-x')^2\right) W(p',x';t) \geq 0$$

(Δ: smearing parameter)

… H is obtained by smearing W with minimum uncertainty in phase space

- **Husimi-Wehrl entropy**

$$S_{\mathrm{H},\Delta}(t) = -\int \frac{dp\,dx}{2\pi\hbar} H_\Delta(p,x;t) \ln H_\Delta(p,x;t)$$

Relation bw KS and HW entropy

Important observation:

Coarse grained quantum mechanical (HW) entropy growth rate agrees with KS entropy.

Ref.) T.Kunihiro, B.Mueller, A.Ohnishi, A.Schaefer, PTP121 (2009)

- $\hat{\mathcal{H}} = \frac{1}{2}\hat{p}^2 - \frac{1}{2}\lambda^2\hat{x}^2$ $\qquad$ $\frac{dS_{\mathrm{H},\Delta}}{dt} \to \lambda \ (t \to \infty) \quad (t \gg \lambda^{-1})$
- $\hat{\mathcal{H}} = \sum_k \frac{1}{2}\left(\hat{p}_k^2 - \lambda_k^2\hat{x}_k^2\right)$ $\qquad$ $\frac{dS_{\mathrm{H},\Delta}}{dt} \xrightarrow{t\to\infty} \sum_k \lambda_k$
- $\mathcal{L} = \int dx \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial t}\right)^2 - \left(\frac{\partial\Phi}{\partial x}\right)^2 + \mu^2\Phi^2\right]$ $\qquad$ $\frac{dS_{H,\Delta}}{dt} \to V\int_\mu^\mu \frac{dp}{2\pi}\lambda_p = \frac{V\mu^2}{4}$

$$\lambda_p \equiv \sqrt{\mu^2 - p^2}$$

- The growth rate of the Husimi-Wehrl entropy is given by the K-S entropy (positive Lyapunov exponent) in the classical dynamics!
- Unstable modes in the classical dynamics plays the essential role for entropy production at quantum level.

→ may account for entropy production in quantum level in HI collisions at RHIC.

Entropy production in classical Yang-Mills dynamics

T.Kunihiro, B.Mueller, A.Ohnishi, A.Schaefere, T.T.Takahashi and A.Yamamoto, PRD**82**, 114015(2010).

- Focusing on **entropy production through the chaotic behavior** in Classical Yang-Mills system.

Two quantities

Distance between two trajectories in phase space

$$D_{EE} = \sqrt{\sum_x \left\{ \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E_i'^a(x)^2 \right\}^2} \qquad D_{FF} = \sqrt{\sum_x \left\{ \sum_{a,i,j} F_{ij}^a(x)^2 - \sum_{a,i,j} F_{ij}'^a(x)^2 \right\}^2}$$

E' & F' …slightly different from E & F at initial time, respectively

Entropy production rate (Kolmogolov-Sinai entropy)

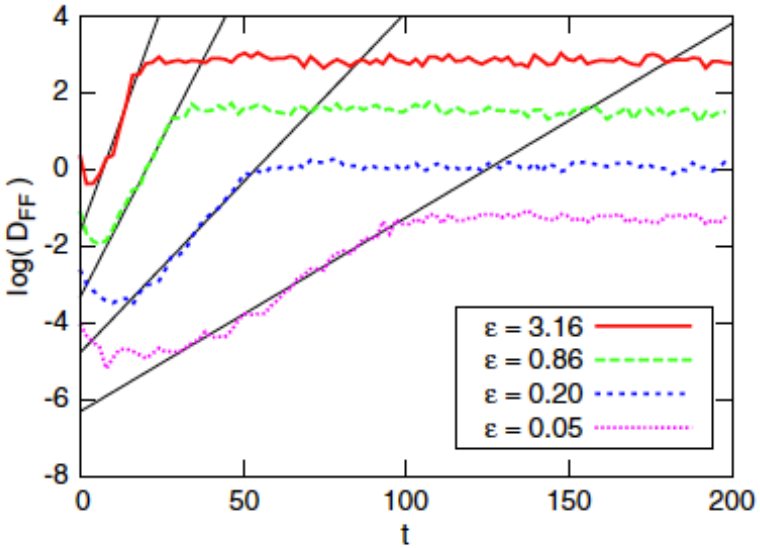
$$\frac{dS}{dt} = S_{\text{KS}} = \sum_{\lambda_i^{\text{ILE}}>0} \lambda_i^{\text{ILE}}$$

… related to chaoticity $|\delta X_i(t)| \propto e^{\lambda_i t}$

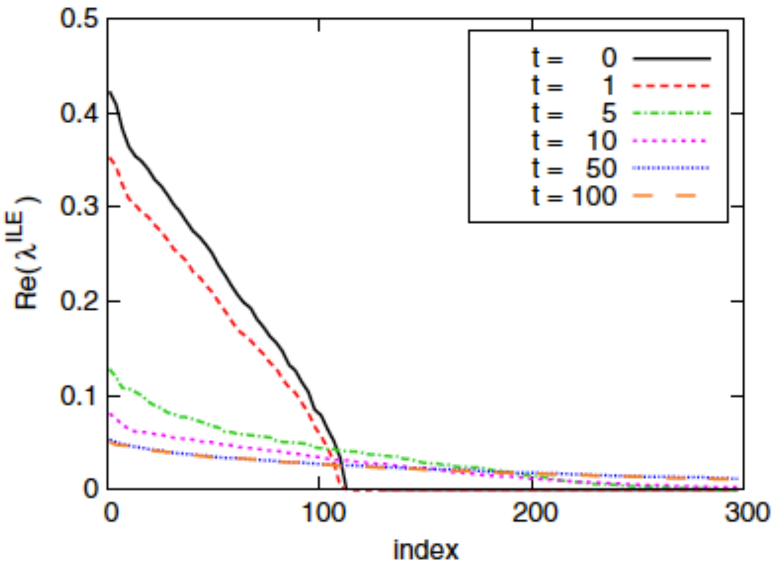
Entropy production in classical Yang-Mills dynamics

T.Kunihiro, B.Mueller, A.Ohnishi, A.Schaefere, T.T.Takahashi and A.Yamamoto, PRD**82**, 114015(2010).

- Entropy production & early thermalization is investigated in CYM with random initial condition



Time evolution of δX
for several initial energy density



Time evolution of Lyapunov exponent

In this study

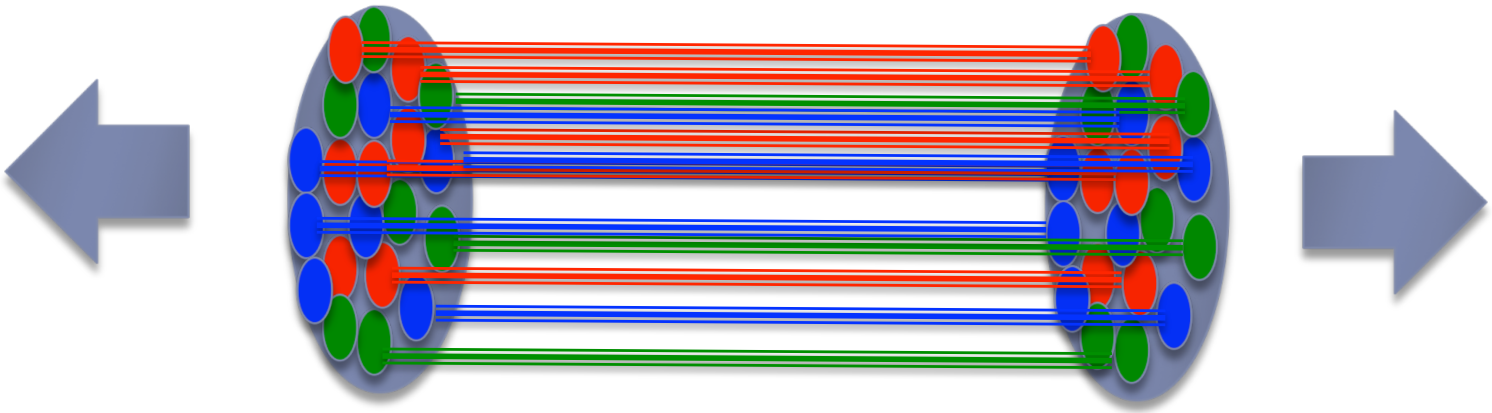
- Study of early thermalization in heavy-ion collisions using classical Yang-Mills eq.
  with CGC-like initial conditions
  in non-expanding plasma

  (initial gauge field has some spatial correlations)

- We focus on the **chaotic behavior** of the systems: distance between two trajectories with slightly different initial conditions & Lyapunov exp.

Color-glass condensate

- **High energy collision… high gluon density**
  → gluons are coherent field rather than particles
  First approximation: classical treatment
- **Initial condition: color-glass condensate (CGC)**
  (L.McLerran and R.Venugopalan 1994; T.Lappi and L.McLerran 2006, …)



Classical Yang-Mills calculation with CGC initial condition
… appropriate for the study of thermalization process

- **Instability … served as a possible mechanism of early thermalization**
  (Weibel instability… S.Mrowczynski, Nielsen-Olesen instability… Fujii, Itakura; Fujii, Itakura, Iwazaki)

1. Modulated initial condition

$$A_i^a(\vec{r}) = \underline{\eta^a(\vec{r})} + \delta_{i2}\left(\epsilon_1 \sin\left(\frac{2x\pi}{N_x}\right) + \epsilon_2 \sin\left(\frac{2nx\pi}{N_x}\right)\sin\left(\frac{2mz\pi}{N_z}\right)\right),$$

$$E_i^a(\vec{r}) = 0,$$

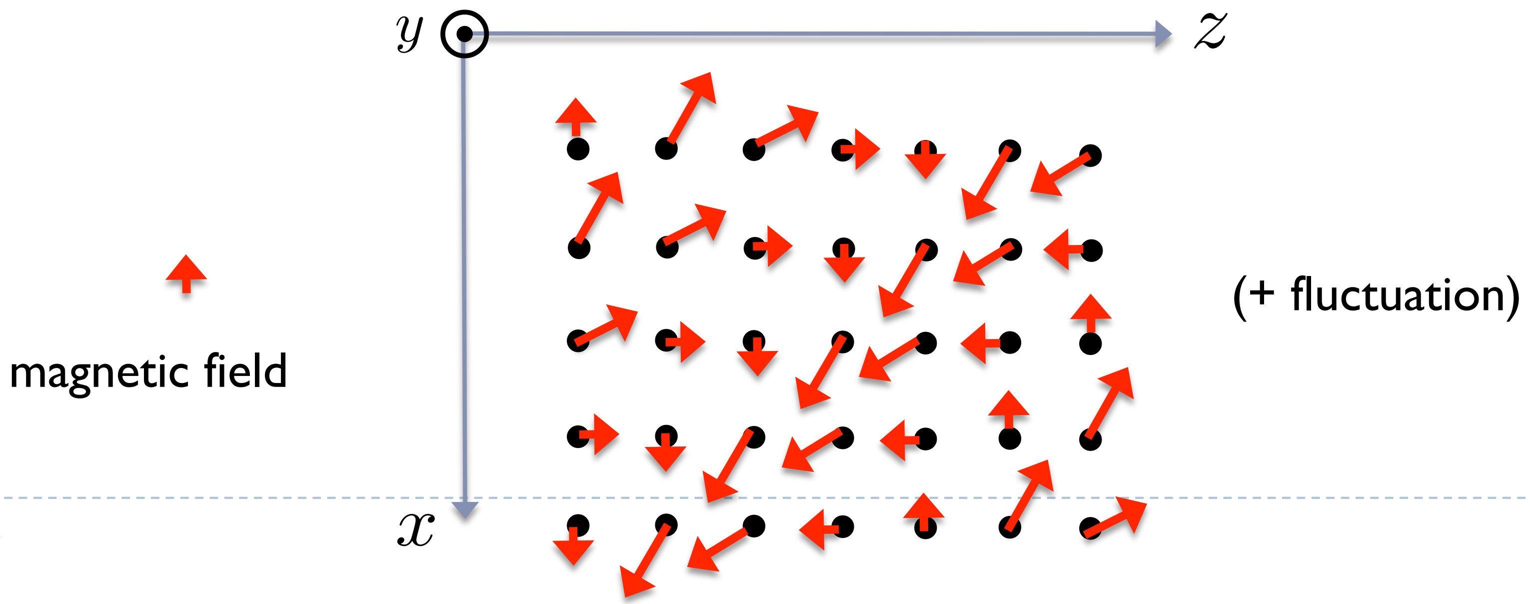
Fluctuation (noise)

Magnetic field from spatial modulation (neglecting noise)

$$B_x = \partial_y A_z - \partial_z A_y + g\epsilon^{1bc} A_2^b A_3^c = -\partial_z A_y = -2m\frac{\pi}{N_z}\epsilon_2 \sin(\frac{2nx\pi}{N_x})\cos(\frac{2mz\pi}{N_z}),$$

$$B_y = \partial_z A_x - \partial_x A_z + g\epsilon^{2bc} A_2^b A_3^c = 0,$$

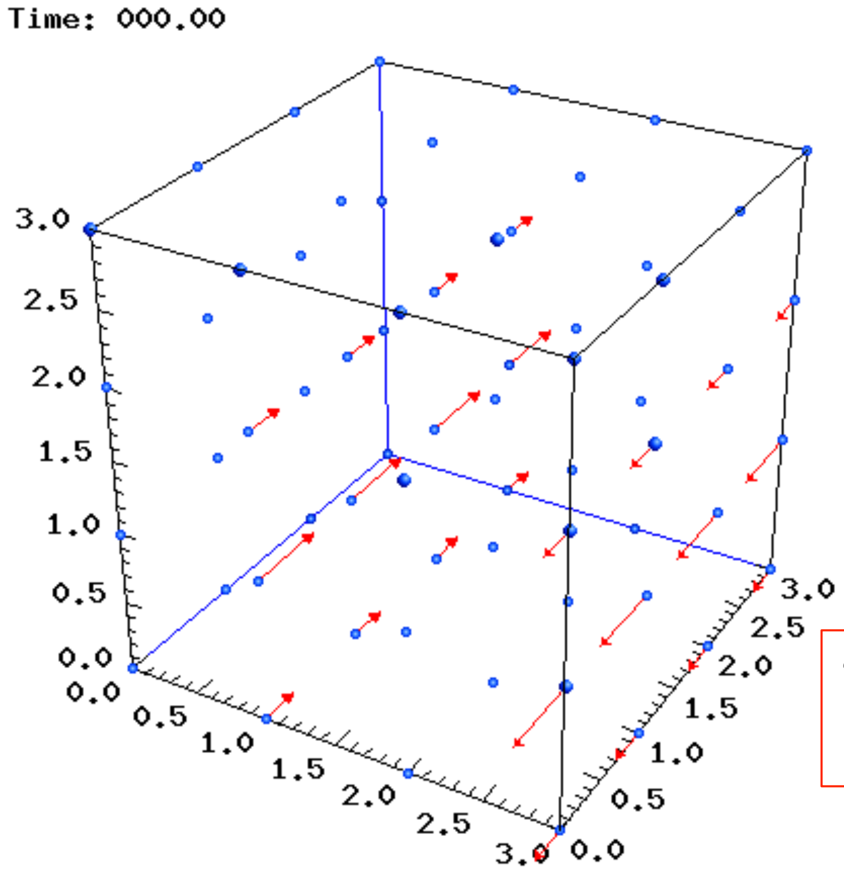
$$B_z = \partial_x A_y - \partial_y A_x + g\epsilon^{3bc} A_2^b A_3^c = \partial_x A_y = \frac{2\pi}{N_x}\epsilon_1 \cos(\frac{2x\pi}{N_x}) + \frac{2n\pi}{N_x}\epsilon_2 \cos(\frac{2nx\pi}{N_x})\sin(\frac{2mz\pi}{N_z}),$$



Results: modulated init. cond.

- Time evolution of gauge fields

SU(2), 4^3



Red arrow: A_i

Green arrow: E_i

"Instability" (chaotic behavior?) seems to occur for large t

Distance between two trajectories

$$D_{EE} = \sqrt{\sum_x \left\{ \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E_i'^a(x)^2 \right\}^2}$$

$$D_{FF} = \sqrt{\sum_x \left\{ \sum_{a,i,j} F_{ij}^a(x)^2 - \sum_{a,i,j} F_{ij}'^a(x)^2 \right\}^2}$$

E' & F'

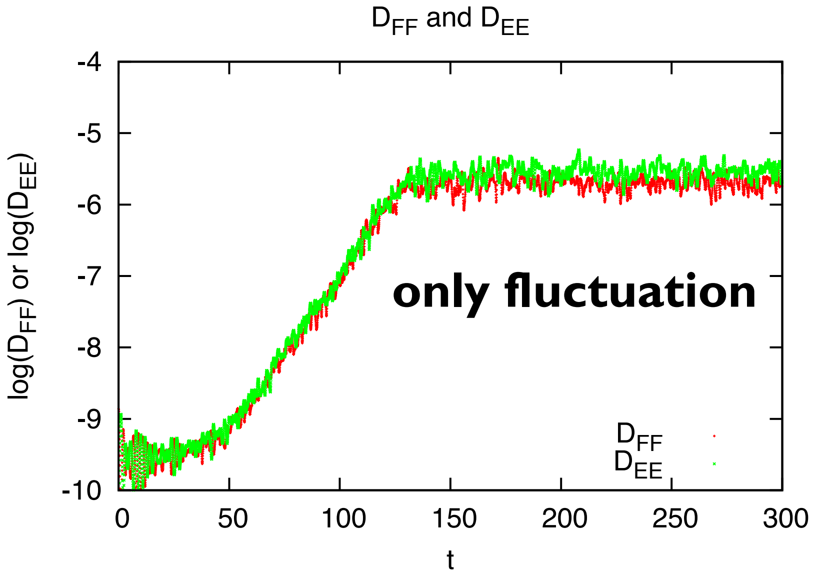
…slightly different from E & F at initial time, respectively

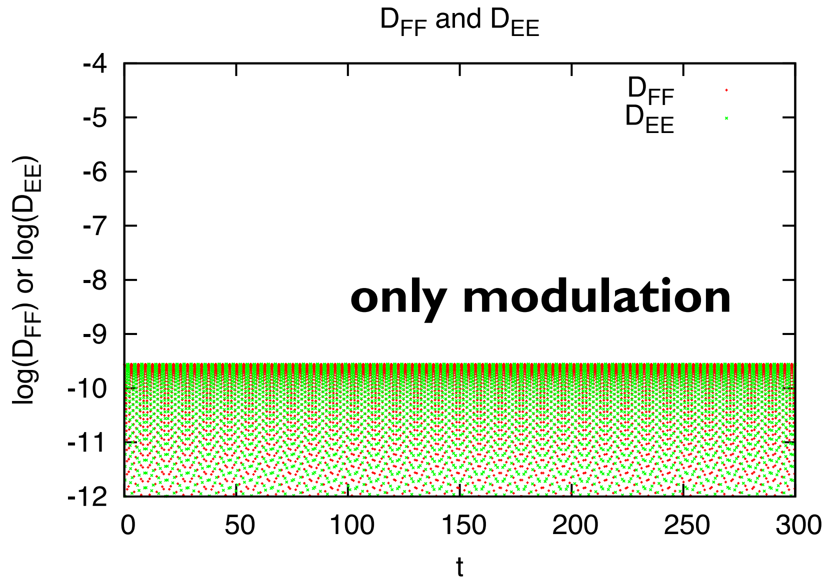
cf) Lyapunov exponent $|\delta X_i(t)| \propto e^{\lambda_i t}$

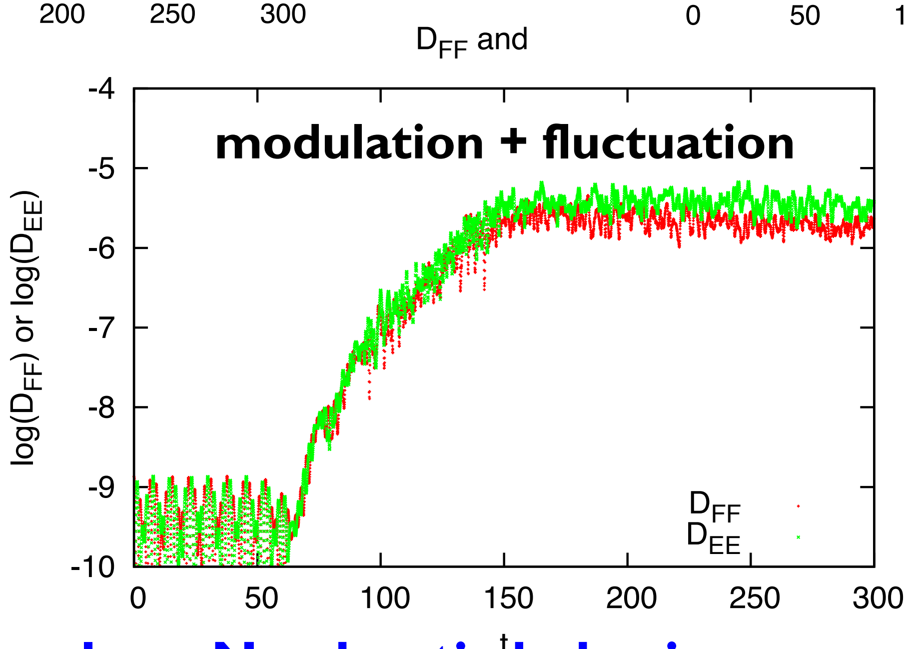
(time evolution of two trajectories from very similar init. cond.)

$D \propto e^{\lambda_D t}$ λ_D :governed by maximum Lyapunov exp.

Results: time dep. of distance

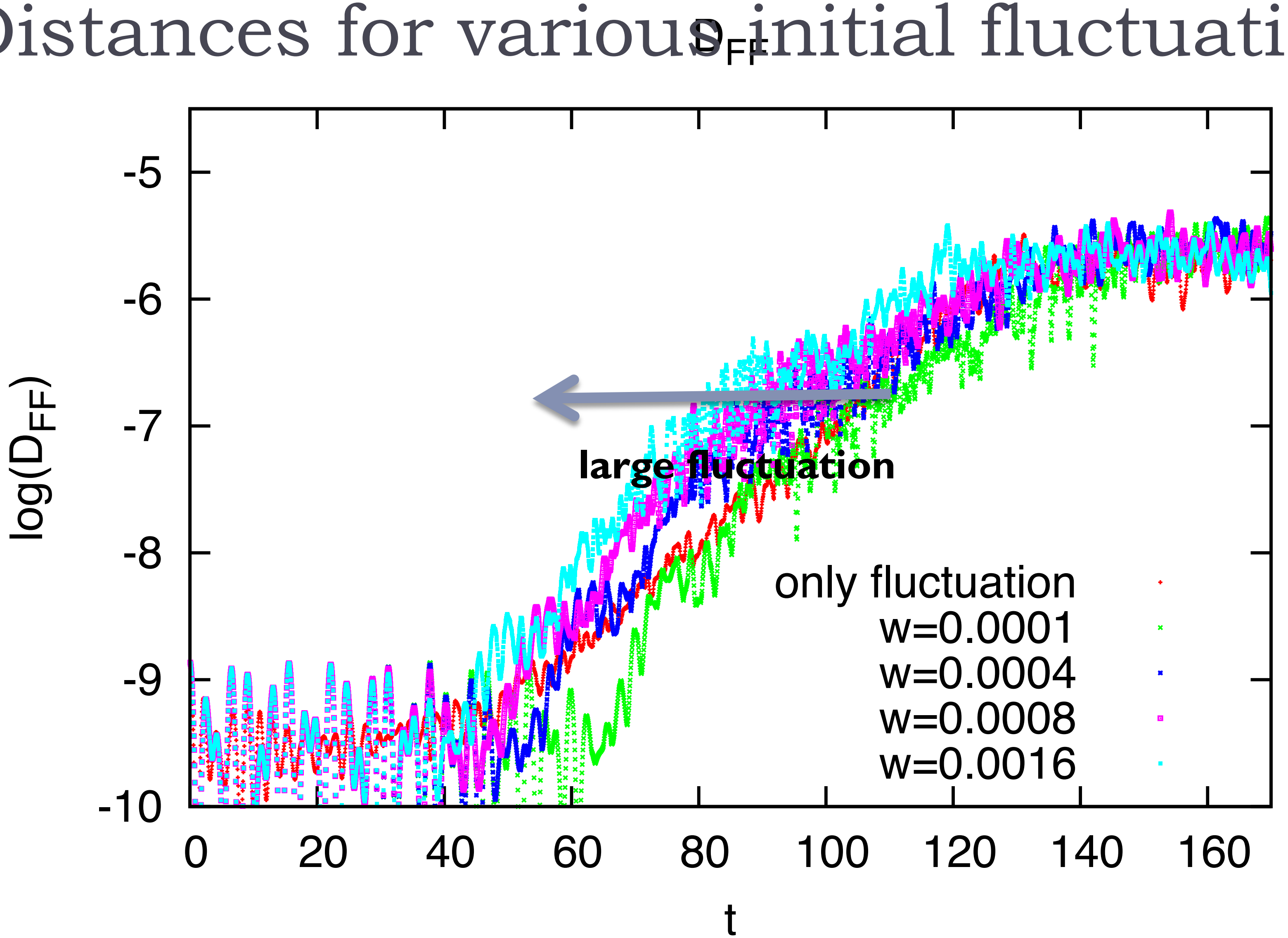






- **Modulation only … No chaotic behavior**
- **Modulation + (tiny) fluctuation…chaotic behavior occurs**

Distances for various initial fluctuation



Onset time of increase of D depends on the ratio of fluctuation to background modulation field

Intermediate Lyapunov exponent

- Time evolution of difference between two trajectories:

$$\delta\dot{X}(t) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} H_{xx} & H_{xp} \\ H_{px} & H_{pp} \end{pmatrix}}_{\text{Hessian } \mathcal{H}} \delta X(t) \qquad (H_{xx})_{ij} = \partial^2 H / \partial x_i \partial x_j$$

- We can formally solve the equation for finite Δt :

$$\delta X(t+\Delta t) = U(t, t+\Delta t)\delta X(t)$$

$$U(t, t+\Delta t) = \mathcal{T}\left[\exp\left(\int_t^{t+\Delta t} \mathcal{H}(t')dt'\right)\right]$$

- Using Trotter formula, U is written as

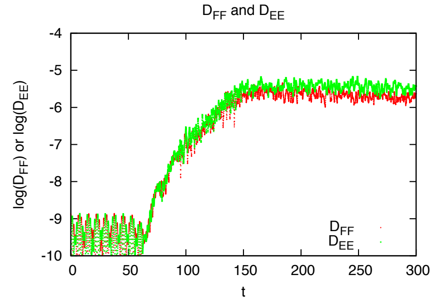
$$U(t, t+\Delta t) \simeq \mathcal{T} \prod_{k=1,N} \exp[\mathcal{H}(t_{k-1})\delta t] \simeq \mathcal{T} \prod_{k=1,N} [1 + \mathcal{H}(t_{k-1})\delta t]$$

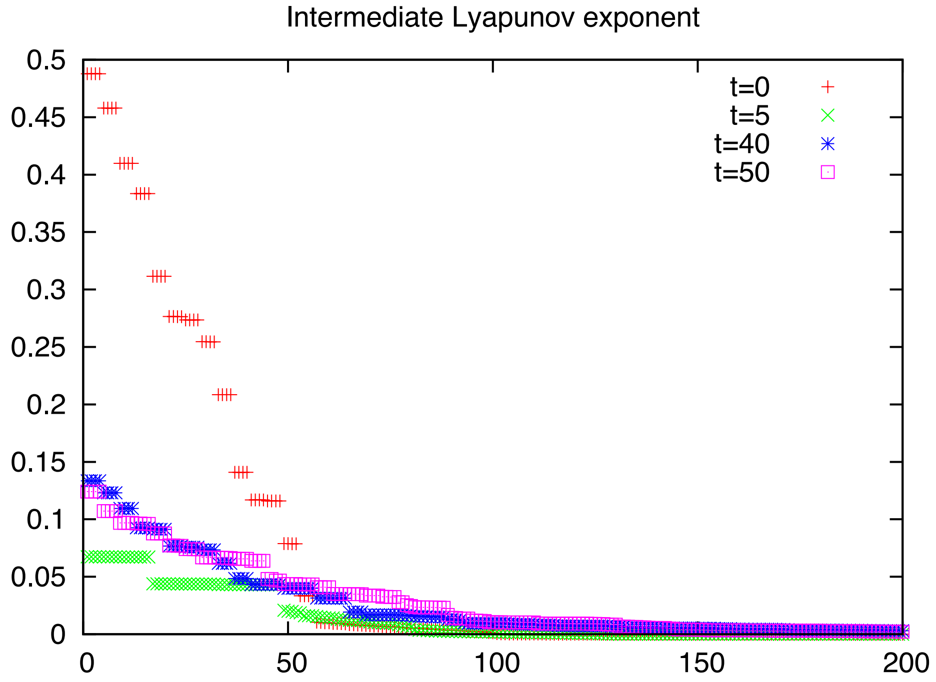
- By diagonalization, ILE is obtained:

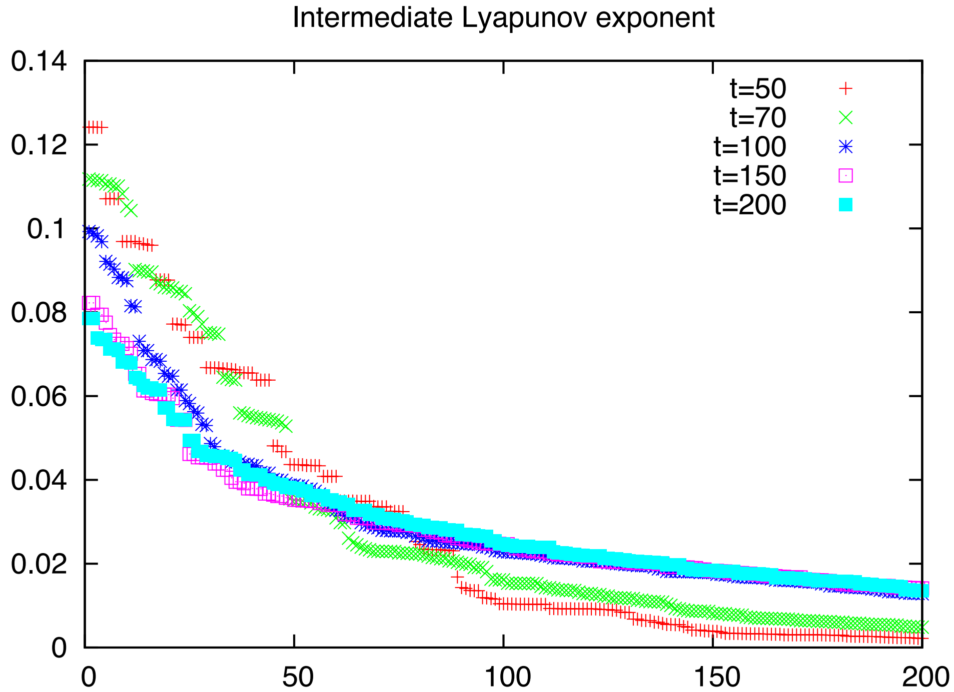
$$U_D(t, t+\Delta t) = \mathrm{diag}(e^{\lambda_1^{\mathrm{ILE}}\Delta t}, e^{\lambda_2^{\mathrm{ILE}}\Delta t}, \cdots)$$

We set $t = 0$

Results: Intermediate Lyapunov exp.



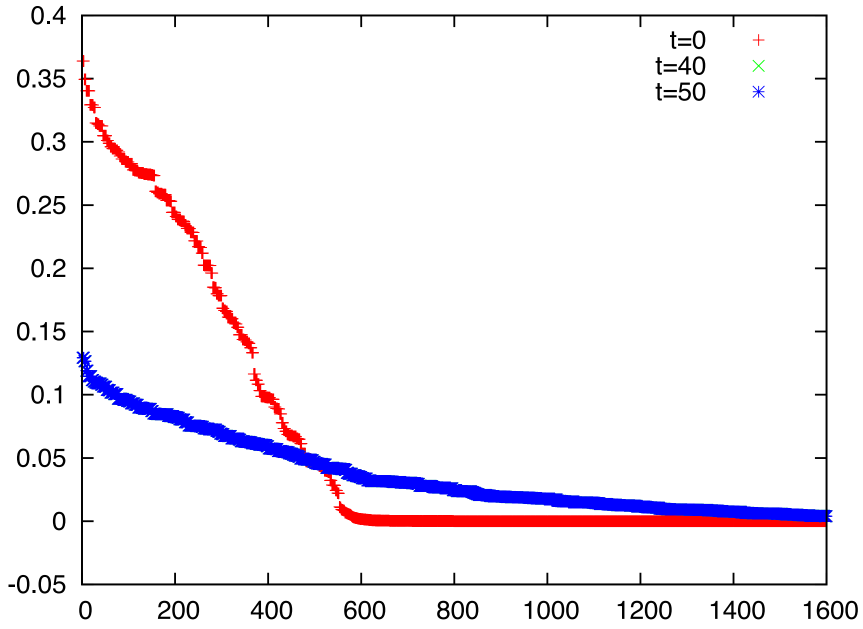


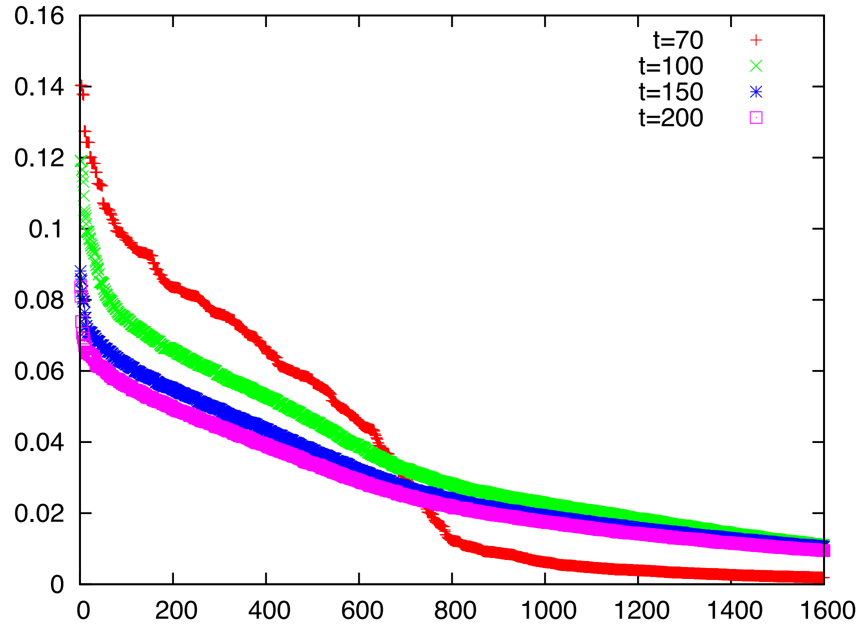


# of ILE = $V*(N_c^2-1)*3*2$

- **Instability spreads to many modes in the very early stage.**
- **Distribution of Lyapunov exp. is stable at large t, and then, significant portion of Lyapunov exponents keeps positive in a robust way → chaotic**

Intermediate Lyapunov exponent ($V=8^3$)





- **For large volume, qualitative behavior does not change:**
  Distribution of Lyapunuov exp. is stable at large t, and then, significant portion of Lyapunov exponents keeps positive in a robust way → chaotic

2. Constant A initial condition

Ref.) J.Berges, S.Scheffler, S.Schlichting and D.Sexty

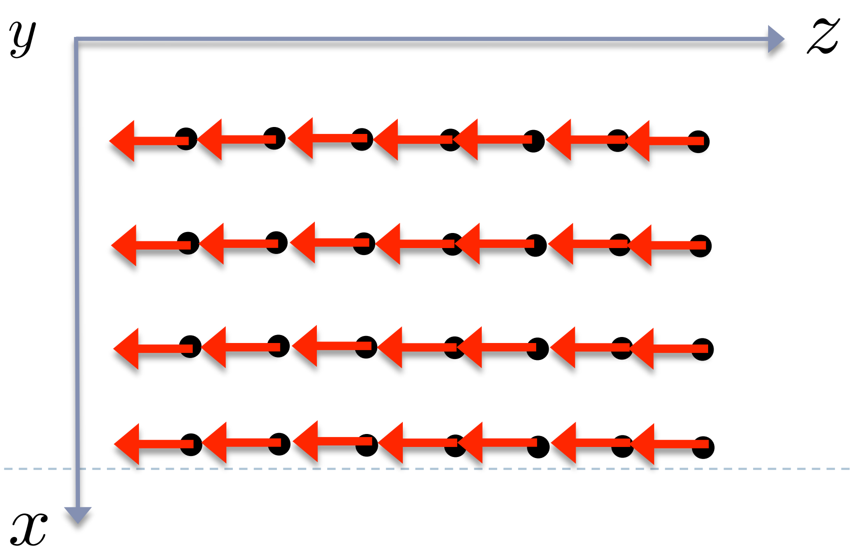
$$A_i^a(\vec{r}) = \eta_i^a(\vec{r}) + (\delta^{a2}\delta_{xi} + \delta^{a3}\delta_{yi})\sqrt{\frac{B}{g}}$$

$$E_i^a(\vec{r}) = 0$$

Magnetic field (neglecting noise)

$$B_3^1 = -F_{21}^1 = g\epsilon^{1bc}A_2^b A_3^c = g(A_2^2 A_1^3 - A_2^3 A_1^2) = -B,$$

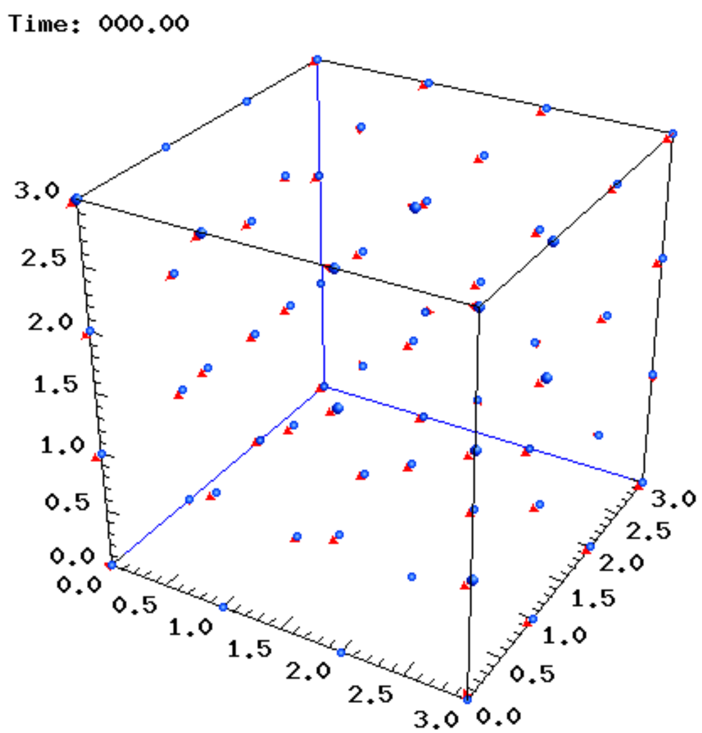
Others: zero



(+ fluctuation)

Results: constant A init. cond.

- Time evolution of gauge fields



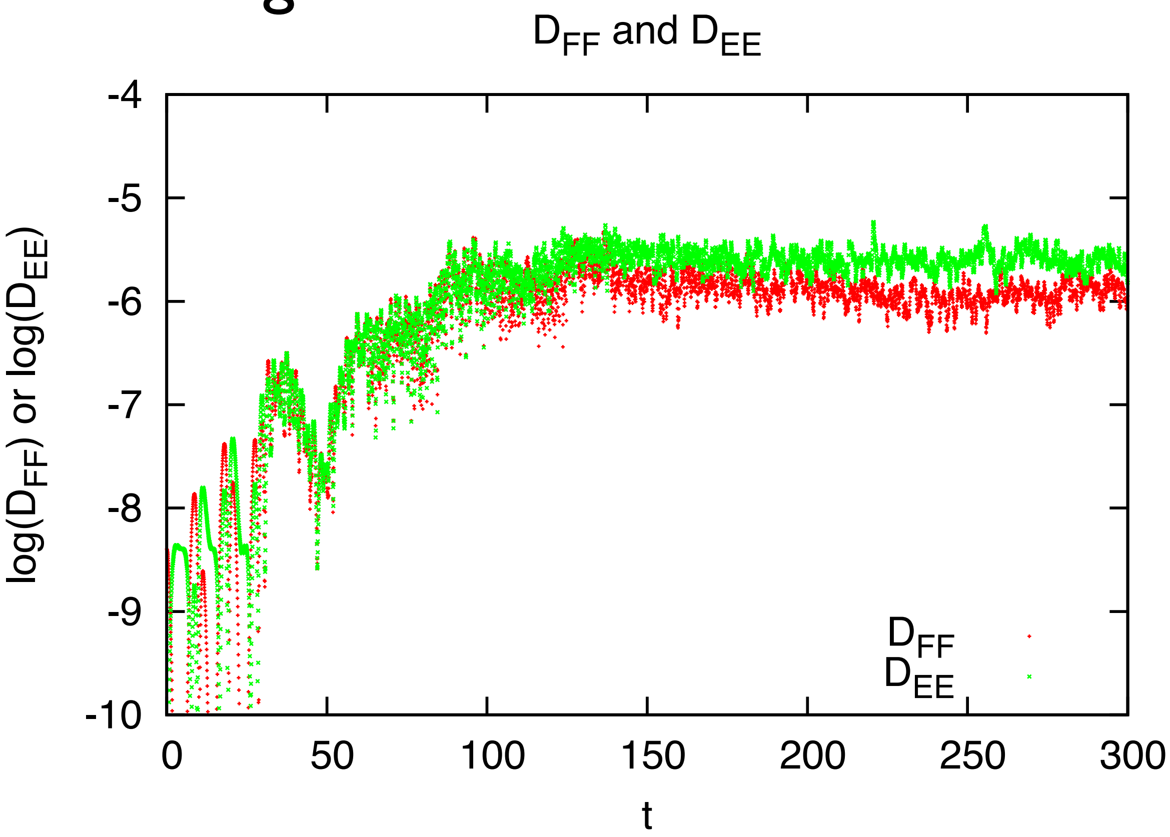
Red arrow: A_i

Green arrow: E_i

"Instability" (chaotic behavior?) seems to occur for large t.

Results: time dep. of distance

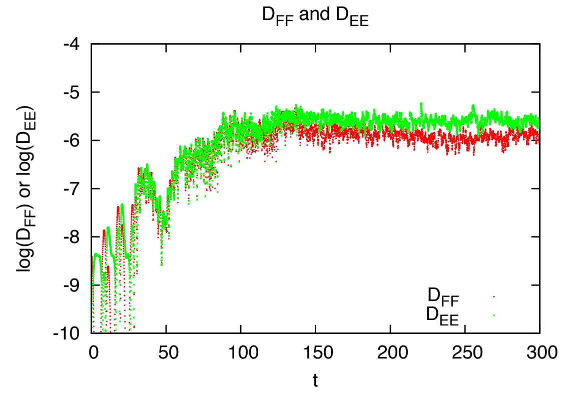
- (2) Constant magnetic field

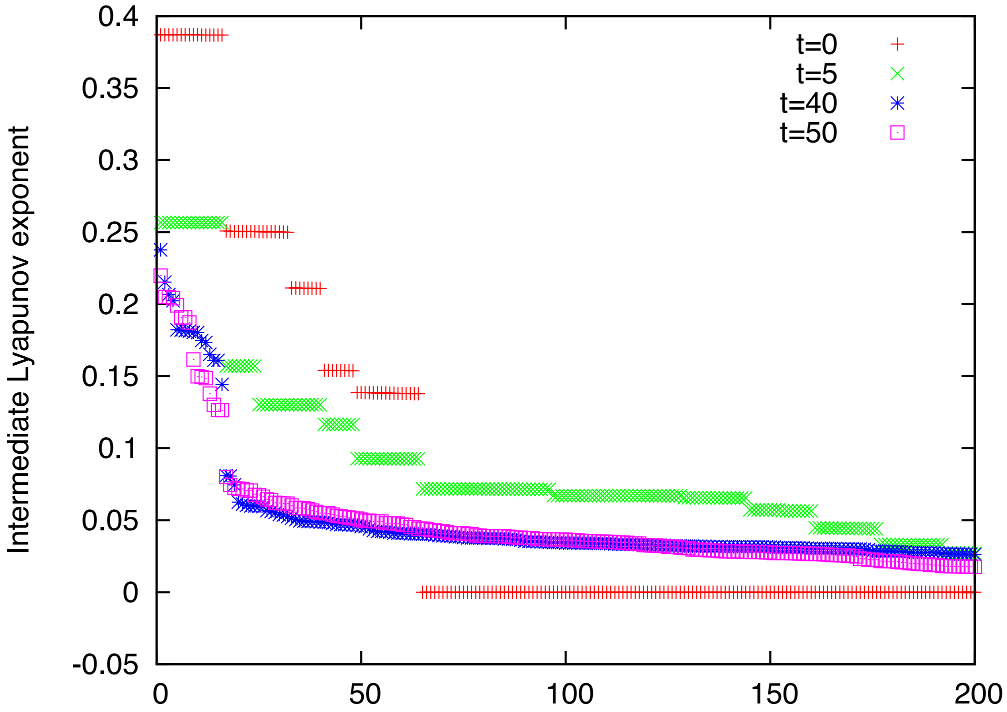


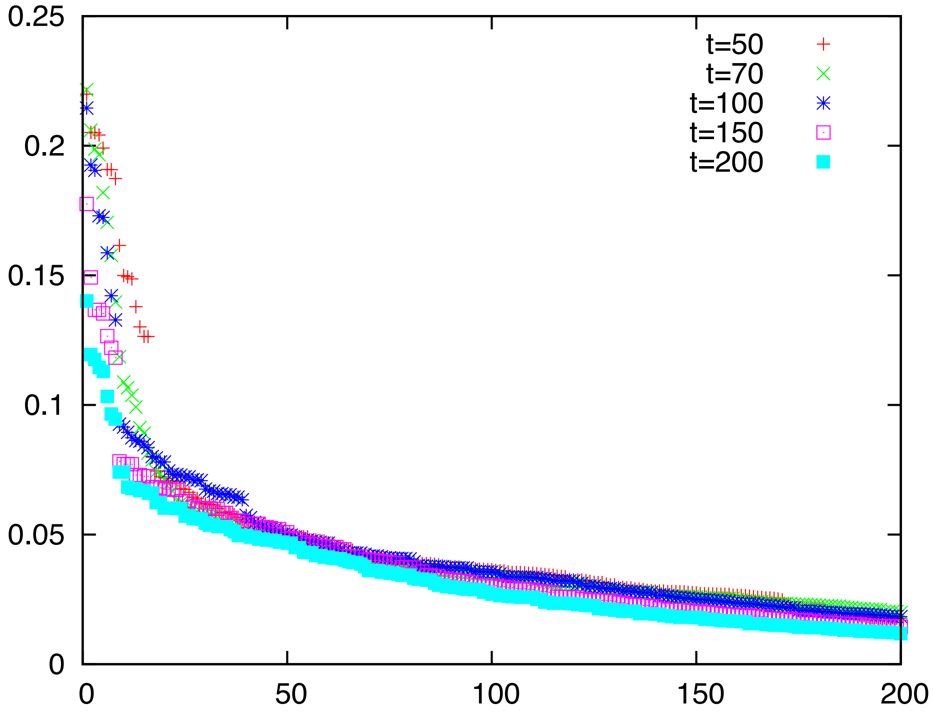
D becomes large (with oscillation)

·Modulation + (tiny) fluctuation…chaotic behavior occurs also in the initial condition

ILE for Constant A

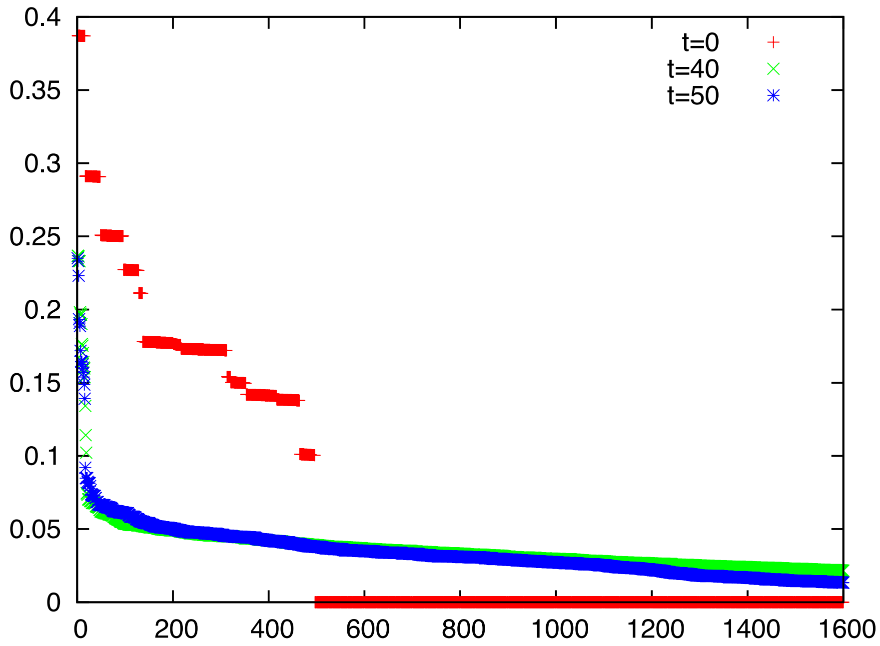


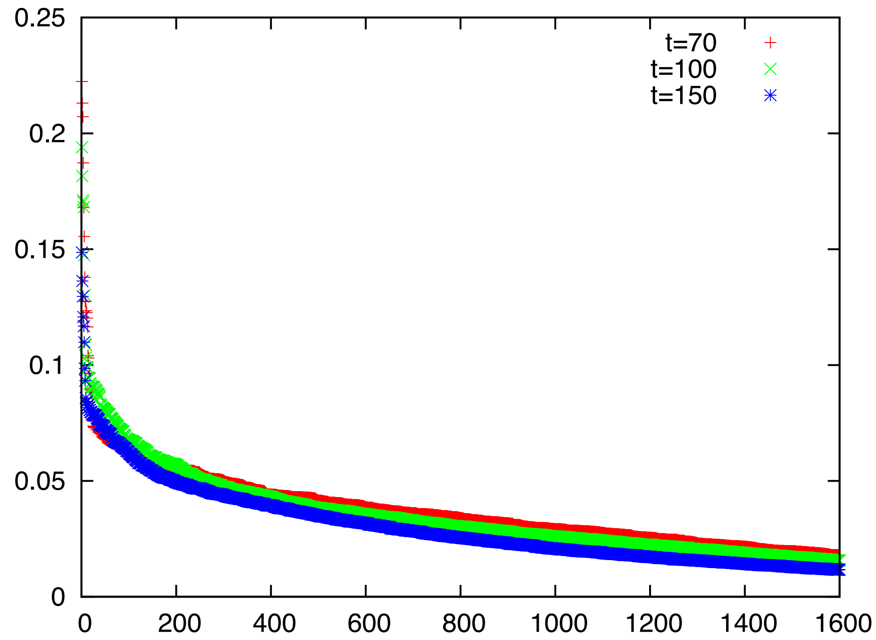




- **Although the distribution of ILE in initial stage is different from that in modulated initial condition (MIC), the distribution in later stage is similar to that in MIC.**

ILE for Constant A ($V=8^3$)





- **For large volume, qualitative behavior does not change: Distribution of Lyapunuov exp. is stable at large t, and then, significant portion of Lyapunov exponents keeps positive in a robust way → chaotic**

Summary and conclusions

- Background magnetic field + (tiny) fluctuation

  → chaotic behavior

- Behavior of intermediate Lyapunov exponent:

  Distribution of Lyapunov exponent seems to be stable at large t, and then, significant portion of Lyapunov exponents keeps positive in a robust way.

・Entropy production proceeds already in the CYM evolution.

・The entropy production in CYM may serve as a possible mechanism of early thermalization.

・Initial fluctuation has important role for the mechanism.

init.	A	E	A'	w	ϵ_1	ϵ_2	mode1	mode2	E_{tot}
CGC0_SU2_L040404	$\neq$	0	$1.005A$	0.1	0	0	#	#	0.01379
CGC1_SU2_L040404	$\neq$	0	$1.005A$	0.09	0.045	0.045	3	3	0.01389
CGC2_SU2_L040404	$\neq$	0	$1.005A$	0.05	0.085	0.085	3	3	0.01395
CGC3_SU2_L040404	$\neq$	0	$1.005A$	0.02	0.095	0.095	3	3	0.01394
CGC4_SU2_L040404	$\neq$	0	$1.005A$	0	0.097	0.097	3	3	0.01411
CGC5_SU2_L040404	$\neq$	0	$1.005A$	0.0001	0.097	0.097	3	3	0.01411
CGC6_SU2_L040404	$\neq$	0	$1.005A$	0.0001	0	0	#	#	1.379×10^{-8}

init.	A	E	A'	w	ϵ_1	ϵ_2	mode1	mode2	E_{tot}
CGC0_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0	0	3	3	3.8624E-008
CGC1_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.097	0.097	3	3	0.03763
CGC2_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.048	0.048	3	3	0.0092155
CGC3_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.024	0.024	3	3	0.0023038
CGC4_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.012	0.012	3	3	0.00057591
CGC5_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.006	0.006	3	3	0.00014397
CGC6_SU3_L040404	$\neq$	0	$1.005A$	0.0001	0.0765	0.0765	3	3	0.02341

Various kind of distance

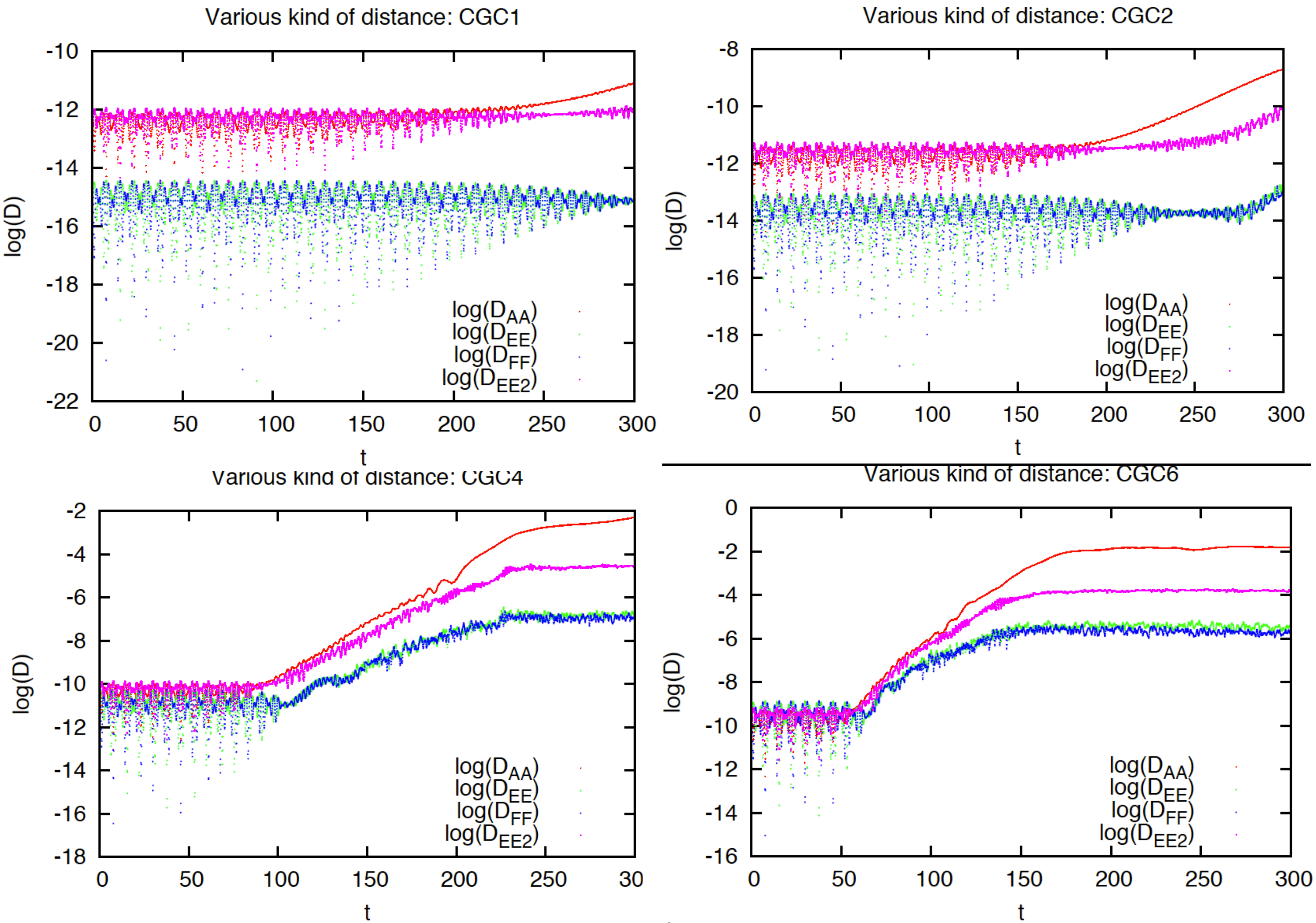
$$D_{EE} = \sum_x \left| \sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E_i'^a(x)^2 \right|,$$

$$D_{FF} = \sum_x \left| \sum_{a,i,j} F_{i,j}^a(x)^2 - \sum_{a,i,j} F_{i,j}'^a(x)^2 \right|,$$

$$D_{EE2} = \sum_x \left\{ \sum_{a,i} E_i^a(x) - \sum_{a,i} E_i'^a(x) \right\}^2,$$

$$D_{AA} = \sum_x \left\{ \sum_{a,i} A_i^a(x) - \sum_{a,i} A_i'^a(x) \right\}^2,$$

Various kind of distances



- Different definitions give different slopes
- D_{EE} & D_{FF} are almost the same