Equilibration of Scalar Fields in an Expanding System

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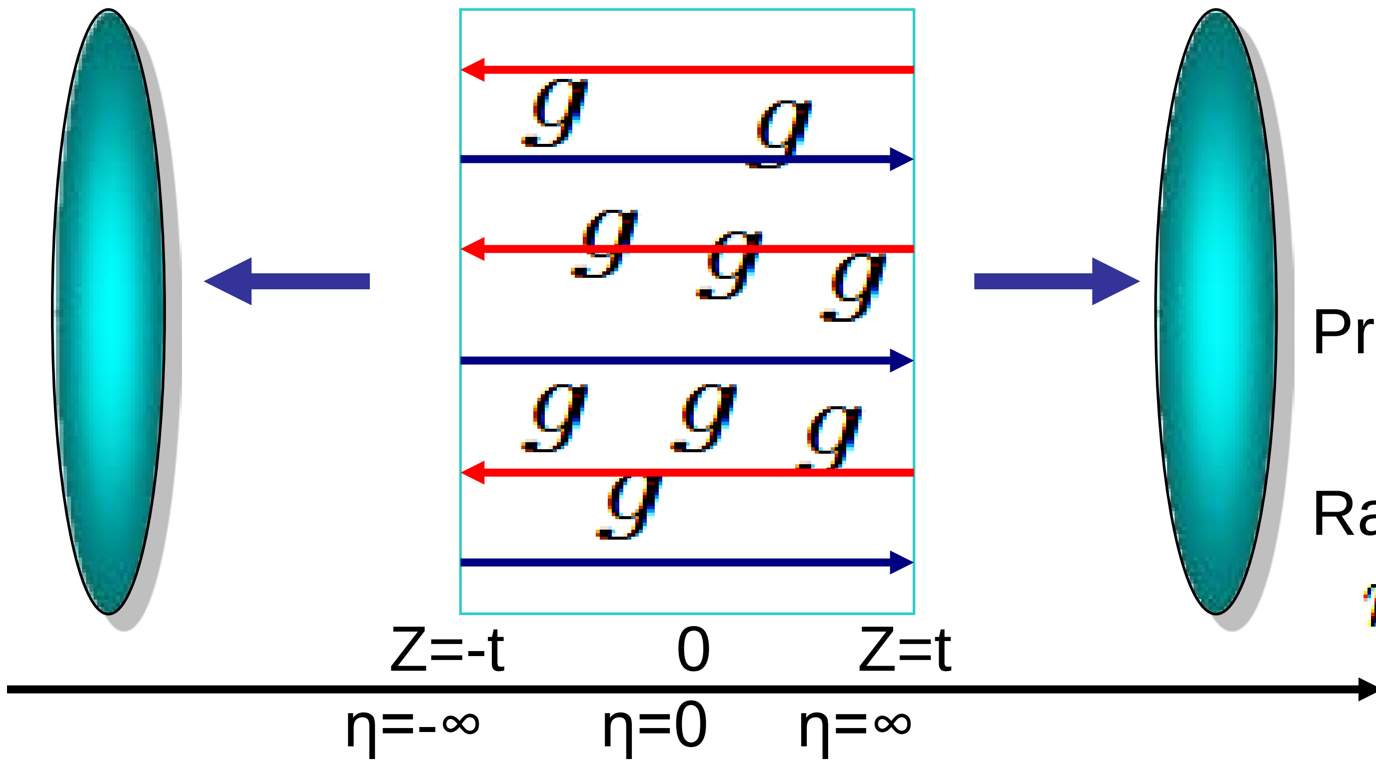
Relativistic Heavy Ion Collision at RHIC and LHC



$\sqrt{s_{NN}}$=0.2 TeV

Au+Au (RHIC)

$\sqrt{s_{NN}}$=2.76TeV

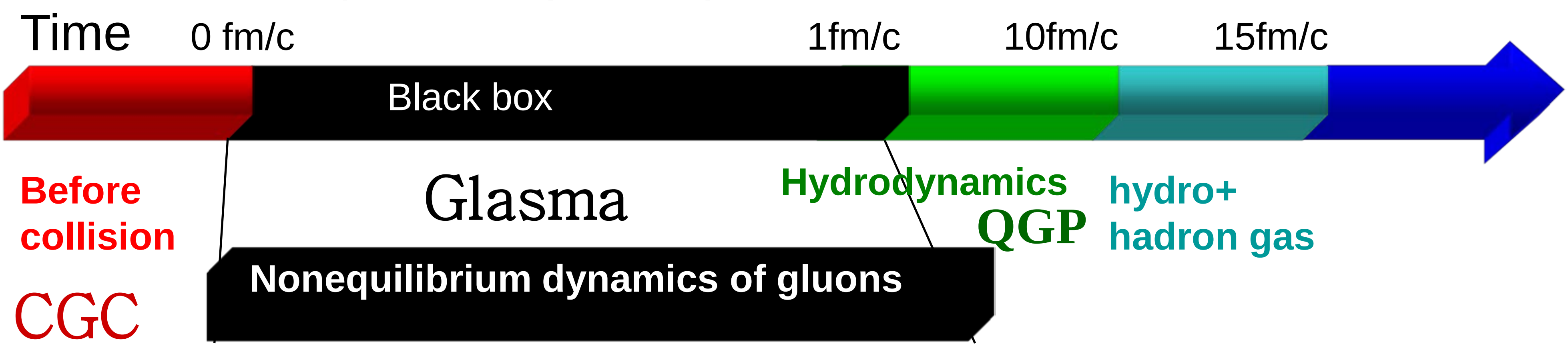
Pb+Pb (LHC)

Proper time

$$\tau = \sqrt{t^2 - z^2}$$

Rapidity

$$\eta = \tanh^{-1}\frac{z}{t}$$



Formation of Quark-Gluon Plasma (QGP)

Success of nearly ideal hydrodynamics after thermalization.
Early Thermalization of gluons (0.6-1fm/c)! (RHIC and LHC)
Kolb and Heinz (2002), Hirano et al. (2010)

Comparative to formation time of partons (1/Qs~0.2fm/c)
Semi-Classical Boltzmann eq. should not be applied,
since 2-3fm/c is predicted for **gg→gg, gg→ggg (Boltzmann).**

Baier, Mueller, Schiff, Son (2001 and 2011)

Decoherence: Muller, Schafer (2006)

New method is needed.

Quantum nonequilibrium processes based on field theory

Application of Kadanoff-Baym eq.
to early thermalization of gluons.

Purpose of this talk

Introduction of time evolution equation for classical field and Kadanoff-Baym equation for quantum fluctuation in O(N) scalar model in an expanding metric.

To show particle production and equilibration in Numerical Analyses.

Comparison of expanding and nonexpanding system.

Rest of this

- **Time Evolution Equation I, II**
- **Initial condition**
- **Comparison of expanding and nonexpanding system**
- **Summary and Remaining Problems**

Time Evolution Equation I

Action of scalar O(N) model

$$S = \int d^4x \sqrt{-g} \left[\frac{g^{\mu\nu}}{2} \partial_\mu \varphi_a \partial_\nu \varphi_a - \frac{m^2}{2} \varphi_a \varphi_a - \frac{\lambda (\varphi_a \varphi_a)^2}{4! N} \right]$$

a=1,…,N

g=diag(1, -τ^2, -1, …)

Interaction term

Equation of motion of classical field

$$\phi_a(t,x) = \langle \hat{\varphi}_a(t,x) \rangle$$

$$\left[\partial_\tau^2 + \boxed{\frac{1}{\tau}\partial_\tau} + m^2 + \frac{\lambda}{6N}\phi^2(\tau) \right] \phi_a(\tau) = 0$$

Or effect of fluctuations

Damping of classical field for an expanding system

Time Evolution Equation II

- Kadanoff-Baym equation: Quantum evolution equation of 2-point Green's function (fluctuations).

statistical (distribution) and spectral functions

$$F_{ab}(x,y) = \frac{1}{2}\langle\{\tilde{\phi}_a(x), \tilde{\phi}_b(y)\}\rangle$$

$$\rho_{ab}(x,y) = \langle[\tilde{\phi}_a(x), \tilde{\phi}_b(y)]\rangle$$

$$F(\tau,\tau',p) \approx \frac{1}{2m_\perp\sqrt{\tau\tau'}}\cos m_\perp(\tau-\tau')\,(2n_p+1)$$

$\tau, \tau' \to \infty$ Boson

$$\rho(\tau,\tau',p) \approx \frac{1}{m_\perp\sqrt{\tau\tau'}}\sin m_\perp(\tau-\tau')$$

$$m_\perp = \sqrt{m^2+p_\perp^2}$$

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)F(x,y) = \int_0^{y^0} dz\Sigma_F(x,z)\rho(z,y) - \int_0^{x^0} dz\Sigma_\rho(x,z)F(z,y)$$

$$\left(-G_0^{-1}+\Sigma_{\rm loc}\right)\rho(x,y) = \int_{x^0}^{y^0} dz\Sigma_\rho(x,z)\rho(z,y)$$

Memory integral

$$G_0^{-1} \equiv -\frac{\partial^2}{\partial\tau^2} - \frac{1}{\tau}\frac{\partial}{\partial\tau} + \frac{1}{\tau^2}\frac{\partial^2}{\partial\eta^2} + \nabla_\perp^2 + m^2$$

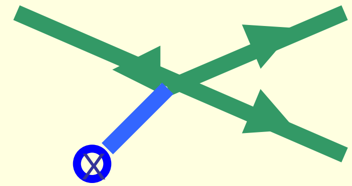
Σ=Self-energies

Self-energies: local $\Sigma_{\rm loc}$ mass shift, nonlocal real Σ_F and imaginary part Σ_ρ

Merit

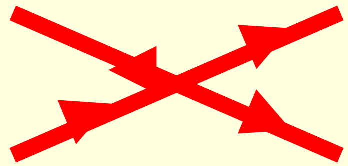
- **Field-Particle Conversion: Particle production from classical field.**

  (Parametric resonance) +



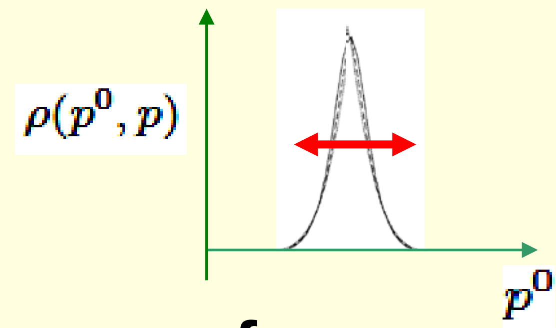
- **Collision of particles → Bose-Einstein distribution**

  binary



Finite decay width

- **Off-shell effect: Memory effects and finite spectral width**



binary collisions (2-to-2) → Rapid Change of distribution functions (Lindner and Muller 2006) +

3-to-1 → entropy production + chemical equilibrium.

Demerit

Numerical simulation needs much memory of computers.

Initial condition

Initial condition: Classical field with vacuum quantum fluctuations (Color Glass Condensate ?)

$$\phi_a(\tau) = \phi(\tau)\delta_{a1} \qquad \phi(\tau_0) = \sqrt{\frac{6N}{\lambda}}\sigma_0 \qquad \begin{aligned}\lambda &= 10 \\ N &= 4\end{aligned}$$

m/σ0=0.1

$$F_{ab} = \mathrm{diag}(F_{\parallel}, F_{\perp}, \ldots, F_{\perp})$$

vacuum

We assume homogeneity in space.

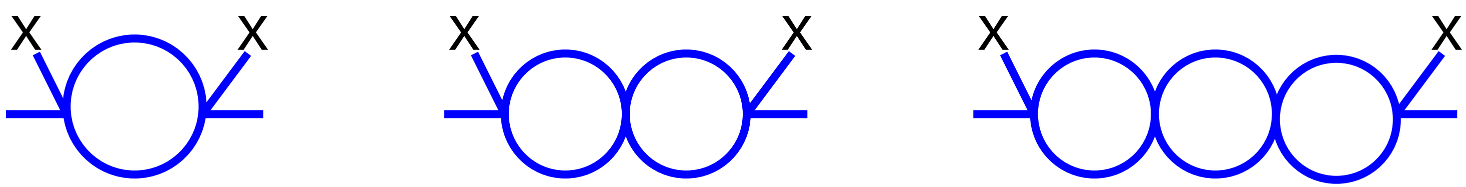
Collision term

Our results (collaboration with Y. Hatta): Quantum collision term.

$\Sigma =$

Normal collision term.

$+$



$+\cdots$

Source induced amplification.

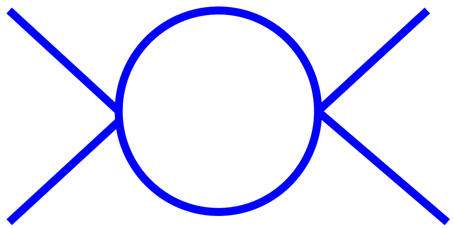
Summation of Next-to-Leading Order of 1/N expansion. This approach covers all evolution of F from O(1) to O(λ^{-1})

Classical Statistical Approximation

Berges et al. (2012)

Gelis et al. (2012)

For example

$\Pi_F =$  $= FF - (1/4)\rho\rho$

Neq~T/p-1/2

F~(1/τ)n_p, ρ~1/τ

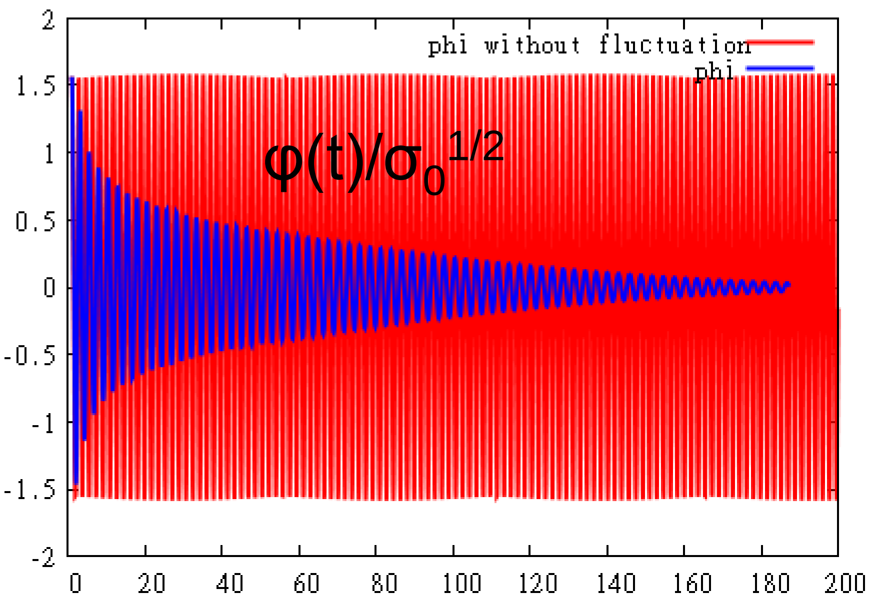
n_p~1/λ,at late time

This approximation is good in the case of dense system FF>>ρρ (Weakly coupled). But if FF~ρρ (Normal coupling), the difference appears.

Comparison of Nonexpanding and Expanding systems (2+1 dim)

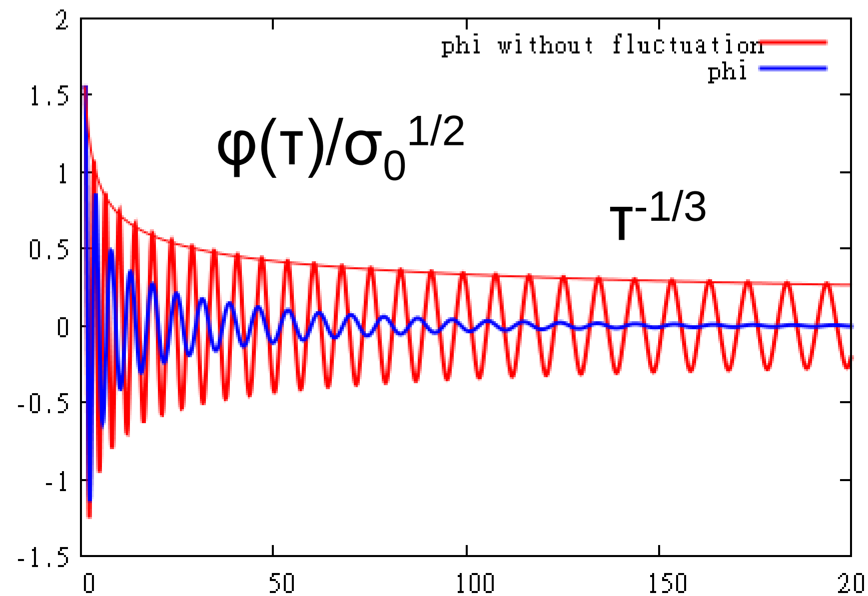
Evolution of classical field

Nonexpanding

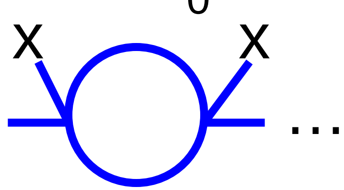


t/t_0

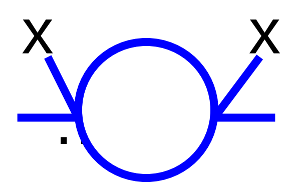
Expanding



τ/τ_0



$$\frac{1}{\tau}\partial_\tau \phi_a \; +$$



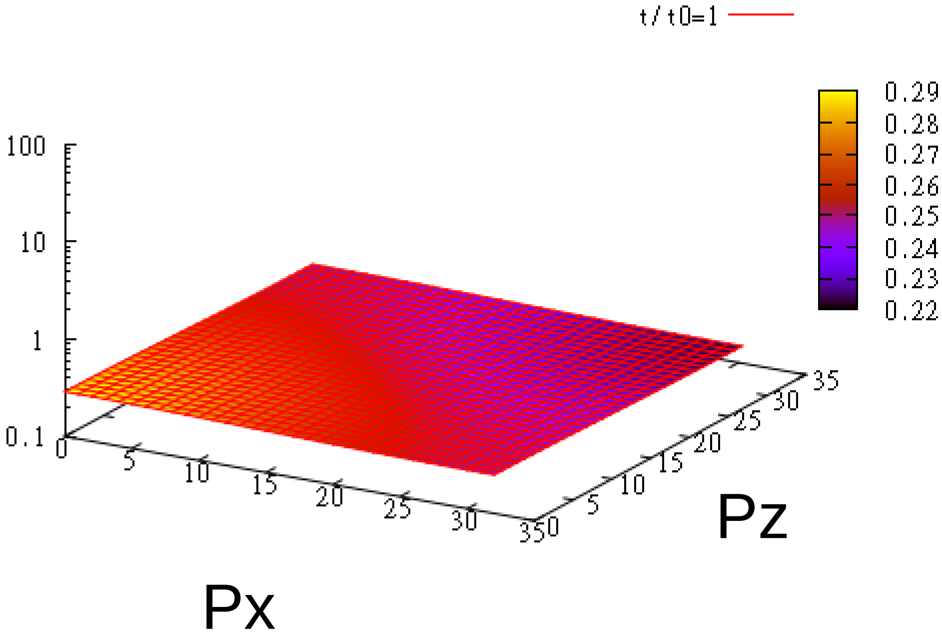
The above term + source induced amplification

Evolution of Green's functions F

Quantum evolution

Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

Evolution of Green's functions F

Quantum evolution

Nonexpanding

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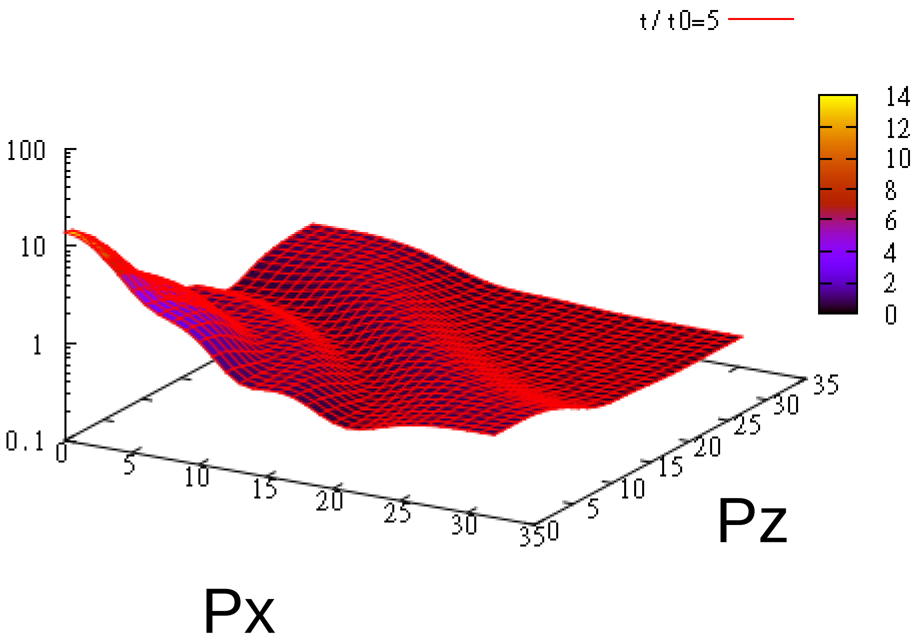


Evolution of Green's functions F

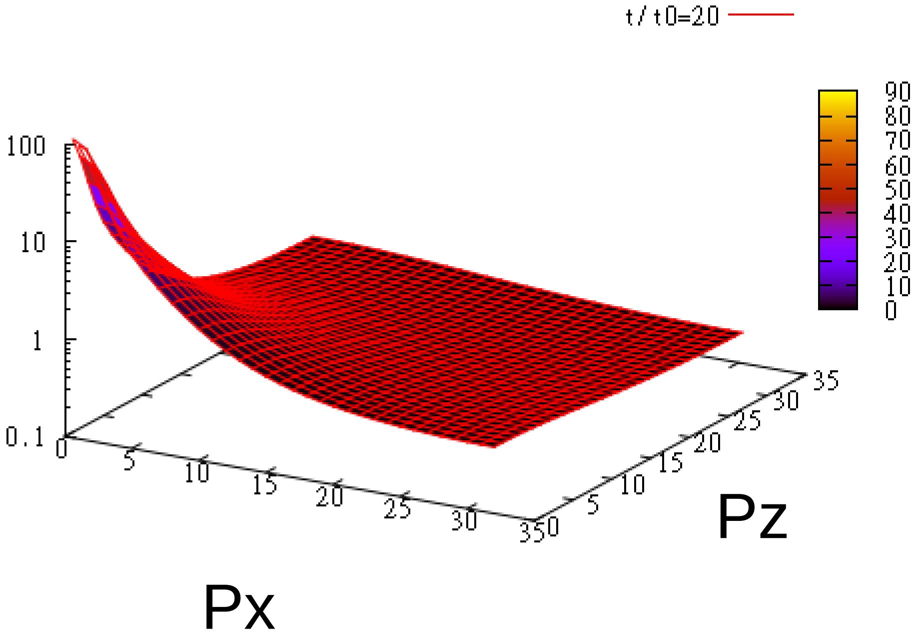
Quantum evolution

Nonexpanding

Expanding

$\sigma_0 \ F_{\|}(t, t, p_T, p_z)$

$F_{\|}(\tau, \tau, p_T, p_\eta)$



Evolution of Green's functions F

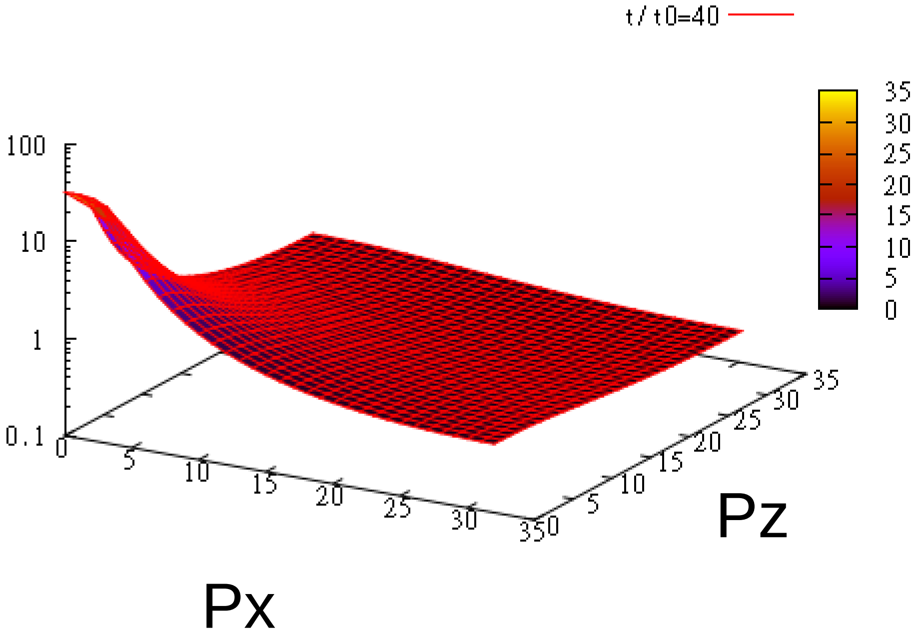
Quantum evolution

Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$

Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$



Evolution of Green’s functions F

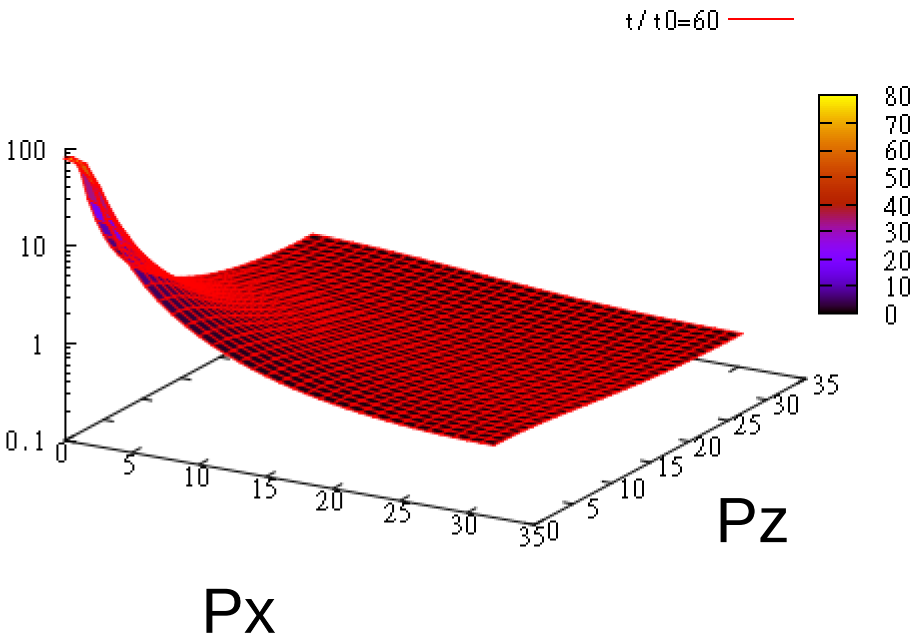
Quantum evolution

Nonexpanding

$\sigma_0 \; F_{||}(t, t, p_T, p_z)$

Expanding

$F_{||}(\tau, \tau, p_T, p_\eta)$



Evolution of Green's functions F

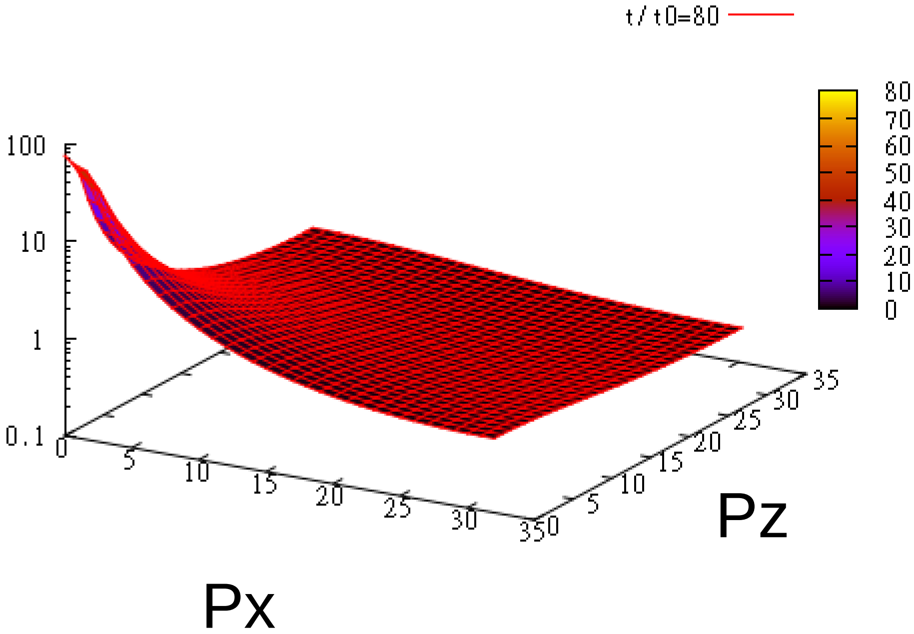
Quantum evolution

Nonexpanding

Expanding

σ$_0$ $F_{\|}(t,t,p_T,p_z)$

$F_{\|}(\tau,\tau,p_T,p_\eta)$



Evolution of Green's functions F

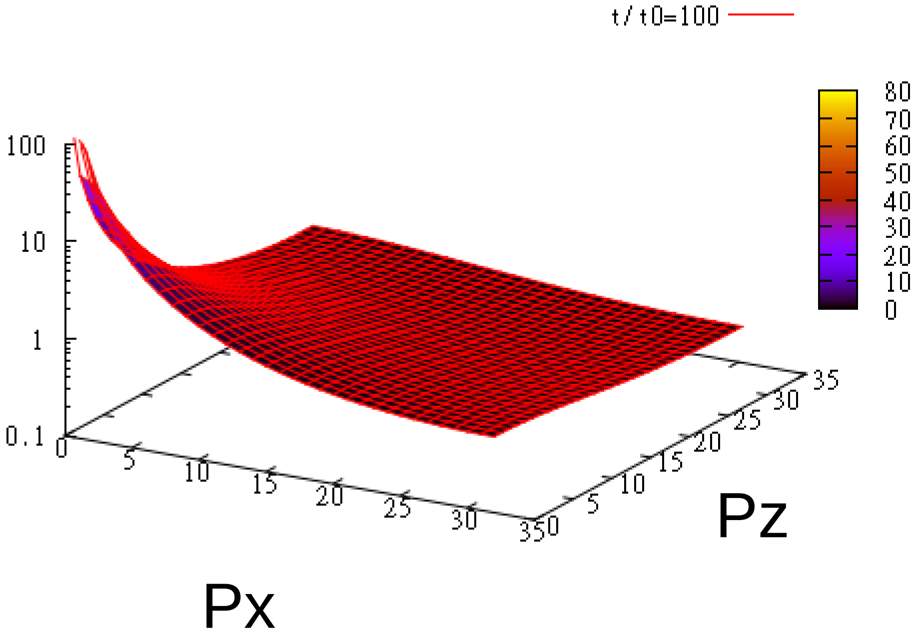
Quantum evolution

Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$

Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$



Evolution of Green's functions F

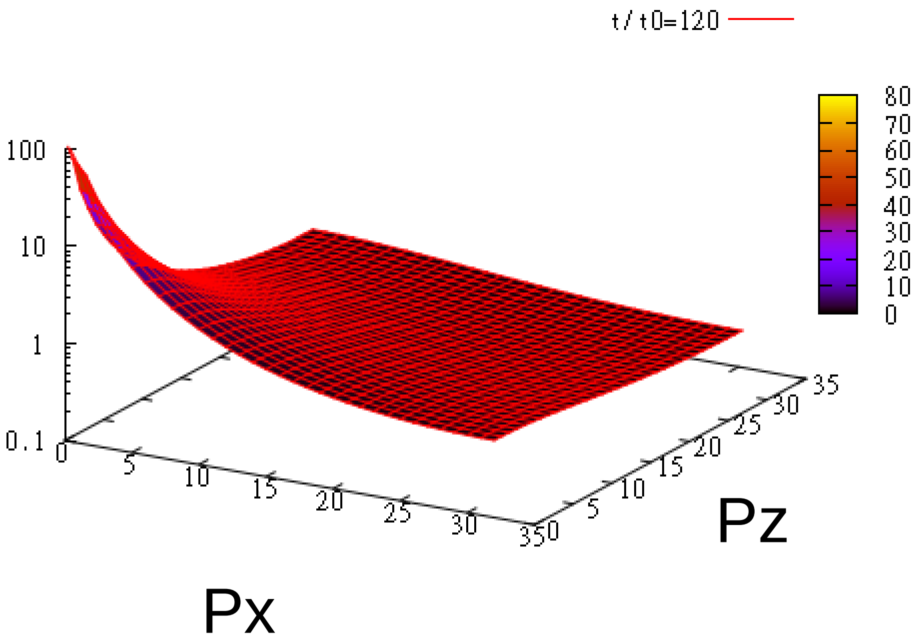
Quantum evolution

Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$

Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$



Evolution of Green's functions F

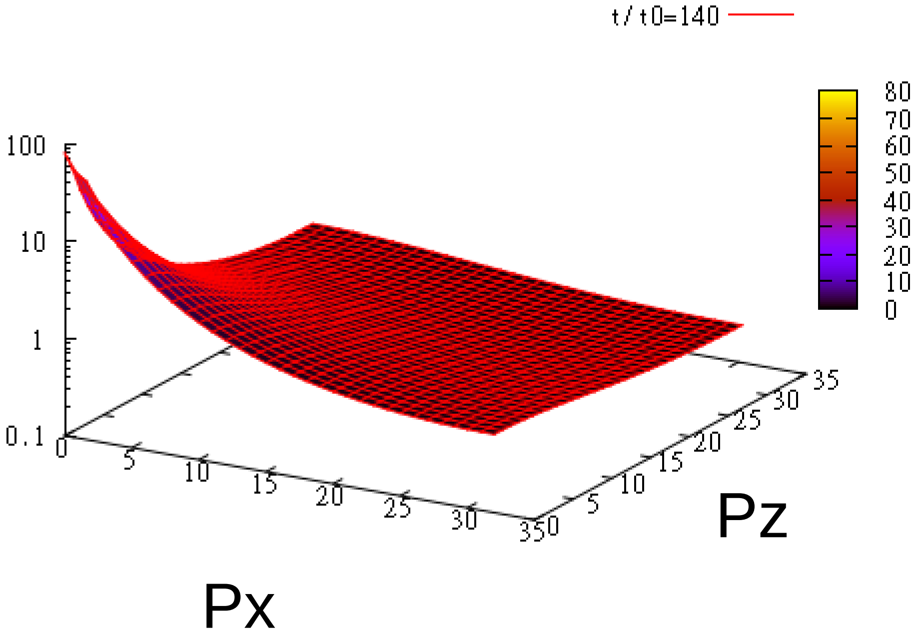
Quantum evolution

Nonexpanding

Expanding

σ$_0$ $F_{\|}(t, t, p_T, p_z)$

$F_{\|}(\tau, \tau, p_T, p_\eta)$

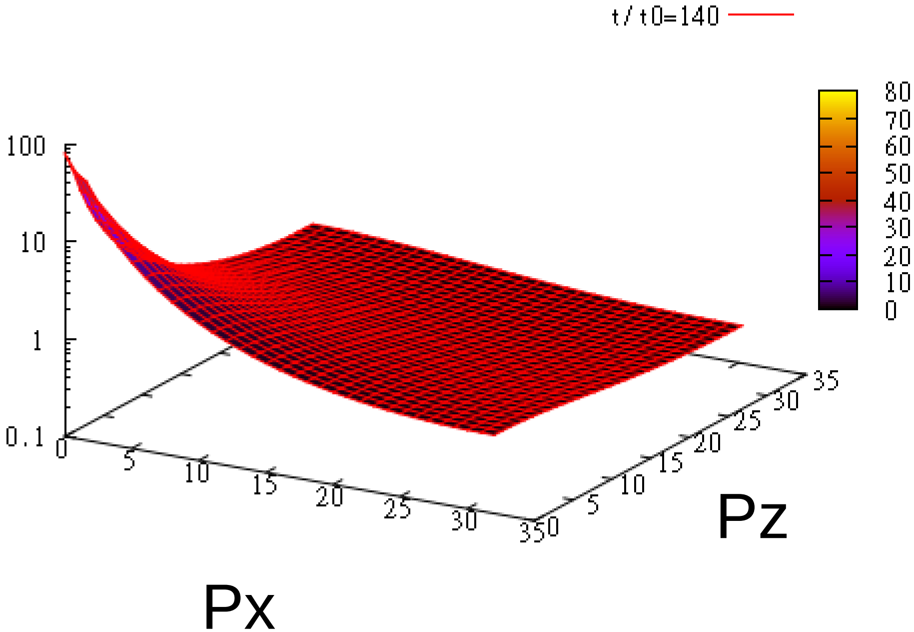


Evolution of Green's functions F

Quantum evolution

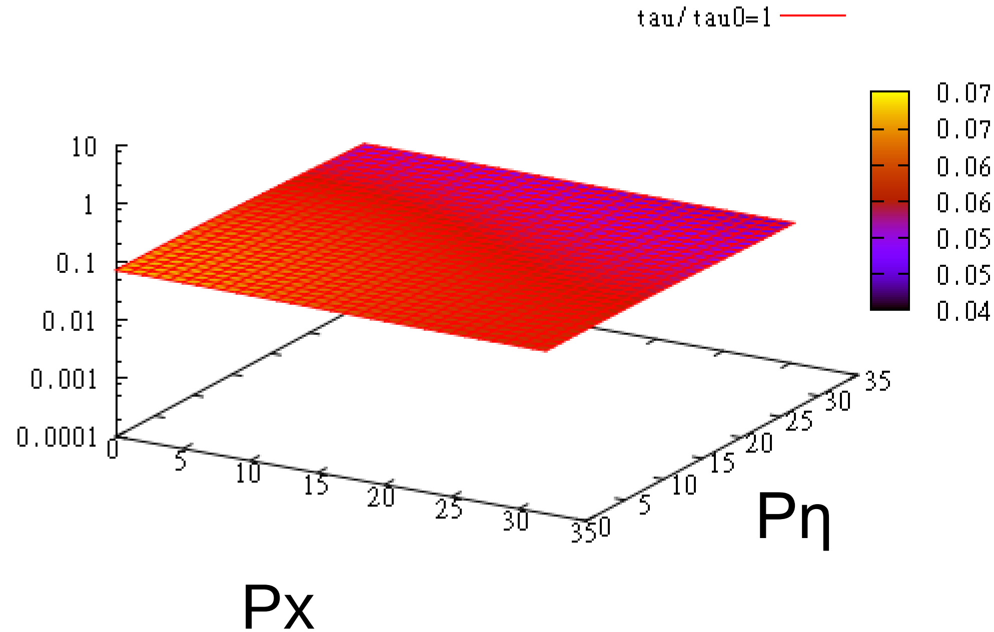
Nonexpanding

$$\sigma_0 \; F_{\|}(t,t,p_T,p_z)$$



Expanding

$$F_{\|}(\tau,\tau,p_T,p_\eta)$$

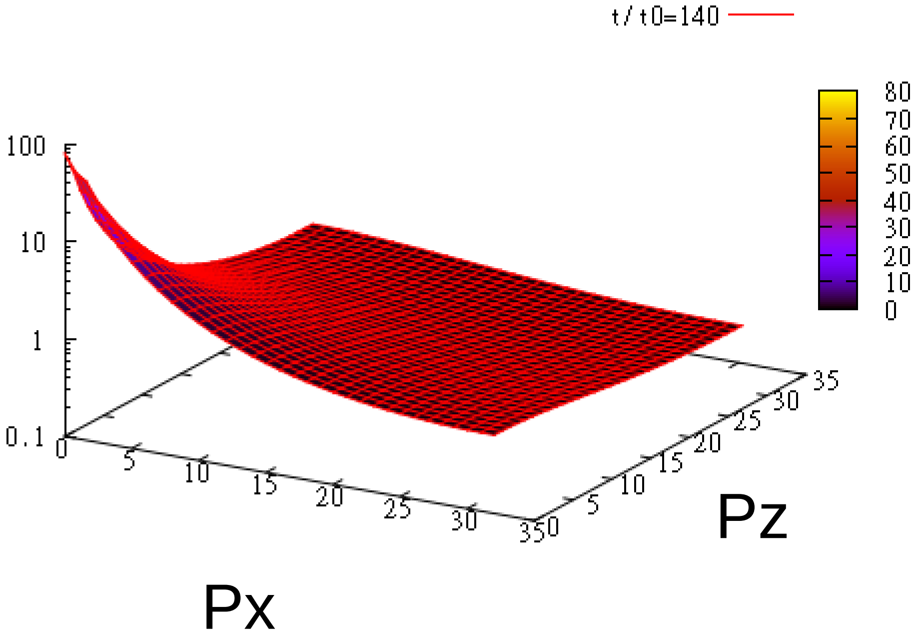


Evolution of Green's functions F

Quantum evolution

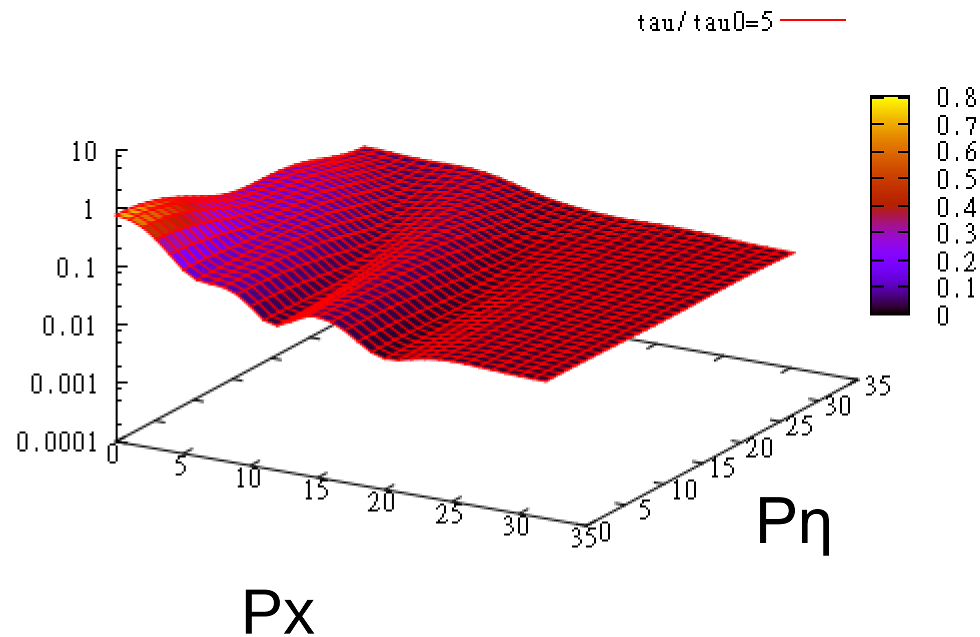
Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

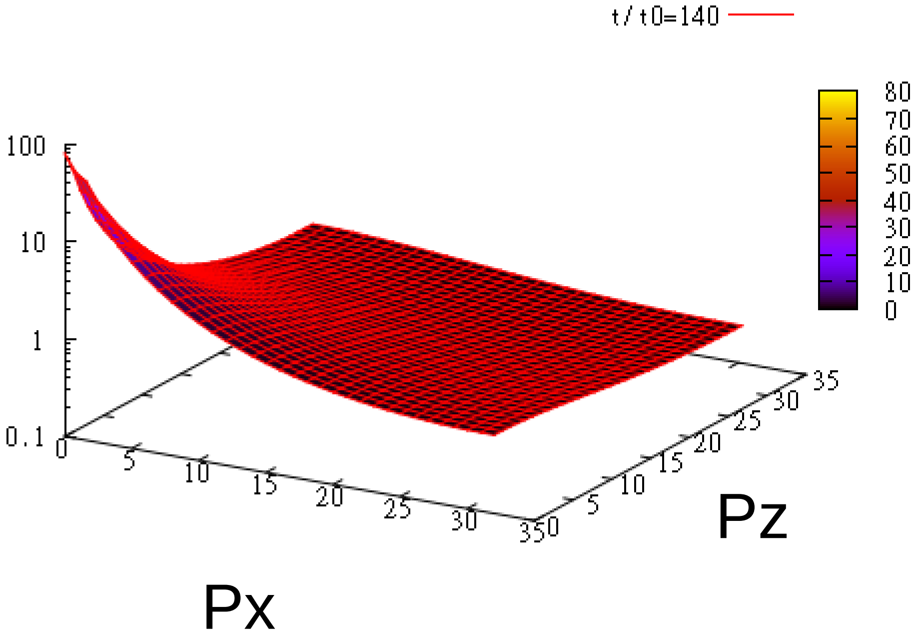


Evolution of Green's functions F

Quantum evolution

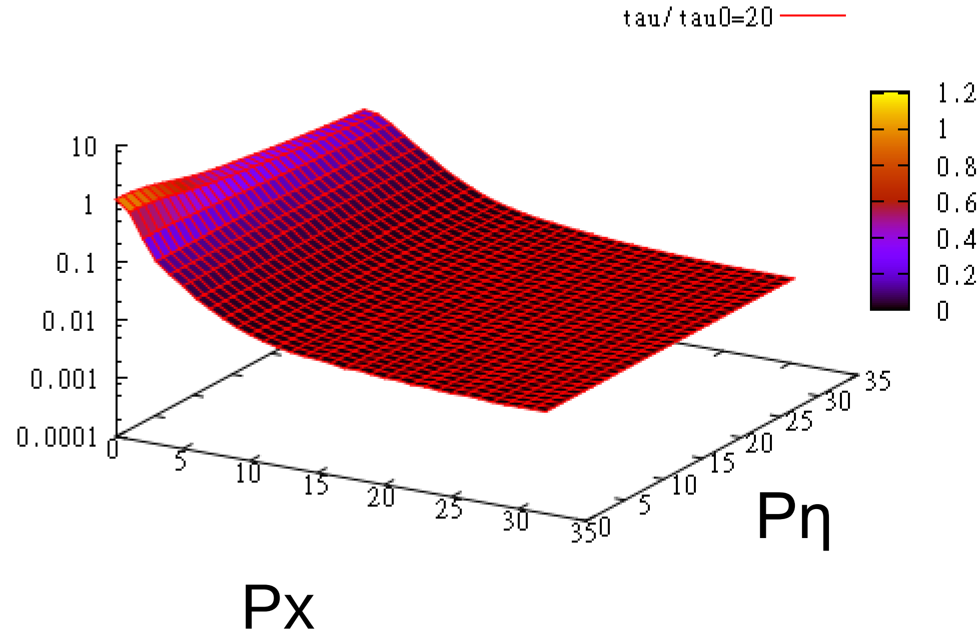
Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

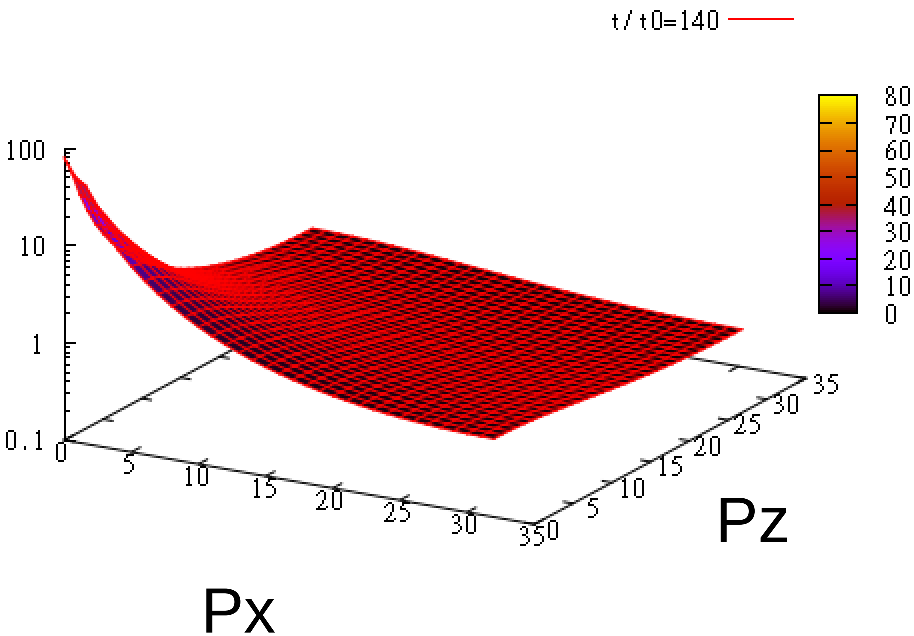


Evolution of Green's functions F

Quantum evolution

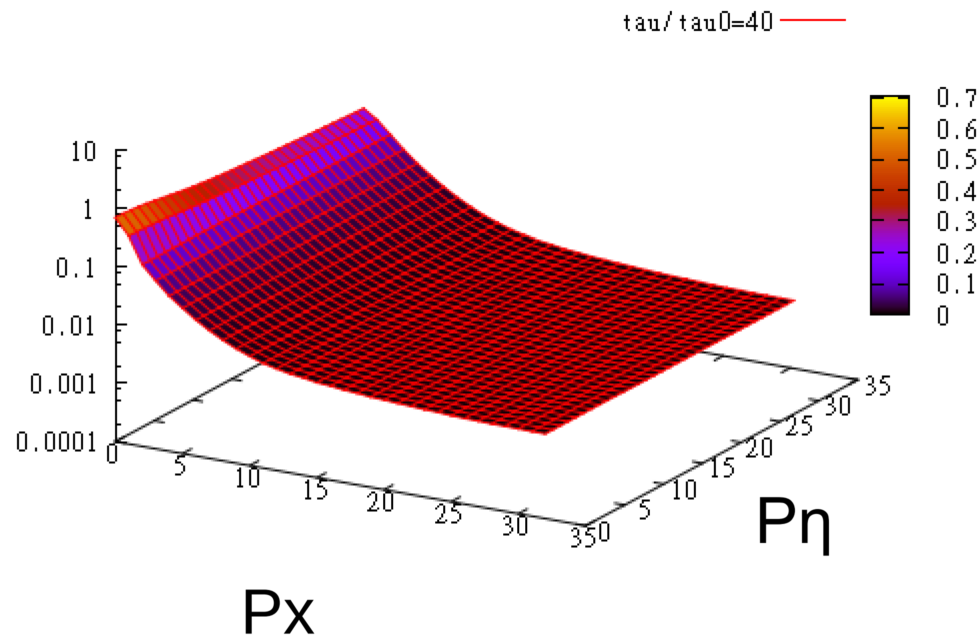
Nonexpanding

$$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$$



Expanding

$$F_{\|}(\tau, \tau, p_T, p_\eta)$$

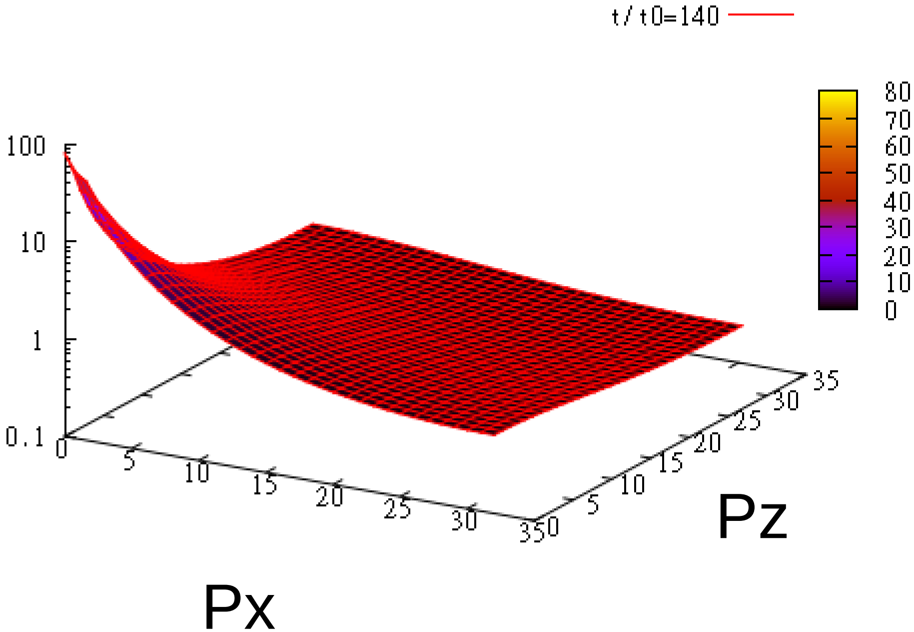


Evolution of Green's functions F

Quantum evolution

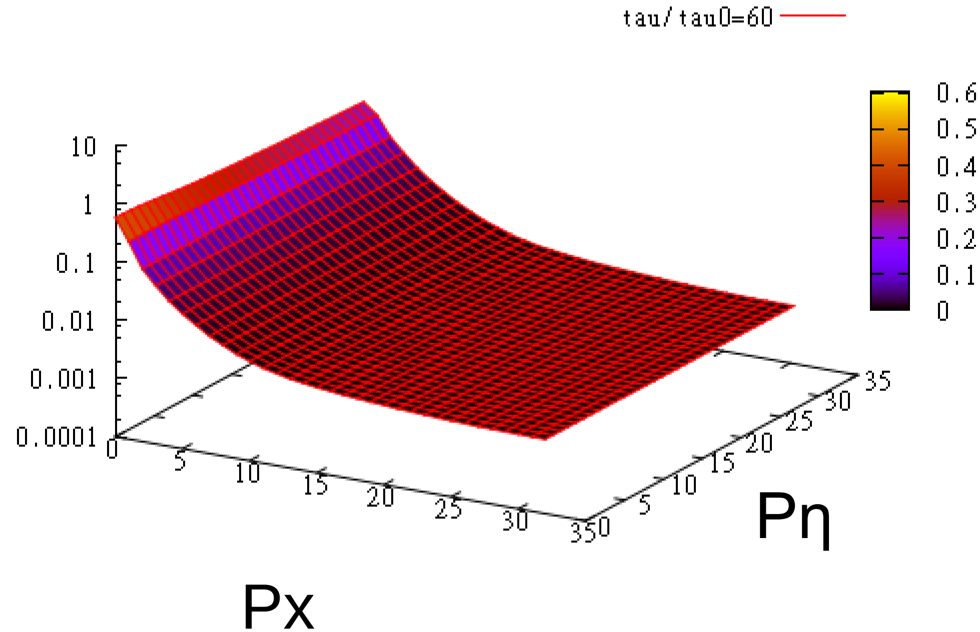
Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

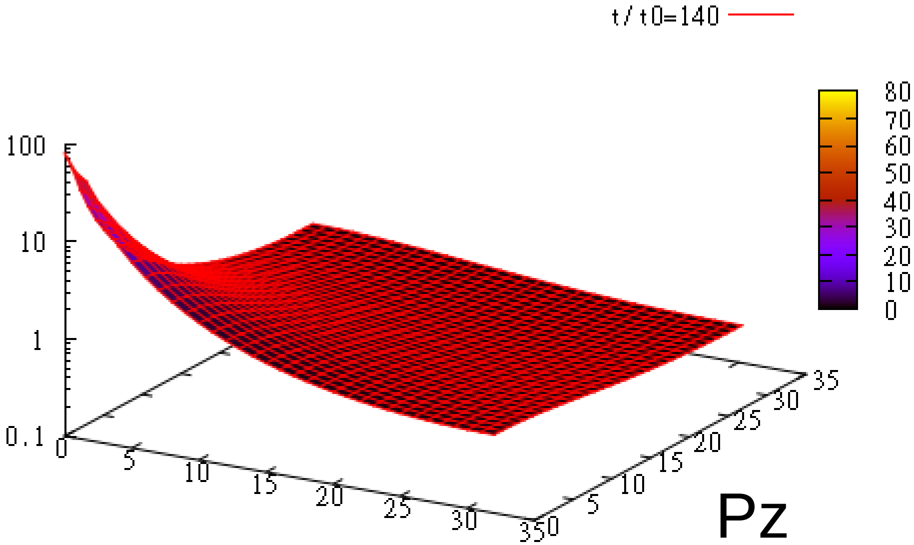


Evolution of Green's functions F

Quantum evolution

Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$

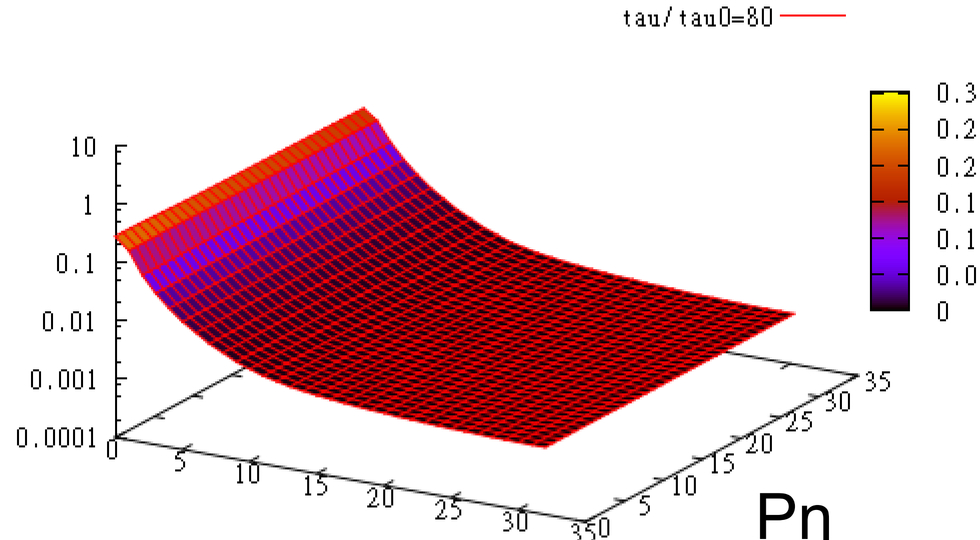


Pz

Px

Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$



Pη

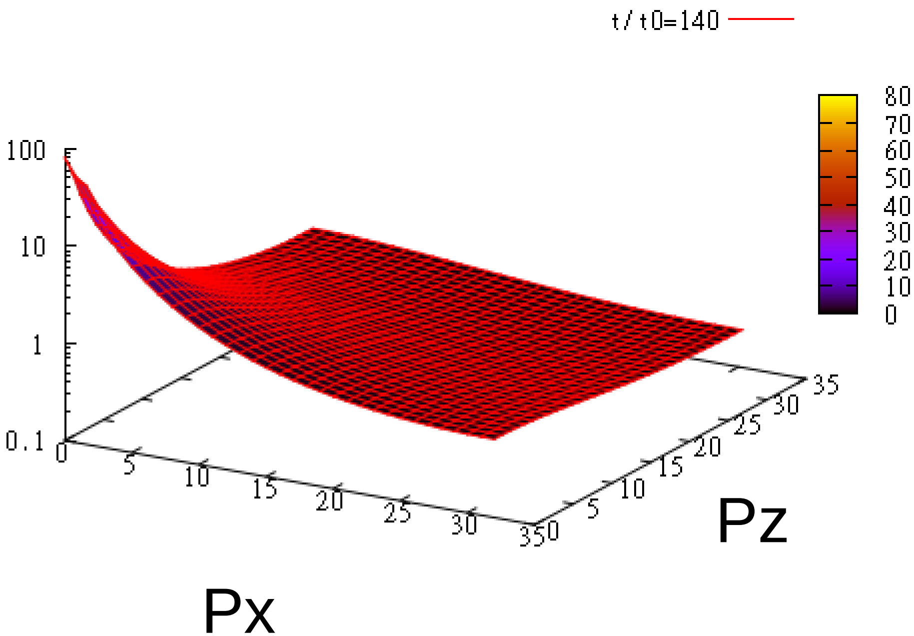
Px

Evolution of Green's functions F

Quantum evolution

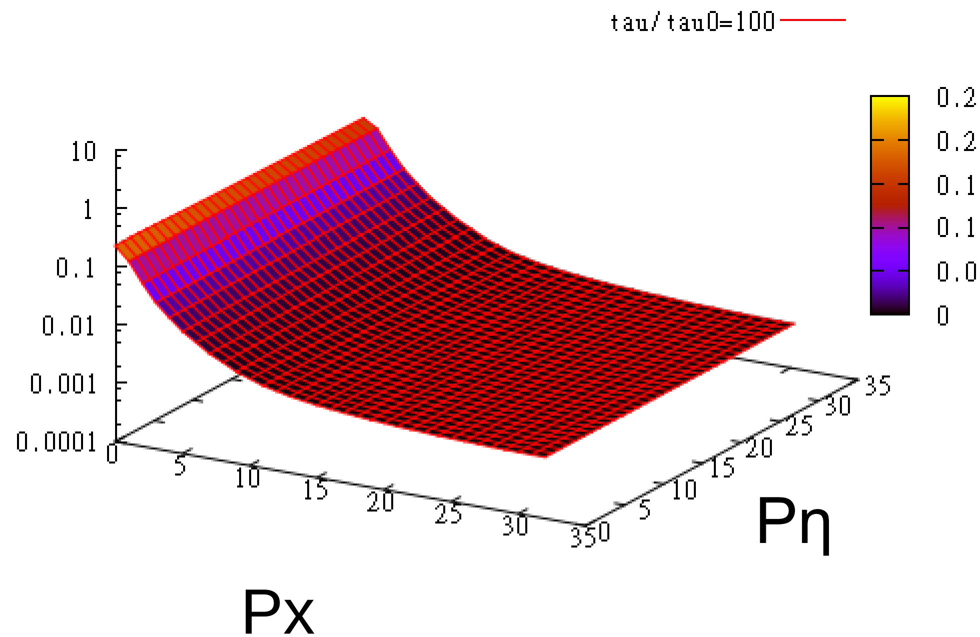
Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

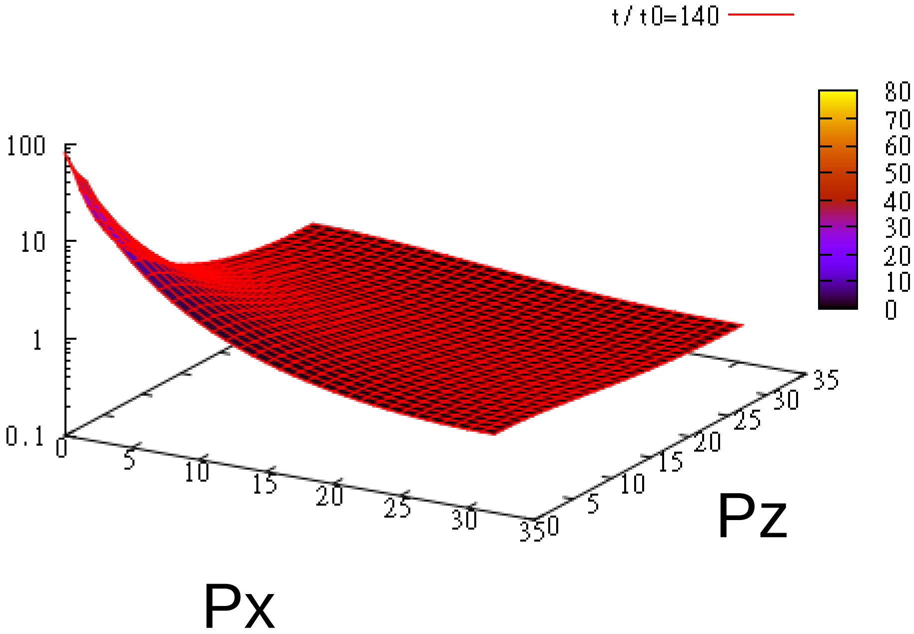


Evolution of Green's functions F

Quantum evolution

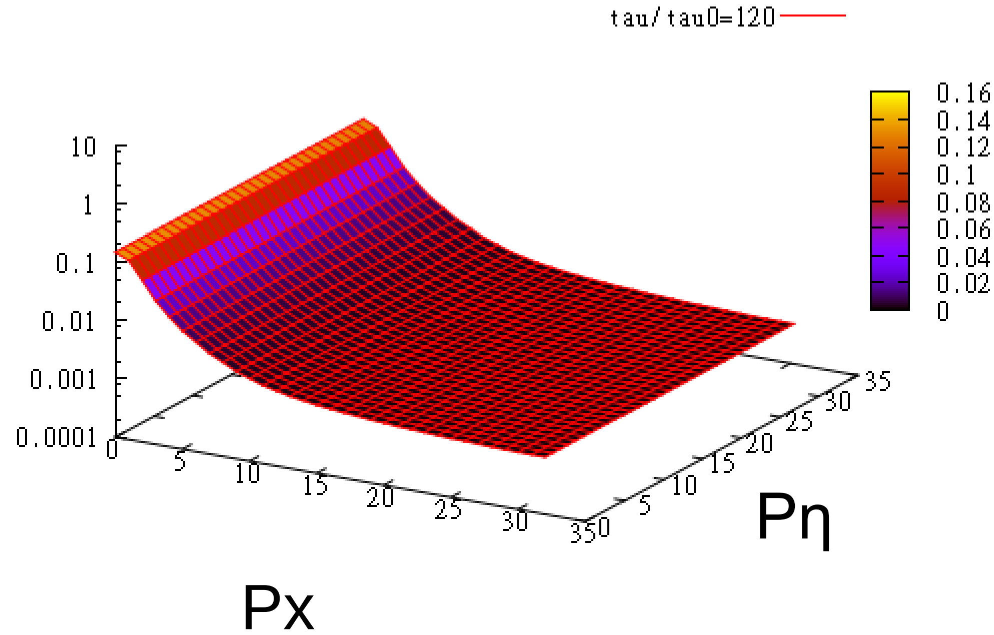
Nonexpanding

$\sigma_0 \; F_{\|}(t, t, p_T, p_z)$



Expanding

$F_{\|}(\tau, \tau, p_T, p_\eta)$

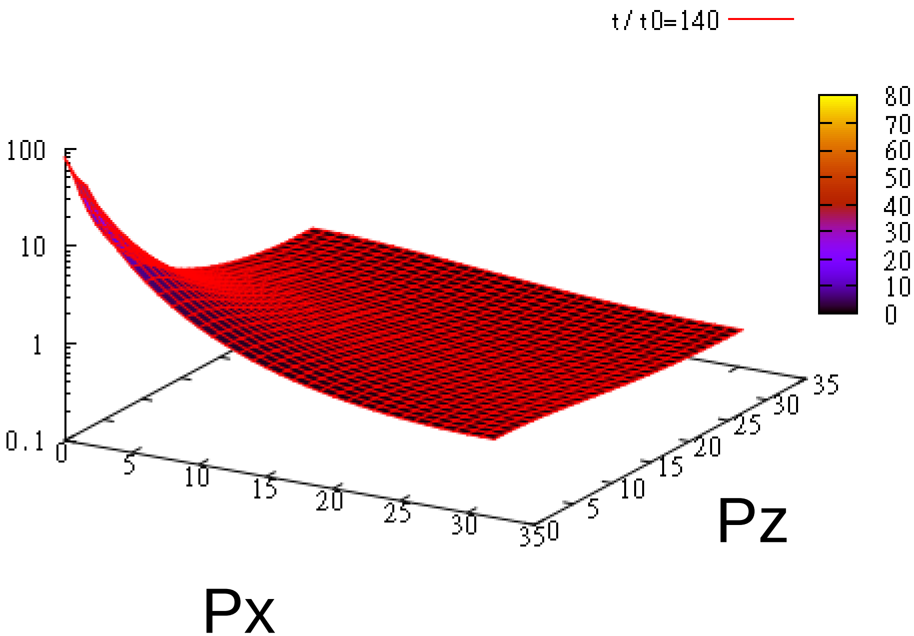


Evolution of Green's functions F

Quantum evolution

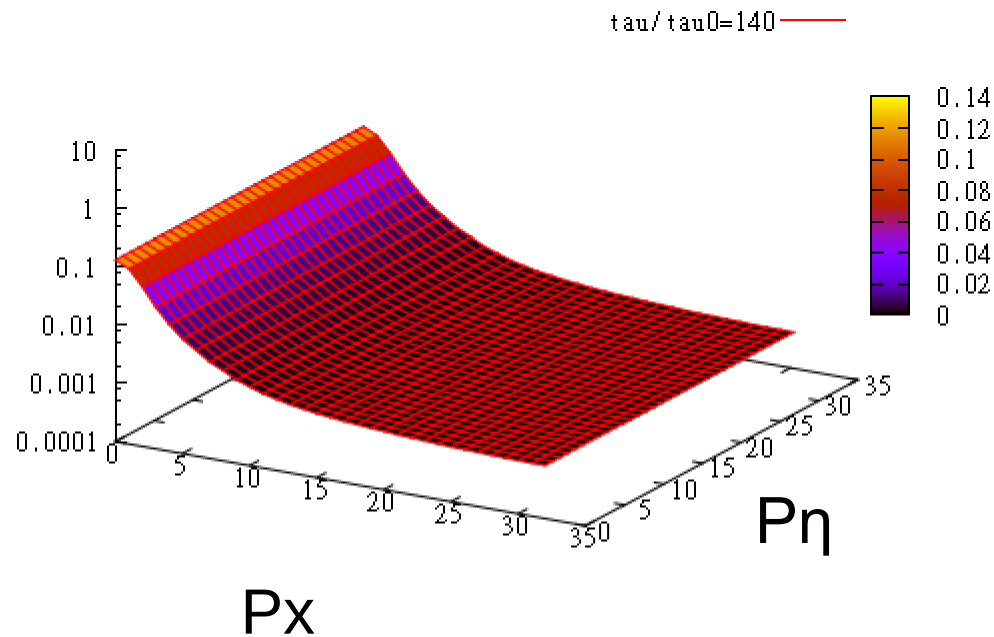
Nonexpanding

$\sigma_0 \; F_{\parallel}(t, t, p_T, p_z)$



Expanding

$F_{\parallel}(\tau, \tau, p_T, p_\eta)$



$$G_0^{-1} \equiv -\frac{\partial^2}{\partial \tau^2} - \frac{1}{\tau}\frac{\partial}{\partial \tau} + \frac{1}{\tau^2}\frac{\partial^2}{\partial \eta^2} + \nabla_\perp^2 + m^2$$

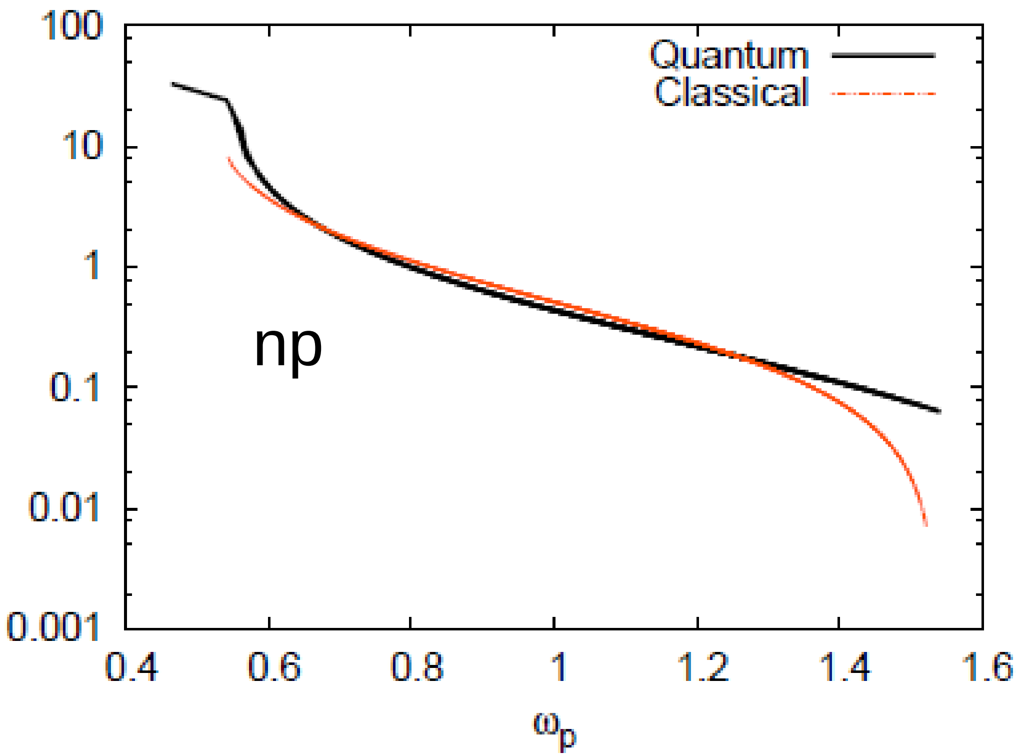
$\to p_\eta^2/\tau^2$

$P_z = P_\eta/\tau$

Late-time distribution function np

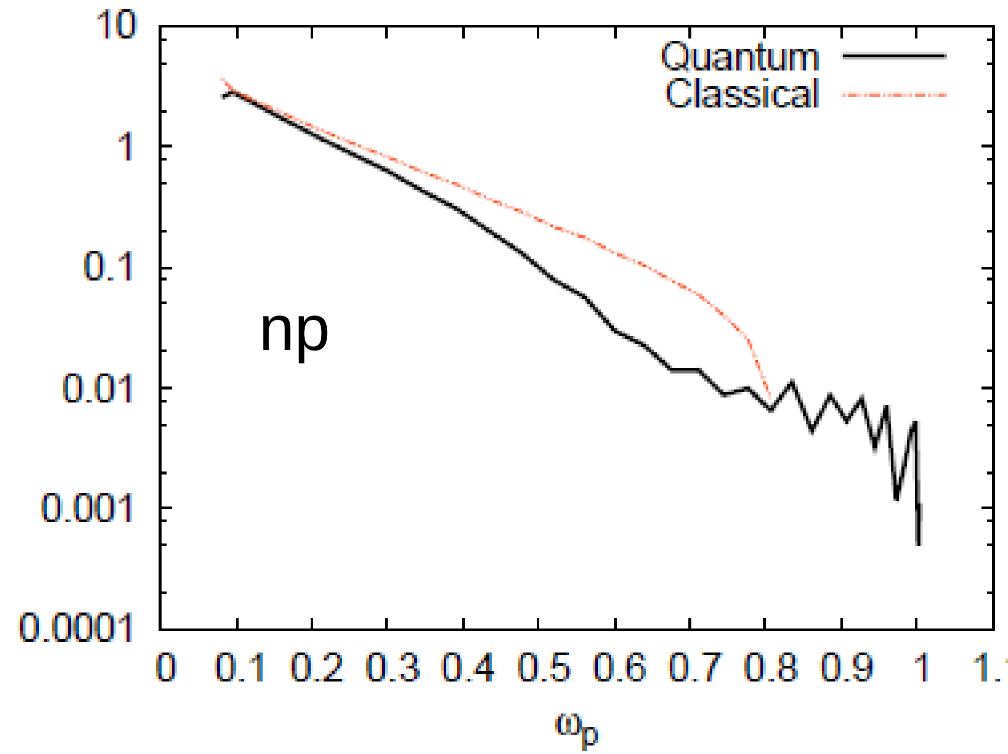
Nonexpanding

t/t0=150



Expanding

τ/τ0=150



Classical statistical approximation: Neq~ T/p-1/2

Summary

- **We have considered the Kadanoff-Baym approach to thermalization of O(N) scalar fields from initial background classical field with longitudinal expansion in 2+1 dimensions.**
- **Field-particle conversion occurs when we include effects of fluctuation. Then classical field damps rapidly due to expansion of the system compared with nonexpanding system.**
- **In both nonexpanding and expanding system, late-time Boltzmann tails are realized in Quantum evolution.**
- **In classical statistical approximation, the late-time distribution is not Bose-Einstein type (Normal coupling)**

Remaining problems

- **Application to non-Abelian gauge theories in expanding system.**
- **Initial condition in an expanding system (Color Glass Condensate with vacuum fluctuations).**
- **Renormalization procedure in an expanding system.**
- **Tuning of program codes.**

F

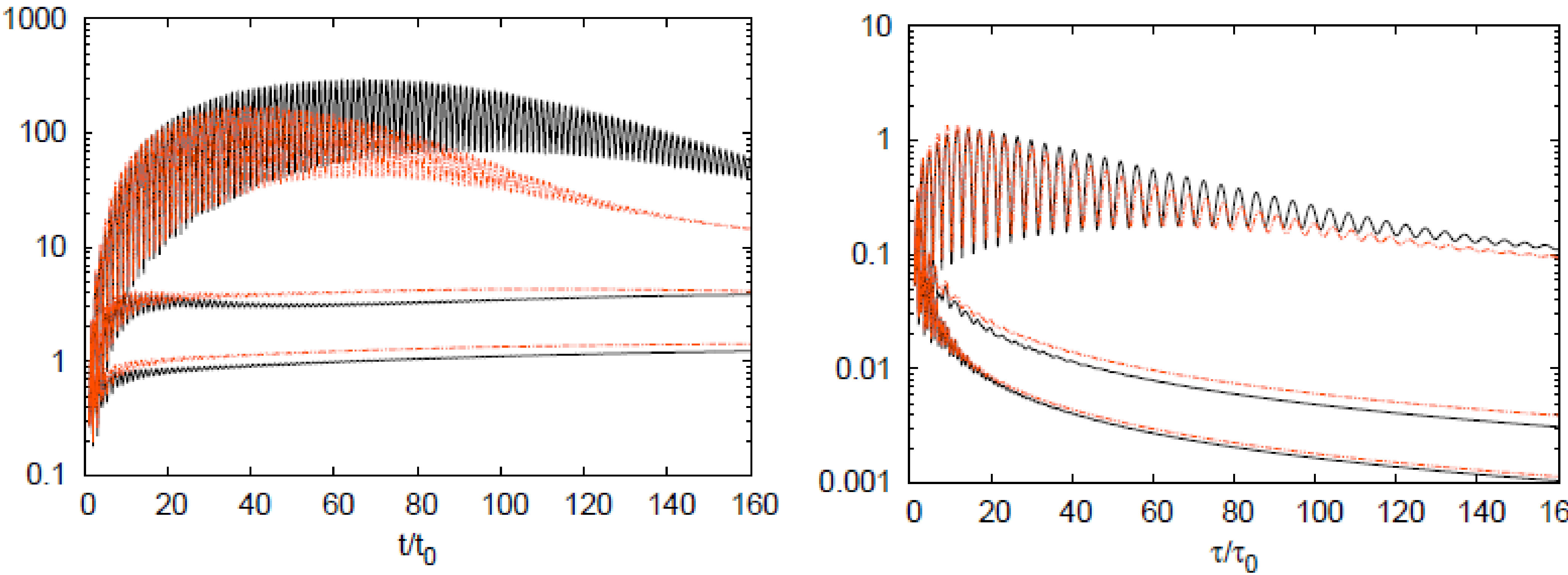
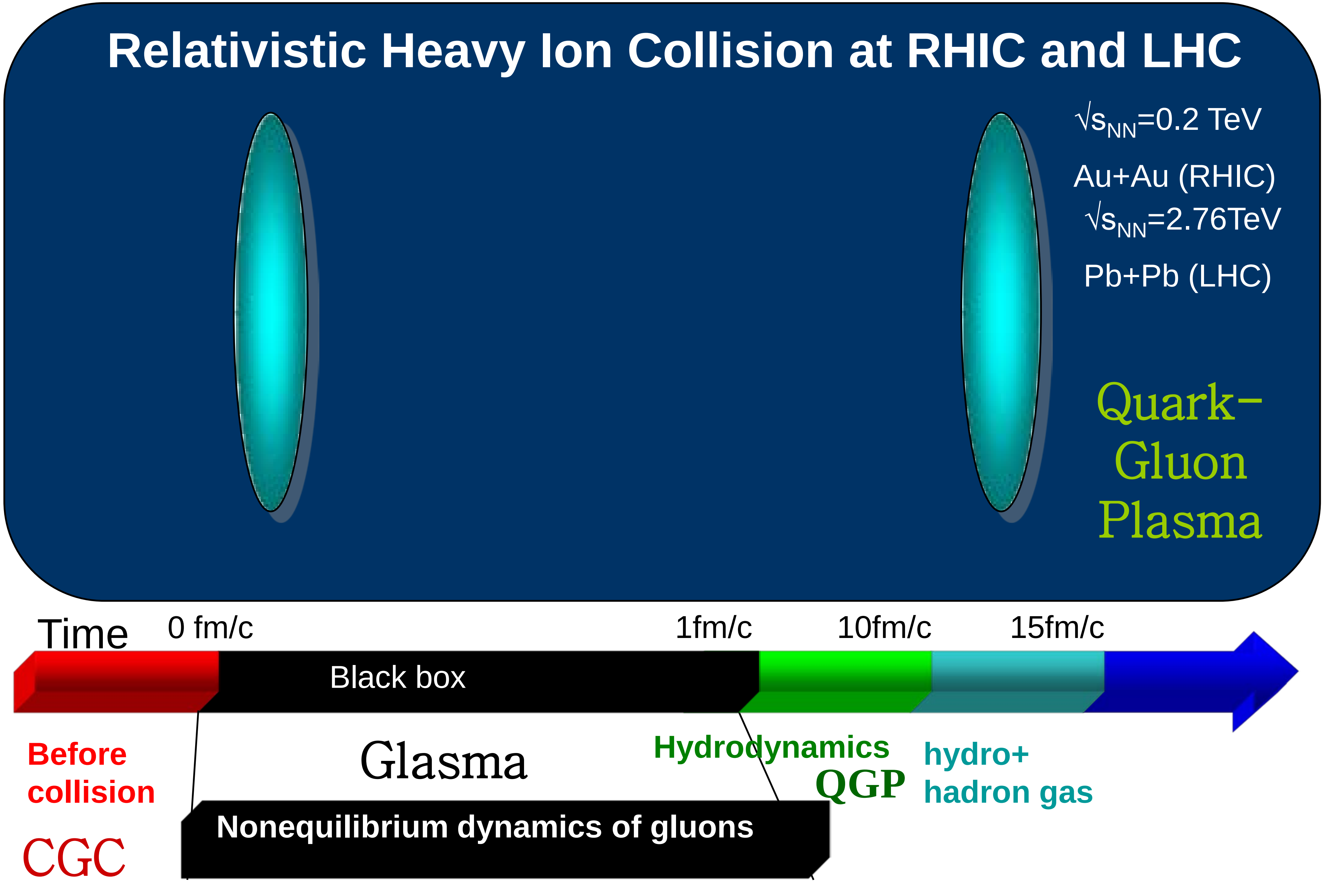
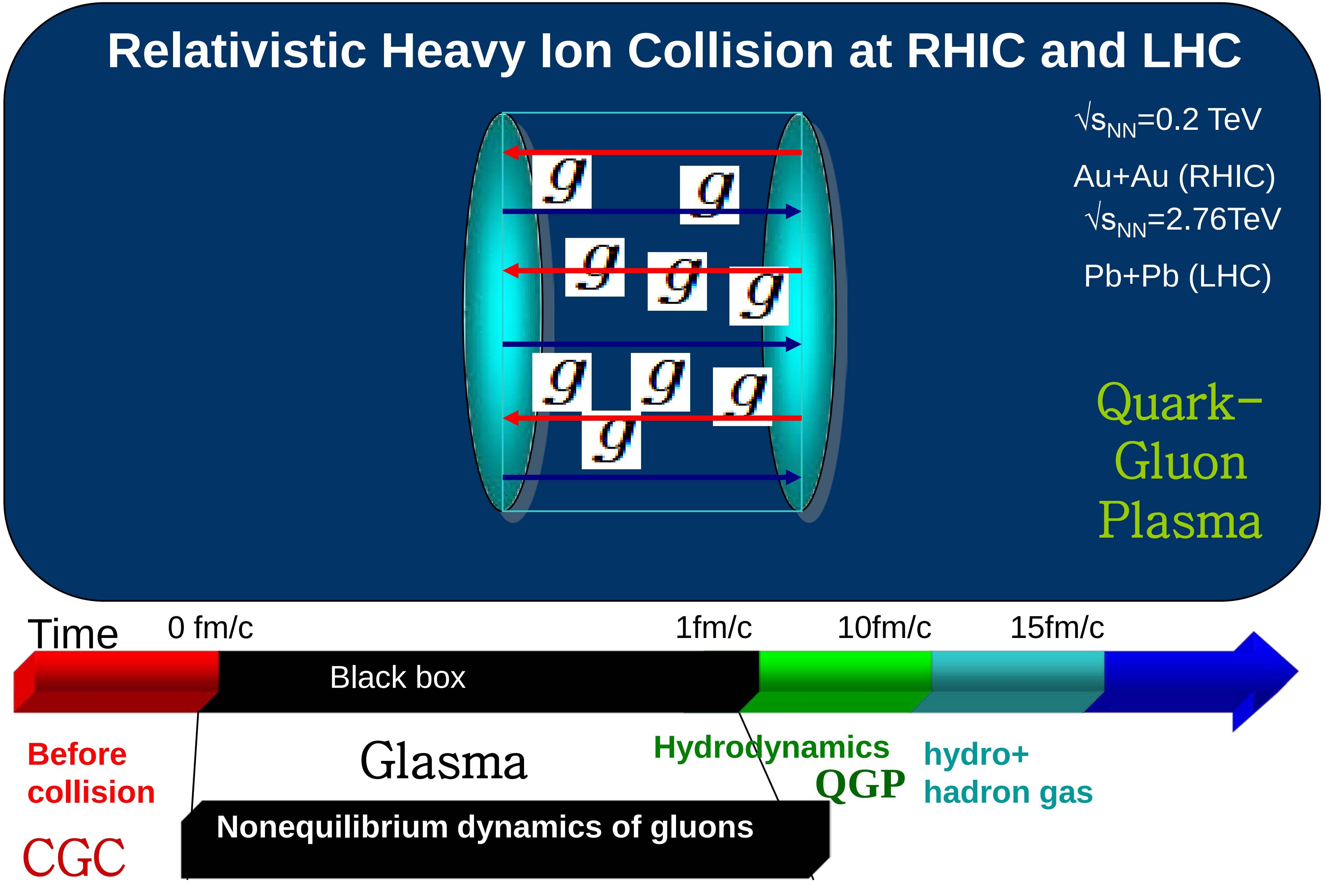


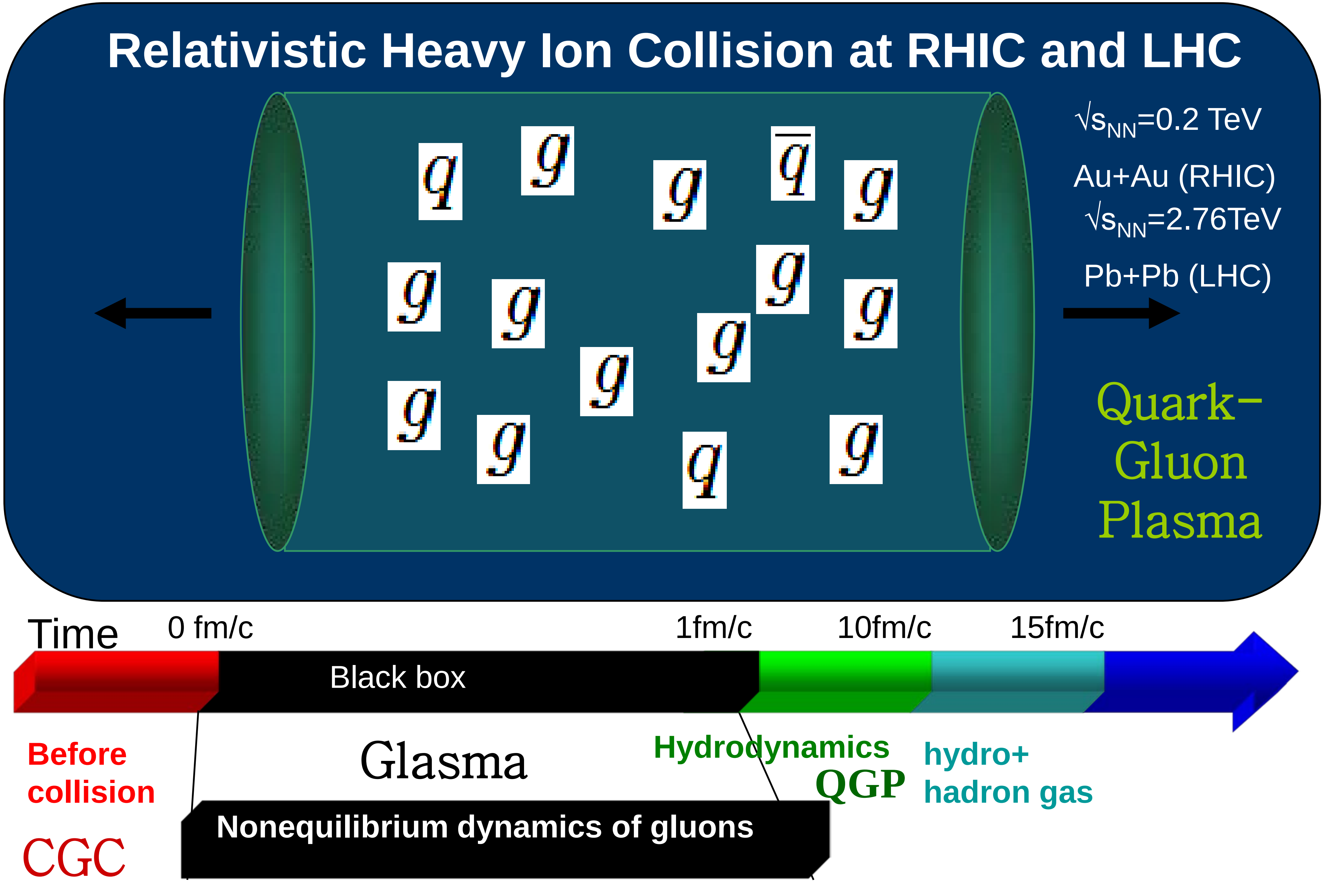
Fig. 4. Time evolution of the statistical function at strong coupling $\lambda = 10$ for three different values of n_T: $n_T = 0$, $n_T = 8$, $n_T = 16$ (from top to bottom).



Relativistic Heavy Ion Collision at RHIC and LHC
√sNN=0.2 TeV
Au+Au (RHIC)
√sNN=2.76TeV
Pb+Pb (LHC)
Quark-
Gluon
Plasma
Time
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before
collision
CGC
Glasma
Nonequilibrium dynamics of gluons
Hydrodynamics
QGP
hydro+
hadron gas



Relativistic Heavy Ion Collision at RHIC and LHC
g
√s_NN=0.2 TeV
Au+Au (RHIC)
√s_NN=2.76TeV
Pb+Pb (LHC)
Quark–
Gluon
Plasma
Time
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before
collision
CGC
Glasma
Nonequilibrium dynamics of gluons
Hydrodynamics
QGP
hydro+
hadron gas

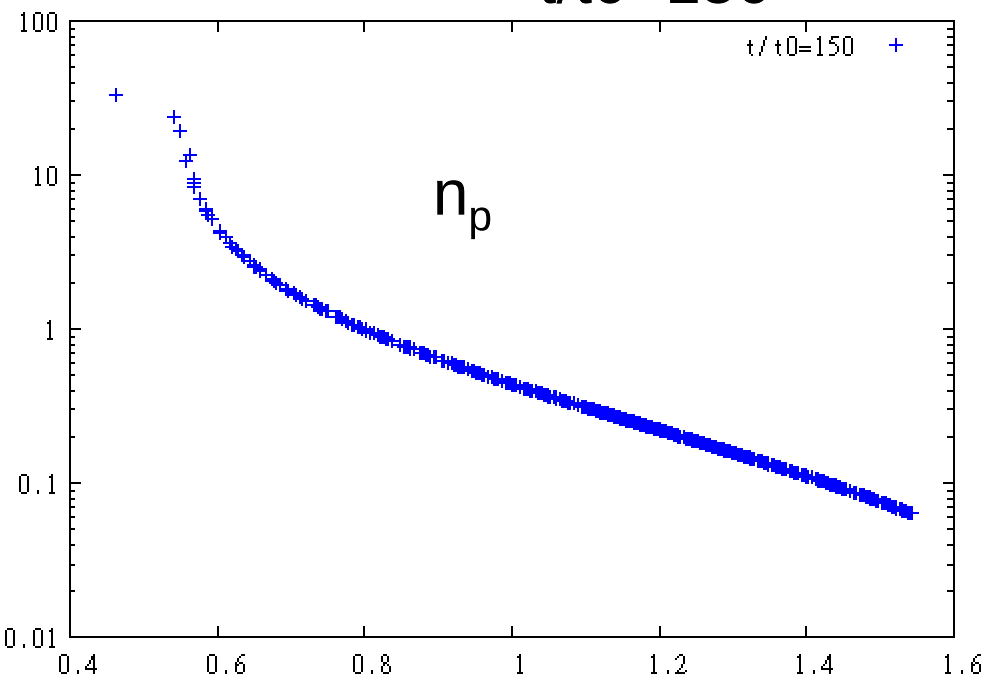


Relativistic Heavy Ion Collision at RHIC and LHC
$\sqrt{s_{NN}}$=0.2 TeV
Au+Au (RHIC)
$\sqrt{s_{NN}}$=2.76TeV
Pb+Pb (LHC)
Quark–Gluon Plasma
Time
0 fm/c
1fm/c
10fm/c
15fm/c
Black box
Before collision
CGC
Glasma
Nonequilibrium dynamics of gluons
Hydrodynamics
QGP
hydro+ hadron gas

Late-time distribution function (Quantum evolution)

Nonexpanding

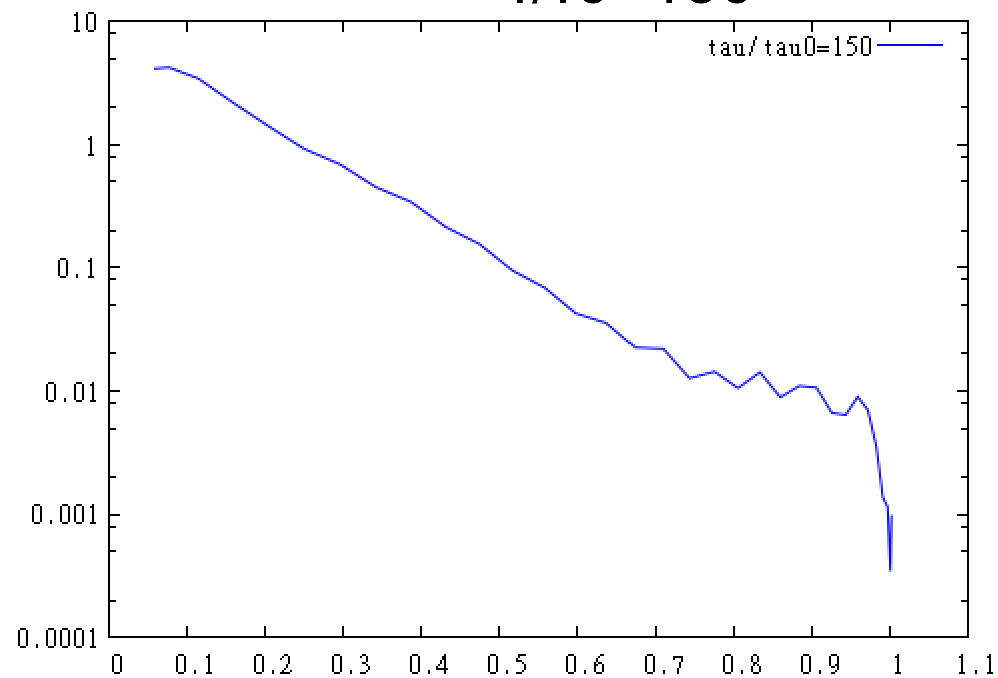
t/t0=150



ε_p

Expanding

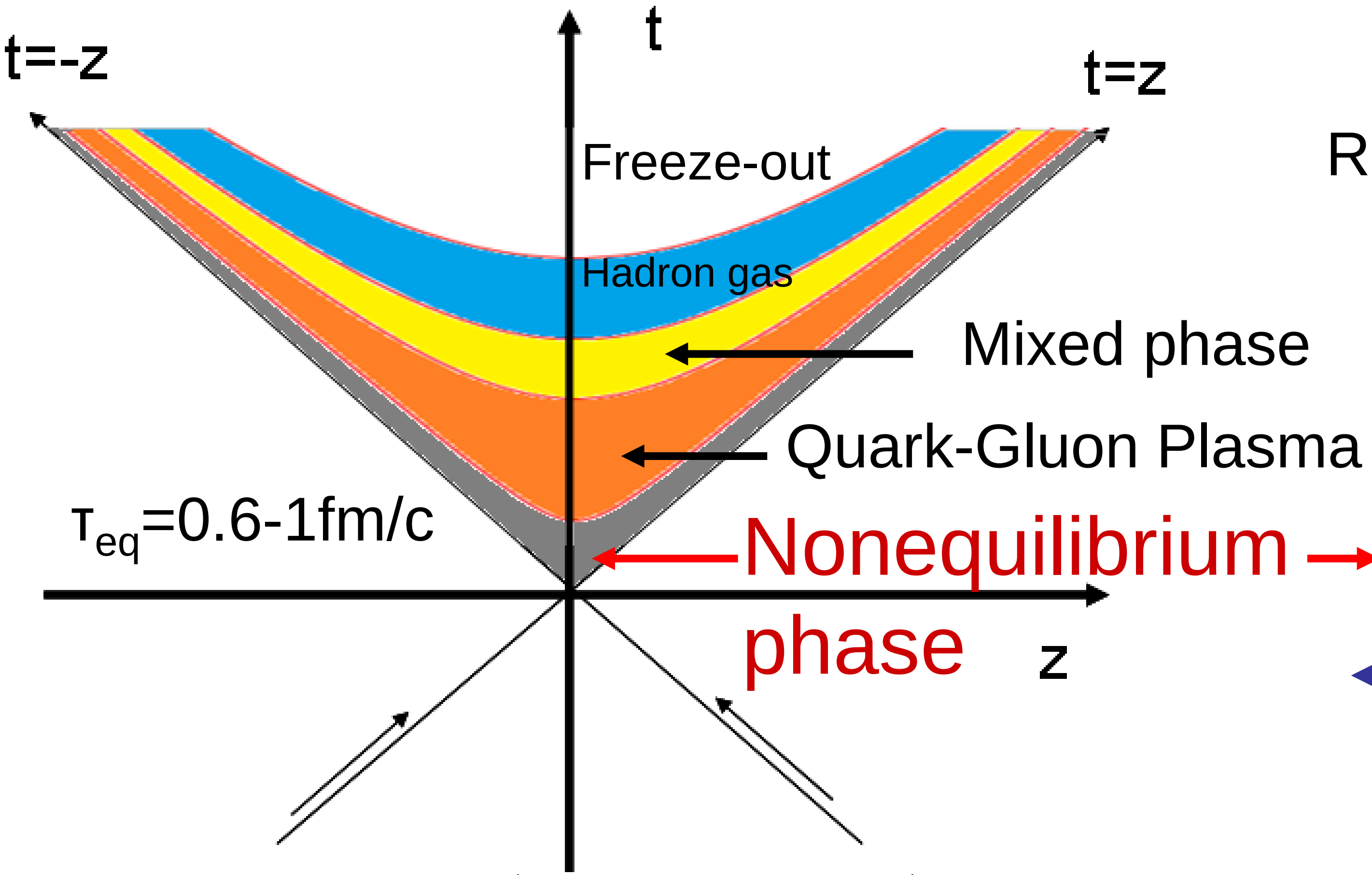
τ/τ0=150



ω_p

$$n_p(\tau) \equiv \tau \omega_p F(\tau, \tau, p)/C - \frac{1}{2}$$

Relativistic Heavy Ion Collison at RHIC and LHC



$\sqrt{s_{NN}}$=0.2 TeV

Au+Au (RHIC)

$\sqrt{s_{NN}}$=2.76TeV

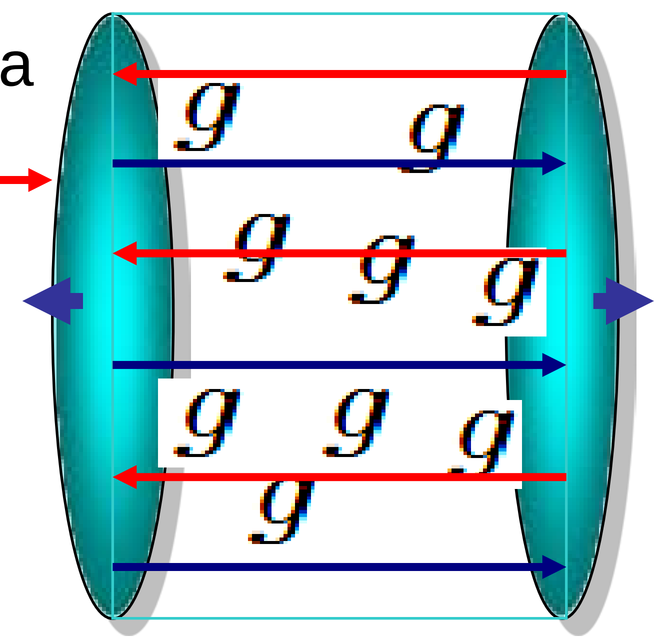
Pb+Pb (LHC)

Proper time

$$\tau = \sqrt{t^2 - z^2}$$

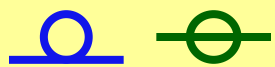
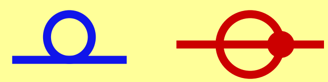
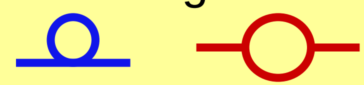
Rapidity

$$\eta = \tanh^{-1} \frac{z}{t}$$



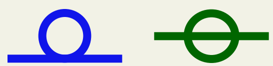
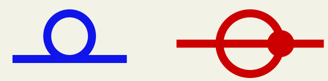
Time irreversibility

Symmetric phase ⟨Φ⟩=0

	$\lambda\Phi^4$	O(N)	SU(N)
Exact 2PI (no truncation)	×	×	×
Truncation	NLO of λ  △	NLO of 1/N  △	LO of g^2  △(TAG)
LO of Gradient expansion **H-theorem**	○	○	○ (TAG)

Numerical Simulation for KB eq.

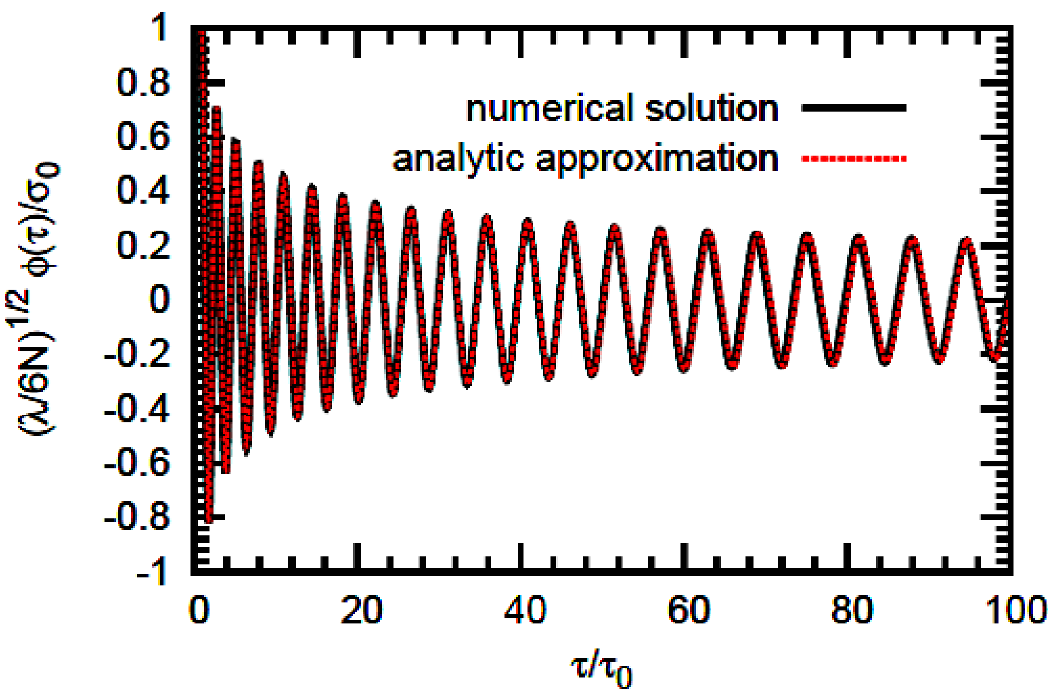
Symmetric phase ⟨Φ⟩=0

	$\lambda\Phi^4$	O(N)	SU(N)
Truncation	NLO of λ 	NLO of 1/N 	LO of g^2 
Others' Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim 3+1 dim	?
Our Code	1+1 dim 2+1 dim 3+1 dim	1+1 dim 2+1 dim 3+1 dim	2+1 dim 3+1 dim

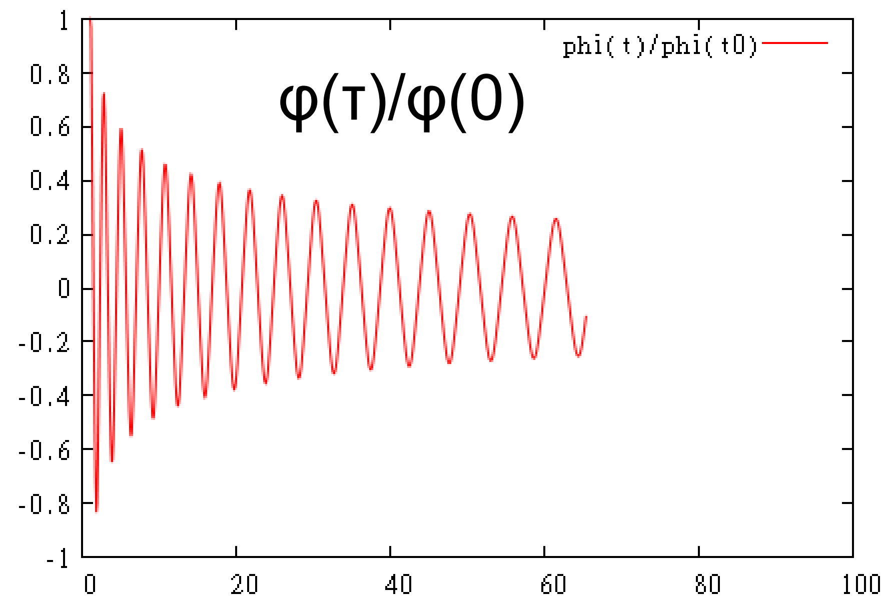
Evolution of classical field and fluctuation

Reproduction of J. Berges, K. Boguslavski, S. Schlichting, hep-ph 1201.3582.

Case without collision term



$\varphi \sim \tau^{-1/3}$



τ/τ_0

Parametric Resonance instability

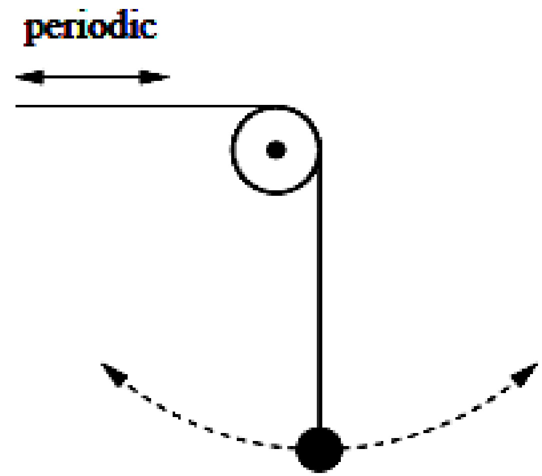
Fluctuation $\ddot{y} + \omega^2(t) y = 0$

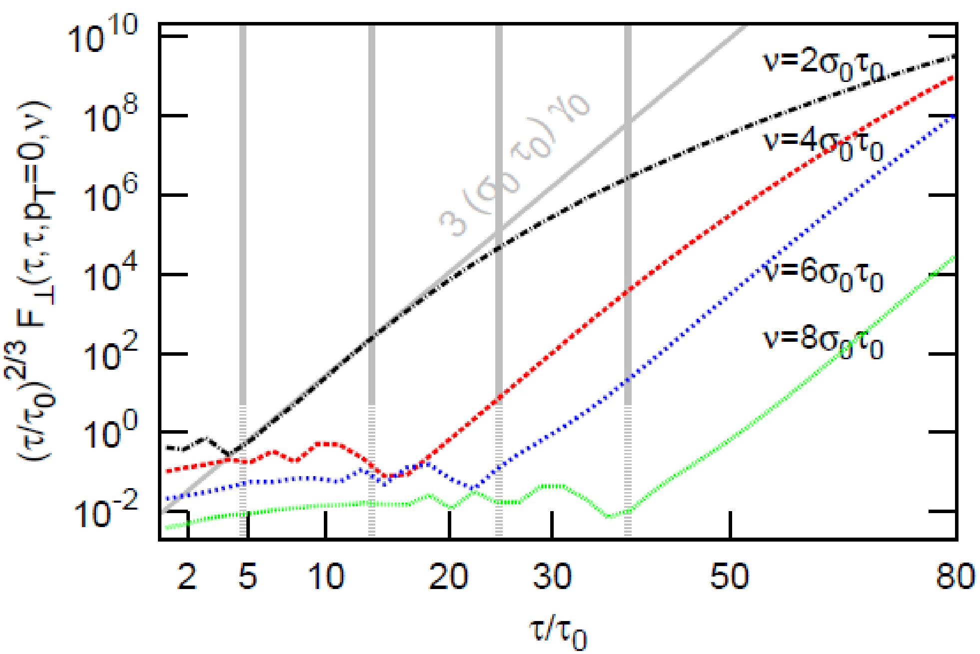
periodic $\omega(t+T) = \omega(t)$

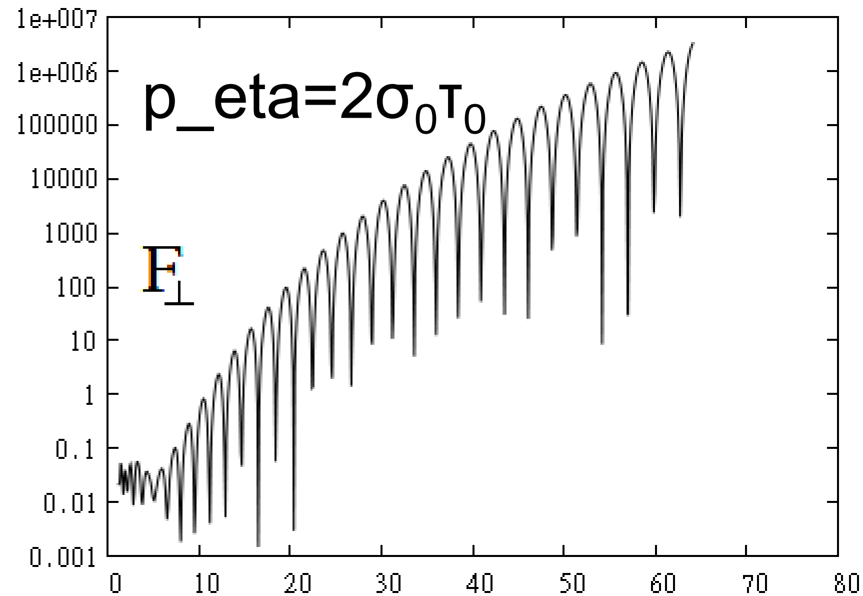
$y(t) = c^{t/T}\Pi(t) \qquad c > 1$

$\omega^2(t) \sim \varphi^2(t) + \ldots$

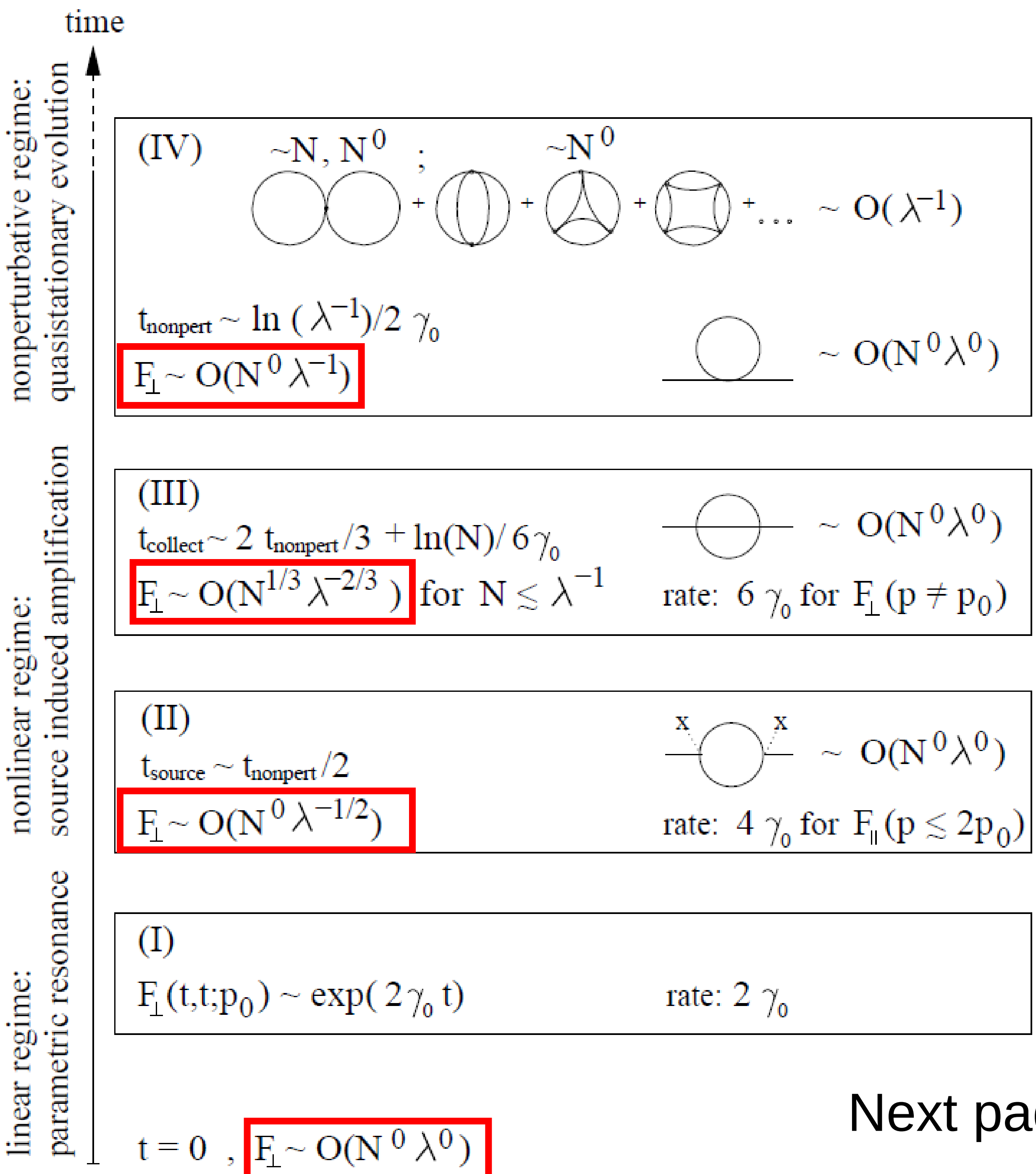
Flat







Curved $\exp(\gamma_0\tau^{2/3})$



Berges 2004.

(Flat metric)

Next page: Our results