Interweaving Chiral Spirals

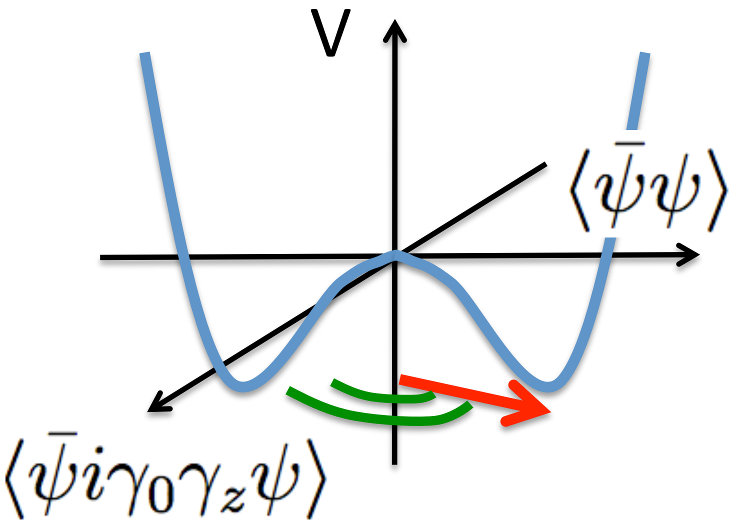
Toru Kojo (Bielefeld U.)

with: **K. Fukushima, Y. Hidaka, L. McLerran, R.D. Pisarski**

space-dep.

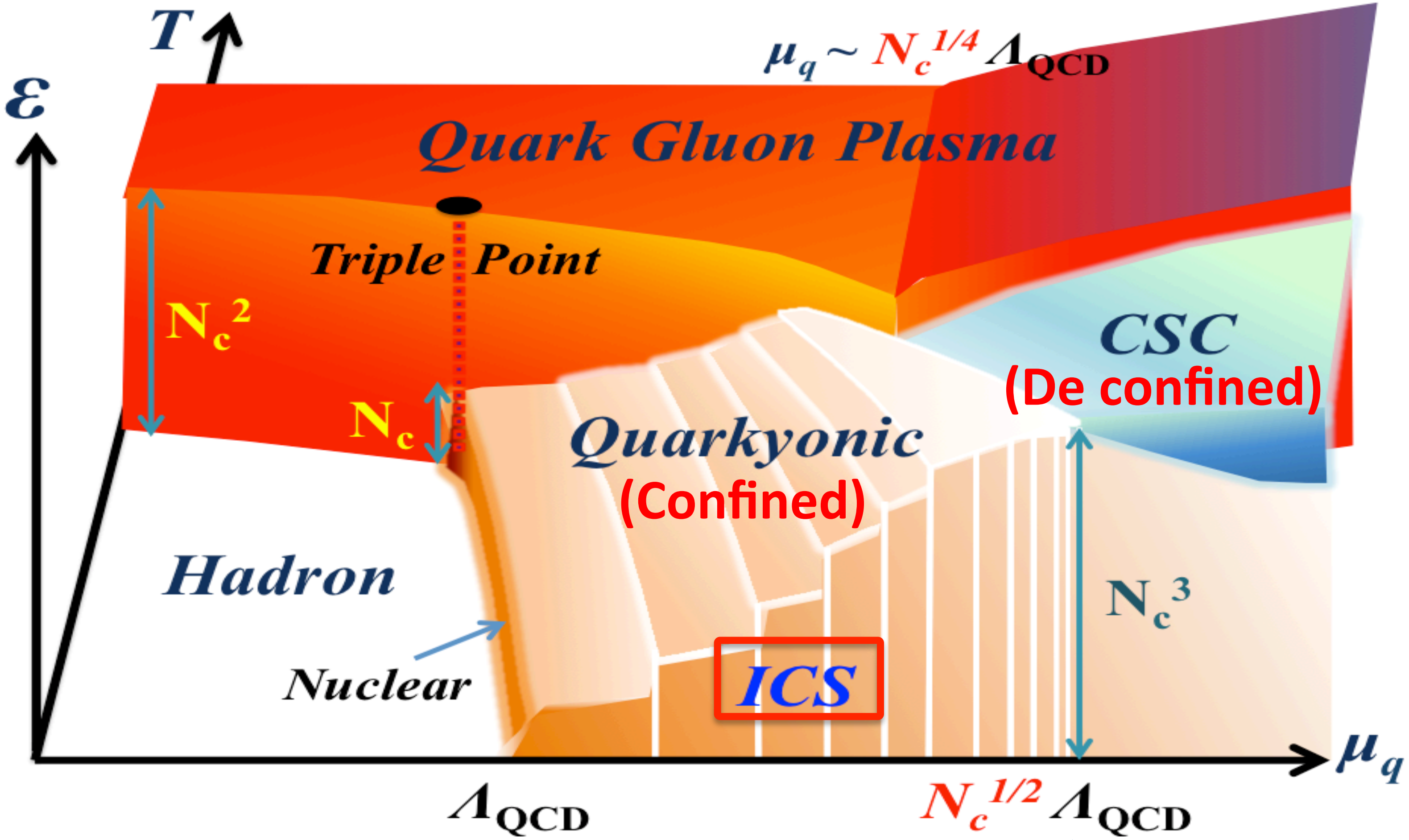
$$\langle\bar\psi\psi\rangle = \Delta\cos(2p_F\underline{z})$$

$$\langle\bar\psi i\gamma_0\gamma_z\psi\rangle = \Delta\sin(2p_F\underline{z})$$



Phase diagram at large Nc

Kojo-Hidaka-Fukushima-McLerran-Pisarski (2011)



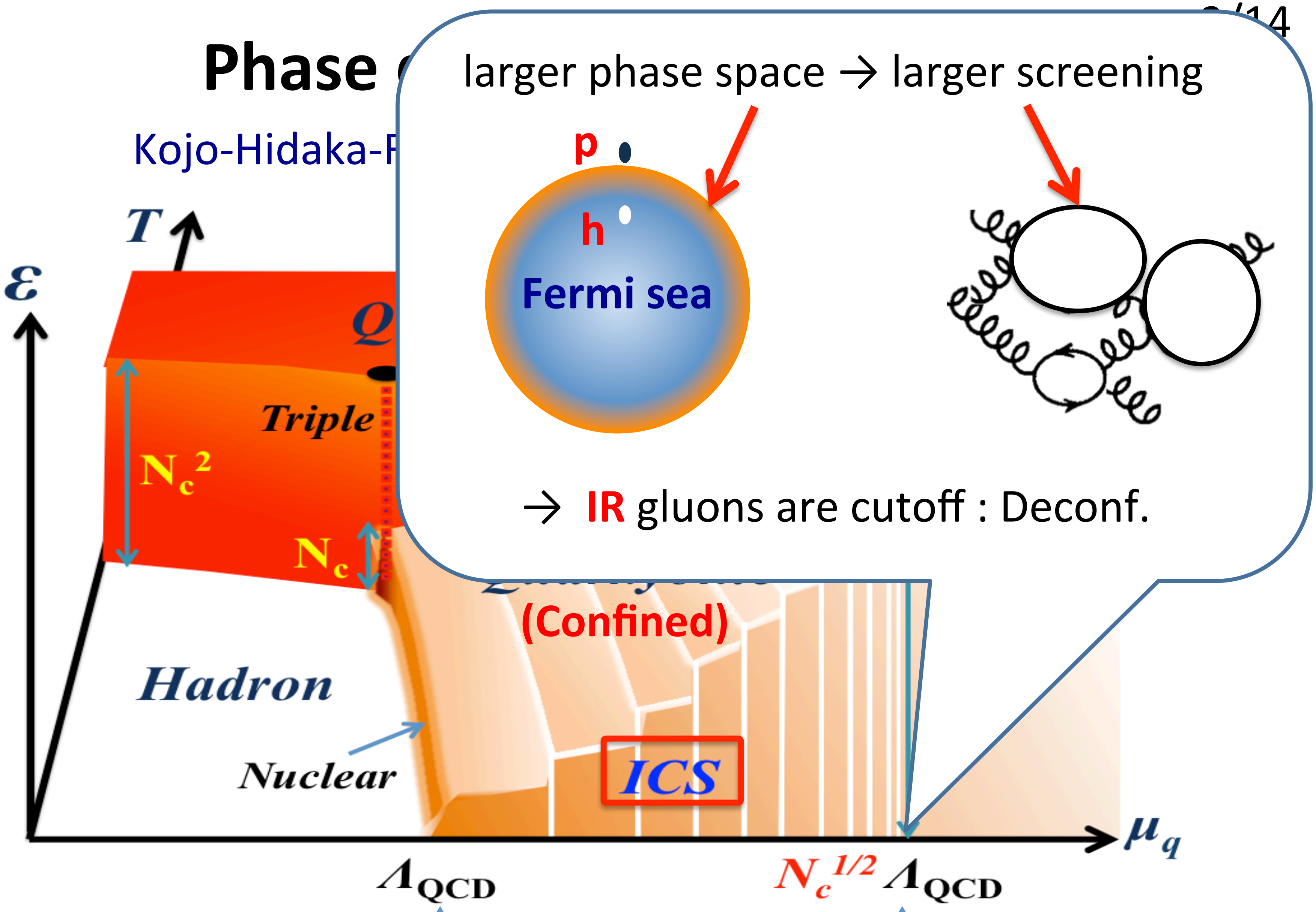
scale of quark matter formation

scale of deconfinement

(large gluon screening)

Phase

Kojo-Hidaka-F



larger phase space → larger screening

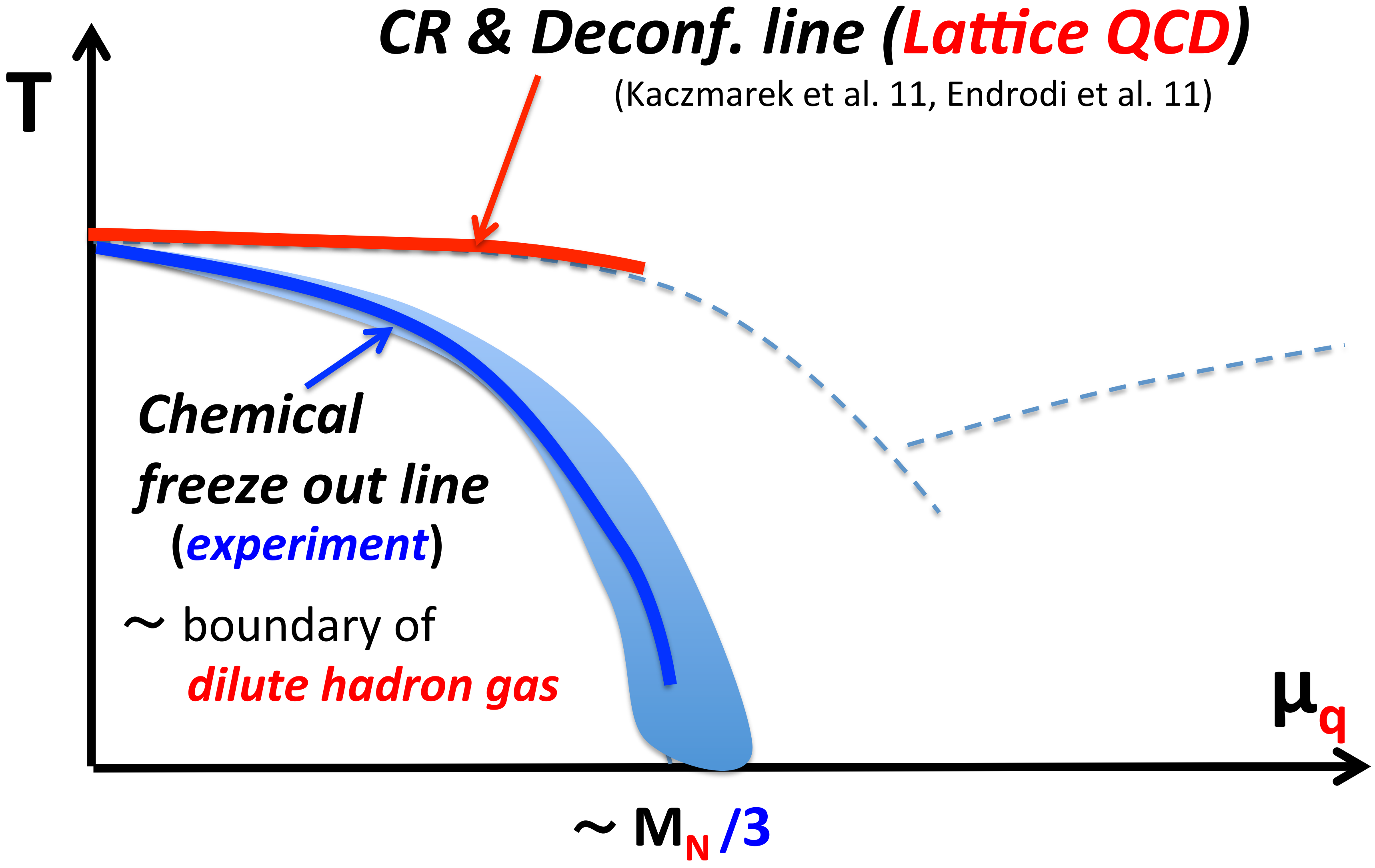
→ **IR** gluons are cutoff : Deconf.

scale of quark matter formation

scale of deconfinement

(large gluon screening)

Chiral Restoration (CR) line



CR & Deconf. line (Lattice QCD)

(Kaczmarek et al. 11, Endrodi et al. 11)

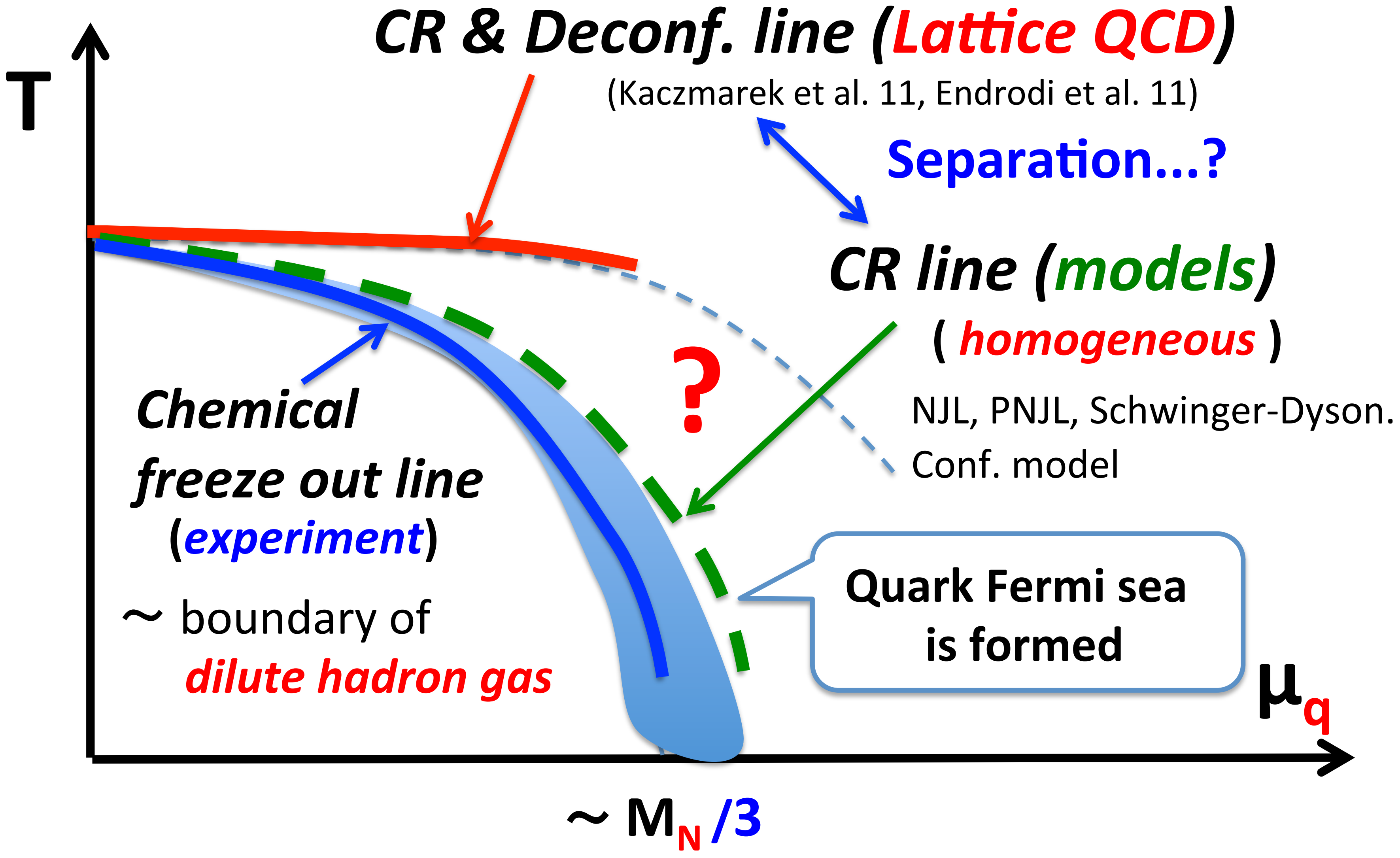
Chemical freeze out line

(experiment)

∼ boundary of

dilute hadron gas

Chiral Restoration (CR) line



CR & Deconf. line (Lattice QCD)

(Kaczmarek et al. 11, Endrodi et al. 11)

Separation...?

CR line (models)

(homogeneous)

NJL, PNJL, Schwinger-Dyson.

Conf. model

Chemical freeze out line

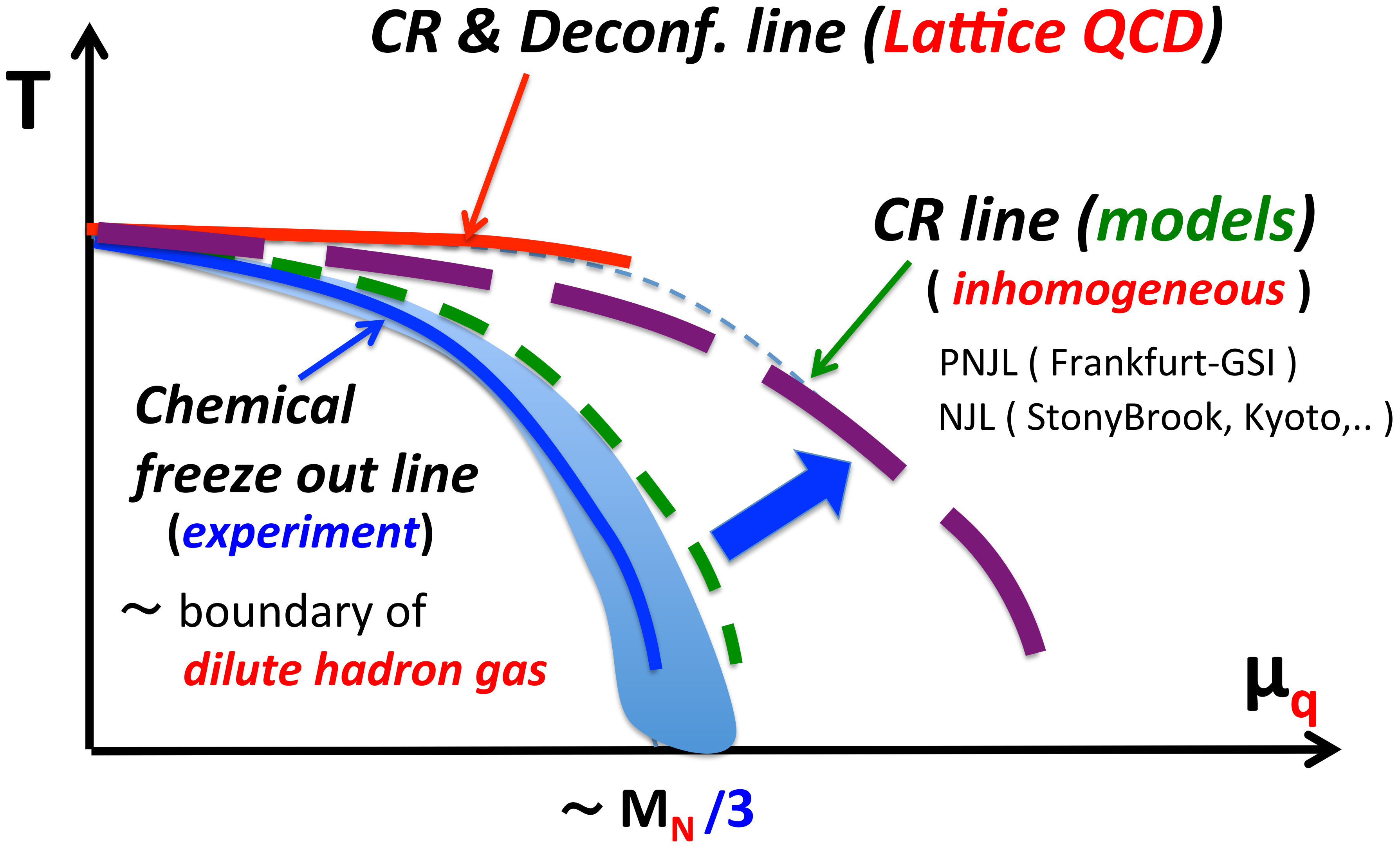
(experiment)

~ boundary of

dilute hadron gas

Quark Fermi sea is formed

With inhomogeneous chiral condensate...



CR & Deconf. line (Lattice QCD)

CR line (models)

(inhomogeneous)

PNJL (Frankfurt-GSI)

NJL (StonyBrook, Kyoto,..)

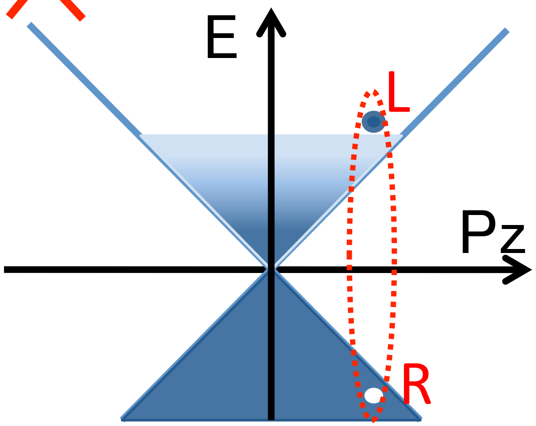
Chemical freeze out line

(experiment)

~ boundary of *dilute hadron gas*

Candidates of Chiral Pairing (T=0)

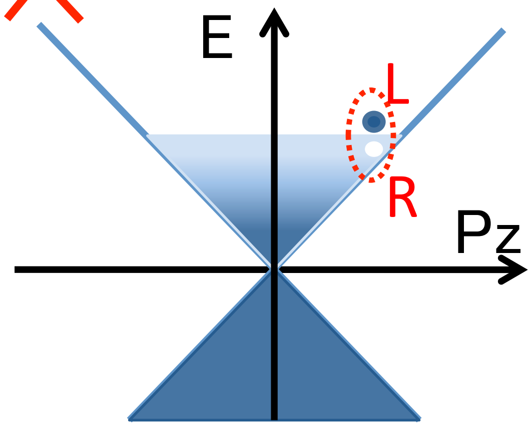
✗ Dirac Type



$P_{Tot}=0$ (uniform)

Kin. suppressed

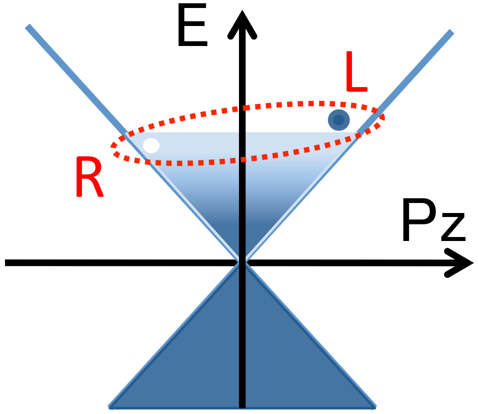
✗ Exciton Type



$P_{Tot}=0$ (uniform)

NOT favored by **int.**

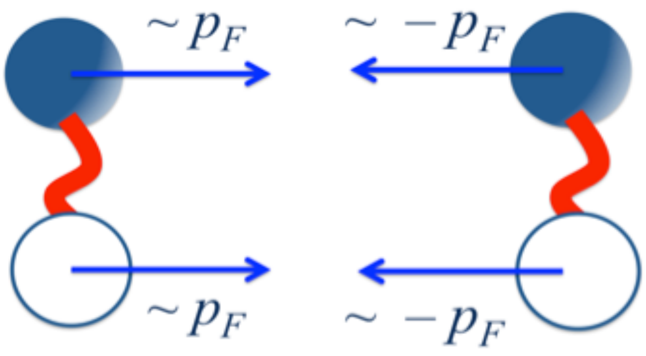
Density wave



$P_{Tot}=2\mu$ (non-uniform)

favored by **int.**

Co-moving pairs condense

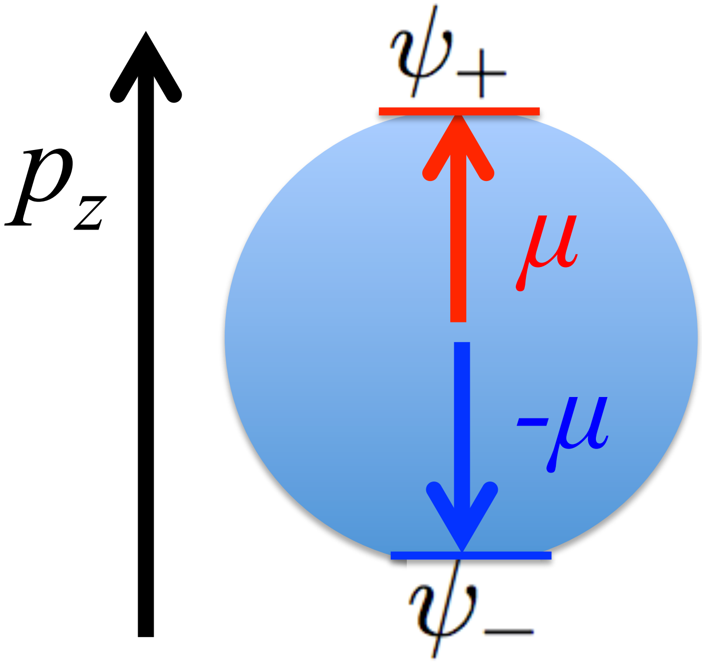


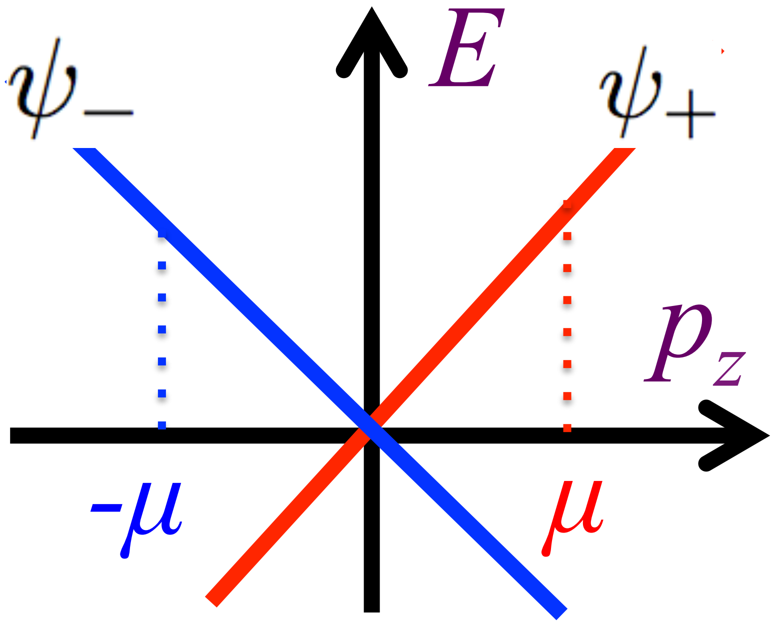
Single Chiral Spiral in z-direction

- ***Projection***: $$\psi_\pm = \frac{1 \pm \underline{\gamma_0 \gamma_z}}{2}\psi$$ ← like (1+1) D chirality

- ***Kin. terms***: $$\mathcal{L}_{kin} \simeq \psi_\pm^\dagger\left(\mathrm{i}(\partial_0 \underline{\mp \partial_z}) + \mu\right)\psi_\pm + \psi_\pm^\dagger \frac{\partial_\perp^2}{2\mu}\psi_\mp$$

(near the Fermi surface) **longitudinal** **transverse**





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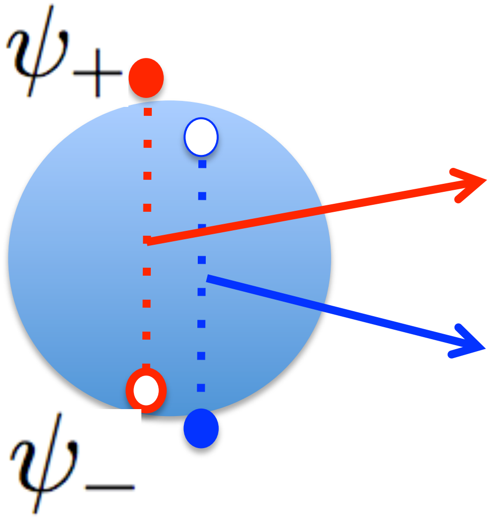


p h

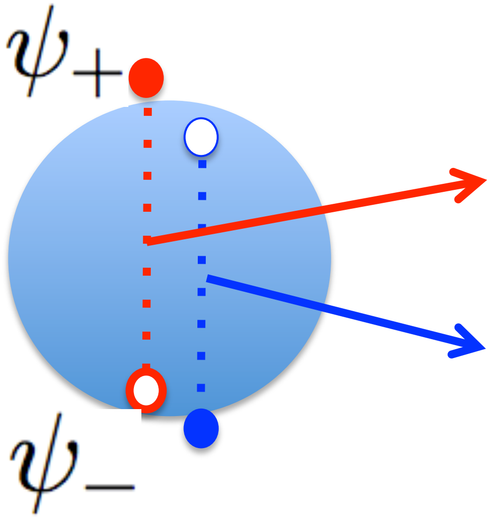
Phase (due to finite mom.)

$$\langle \bar{\psi}_+ \psi_- \rangle = \Delta\, \mathrm{e}^{2\mathrm{i}p_F z}$$

$$\langle \bar{\psi}_- \psi_+ \rangle = \Delta\, \mathrm{e}^{-2\mathrm{i}p_F z}$$

($\Delta \sim \Lambda_{\mathrm{QCD}}{}^3$)

Single Chiral Spiral in z-direction



$p \quad h$

Phase (due to finite mom.)

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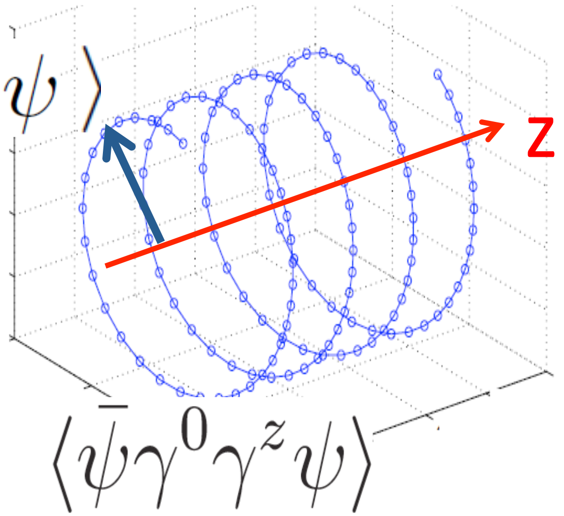
($\Delta \sim \Lambda_{QCD}{}^3$)

$$\psi_\pm = \frac{1 \pm \underline{\gamma_0 \gamma_z}}{2} \psi$$

Linear combination :

space-dep. $\langle \bar{\psi}\psi \rangle$



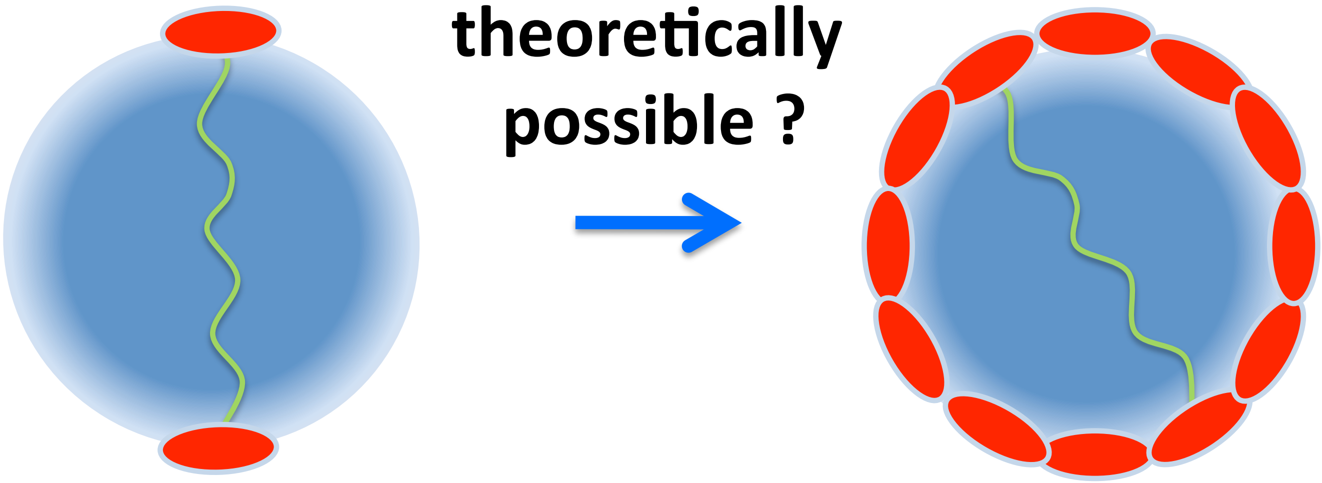
Sum : $\langle \bar{\psi}\psi \rangle = \Delta \cos(2p_F \underline{z})$

P-odd

diff : $\langle \bar{\psi} i \gamma_0 \underline{\gamma_z} \psi \rangle = \Delta \sin(2p_F \underline{z})$

2 CDWs → Single Chiral Spiral

Interweaving Chiral Spirals



theoretically possible ?

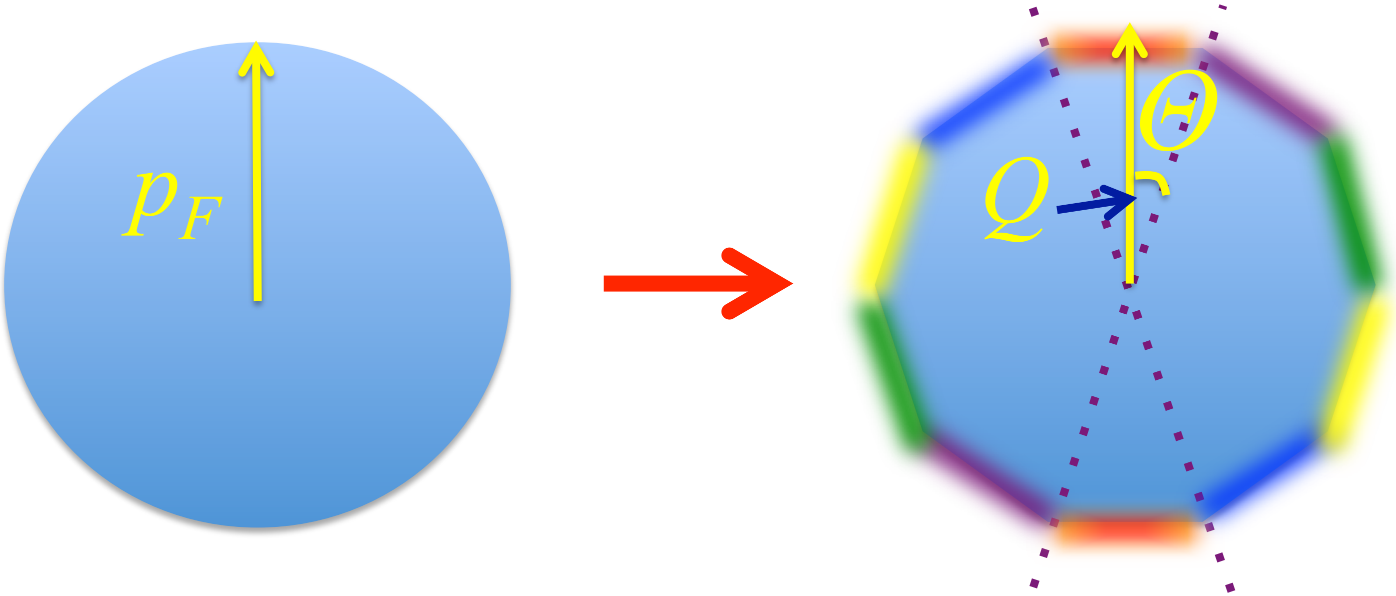
Single Chiral Spiral

Multiple Chiral Spirals

Our study: (2+1) D Example

1, Divide the Fermi surface into N_p patch domains

(N_p: variational parameter)



2, *Each* patch domain has one CS → Compute it.

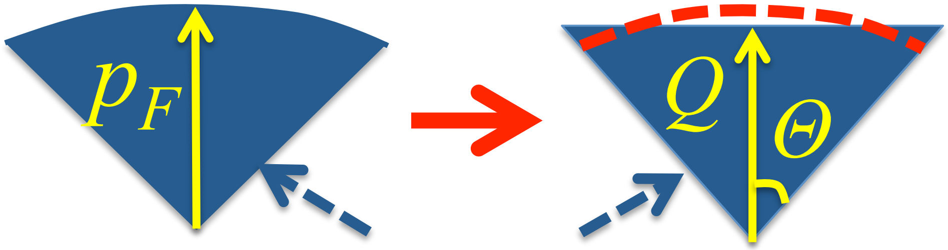
3, Compute interactions b.t.w. CSs.

4, Optimize $N_p \sim 1/\Theta$ → Find the ground state

Energetic gain v.s. cost

- **Cost** : **Deformation**

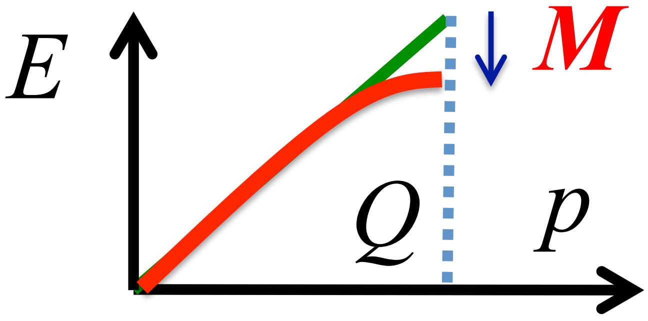
  (dominant for **large** Θ)



equal vol. (particle num.)

- **Gain** : **Mass gap origin**

Condensation effects

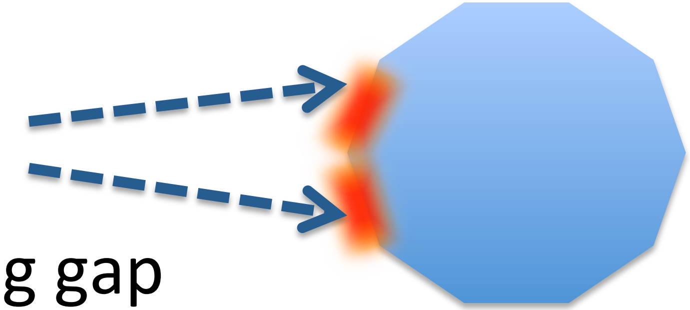


- **Cost** : **Interactions among CSs**

  (dominant for **small** Θ)

(**Model dep. *!!***)

Condensate – Condensate int.



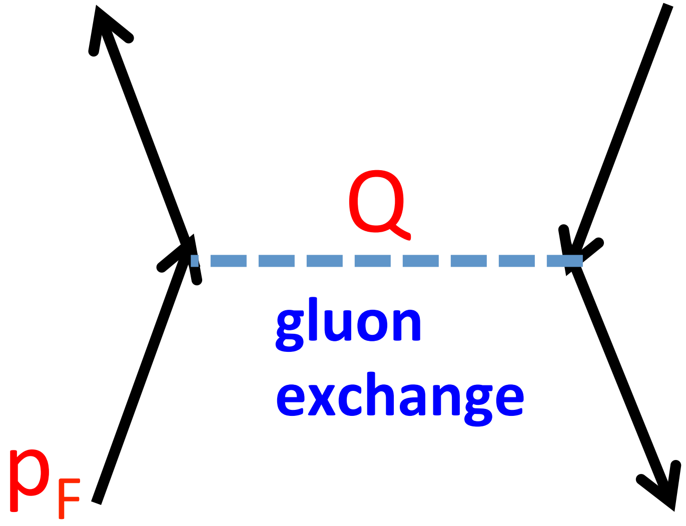
→ destroy one another, reducing gap

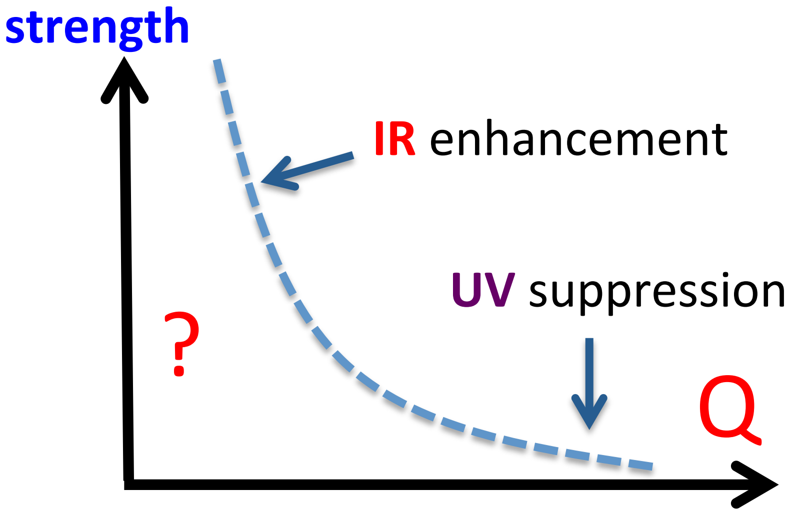
A schematic model

Strength of interactions is determined by

Momentum transfer, **NOT** by **quark momenta.**

→ **Even at high density, int. is strong** for **some processes**.





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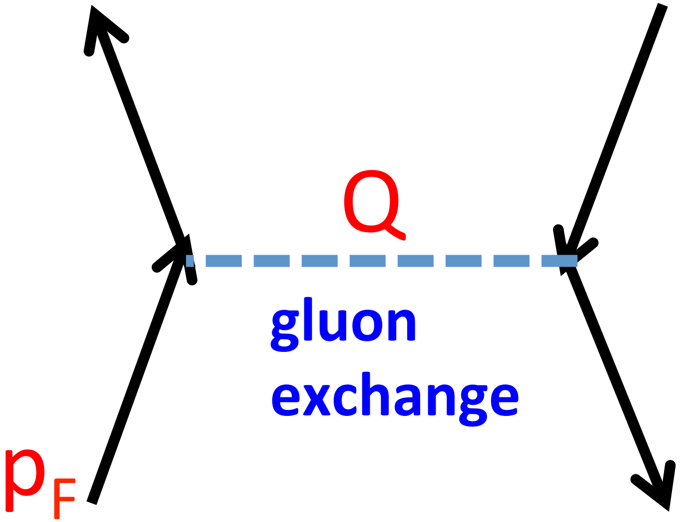
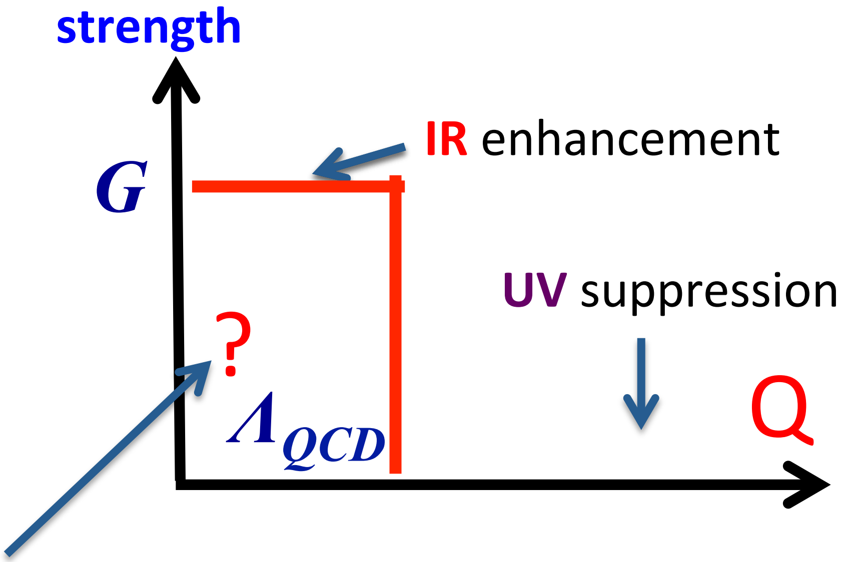




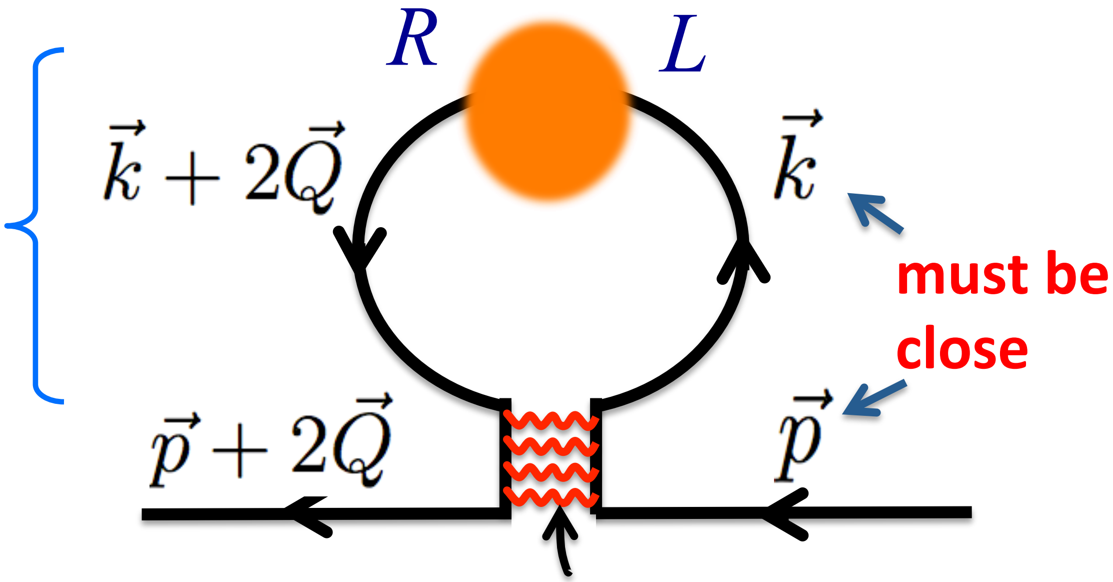
- Detailed form in the **deep IR region:** **We don’t care**

Consequences of the model

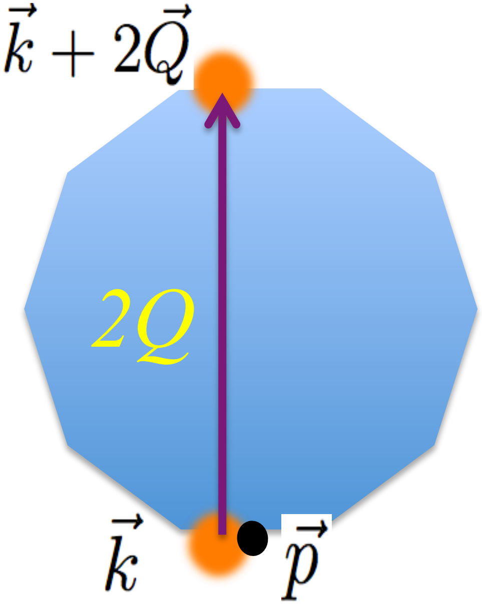
Contributions to the mass gap at leading Nc:

Inhomogeneous condensate

$$\langle \bar{\psi}_R(\vec{k} + \underline{2\vec{Q}})\psi_L(\vec{k})\rangle$$



Strong for small mom. transfer

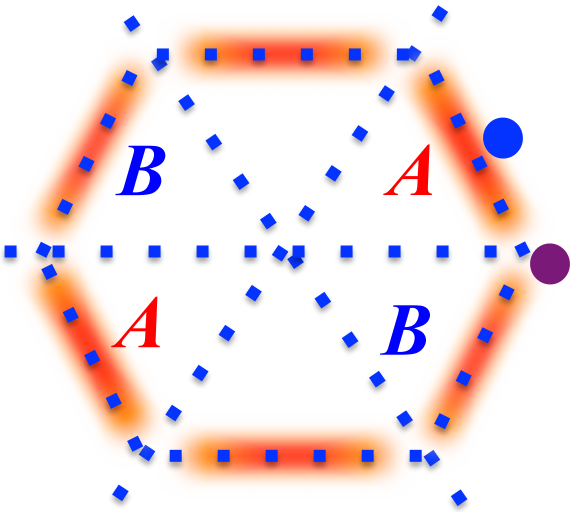


Quark-condensate int. is *Local* in *momentum space* !

Condensate & gap distributions

Condensate contribute to the quark mass gap

only if their momentum domains are close one another.



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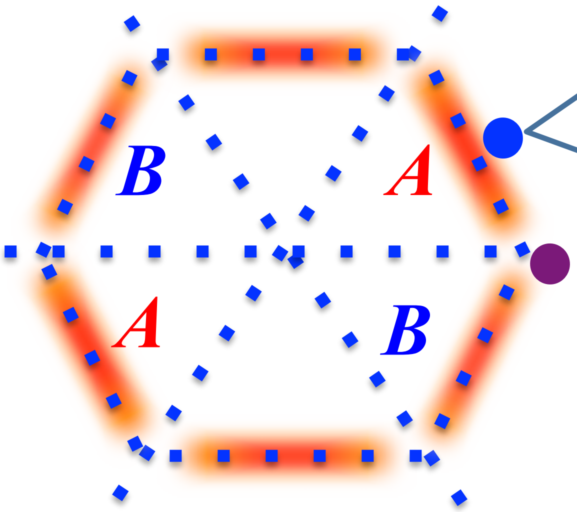
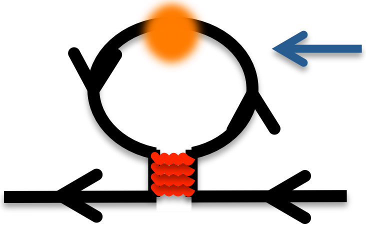


Quarks Away from the patch boundary:



feel only **one** CS (A)
condensate in the gap eq.

The gap eq. can be solved within one patch treatment.

Condensate & gap distributions

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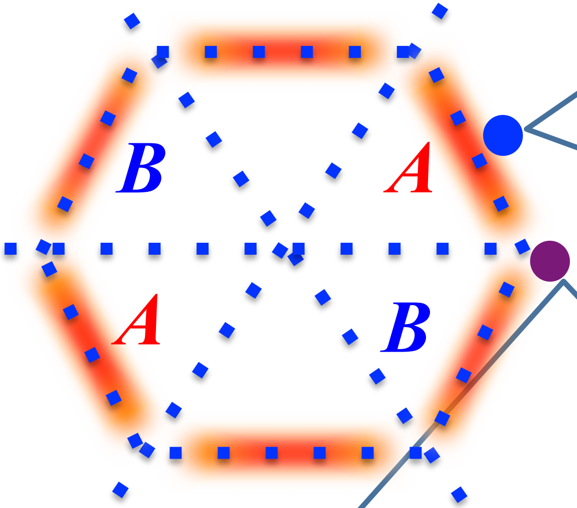
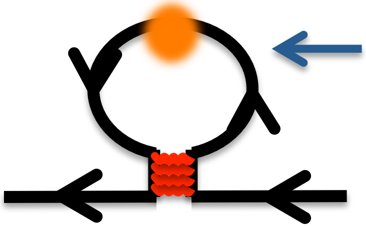


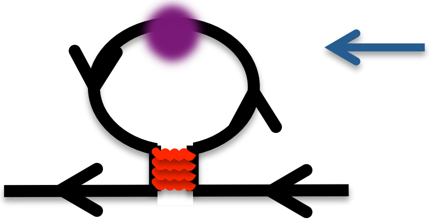
Quarks Away from the patch boundary:



feel only **one** CS (A)
condensate in the gap eq.

The gap eq. can be solved within one patch treatment.

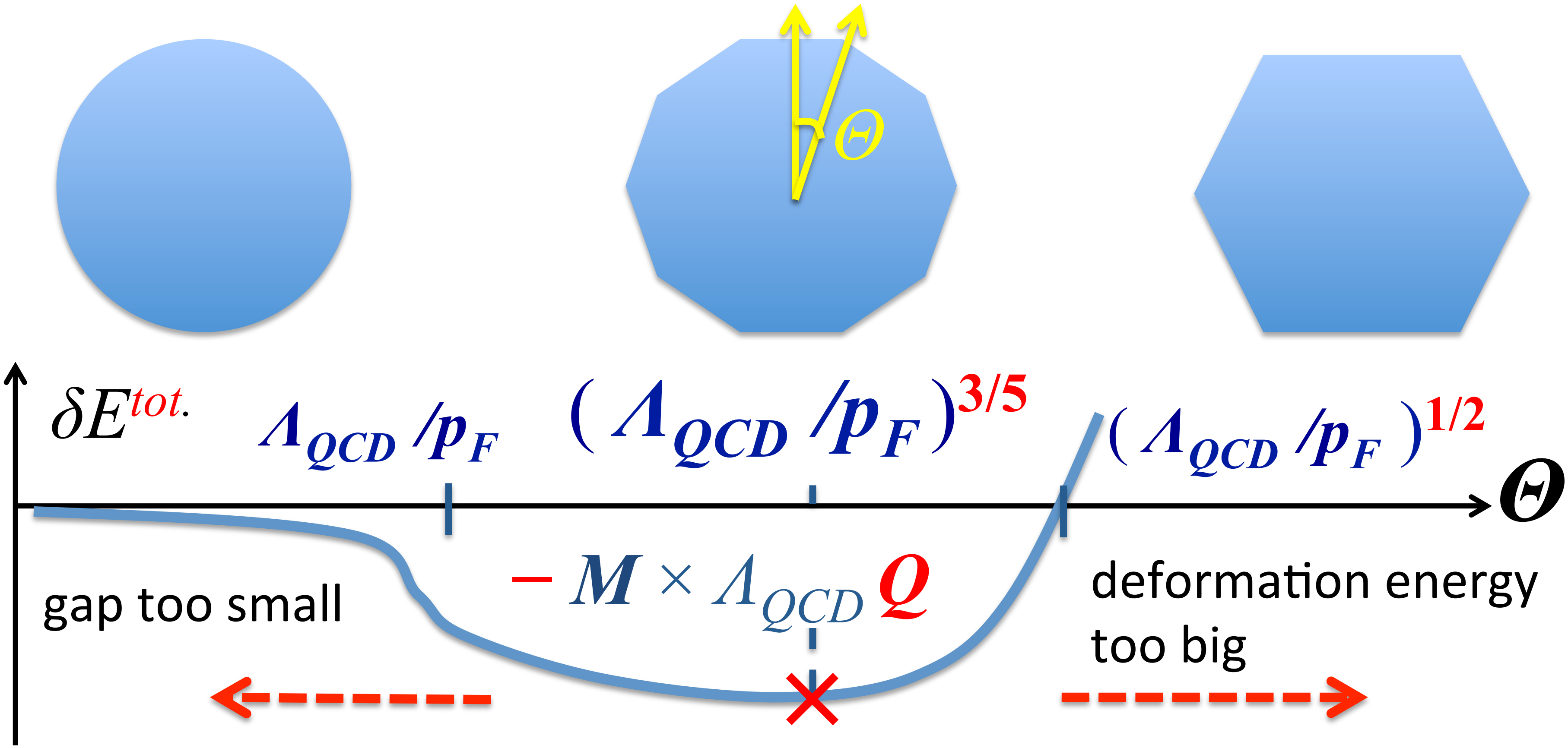
Quarks Near the patch boundaries:



feel **Two** CSs (A & B)
The gap eq. involve **Two** CSs background.

Results: **reduction** of the gap & condensate

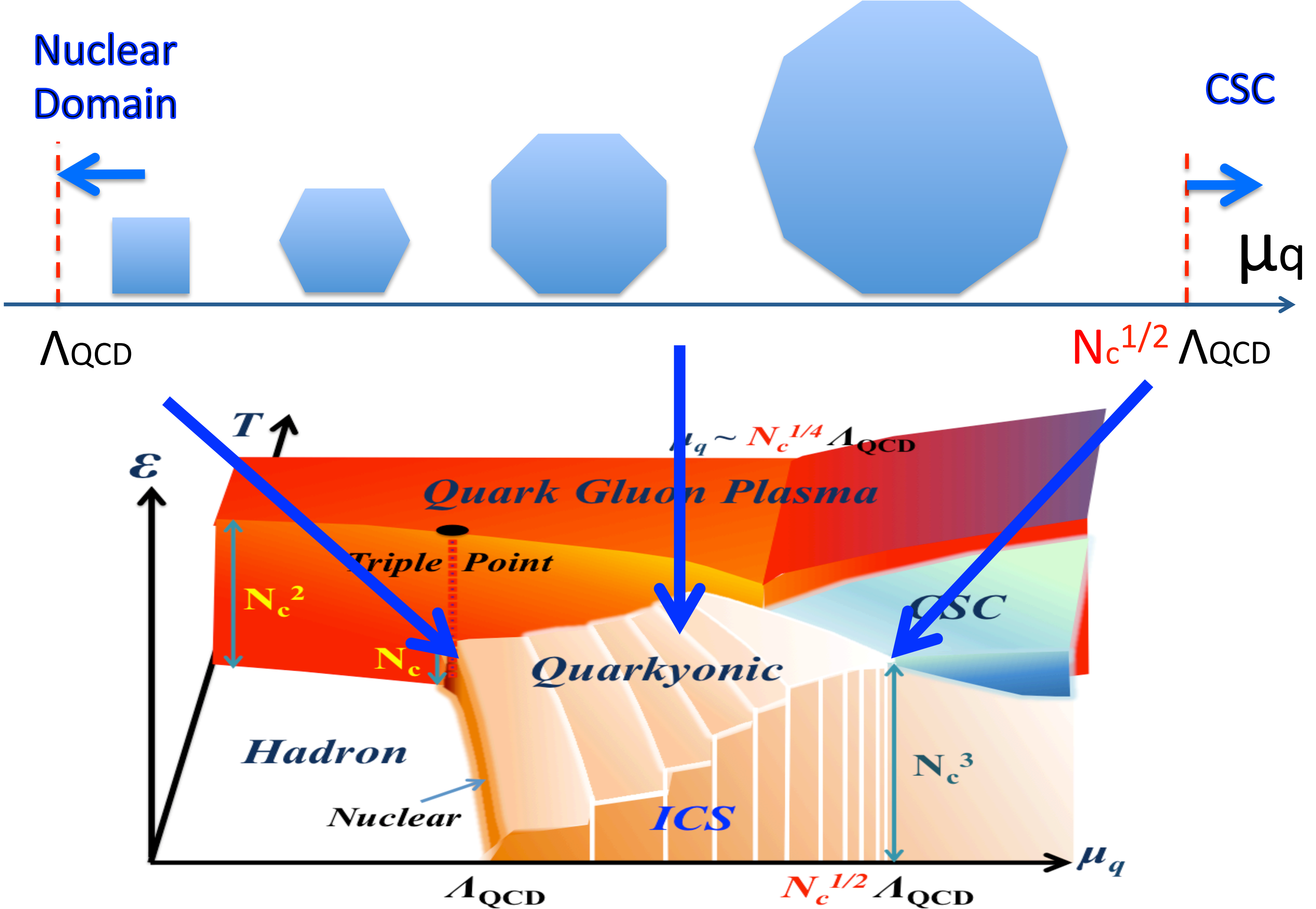
Energy Landscape (for fixed p_F)



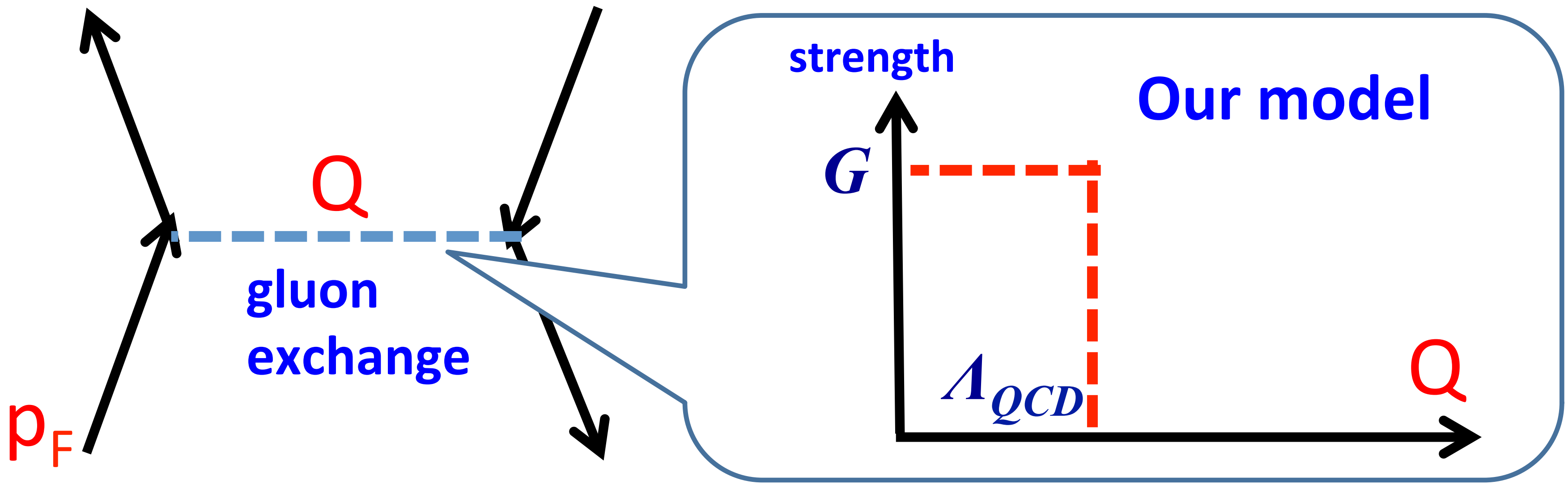
$$\boxed{N_p \sim 1/\Theta \sim (p_F/\Lambda_{QCD})^{3/5}}$$

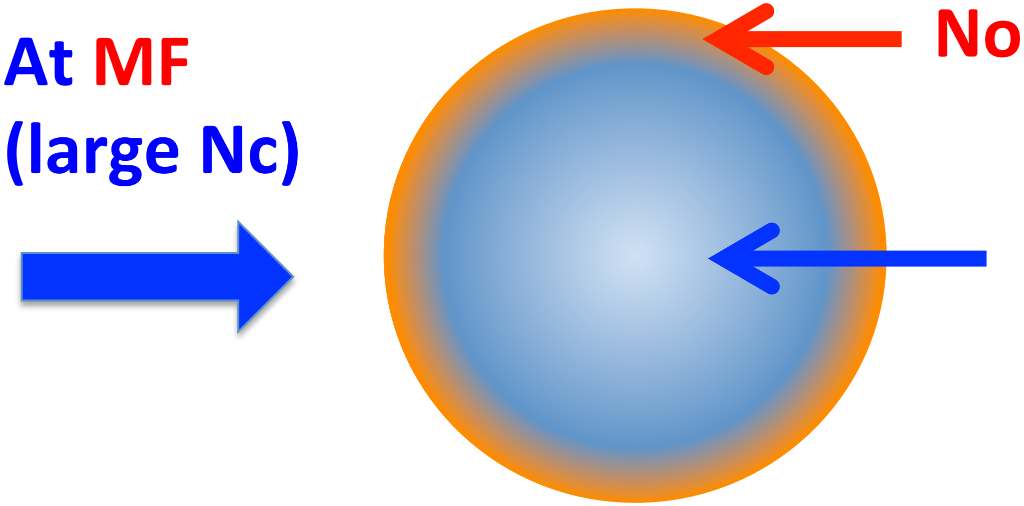
- **Patch num. depends upon density.**

Sequential Phase transitions



Model & consequences





At MF
(large Nc)

Non-pert. quarks (gapped)

Free quarks
(Chiral symmetric)

References

Quarkyonic Chiral Spirals (QCS)

Kojo-Hidaka-McLerran-Pisarski (NPA 843 (2010) 37)

Covering the Fermi surface with patches of QCSs

Kojo-Pisarski-Tsvelik (PRD 82 (2010) 074015)

A (1+1) dimensional example of Quarkyonic matter

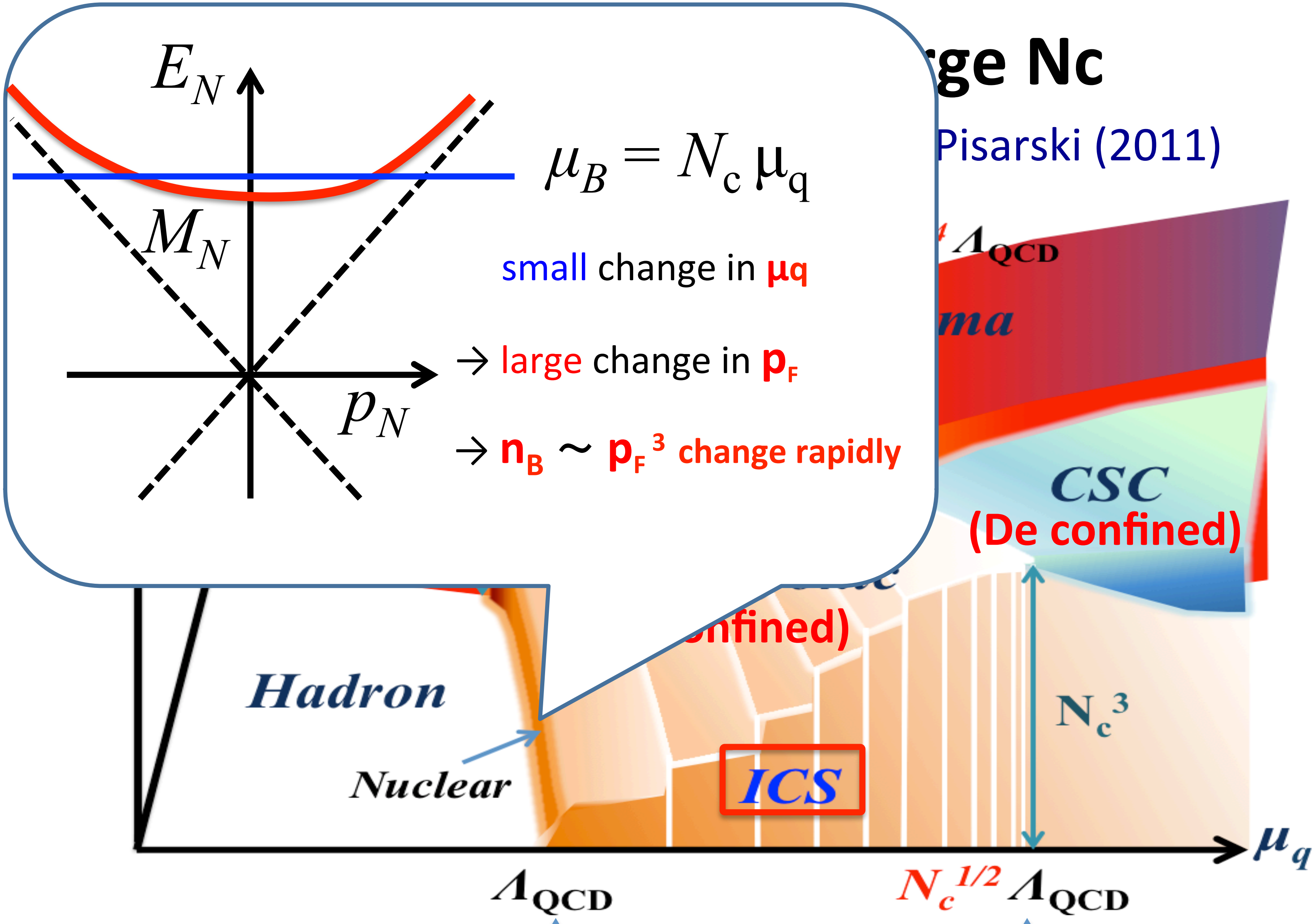
Kojo (NPA 877 (2012) 70)

Interweaving Chiral Spirals (ICS)

Kojo-Hidaka-Fukushima-McLerran-Pisarski (NPA 875 (2012) 94)

rge Nc

Pisarski (2011)



$$\mu_B = N_c\,\mu_q$$

small change in μq

→ large change in p_F

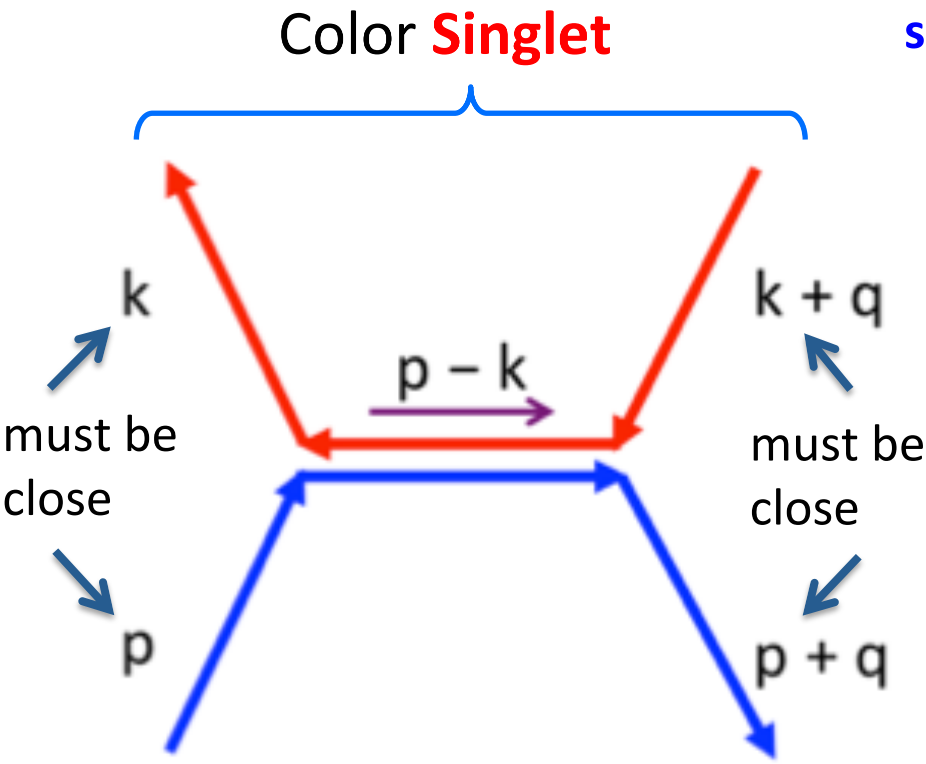
→ $n_B \sim p_F^{\ 3}$ change rapidly

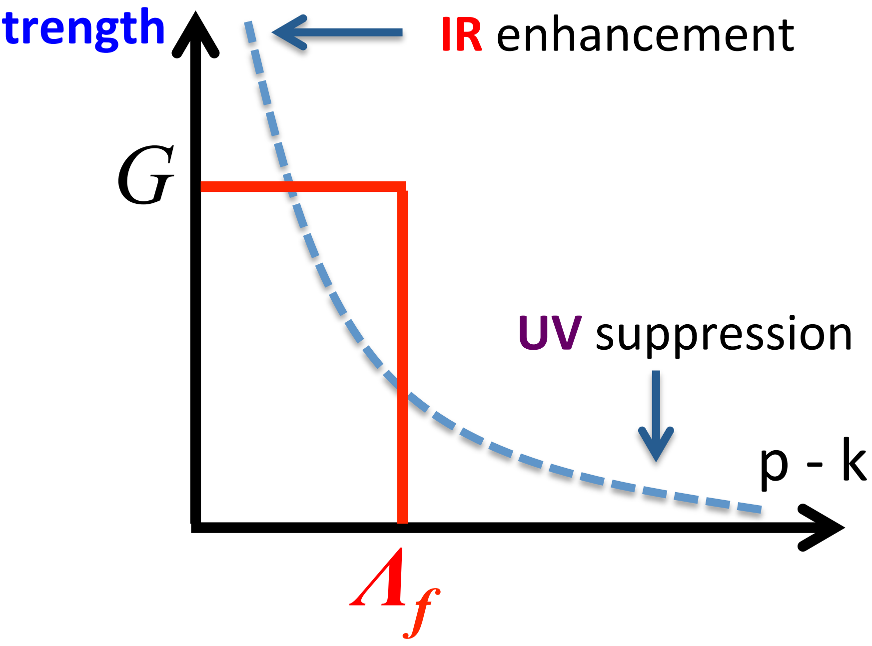
scale of quark matter formation

scale of deconfinement

(large gluon screening)

A crude model with asymptotic freedom



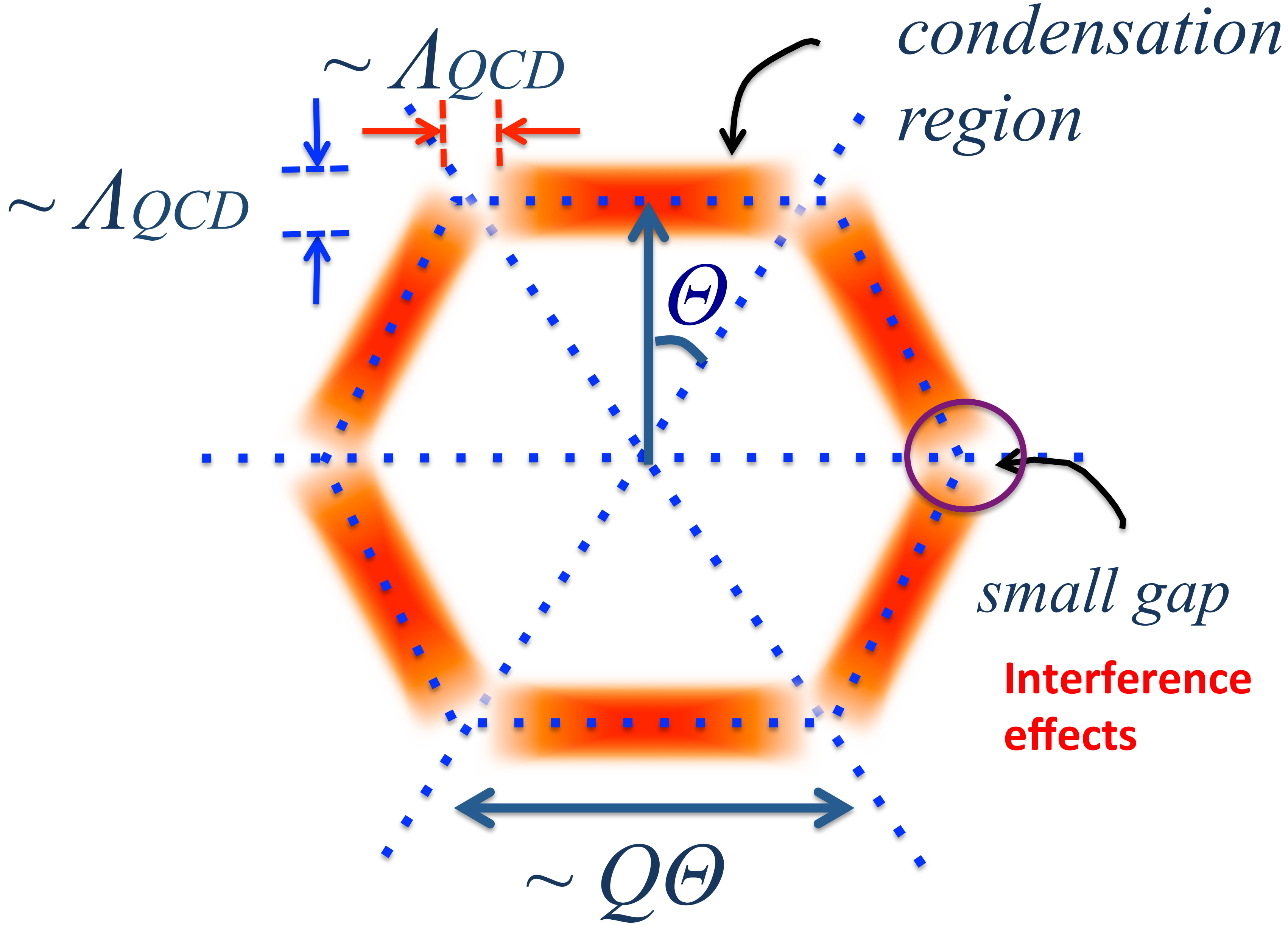


$$\theta_{p,k} \equiv \theta\Big(\Lambda_f^2 - (\vec{p}-\vec{k})^2\Big)$$

- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \underline{\big(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\big)}\,\underline{\big(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\big)}\,\theta_{p,k}$$

Gap distribution will be



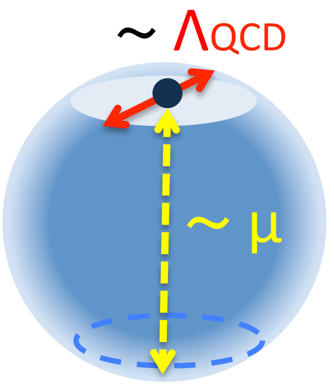
Interference effects

Dim. reduction of integral eqs.

1, Virtual flucts. are limited within small mom. domain.

2, Quark energies are insensitive to small Δk_T.

(due to flatness of Fermi surface in trans. direction)



e.g.) Schwinger-Dyson eq.

insensitive to kT

$$\not{\Sigma}(p) + \Sigma_m(p) = \int \frac{dk_4 dk_z d^2\vec{k}_T}{(2\pi)^4}\gamma_4 S(k)\gamma_4 \frac{\sigma}{|\vec{p}-\vec{k}|^4}$$

$$\xrightarrow{\text{factorization}} \int \frac{dk_4 dk_z}{(2\pi)^2}\gamma_4 S(k_4, k_z, \underline{\vec{0}_T})\gamma_4 \otimes \underbrace{\int \frac{d\vec{k}_T}{(2\pi)^2}\frac{\sigma}{|\vec{p}-\vec{k}|^4}}_{\text{smeared gluon propagator}}$$

- At leading order:

Dimensional reduction of Non-pert. self-consistent eqs :

4D “QCD” in Coulomb gauge ⟷ 2D QCD in $A_1=0$ gauge

(confining model)

Dictionary: μ = 0 & μ ≠ 0 in (1+1)D

- μ ≠ 0 2D QCD can be mapped onto μ = 0 2D QCD

$$\Phi = \exp\left(-i\mu z\Gamma^5\right)\Phi'$$

: Chiral rotation

(Opposite shift of mom. for (+, -) moving states)

$$\overline{\Phi}\left[i\Gamma^\mu\partial_\mu + \mu\Gamma^0\right]\Phi \to \overline{\Phi}' i\Gamma^\mu\partial_\mu\Phi'$$

(μ ≠ 0) (μ = 0)

(due to special geometric property of 2D Fermi sea)

- Dictionary between μ = 0 & μ ≠ 0 condensates:

μ = 0 μ ≠ 0

$$\langle\overline{\Phi}'\Phi'\rangle \to \cos(2\mu z)\,\langle\overline{\Phi}\Phi\rangle - \sin(2\mu z)\,\langle\overline{\Phi}i\Gamma^5\Phi\rangle$$

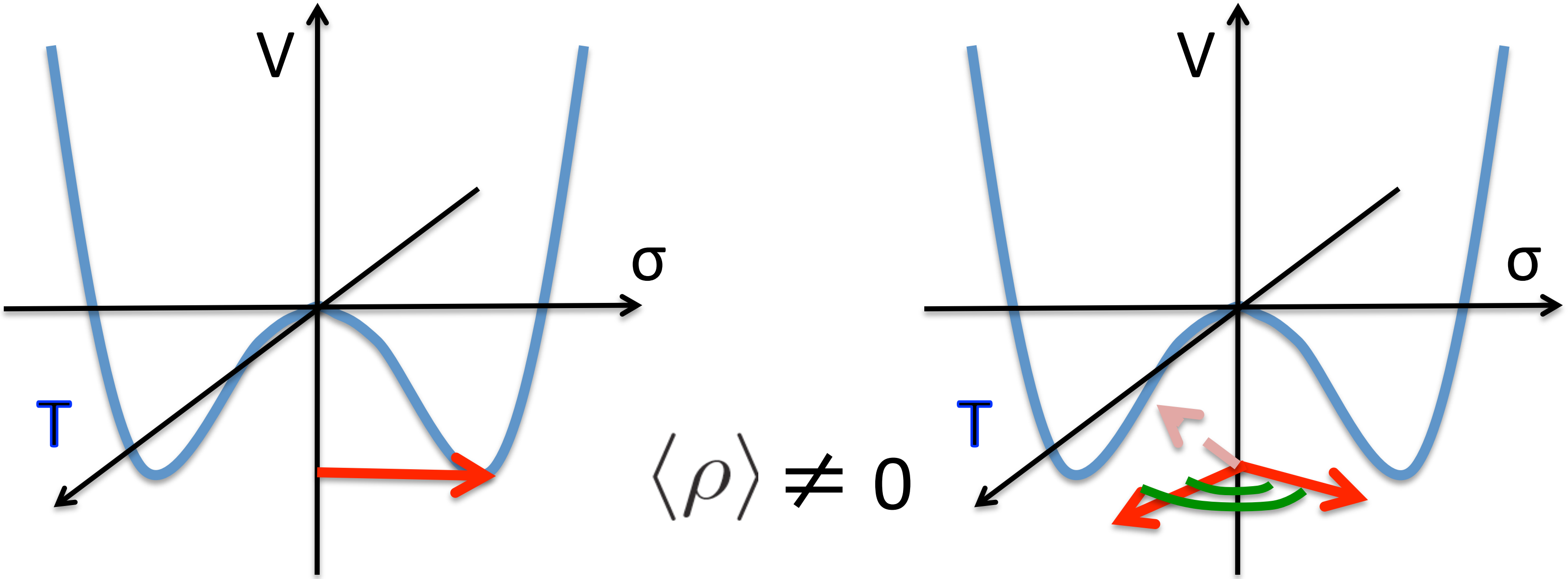
$$\underset{(=0)}{\langle\overline{\Phi}'\Gamma_0\Phi'\rangle} \to \underset{(=0)}{\langle\overline{\Phi}\Gamma_0\Phi\rangle} + \underline{\frac{\mu}{2\pi}}$$

induced by anomaly

"correct baryon number"

Coleman's theorem ?

- Coleman's theorem: No Spontaneous sym. breaking in 2D



$\langle \rho \rangle \neq 0$

$\langle e^{i\theta} \rangle \neq 0$ (SSB)

$\langle e^{i\theta} \rangle = 0$ (No SSB)

IR divergence in (1+1)D phase dynamics

- Phase fluctuations belong to:

Excitations
(physical pion spectra)

ground state properties
(No pion spectra)

Quasi-long range order & large Nc

- Local order parameters:

gapless modes

gapped modes

$$\langle\bar{\Psi}_+\Psi_-\rangle \sim \langle e^{i\sqrt{4\pi/N_{\mathrm{c}}N_{\mathrm{f}}}\phi}\rangle \otimes \langle \mathrm{tr}g\rangle \otimes \langle \mathrm{tr}h\rangle$$

▼ 0 ▼ 0 ▼ finite

due to IR divergent phase dynamics → 0

But this does not mean the system is in the usual symmetric phase!

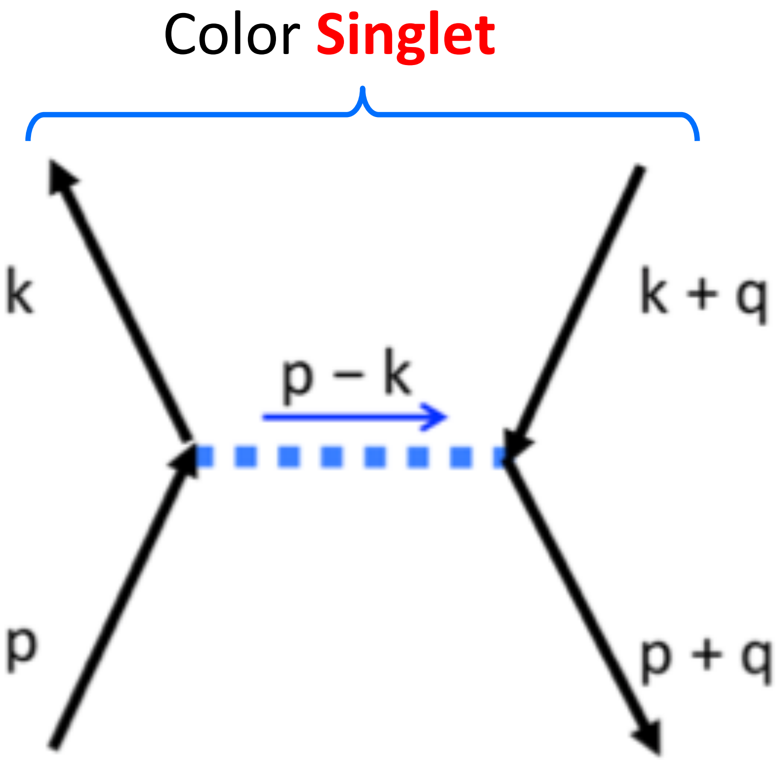
- Non-Local order parameters:

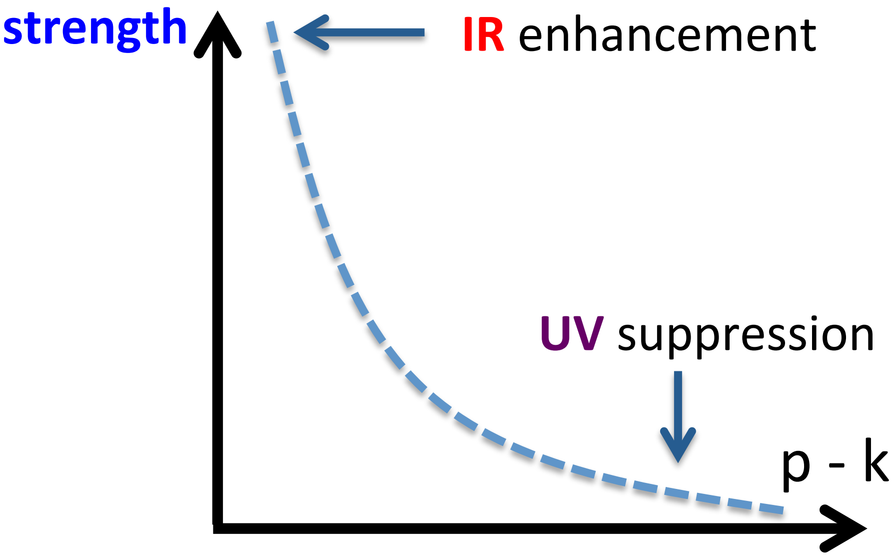
$$\langle\bar{\Psi}_+\Psi_-(x)\bar{\Psi}_-\Psi_+(0)\rangle \sim \begin{cases} \xcancel{e^{-m|x|}} & \text{: symmetric phase} \\ \xcancel{\langle\bar{\Psi}_+\Psi_-\rangle^2} & \text{: long range order} \\ |x|^{\underline{-C/N_{\mathrm{c}}}} & \text{: quasi-long range order} \end{cases}$$

(including disconnected pieces)

(power law)

A crude model with asymptotic freedom





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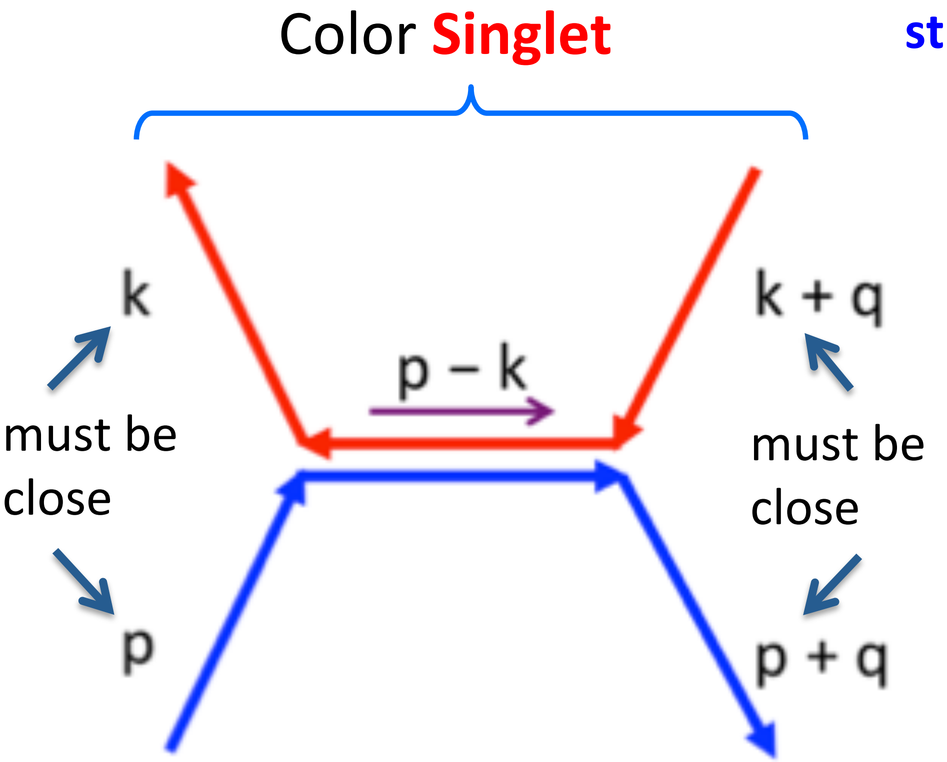
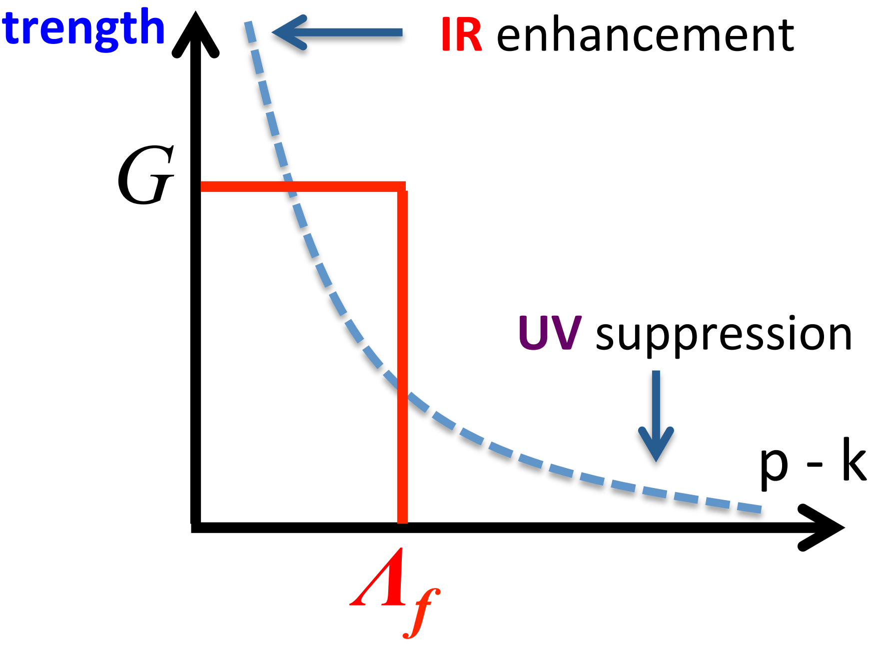
A crude model with asymptotic freedom





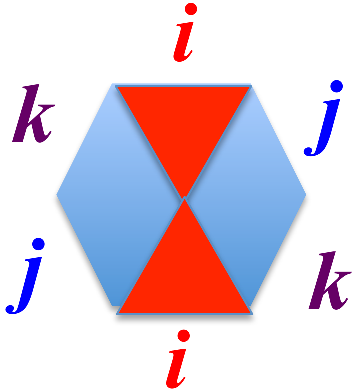
$$\theta_{p,k} \equiv \theta\Big(\Lambda_f^2 - (\vec{p}-\vec{k})^2\Big)$$

- ex) **Scalar** - **Scalar** channel

$$\frac{G}{N_c}\int dx^0 \int_{q,p,k} \underline{\big(\bar{\psi}(\vec{p}+\vec{q})\psi(\vec{p})\big)}\,\underline{\big(\bar{\psi}(\vec{k})\psi(\vec{k}+\vec{q})\big)}\,\theta_{p,k}$$

Picking out one patch Lagrangian

$\underline{\psi_i}$: momentum belonging to ***i*** -th patch



- **Kin. terms:** trivial to decompose

$$\mathcal{L}^{kin} \rightarrow \sum_i \bar{\psi}_i i \partial\!\!\!/ \psi_i \equiv \sum_i \mathcal{L}_i^{kin}$$

- **Int. terms:** Different patches can couple

$$\frac{G}{N_c} \sum_{i,j,k,l} \left((\bar{\psi}_i \psi_j)(\bar{\psi}_k \psi_l) + (\bar{\psi}_i i\gamma_5 \psi_j)(\bar{\psi}_k i\gamma_5 \psi_l) \right)$$

All fermions belong to the ***i*** -th patch

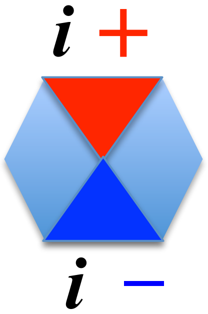
Patch - Patch int.

$$\mathcal{L} = \sum_i \mathcal{L}_i^{1patch} + \Delta\mathcal{L}$$

Dominant terms in One Patch, 1

“(1+1) D” “**chirality**” in ***i -th*** patch

$$\Gamma_{i5} \equiv \gamma_0 \gamma_{i\|} \qquad \psi_{i\pm} \equiv \frac{1 \pm \Gamma_{i5}}{2}\psi_i$$



eigenvalue: **Moving direction**

- **Fact :** “Chiral” **Non** - sym. terms → suppressed by ***1/Q***

ex) free theory

- **Longitudinal Kin. (Sym.)**

$$\psi^\dagger_{\underline{i+}} i(\partial_0 - \partial_{i\|})\psi_{\underline{i+}}$$

- **Transverse Kin. (Non-Sym.)**

$$\bar{\psi}_{\underline{i+}} i \not{\partial}_\perp \psi_{\underline{i-}}$$

excitation energy

$$\epsilon^{\text{free}}(\delta\vec{p}) = |\delta p_\||+\frac{\delta p_\|^2 + p_\perp^2}{\underline{2Q}} + \cdots$$

momentum **measured from Fermi surface**

Dominant terms in One Patch, 2

$$(\bar{\psi}\psi)^2$$

"Chiral" sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2+(\bar{\psi}i\Gamma_5\psi)^2\Big)$$

IR dominant

(must be **resummed** → **MF**)

Non - sym. part

$$\frac{1}{2}\Big((\bar{\psi}\psi)^2-(\bar{\psi}i\Gamma_5\psi)^2\Big)$$

1/Q suppressed

(can be treated in **Pert**.)

- **IR** dominant : **Unperturbed Lagrangian**

  Longitudinal Kin. + "Chiral" sym. 4-Fermi int.

  → Gap eq. can be reduced to (1+1) D

  (P_T - factorization)

- **IR** suppressed : **Perturbation**

  Transverse Kin. + Non - sym. 4-Fermi int.

Quick Summary of 1-Patch results

At leading order of Λ_{QCD}/μ

- **Integral** eqs. such as Schwinger-Dyson, Bethe-Salpeter, can be reduced from (2+1) D to (1+1) D. cf) *kT factorization*
- **Chiral Spirals** emerge, generating **large** quark mass gap. (even larger than vac. mass gap)
- **Quark num**. is **spatially uniform**. (in contrast to chiral density)

Pert. corrections

- **Quark num**. **oscillation**.
- **CSs : Plane wave → Solitonic**

approach to **Baryonic Crystals**

Consequences of form factor. 2

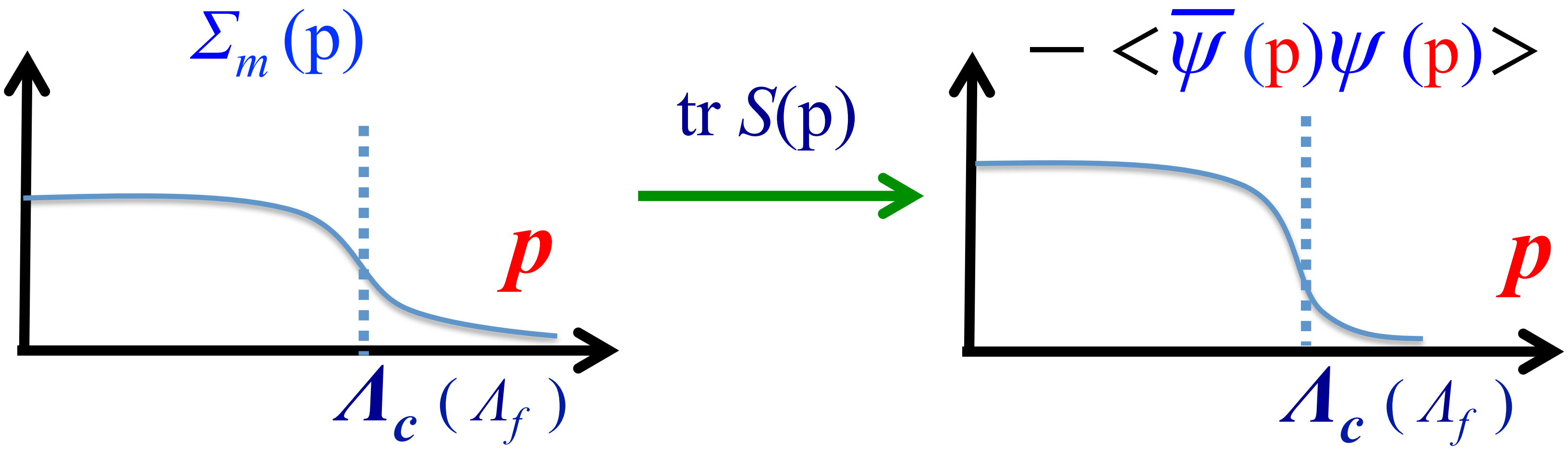
Dominant contributions to condensates : Low **energy** modes

(for vacuum)

$$\Sigma_m(\vec{p}) = \int \frac{d\vec{k}}{(2\pi)^2} \frac{\Sigma_m(\vec{k})}{\boxed{2\epsilon(\vec{k})}} \theta_{p,k}$$

When $p \longrightarrow \infty$:

- k must also go to ∞, so $\varepsilon(k) \longrightarrow \infty$.
- Phase space is **finite** : Nothing compensates denominator.

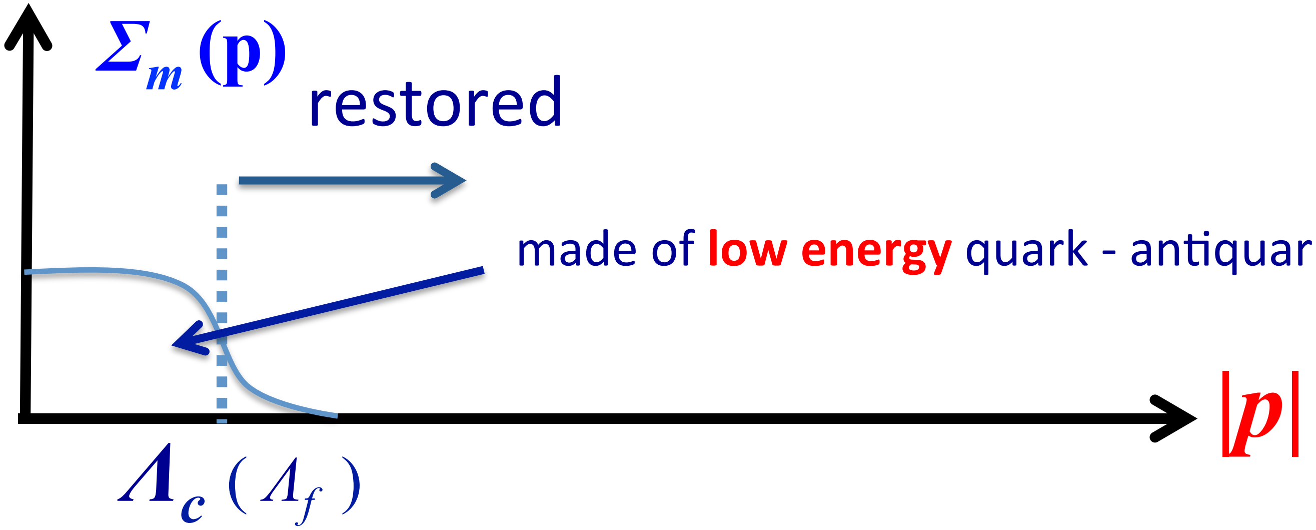


Remark)

- finite density: Low **energy** modes appear **near the Fermi surface**.

Relevant domain of Non-pert. effects

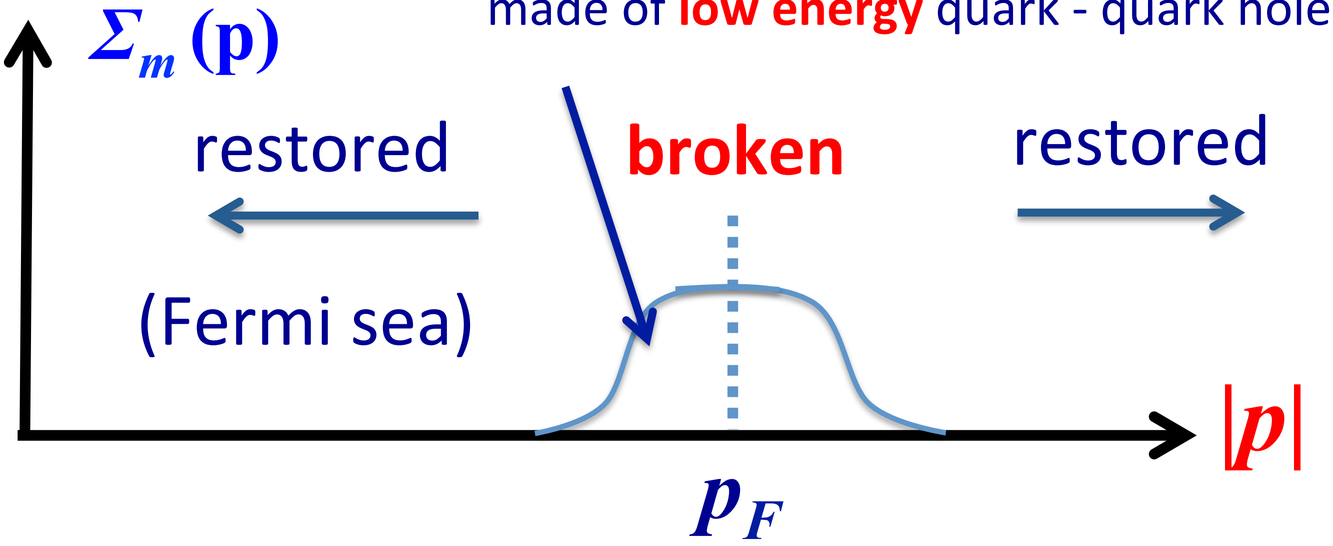
Vac.



restored

made of **low energy** quark - antiquark

Finite Density



made of **low energy** quark - quark hole

restored

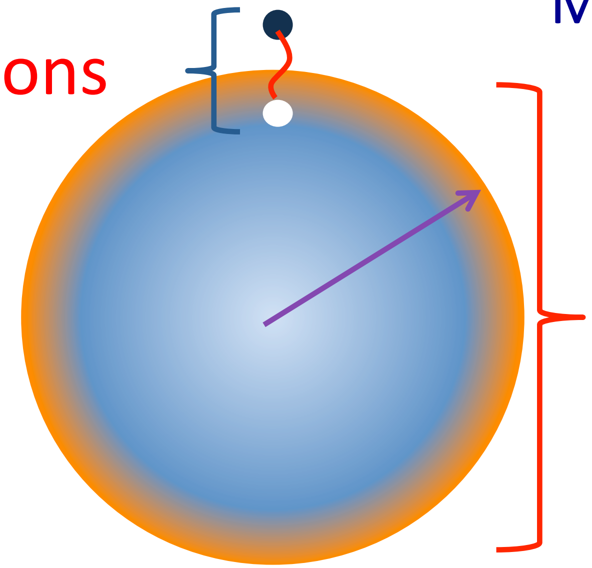
(Fermi sea)

broken

restored

Quarkyonic Matter: Basic picture

McLerran-Pisarski (2007)



hadronic excitations

- Transport properties
- Phase structures
  (condensation)
- etc.

Bulk properties:
(EOS, Pressure, etc.)

Weakly int. quarks

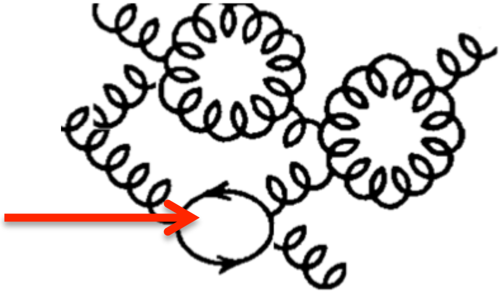
- Pauli-blocking.
- forming color singlet B.G.

Quark Fermi sea + baryonic Fermi surface → Quarkyonic
(hadronic)

- **Gluon sector is modified when screening becomes large:**

$\mu \sim \Lambda_{QCD}$

small
fraction



$\mu \sim Nc^{1/2} \Lambda_{QCD}$

large
fraction