Beyond the ladder analysis of chiral and color symmetry breaking using the non-perturbative renormalization group

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Thermal Quantum Field Theory and Their Application @ YITP

Chiral and color symmetry breakdown

QCD (SU(3) gauge theory with small current quark mass)

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}{\rm tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \sum_{{\rm f}=u,d,\cdots}^{N_{\rm f}} \bar{\psi}_{\rm f}\left(i\partial\!\!\!/ - g_{\rm s}A\!\!\!/ - m_{\rm f}\right)\psi_{\rm f}$$

$m_{\rm f} \ll$ hadron mass scale, approximate chiral symmetry

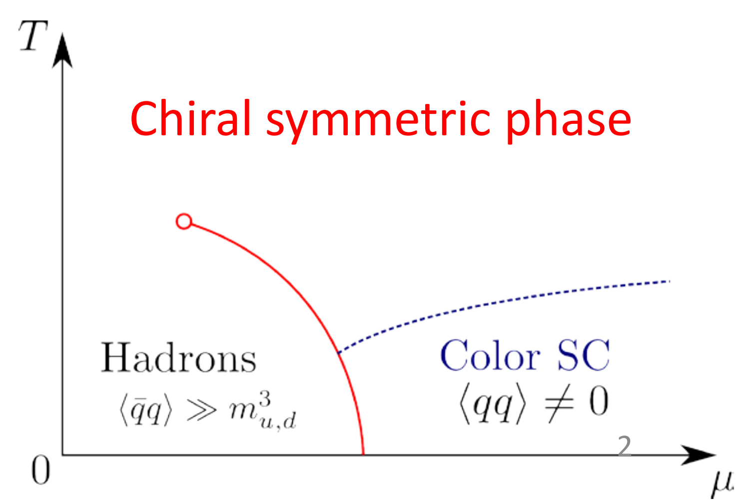
Dynamical chiral symmetry breaking (DχSB)

$\langle\bar{\psi}\psi\rangle \neq 0$ ⇒ Hadron masses $\gg 3m_{u,d,s}$

Color symmetry breaking (color super conductivity)
at low temperature and high density

Diquark condensates

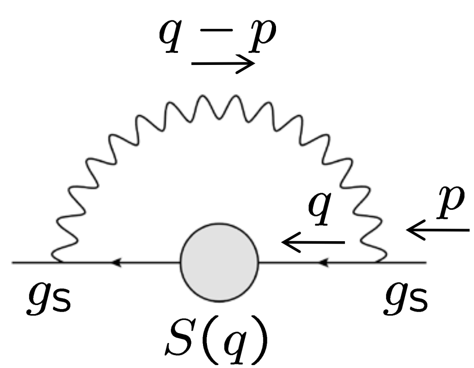
$$\left\langle\left(\bar{\psi}^C\right)_i^a \gamma^5 \psi_j^b\right\rangle \sim \varepsilon_{ij}\epsilon^{abc}$$



Ladder Schwinger-Dyson equation

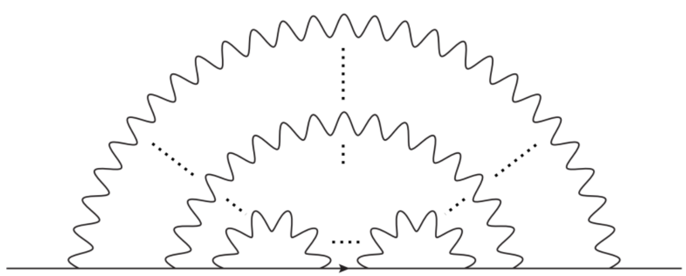
Ladder approximated self-consistency equation

$$-i\Sigma(p) =$$

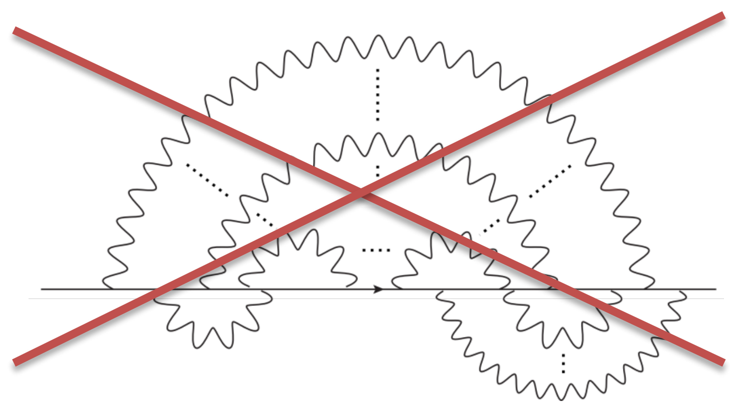


$$S(q) = \frac{i}{\not{q} - \Sigma(q)}$$

Ladder diagrams



Non-ladder diagrams



Strong gauge dependence

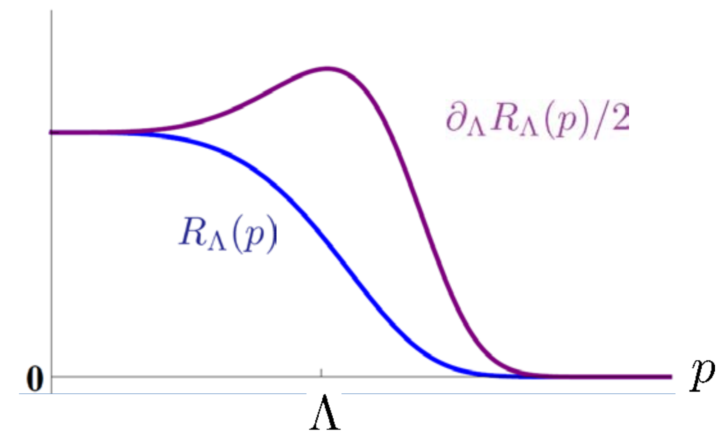
Non-Perturbative Renormalization Group (NPRG)

Effective action with infrared cutoff: $\Gamma_\Lambda[\Phi]$

Propagator with the regulator function $R_\Lambda(p)$ suppresses the mode lower than the cutoff scale Λ.

$$\frac{1}{p^2 + R_\Lambda(p)}$$

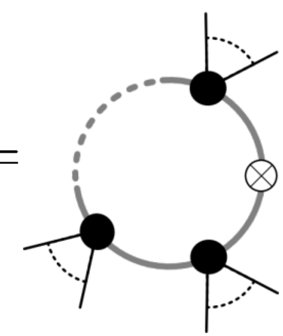
Regulator function $R_\Lambda(p)$ (ex.)



Wetterich-type flow equation

$$\partial_\Lambda \Gamma_\Lambda[\Phi] = \frac{1}{2}\mathrm{STr}\frac{1}{\Gamma_\Lambda^{(2)} + R_\Lambda}\partial_\Lambda R_\Lambda$$

$$\left(\Gamma_\Lambda^{(2)}\right)_{ij}(p,q) = \frac{\delta^2 \Gamma_\Lambda[\Phi]}{\delta\Phi_i(-p)\delta\Phi_j(q)}$$

$\partial_\Lambda \Gamma_\Lambda[\Phi] =$  $\partial_\Lambda R_\Lambda$

Solve the non-perturbative flow equation

Initial condition: $\Gamma_{\Lambda \to \Lambda_0} \to S_{\mathrm{bare}}$

$\Rightarrow$ $\Gamma_{\Lambda \to 0} = \Gamma$

Generate the infinite number of effective operators

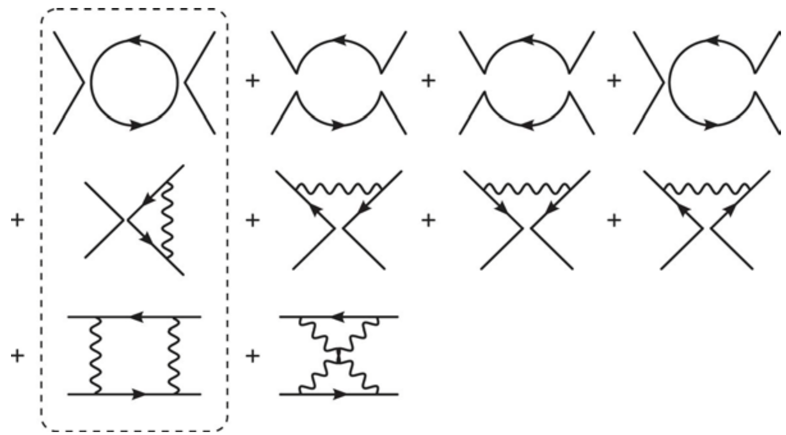
NPRG and Dynamical Chiral Symmetry Breaking (DχSB) in QCD

- Effective action of QCD

$$\Gamma_\Lambda[\Phi] = \int_x \left\{ \frac{Z_F}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}\left(Z_\psi \partial\!\!\!/ + i\bar{g}_s A\!\!\!/\right)\psi - \underline{V(\psi,\bar{\psi};\Lambda)} \right\}$$

$V(\psi,\bar{\psi};\Lambda)$: effective fermion potential

- Generation of 4-fermi operators



the gauge interactions generate the 4-fermi operator, which brings about the DχSB at low energy scale, just as the Nambu-Jona-Lasinio model does.

- Field operator expansion (Derivative expansion)

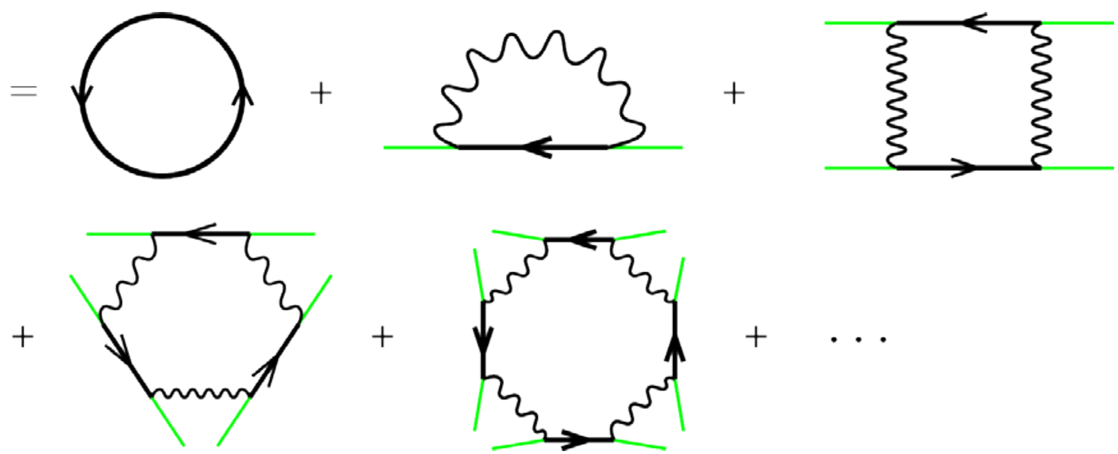
$$V(\psi,\bar{\psi};\Lambda) = \boxed{\frac{G_2}{2}\left((\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right)} + \frac{G_V}{2}\left((\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_5\gamma_\mu\psi)^2\right) + \frac{G_4}{4}\left((\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2\right)^2 + \cdots$$

$$\Rightarrow V(\sigma;\Lambda),\ \sigma = \bar{\psi}\psi$$

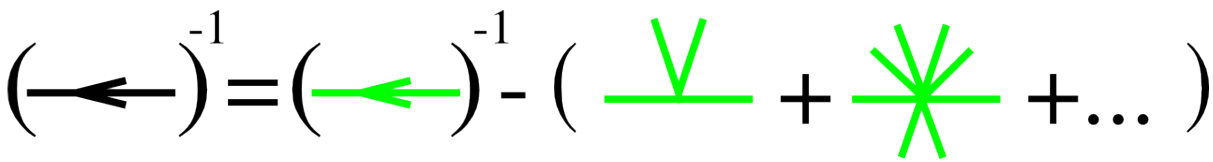
Ladder Approximation

- Limit the NPRG β function to the ladder-type diagrams.

$\partial_t V(\sigma;t) =$



Quark propagator dressed by multi-fermion operators



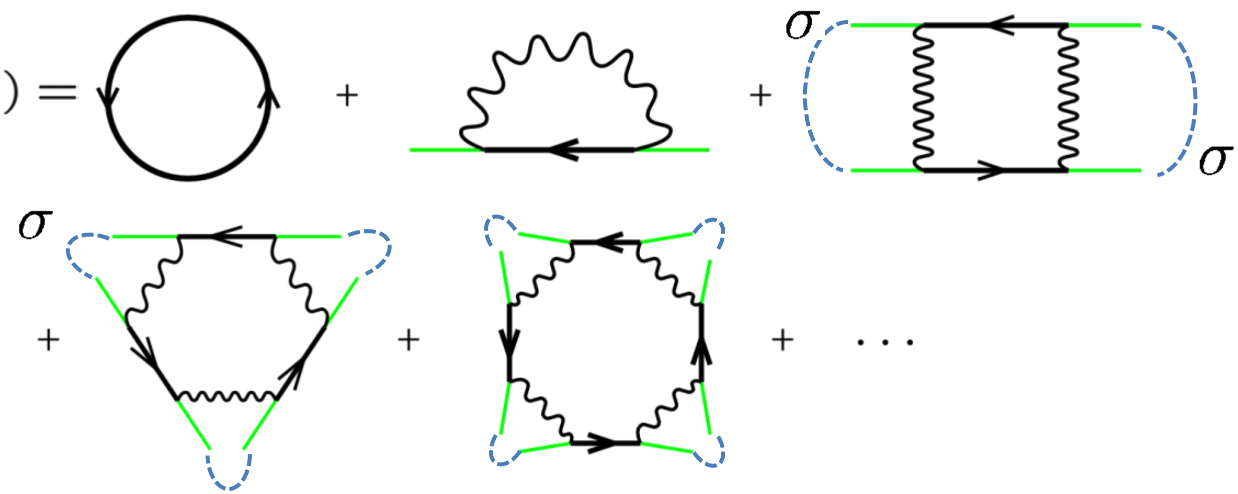
$$= i\not{p} - M(\sigma;\Lambda)$$

Mass function: $M(\sigma;\Lambda) = \partial_\sigma V(\sigma;\Lambda)$

Summation of the infinite diagrams

- Extract the scalar-type operators $\sigma^n = (\bar{\psi}\psi)^n$, which are central operators for DχSB.

$$\Lambda\frac{\partial}{\partial\Lambda}V(\sigma;\Lambda) =$$



- **Ladder-Approximated NPRG equation**

$$\Lambda\frac{\partial}{\partial\Lambda}V(\sigma;\Lambda) = \frac{\Lambda^4}{4\pi^2}\ln\left[1+\frac{1}{\Lambda^2}\left(m+\frac{(3+\xi)C_2 g_s^2}{4\Lambda^2}\cdot\sigma\right)^2\right] \qquad C_2 = \sum_{a=1}^{N_c^2-1} T^a T^a$$

ξ: gauge fixing parameter in the covariant gauge

(ingore the anomalous deimesion of quark field)

> This NPRG equation has the results equivalent to the improved ladder Schwinger-Dyson equation.

RG Flow of the mass function

$$M(\sigma;\Lambda) = \partial_\sigma V(\sigma;\Lambda)$$

$$\Lambda\frac{\partial}{\partial\Lambda}M(\sigma;\Lambda) = \frac{\Lambda^4}{2\pi^2}\frac{M+3\Lambda^{-2}C_2\alpha_s\sigma}{\Lambda^2+(M+3\Lambda^{-2}C_2\alpha_s\sigma)^2}\left(\underline{\partial_\sigma M}+3\Lambda^{-2}C_2\alpha_s\right)$$

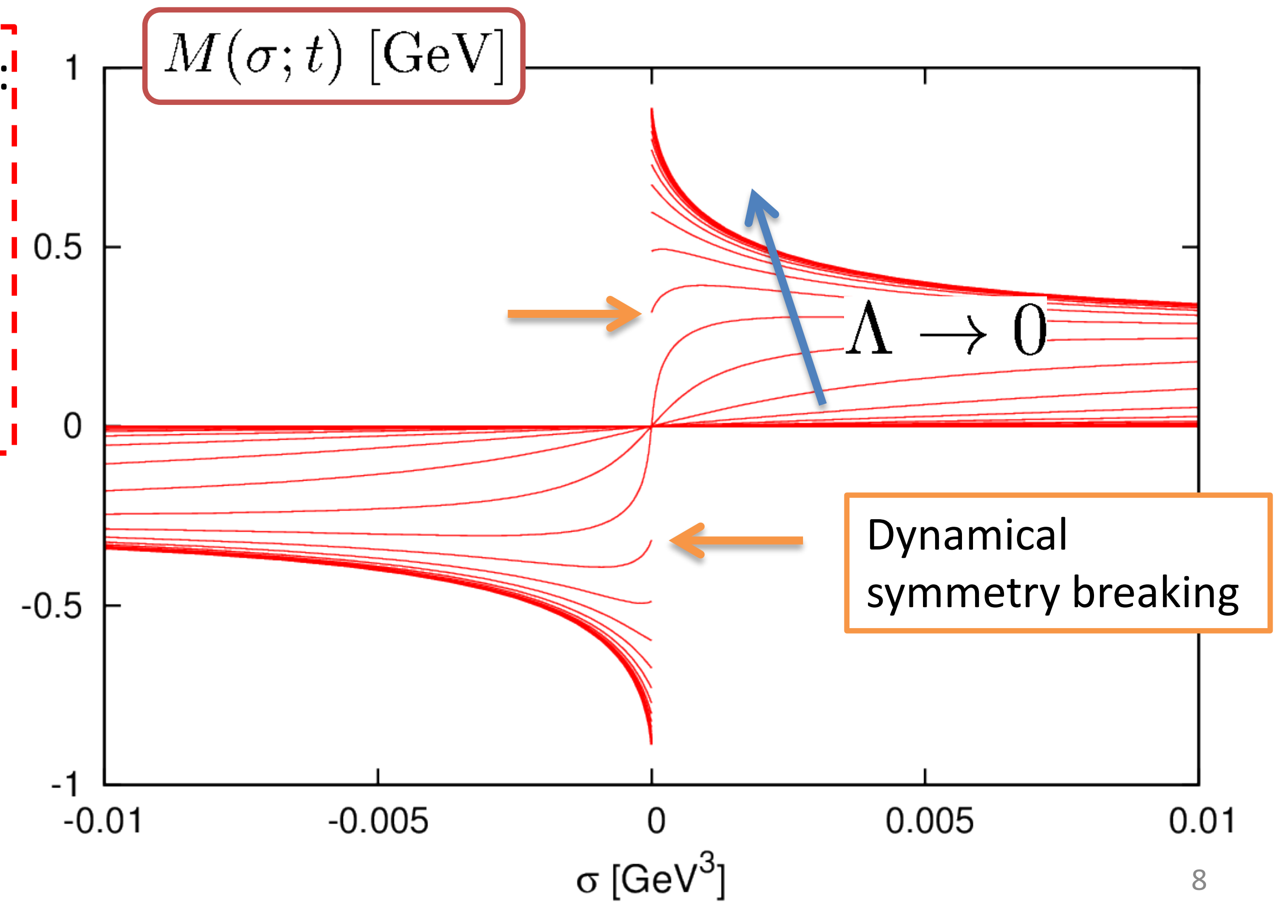
$\xi = 0$: Landau gauge

Discrete chiral symmetry:

$$M(-\sigma) = -M(\sigma)$$

γ_5 discrete transformation

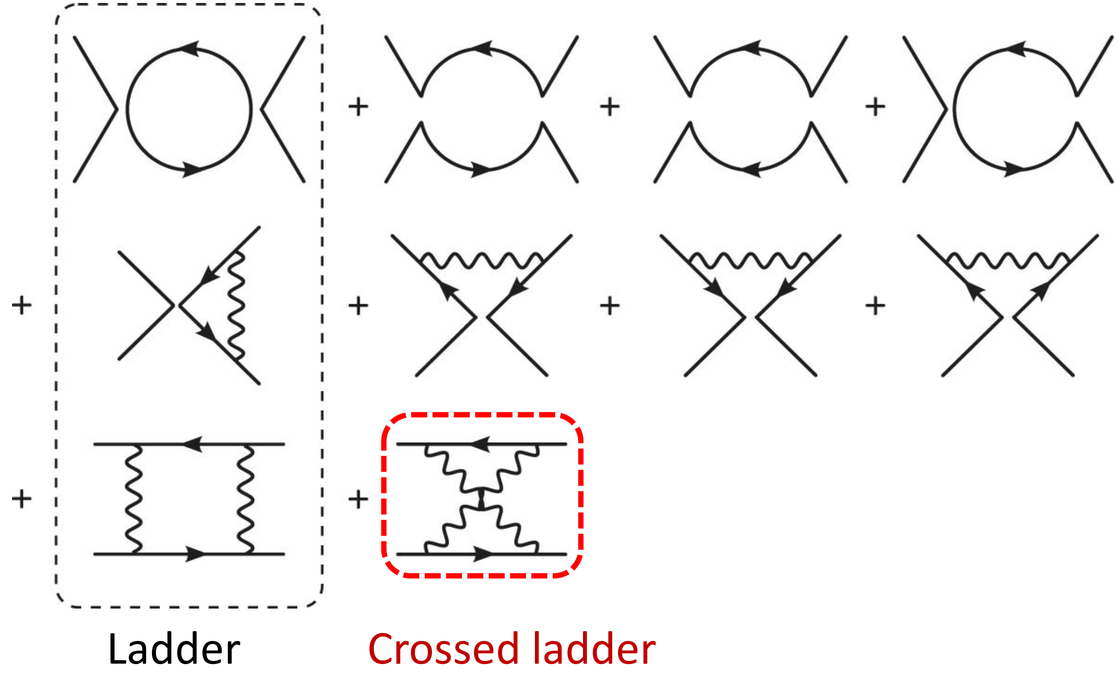
$$\psi \to \gamma_5\psi,\ \bar{\psi} \to -\bar{\psi}\gamma_5$$
$$\sigma(=\bar{\psi}\psi) \to -\sigma$$



Dynamical mass:

$$m_{\mathrm{dyn.}} = \lim_{\sigma\to+0} M(\sigma)$$

Approximation beyond "the Ladder"

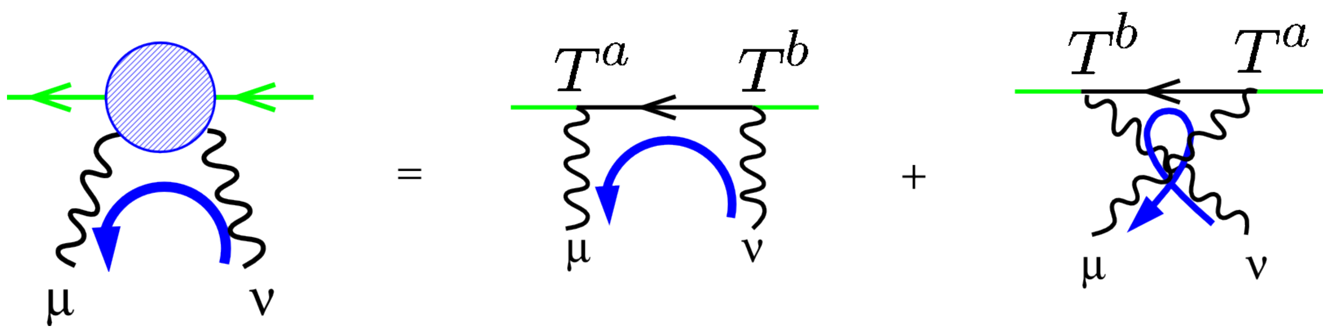


- Crossed ladder diagrams play important role in cancelation of gauge dependence.

- Take into account of this type of non-ladder effects for all order terms in σ.

Approximation beyond “the Ladder”

- Introduce the following corrected vertex to take into account of the non-ladder effects.



Ignore the commutator term.

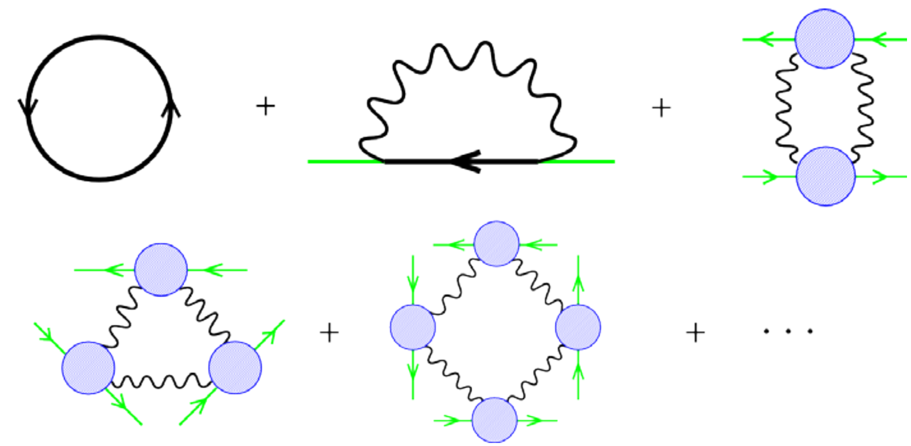
$$T^aT^b + T^bT^a = 2T^aT^b + \cancel{[T^a, T^b]}$$

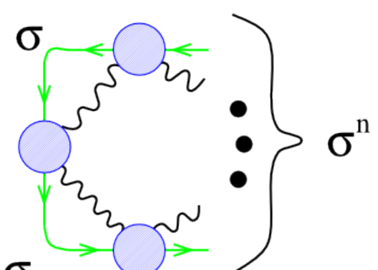
K.-I. Aoki, K. Takagi, H. Terao and M. Tomoyose (2000)

NPRG Eq. Beyond Ladder Approximation

- NPRG eq. described by the infinite number of ladder-form diagrams using the corrected vertex.

$$\partial_t V(\sigma;t) =$$





$$= \frac{\Lambda^4}{4\pi^2}\frac{(-1)^{n+1}}{n}\left(\frac{C_2 g_s^2}{2\Lambda^2(\Lambda^2+m^2)}\right)^n \times \left[(\xi m)^n + \sum_{k=0}^{[\frac{n}{2}]}(-1)^k \binom{n}{2k}(2+4^k)m^{n-2k}\Lambda^{2k}\right]\sigma^n$$

Partial differential Eq. equivalent to this beyond the ladder approximation

Non-ladder extended NPRG eq.

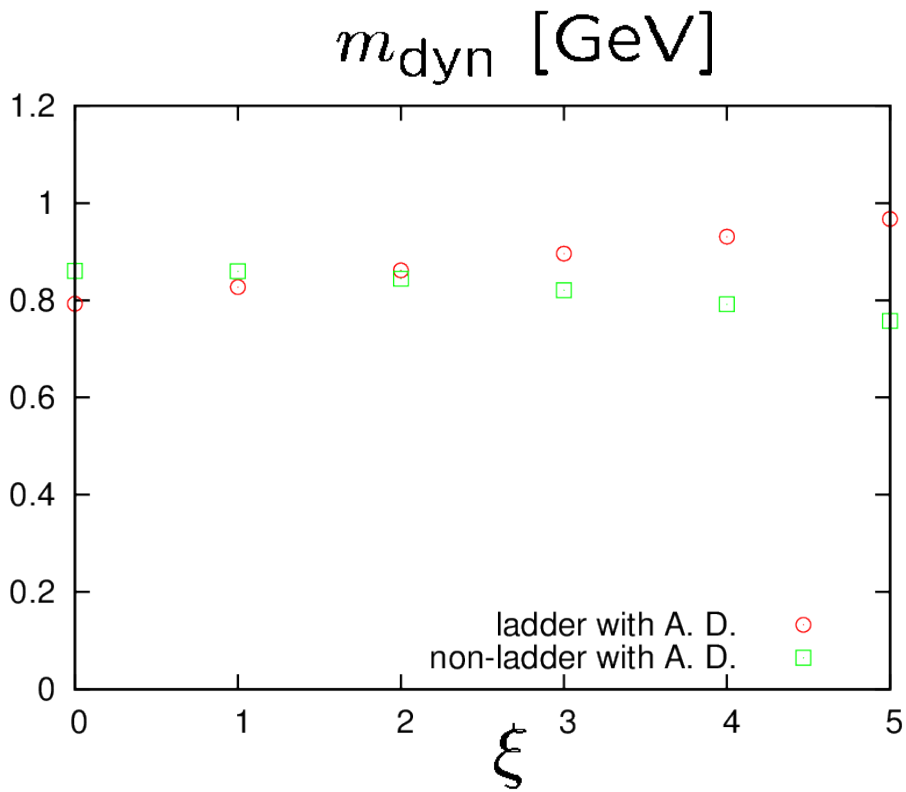
$$\partial_t V(\sigma;t) = \frac{\Lambda^4}{4\pi^2}\log\left[1+\frac{1}{\Lambda^2}\left(m+\frac{C_2 g_s^2}{2\Lambda^2}\cdot\sigma\right)^2\right]$$
$$+\frac{\Lambda^4}{8\pi^2}\log\left[\frac{\Lambda^2+\left(m+\frac{C_2 g_s^2}{2\Lambda^2}\cdot\sigma\right)^2}{\Lambda^2+m^2}+\frac{3\Lambda^2}{(\Lambda^2+m^2)^2}\left(\frac{C_2 g_s^2}{2\Lambda^2}\cdot\sigma\right)^2\right]$$
$$+\frac{\Lambda^4}{4\pi^2}\log\left[1+\xi\frac{m}{\Lambda^2+m^2}\frac{C_2 g_s^2}{2\Lambda^2}\cdot\sigma\right]$$

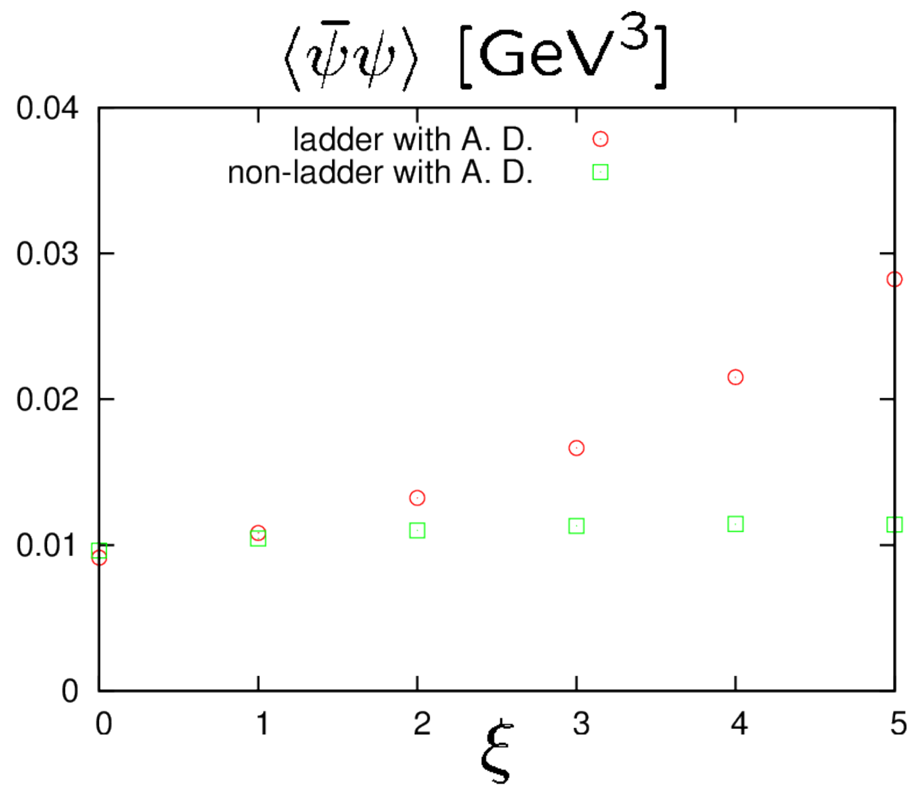
$$m(\sigma;t) = \partial_\sigma V(\sigma;t)$$

Ladder-approximated NPRG eq.

$$\partial_t V(\sigma;t) = \frac{\Lambda^4}{4\pi^2}\ln\left[1+\frac{1}{\Lambda^2}\left(m+\frac{(3+\xi)C_2 g_s^2}{4\Lambda^2}\cdot\sigma\right)^2\right]$$

Gauge dependence of the order parameters





$\langle\bar{\psi}\psi\rangle$ is an observable.

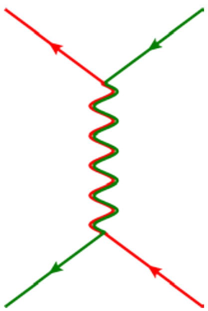
The non-ladder extended result respects to the gauge invariance (almost).

In the Landau gauge, $\xi = 0$, the ladder result of chiral condensates coincides with the non-ladder extended one.

Color superconductivity
(Spontaneous Color symmetry breaking)

- Diquark channel exchanging one gluon

$T \sim 0 \quad \mu \neq 0$



$= \bar{3}_a + 6_s$

attractive repulsive

Diquark condensates

$$\left\langle \left(\bar{\psi}^C\right)_i^a \gamma^5 \psi_j^b \right\rangle \sim \varepsilon_{ij}\epsilon^{abc}$$

$$\sum_{A=1}^{N_c^2-1} T^A_{a'a} T^A_{b'b} = -\frac{N_c+1}{4N_c}\left(\delta_{aa'}\delta_{b'b} - \delta_{ab'}\delta_{a'b}\right) + \frac{N_c-1}{4N_c}\left(\delta_{aa'}\delta_{b'b} + \delta_{ab'}\delta_{a'b}\right)$$

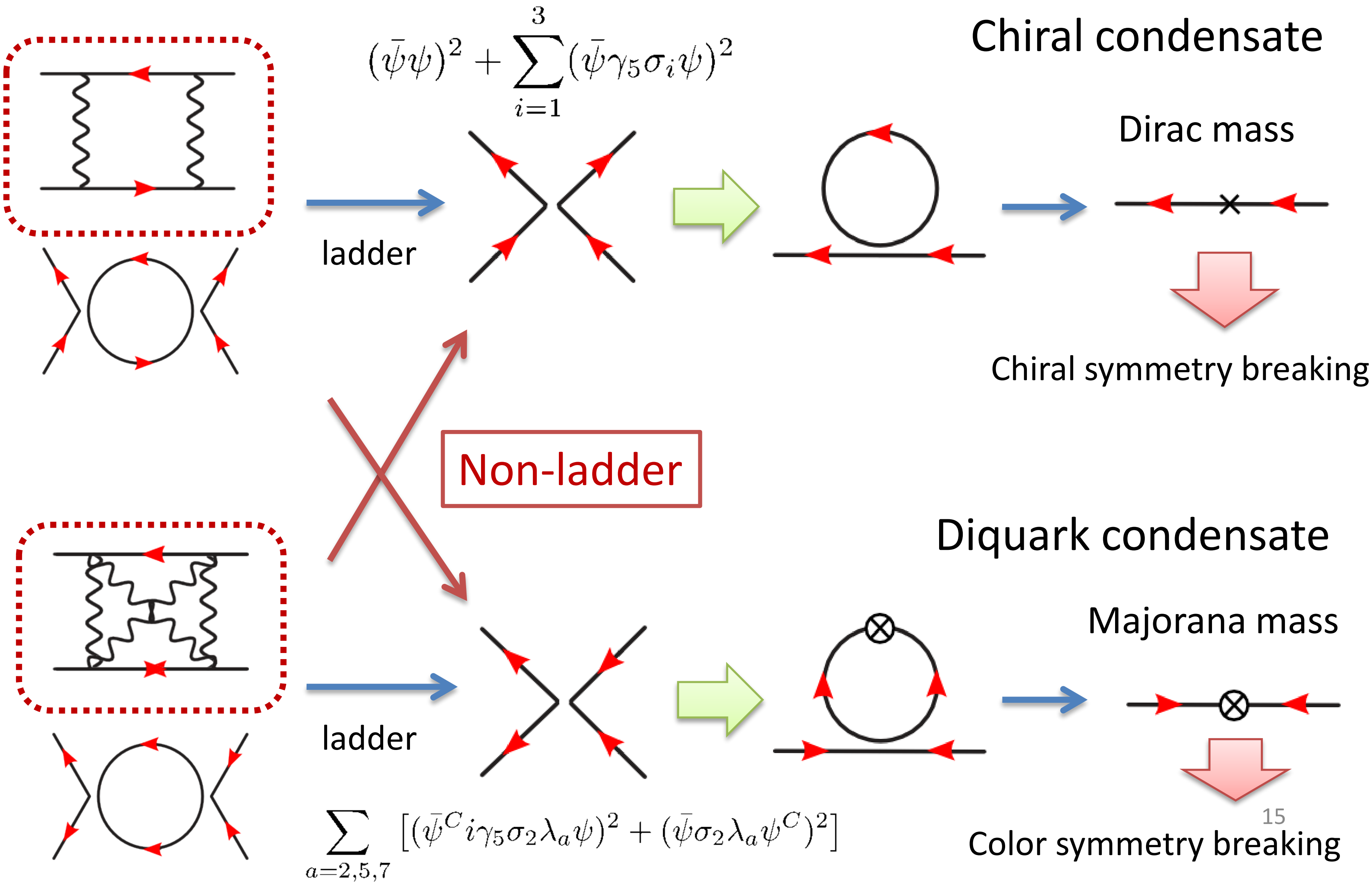
- two flavor color superconductivity (2SC)

Symmetry breaking pattern: $SU(3)_c \to SU(2)$

Diquark condensates: $\langle \bar{\psi}^C i\gamma_5 \sigma_2 \lambda_a \psi \rangle, \ (a = 2, 5, 7)$

σ_i: Pauli matrices in flavor space
λ_a: Gell-Mann matrices in color space

2SC from the view point of NPRG



2-flavor color superconductivity (2SC)

- Extract the scalar bilinear operators

$$V(\psi, \bar{\psi}) \Rightarrow V(\sigma, \Delta) \qquad \begin{aligned} \sigma &= \bar{\psi}\psi \\ \Delta &= \bar{\psi}^C i\sigma_2\lambda_2\psi + \bar{\psi} i\sigma_2\lambda_2\psi^C \end{aligned}$$

- Nambu-Gorkov spinor: $\Psi = \begin{pmatrix} \psi \\ \psi^C \end{pmatrix}$

- Propagator and vertex: $\mathcal{S} = \begin{pmatrix} S_+ & T_- \\ T_+ & S_- \end{pmatrix}$, $\Gamma^a_\nu = \begin{pmatrix} T^A\gamma_\nu & 0 \\ 0 & -T^{A^T}\gamma_\nu \end{pmatrix}$

$$S_\pm^{-1}(p) = i\not{p} \pm \mu\gamma_0 - M - \underline{\Theta} \cdot \frac{\Phi^2}{-i\not{p} \pm \mu\gamma_0 - M},$$

$$T_\pm(p) = i\sigma_2\lambda_2 \frac{\Phi}{-i\not{p} \pm \mu\gamma_0 - M} S_\pm$$

μ: density

Projection operator in the color space: $\Theta = \mathrm{diag}(1, 1, 0)$

Dirac-type mass function: $M(\sigma, \Delta) = \partial_\sigma V(\sigma, \Delta)$

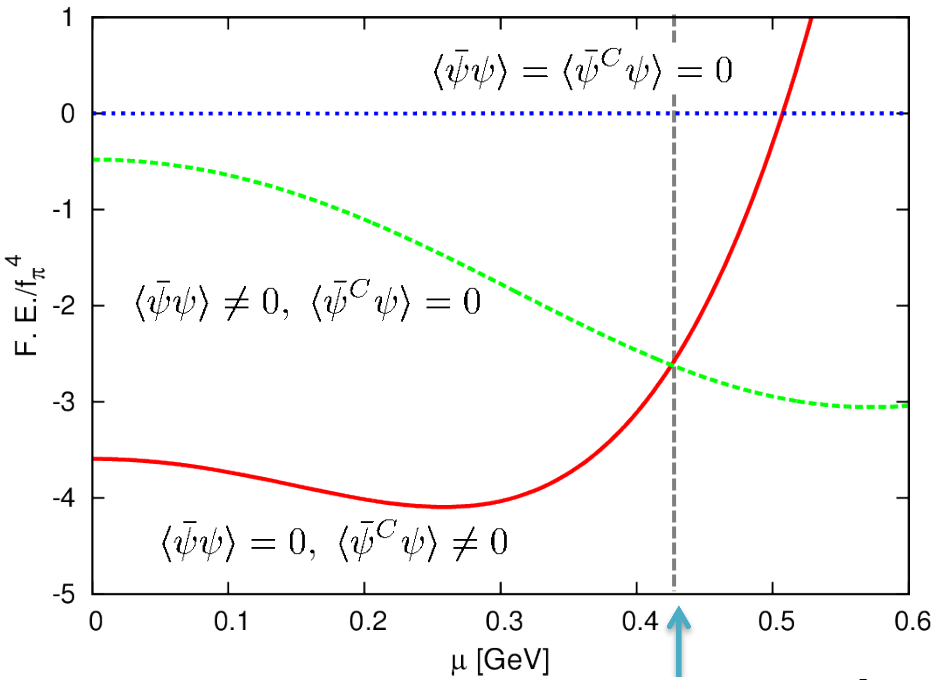
Majorana-type mass function: $\Phi(\sigma, \Delta) = \partial_\Delta V(\sigma, \Delta)$

Result of the phase transition in 2SC at $T = 0$ using the ladder approximation and LPA

$$\partial_t V(\sigma,\Delta) = \left(1-\frac{1}{N_c}\right)\left[\frac{\Lambda^4}{4\pi^2}\log\frac{D+\sqrt{D^2+4\mu^2\Lambda^2}}{2\Lambda^2} - \frac{\mu^2\Lambda^6}{2\pi^2}\left(D+\sqrt{D^2+4\mu^2\Lambda^2}\right)^{-2}\right]$$
$$+\frac{1}{N_c}\left[\frac{\Lambda^4}{4\pi^2}\log\frac{D_0+\sqrt{D_0^2+4\mu^2\Lambda^2}}{2\Lambda^2} - \frac{\mu^2\Lambda^6}{2\pi^2}\left(D_0+\sqrt{D_0^2+4\mu^2\Lambda^2}\right)^{-2}\right]$$

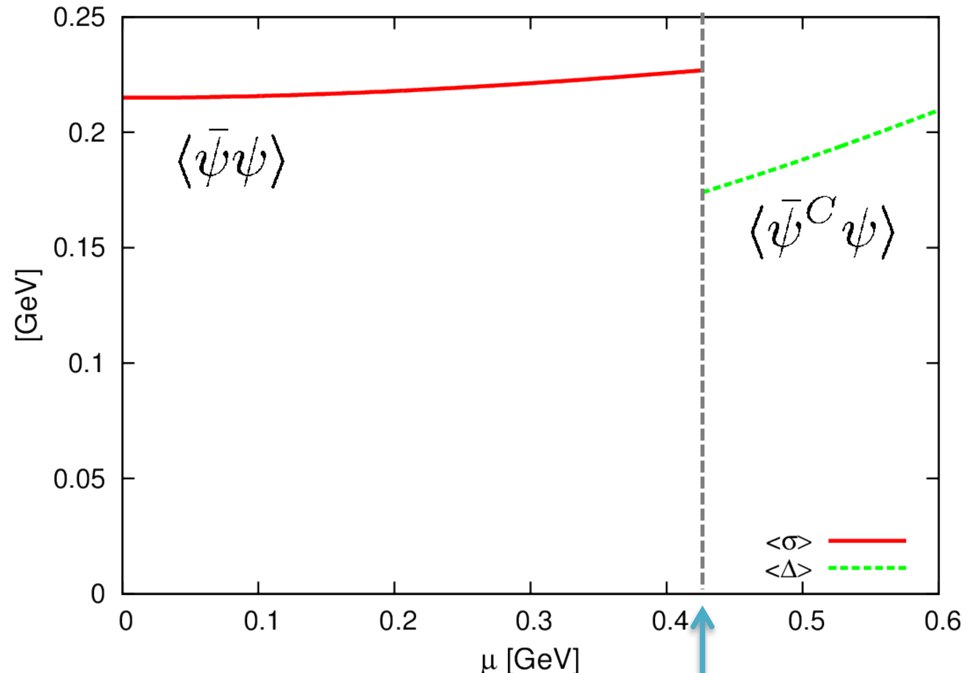
$$D = [(\Lambda^2-\mu^2+M^2+\Phi^2)^2+4\mu^2\Phi^2]^{1/2} \qquad D_0 = |\Lambda^2-\mu^2+M^2|$$

Free energy



$\mu_c = 0.426$ [GeV]

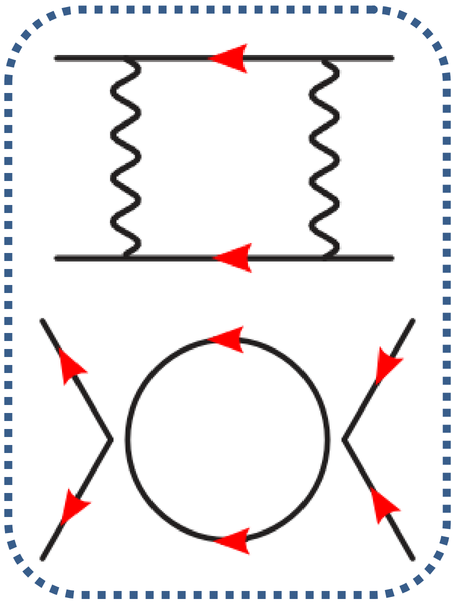
Chiral and diquark condensates



Singularities of the non-ladder β functions using the derivative expansion

Propagator ($\phi = 0$): $$\frac{1}{i\not{p} + \mu\gamma_4 + m} = \frac{-i\not{p} - \mu\gamma_4 + m}{p^2 - \mu^2 - 2i\mu p_4 + m^2}$$

Zero-external-momentum amplitude



The loop of the quarks irrelevant to the diquark condensates:

$$\frac{1}{(\Lambda^2 - \mu^2 + m^2)^2}$$

The loop of the quarks coupled to the diquark condensates:

$$\frac{1}{(\Lambda^2 - \mu^2 + m^2 + \phi^2)^2 + 4\mu^2\phi^2}$$

m : Dirac mass

ϕ : Majorana mass

Summary and prospects

- The NPRG analysis using the non-ladder extended approximation (at zero temperature and zero density) respect the gauge independence (almost), and a great improvement is compared with the ladder approximation.
- In the Landau gauge, however, the gauge dependent ladder result of the chiral condensates coincides with the (almost) gauge independent non-ladder extended one.
- The ladder NPRG analysis has been applied to the 2-flavor color superconductivity (2SC). The results are consistent with those of the ladder Schwinger-Dyson equation. However the non-ladder extended NPRG has a problem of the β functions singularities.
- Prospects
  - Improve the derivative expansion to avoid the singularities.
  - Apply the NPRG analysis to other patters of symmetry breaking, color-flavor locked SC and color-spin locked SC, where the singularities of the β functions do not appear because the color symmetry completely breaks.

Backup slides

Extract the Scalar-type operators

- Chiral symmetric scalar 4-fermi operator

$$SU(3)_L \times SU(3)_R \times U(1)_V.$$

$$\boxed{\rho = \frac{1}{2}\sum_{I=0}^{N_\mathrm{f}^2-1}\left[\left(\bar{\psi}\lambda^I\psi\right)^2 + \left(\bar{\psi}\lambda^I i\gamma_5\psi\right)^2\right]}$$

$\lambda_0 = \sqrt{\frac{2}{N_\mathrm{c}}}, \quad \lambda_a \ (0 \sim a)$: Gell-Mann matrices

- Project the operator space onto the subspace spanned by polynomials in the scalar operator ρ:

$$V(\psi, \bar{\psi}) \Rightarrow V(\rho)$$

- Determining the coefficients of the powers of ρ is equivalent to count the coefficients of the powers of $(\bar{\psi}\lambda_0\psi)^2$.

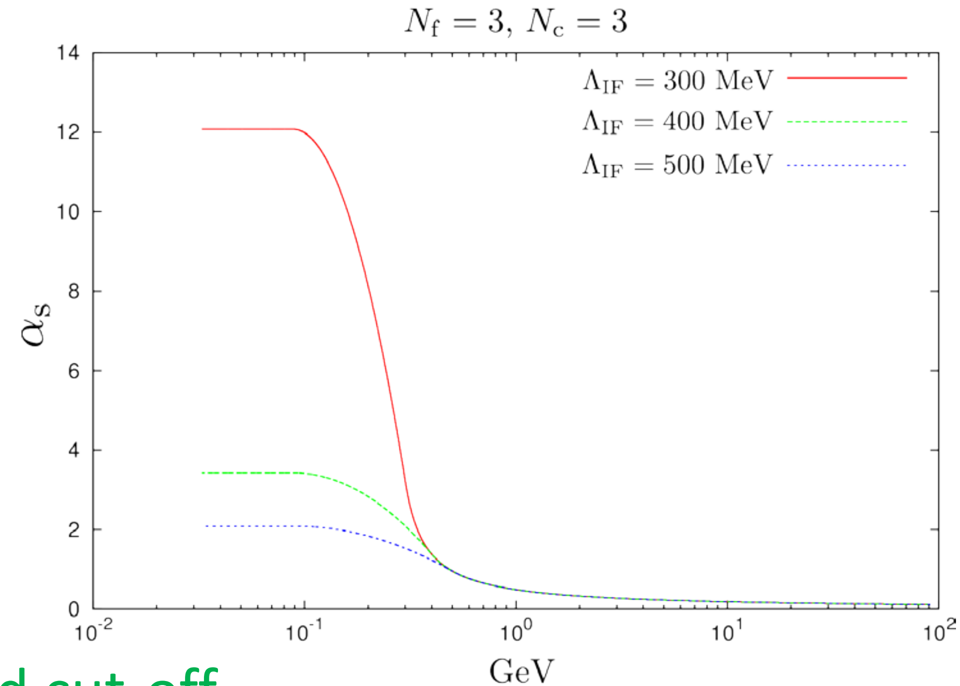
$$V(\psi, \bar{\psi}) \Rightarrow V(\sigma), \ \sigma = \bar{\psi}\psi$$

Running of gauge coupling constant

1-loop perturbative RGE

$$\frac{d}{dt}\alpha_s = \frac{\beta_0}{2\pi}\alpha_s^2 \qquad \alpha_s = g_s^2/4\pi \qquad \beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$$

To take account of the quark confinement , we set a infrared cut-off for the gauge coupling constant.



1-loop perturbative RGE + Infrared cut-off

$$\alpha_s(\Lambda) = \begin{cases} \dfrac{4\pi}{\beta_0 \log(\Lambda^2/\Lambda_{QCD}^2)} & (\Lambda > \Lambda_{IF}) \\ \dfrac{4\pi}{\beta_0 \log(\Lambda_{IF}^2/\Lambda_{QCD}^2)} + \dfrac{4\pi}{\beta_0}\dfrac{[\log(\Lambda_1/\Lambda)]^2 - [\log(\Lambda_1/\Lambda_{IF})]^2}{\log(\Lambda_1/\Lambda_{IF})[\log(\Lambda_{IF}^2/\Lambda_{QCD}^2)]^2} & (\Lambda_1 < \Lambda < \Lambda_{IF}) \\ \dfrac{4\pi}{\beta_0 \log(\Lambda_{IF}^2/\Lambda_{QCD}^2)} - \dfrac{4\pi}{\beta_0}\dfrac{\log(\Lambda_1/\Lambda_{IF})}{[\log(\Lambda_{IF}^2/\Lambda_{QCD}^2)]^2} & (\Lambda < \Lambda_1) \end{cases}$$

$$\log\left(\Lambda_1/\Lambda_{QCD}\right) = -1.0$$

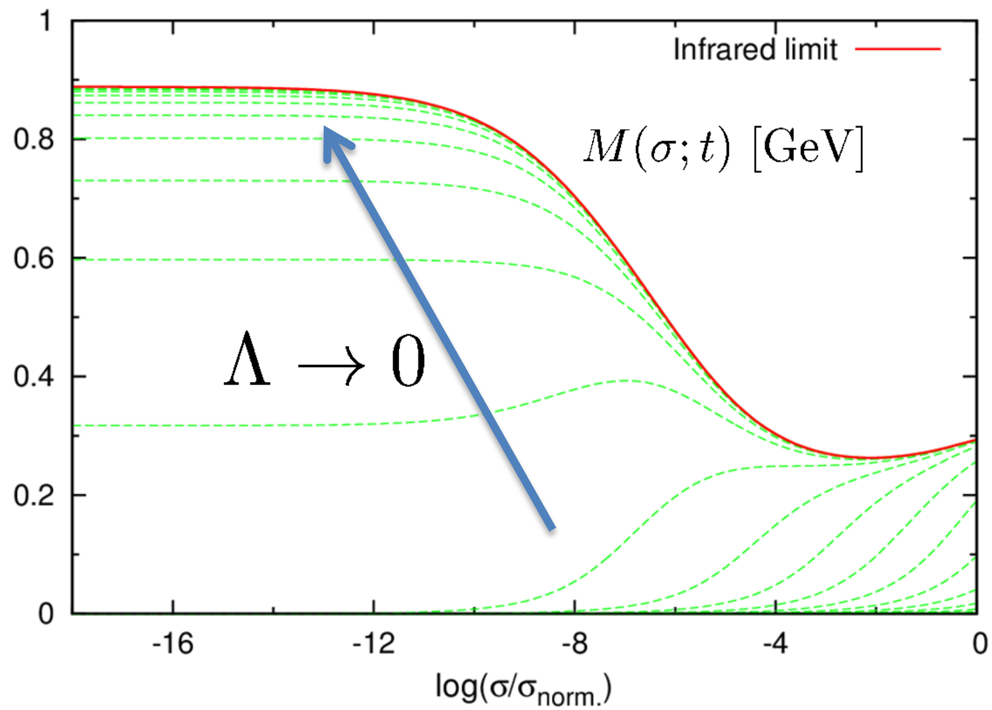
$$\boxed{\Lambda_{QCD} = 484\text{MeV}}$$

RG Flow of the mass function

$$\partial_t M(\sigma;t) = \frac{\Lambda^4}{2\pi^2} \frac{M + 3\Lambda^{-2}C_2\alpha_s\sigma}{\Lambda^2 + (M + 3\Lambda^{-2}C_2\alpha_s\sigma)^2} \left(\underline{\partial_\sigma M} + 3\Lambda^{-2}C_2\alpha_s(t)\right)$$

$$M(\sigma;\Lambda) = \partial_\sigma V(\sigma;\Lambda)$$

- Avoid the singular point, $\partial_\sigma M(\sigma;t)|_{\sigma=0,t=t'} = \infty$
- Reparameterization: $\sigma \to x = \log\sigma$
- Numerical result



- Weak solution of the partial differential equation….

Collective field

Aoki, Morikawa, Sumi, Terao & Tomoyose (2000);
Shimizube & Sumi (2006).

1. Lower the scale Λ of the effective action Γ_Λ to the chiral symmetry breaking scale Λ_C

2. Introduce collective field ϕ to describe composite field $\bar{\psi}\psi$

$$\Gamma_{\Lambda_c}[\Phi] = \int_x \left\{ \frac{Z_F}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \bar{\psi}\left(Z_\psi \partial\!\!\!/ + i\bar{g}_s A\!\!\!/\right)\psi - \underline{\bar{G}_2(\bar{\psi}\psi)^2} - \bar{G}_4(\bar{\psi}\psi)^4 - \cdots \right\}$$

$$\longrightarrow \quad -\frac{1}{2}\phi^2 - y\phi\bar{\psi}\psi - \frac{1}{2}(\bar{G}_2 - y^2)(\bar{\psi}\psi)^2$$

$\phi \sim y\bar{\psi}\psi$ (Equation of motion)

$\Lambda_C \sim 1\mathrm{GeV}$: Chiral symmetry breaking scale

3. Effective potential including the fermions and the boson

$$U(\psi, \bar{\psi}, \phi; \Lambda) = U_0(\phi; \Lambda) - m(\phi; \Lambda)\bar{\psi}\psi + V(\psi, \bar{\psi}; \Lambda)$$

$$\phi(x) = \phi + \cancel{\delta\phi}(x) \longleftarrow$$ Ignore the fluctuation

Scale dependence of collective field potential

collective field potential $U_0(\phi;\Lambda)$