場の理論と物性論における トポロジカル量子現象

～スカーミオンを中心に～

基研研究会 熱場の量子論とその応用
2012年8月22日(水)～8月24日(金)

2012/8/23

Muneto Nitta (新田宗土)
Keio U. (慶應義塾大学)

Topological Quantum Phenomena in
Condensed Matter with Broken Symmetries

Keio University
1858
CALAMVS
GLADIO
FORTIOR

共同研究者 cond-mat.

① スピノールBEC
川口由紀, 小林信吾, 上田正仁（東大本郷）,
小林未知数（東大駒場）, 内野瞬（スイス）

② 多成分BEC
笠松健一 (近畿大), 竹内宏光（広島大）,
坪田誠（大阪市大）, 衛藤稔（山形大）

③ BECにおける人工ゲージ場
川上巧人, 水島健, 町田一成（岡山大）

④ フェルミ気体・超伝導
高橋大介(東大駒場), 土屋俊二（東京理大）,
吉井涼輔（京大基研）, Giacomo Marmorini（理研）

⑤ 非可換統計
安井繁宏, 板倉 数記（KEK）, 広野雄士（東大/理研）

ボゾン系（①–③）
フェルミオン系（④–⑤）

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

§ 4-1 Brane annihilation (4p+22p)

§ 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

  § 4-1 Brane annihilation (4p+22p)

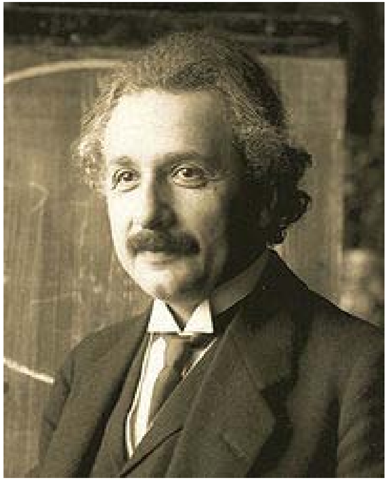
  § 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

1924 Bose Einstein Condensation(BEC), Bose & Einstein

BEC occurs when *de Broglie wave length λ* of particles is comparable with the mean distance.

$$E = \frac{\hbar^2}{2m}\lambda_T^{-2} \approx k_B T \qquad \lambda_T = \sqrt{\frac{h^2}{2\pi m k_B T}}$$



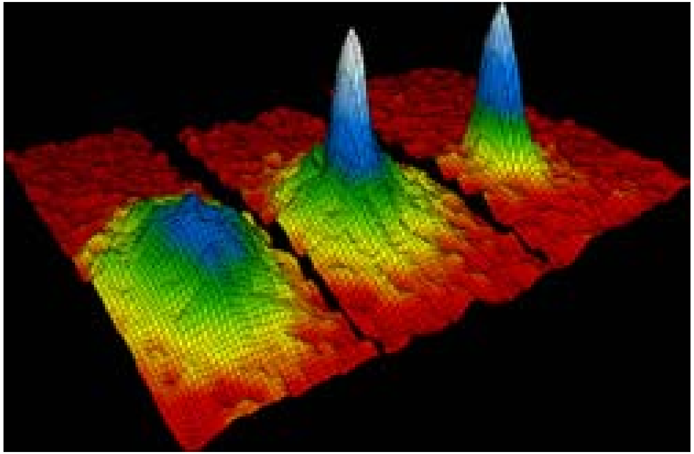
Transition temperature

$$T_0 = \frac{3.31\hbar^2}{m k_B}\left(\frac{N}{V}\right)^{2/3}$$

Number of condensates

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_0}\right)^{3/2}$$

``Pure" BEC (99% is BEC)



Cold atomic gases

1995 cold atomic bose gas

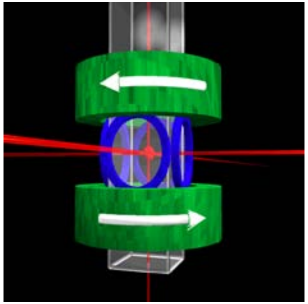
^{87}Rb, ^{23}Na, ^{7}Li

Cornell (Colorado), Ketterle(MIT)

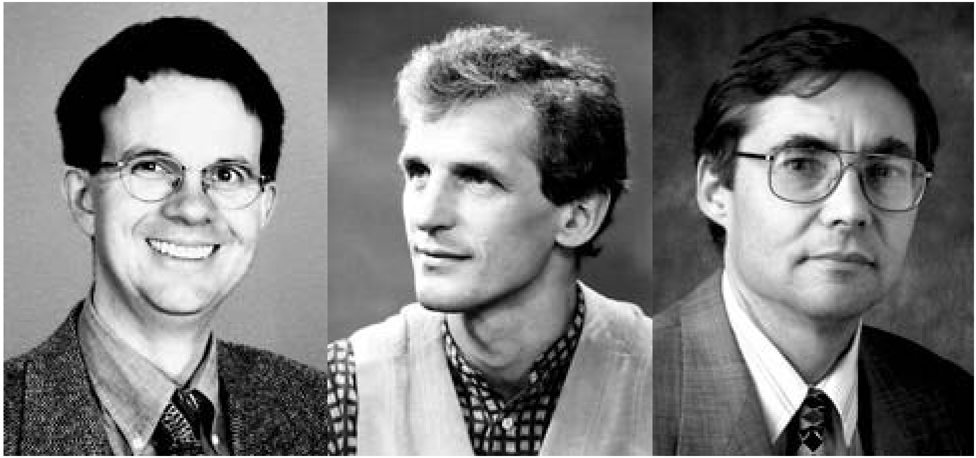
& Wieman (Colorado)

2003 cold atomic fermion gas

JILA(Colorado), MIT



doppler laser cooling
magneto-optical trap
evaporative cooling



Temperature ~ 10^{-6},10^{-7} K

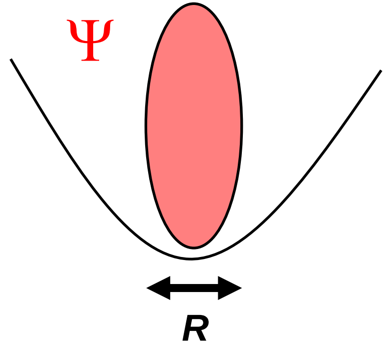
Number ~ 10^{6}, Size ~ 10^{-3}cm

trapping potential

$$V = \frac{1}{2} M \omega^2 r^2$$

ω **: frequency**

M **: mass of atoms**



$$\frac{M\omega^2 R^2}{2} \cong \frac{p^2}{2M} \cong \frac{3}{2} k_B T$$

$$\lambda_T = \frac{\hbar}{p} \cong \frac{\hbar}{M\omega R} \approx \frac{R}{N^{1/3}} \longrightarrow R \cong \sqrt{\frac{\hbar}{M\omega}} N^{1/6}$$

de Broglie wave length

mean particle distance

transition temperature

$$T \cong \frac{\hbar \omega N^{1/3}}{k_B} \approx 10^{-6} [K]$$ **for** $\omega \approx 10^3 [Hz], N \approx 10^6$

Scalar BEC, ^{4}He superfluid

Bogoliubov theory for weakly interactive Bose gas
(with point interaction)

$$H = \int dV \left[\psi^\dagger \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}} - \mu \right) \psi + \frac{1}{2} g\, \psi^{\dagger 2} \psi^2 \right]$$

$$[\psi(r), \psi^\dagger(r')] = \delta(r - r')$$

$V(r) = g\delta(r)$ **point interaction**

mean field approximation

$$\psi(x) = \Psi(x) + \phi(x)$$

wave function for condensation ($\Psi(x)$)

fluctuation (phonon) : non-condensed component ($\phi(x)$)

Scalar BEC, ^{4}He superfluid

Gross-Pitaevskii (nonlinear Schrödinger) Equation

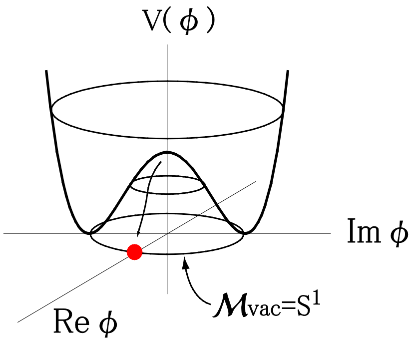
$$i\hbar \frac{\partial \psi}{\partial t} = \left[-\frac{\hbar^2}{2M}\nabla^2 + V_{\text{ext}} - \mu + g|\psi|^2 \right]\psi = \frac{\delta E}{\delta \psi^*} \qquad g \equiv \frac{4\pi\hbar^2 a_s}{M}$$

μ : chemical potential M : mass of atoms a_s : s-wave scattering length

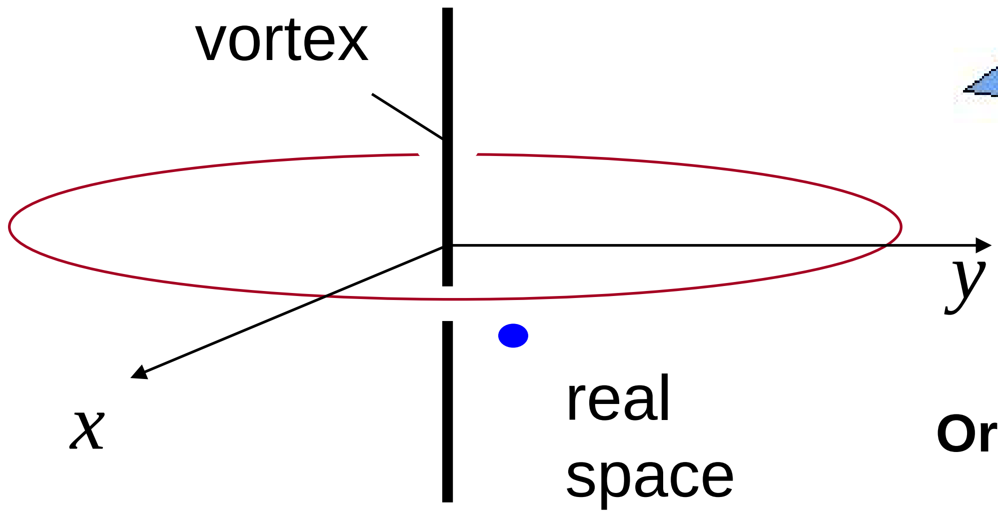
$V_{\text{ext}}(r)$: trapping potential $V_{\text{ext}} = \frac{1}{2}M\omega^2 r^2$

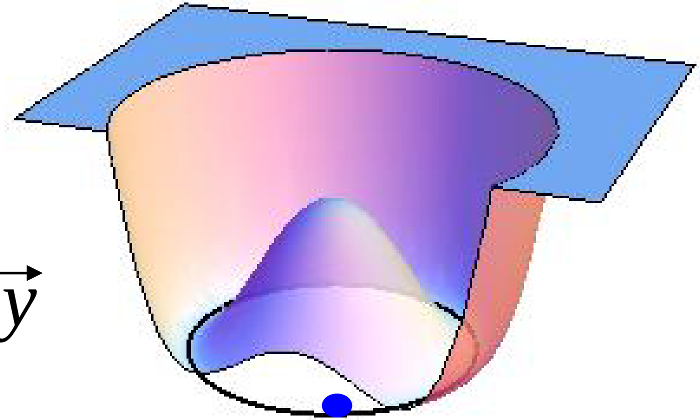
Gross-Pitaevskii energy functional

$$E[\psi] = \int d^3\mathbf{r} \left\{ \frac{\hbar^2}{2M}|\nabla\psi|^2 + (V_{\text{ext}} - \mu)|\psi|^2 + \frac{g}{2}|\psi|^4 \right\}$$



For d=1 with V_{ext}=0, it is *integrable*. [Zakharov-Manakov ('74)]
It is used in optics and water waves. Examples are bright soliton and dark soliton.





Order Parameter Space(OPS) = U(1)

Superconductors under magnetic field

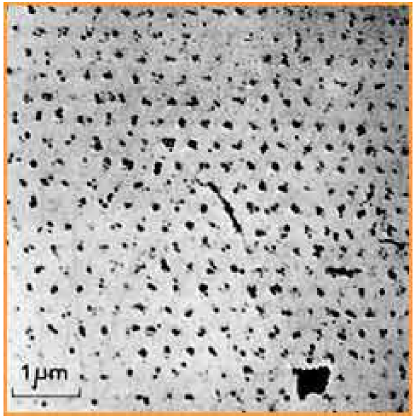
Flux quantization $k \in \pi_1[U(1)] \cong \mathbf{Z}$

$$\Phi = \frac{hc}{2e}k = \Phi_0 k \quad \Phi_0 = \frac{hc}{2e} = 2.07 \times 10^{-15}[\text{weber}]$$

Gallery of Abrikosov Lattices in Superconductors

@ Oslo Superconductivity Lab http://www.fys.uio.no/super/vortex/

U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart
Physics Letters 24A, 526 (1967)

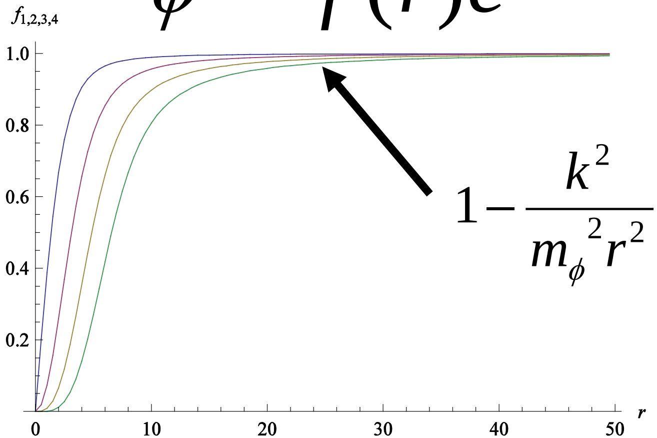


quantization of circulation

$$k \in \pi_1[U(1)] \cong \mathbf{Z} \qquad \oint d\mathbf{r}\cdot\mathbf{v}_{\rm eff} = \frac{\hbar}{M}k$$

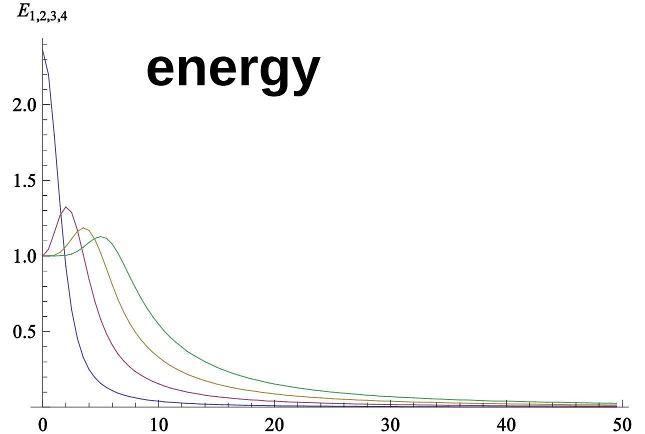
$$\mathbf{v}_{\rm eff} = \frac{1}{2i}\frac{\Psi^*\nabla\Psi - \Psi\nabla\Psi^*}{\Psi^*\Psi}$$

$$\psi = f(r)e^{ik\theta}$$



$$1-\frac{k^2}{m_\phi^2 r^2}$$

energy



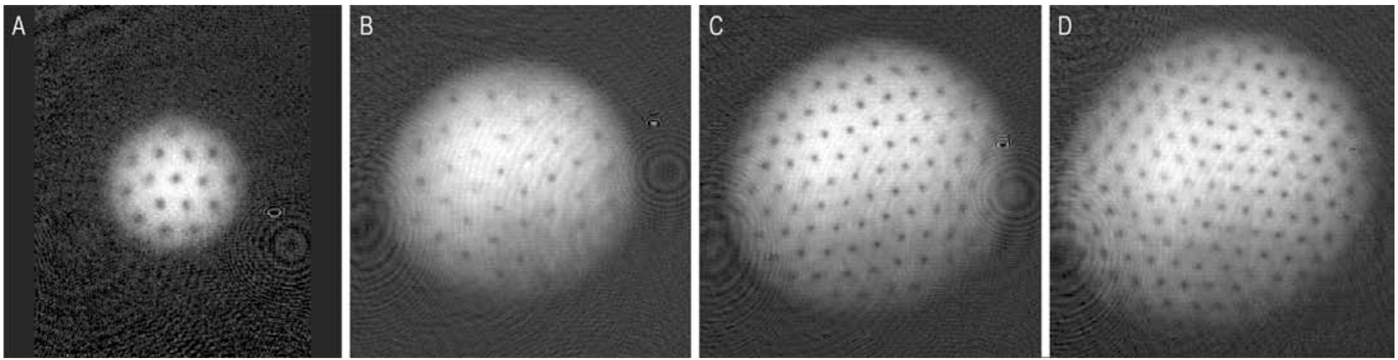
tension $T = 2\pi v^2 k^2 \log\Lambda$ **system size** Λ

Inter-vortex force $F = \dfrac{4\pi v^2}{R}$ **distance** R

Vortex nucleation under rotation

Rotation in rotating frame $\nabla \to \nabla - i\frac{M}{\hbar}\mathbf{\Omega}\times\mathbf{r}$

$$E[\psi] = \int d^3\mathbf{r}\left\{\frac{\hbar^2}{2M}\left|\left(\nabla - i\frac{M}{\hbar}\mathbf{\Omega}\times\mathbf{r}\right)\psi\right|^2 + (V-\mu)|\psi|^2 + \frac{g}{2}|\psi|^4\right\}$$

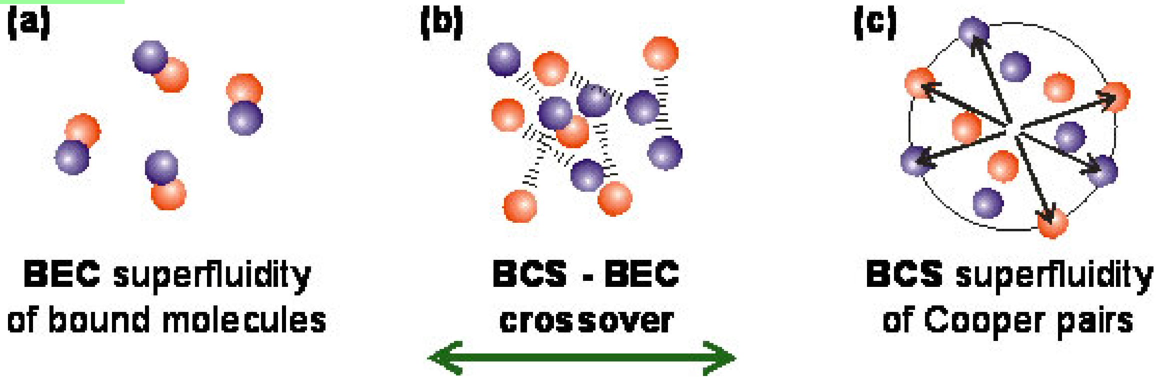


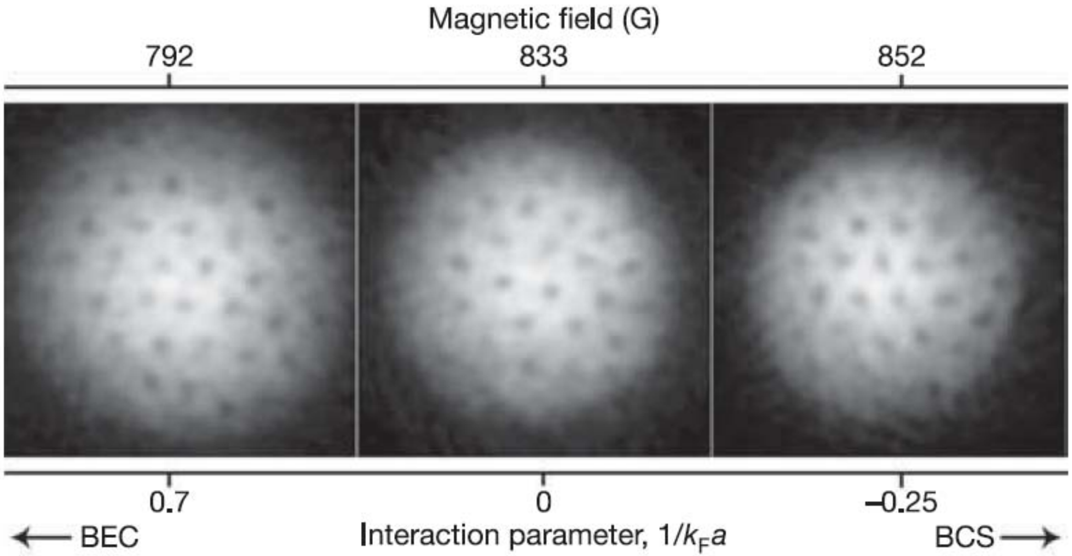
Abo-Shaeer, Raman, Vogels, Ketterle, Science 292, 476-479 (2001)

A proof of superfluidity

BEC/BCS Crossover

Fermions with pseudo spin





Zwierlein, Abo-Shaeer, Schirotzek, Schunck & Ketterle
Nature 435, 1047-1051 (23 June 2005)

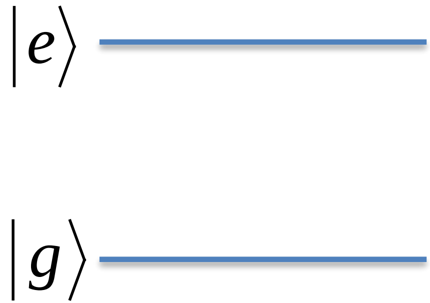
A proof of superfluidity in all range of BES/BCS

Artificial Gauge Field

A review: J.Dalibard et.al., Rev. Mod. Phys. 83, 1523–1543 (2011)

Two-state model

Two states $\{|g\rangle, |e\rangle\}$



Hamiltonian

$$H = \left(\frac{P^2}{2M} + V\right)\hat{1} + U$$

$$\boldsymbol{P} = -i\hbar\boldsymbol{\nabla}$$

coupling

$$U = \frac{\hbar\Omega}{2}\begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta \end{pmatrix}$$

Eigenstates of *U* = Dressed states

$$|\chi_1\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix} \quad |\chi_2\rangle = \begin{pmatrix} -e^{-i\phi}\sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}$$

$\hbar\Omega/2$ **eigenvalues** $-\hbar\Omega/2$

Full state

$$|\Psi(\boldsymbol{r},t)\rangle = \sum_{j=1,2} \psi_j(\boldsymbol{r},t)|\chi_j(\boldsymbol{r})\rangle$$

$$\boldsymbol{\nabla}[\psi_j|\chi_j\rangle] = [\boldsymbol{\nabla}\psi_j]|\chi_j\rangle + \psi_j|\boldsymbol{\nabla}\chi_j\rangle$$

Born-Oppenheimer approximation

$$\boldsymbol{P}|\Psi\rangle = \sum_{j,l=1}^{2}[(\delta_{j,l}\boldsymbol{P} - \boldsymbol{A}_{jl})\psi_l]|\chi_j\rangle$$

Gauge field

$$\mathbf{A}_{jl} = i\hbar\langle\chi_j|\nabla\chi_l\rangle$$

Neglecting ψ_2, EOM of ψ_1

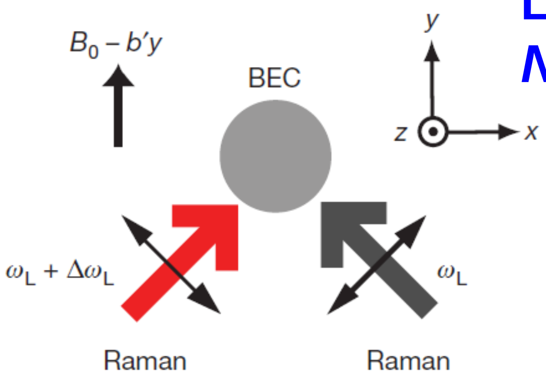
$$i\hbar\frac{\partial\psi_1}{\partial t} = \left[\frac{(\boldsymbol{P}-\boldsymbol{A})^2}{2M} + V + \frac{\hbar\Omega}{2} + W\right]\psi_1$$

Gauge fields as Berry phase

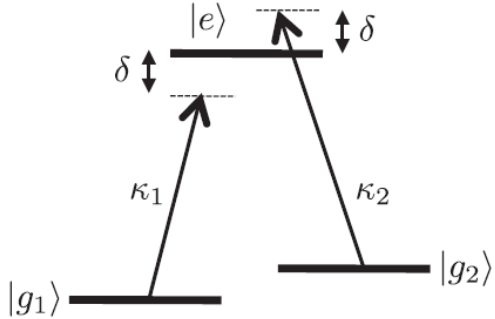
$$\boldsymbol{A}(\boldsymbol{r}) = i\hbar\langle\chi_1|\boldsymbol{\nabla}\chi_1\rangle = \frac{\hbar}{2}(\cos\theta - 1)\boldsymbol{\nabla}\phi. \quad \boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla}\times\boldsymbol{A} = \frac{\hbar}{2}\boldsymbol{\nabla}(\cos\theta)\times\boldsymbol{\nabla}\phi$$

Synthetic magnetic fields for ultracold neutral atoms

Lin, Compton, Jimenez-Garcia, Porto & Spielman, *Nature* 462, 628-632 (3 December 2009)



3 states



interaction

$$U = \frac{\hbar}{2}\begin{pmatrix} -2\delta & \kappa_1^* & 0 \\ \kappa_1 & 0 & \kappa_2 \\ 0 & \kappa_2^* & 2\delta \end{pmatrix}$$

1 dark state $$|D\rangle = (\kappa_2|g_1\rangle - \kappa_1|g_2\rangle)/\kappa$$

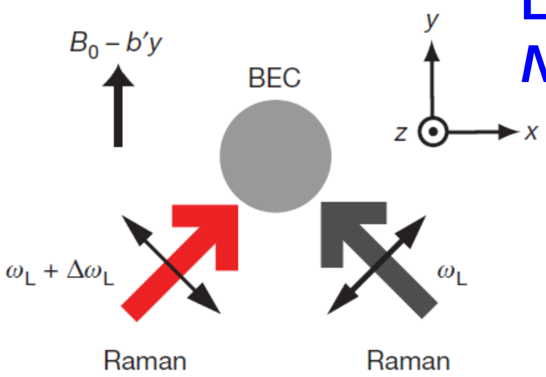
2 bright states $$|\pm\rangle = (|B\rangle \pm |e\rangle)/\sqrt{2} \qquad |B\rangle = (\kappa_1^*|g_1\rangle + \kappa_2^*|g_2\rangle)/\kappa$$

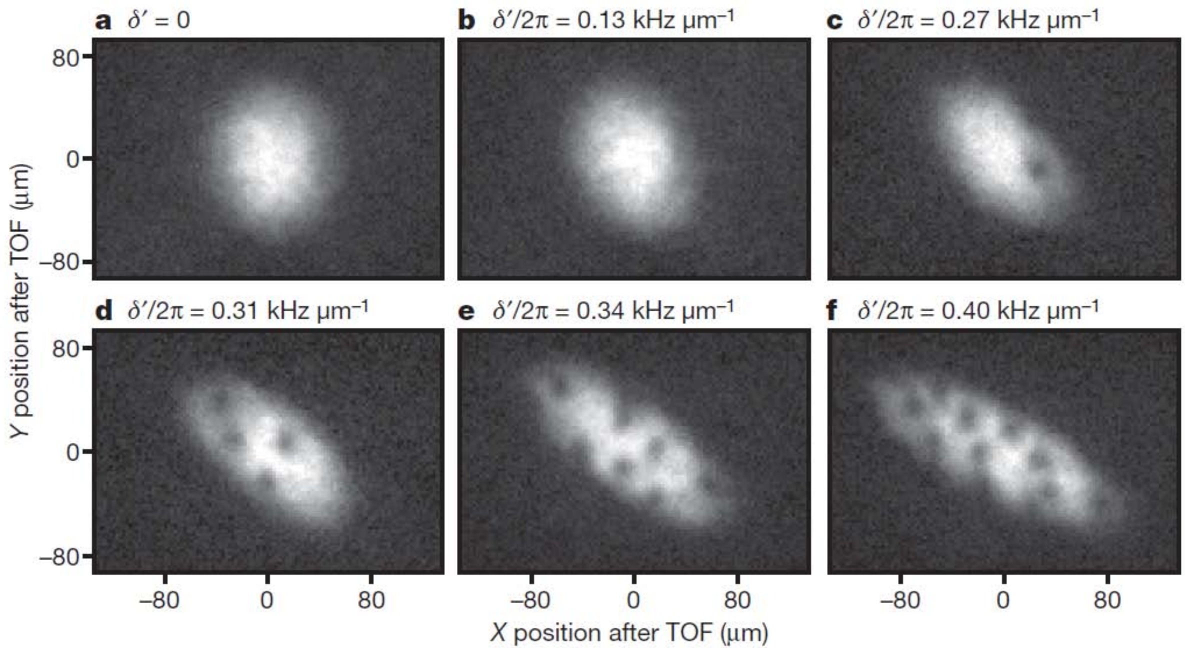
$$|\Psi(\boldsymbol{r})\rangle = \sum_{X=D,\pm} \psi_X(\boldsymbol{r})|X(\boldsymbol{r})\rangle \longrightarrow |\Psi(\boldsymbol{r})\rangle \approx \psi_D(\boldsymbol{r})|D(\boldsymbol{r})\rangle$$

Adiabatic approx

Synthetic magnetic fields for ultracold neutral atoms

Lin, Compton, Jimenez-Garcia, Porto & Spielman, *Nature* 462, 628-632 (3 December 2009)





A proof of artificial magnetic field

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

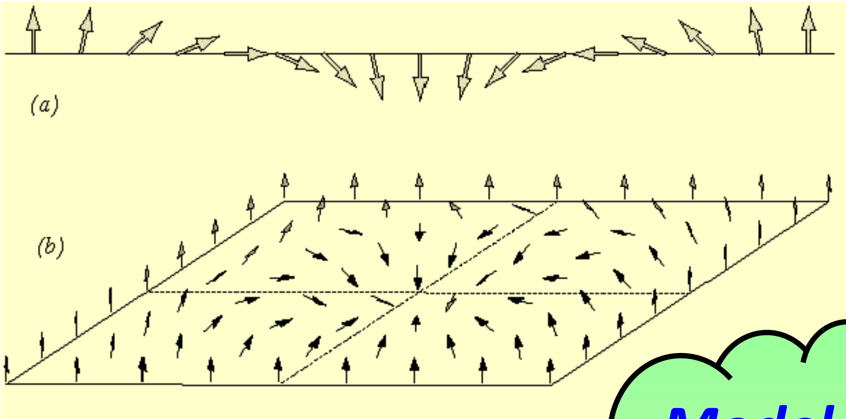
§ 4 3D Skyrmions in BECs

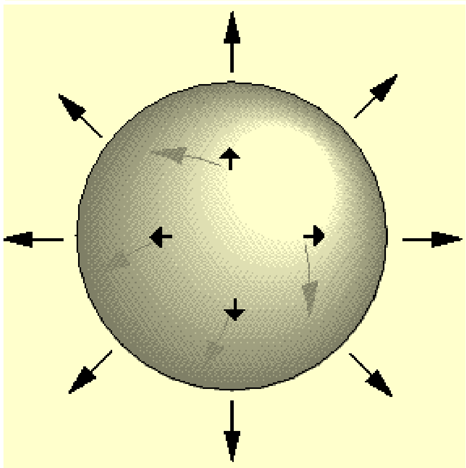
§ 4-1 Brane annihilation (4p+22p)

§ 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

What is a Skyrmion?





3dim hedgehog

Model of nucleon in HEP

1D Skyrmion

=Sine-Gordon kink

$$\pi_1(S^1) = \mathbf{Z}$$

2D Skyrmion

$$\pi_2(S^2) = \mathbf{Z}$$

3D Skyrmion

$$\pi_3(S^3) = \mathbf{Z}$$

T.H.R. Skyrme

A Nonlinear theory of strong interactions
Proc.Roy.Soc.Lond. A247 (1958) 260-278
A Unified Field Theory of Mesons and Baryons
Nucl.Phys. 31 (1962) 556-569

1D Skyrmion

O(2) model (=sine-Gordon model)

Bogomol'nyi completion

$$2E = \int dx \left[(\partial_x \theta)^2 + \cos\theta \right] = \int dx \left[(\partial_x \theta)^2 + \sin^2\left(\frac{\theta}{2}\right) - 1 \right]$$

$$= \int dx \left[\left(\partial_x \theta \mp \sin\left(\frac{\theta}{2}\right) \right)^2 \pm 2\partial_x \theta \sin\left(\frac{\theta}{2}\right) - 1 \right] \geq T_{1D}$$

Bogomol'nyi-Prasad-Sommerfield (BPS) equation

$$\partial_x \theta \mp \sin\left(\frac{\theta}{2}\right) = 0$$

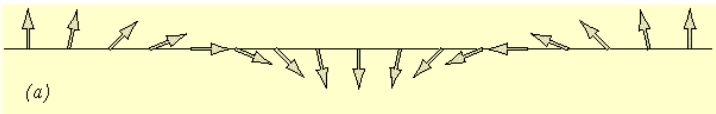
Sine-Gordon kink

SG Topological charge

$$T_{1D} = \pm \int dx \partial_x \theta \sin(\theta/2)$$

$$= \mp 2 \int dx \partial_x \cos(\theta/2)$$

$$= \mp 2 \left[\cos(\theta/2) \right]_{x=-\infty}^{x=+\infty}$$



$$k \in \pi_1(S^1) = \mathbf{Z}$$

O(3) sigma model

1. (Truncated model of) 2component BECs
2. Ferromagnet

$$E=\frac{1}{2}(\nabla S)^2$$

$\mathbf{S}(x)=(S_1,S_2,S_3)$

$\mathbf{S}^2=1$

Target space = S^2

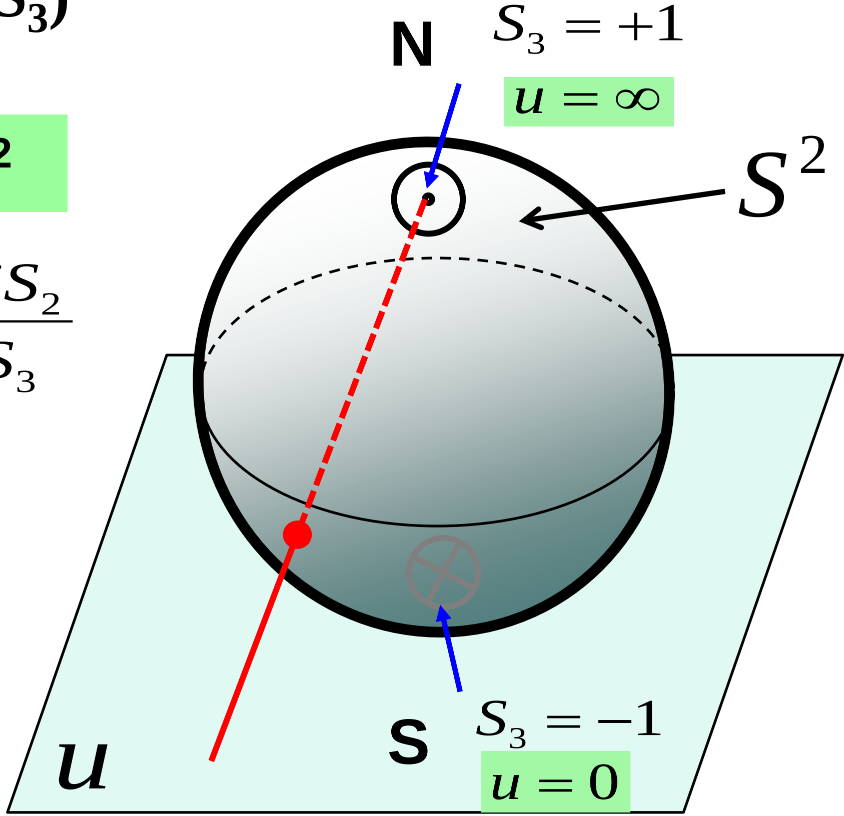
Stereographic coordinate u

$$u=\frac{S_1-iS_2}{1-S_3}$$

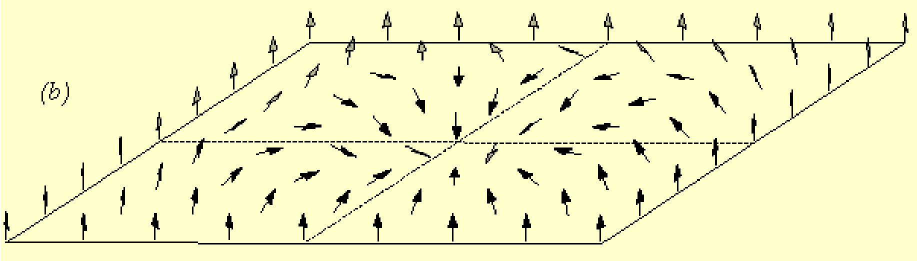
equivalent to

CP¹ model

$$E=\int d\mathbf{r}\,\frac{\sum_\alpha |\partial_\alpha u|^2}{\left(1+|u|^2\right)^2}$$



2D Skyrmion (=lump, sigma model instanton)



Bogomol'nyi completion

$$E = \int d^2x \frac{\sum_\alpha |\partial_\alpha u|^2}{\left(1+|u|^2\right)^2}$$

$$= \int d^2x \left[\frac{|\partial_x u \mp i\partial_y u|^2}{\left(1+|u|^2\right)^2} \pm \frac{i\left(\partial_x u^* \partial_y u - \partial_y u^* \partial_x u\right)}{\left(1+|u|^2\right)^2} \right] \geq |T_L|$$

BPS equation

$$\partial_x u \mp i\partial_y u = 0$$

$$\bar{\partial}_{\bar{z}} u = 0 \qquad z \equiv x + iy$$

2D Skyrme topological charge

$$T_L = \pm \int d^2x \frac{i\left(\partial_x u^* \partial_y u - \partial_y u^* \partial_x u\right)}{\left(1+|u|^2\right)^2}$$

$$= 2\pi k$$

$$k \in \pi_2(S^2) = \mathbf{Z}$$

2D Skyrmion

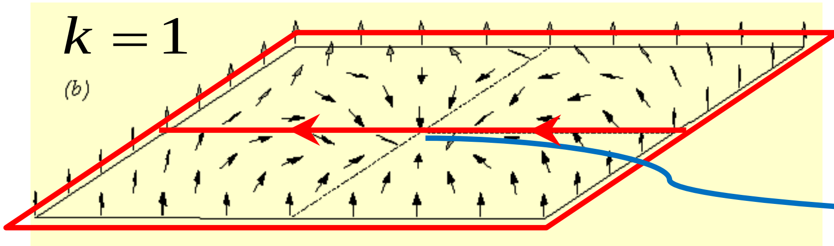
BPS equation

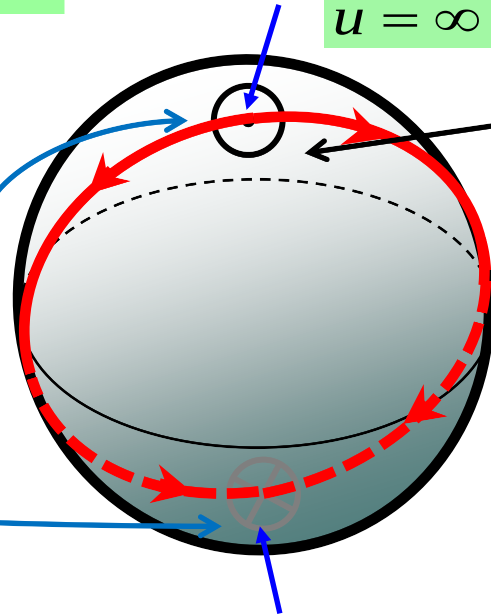
$$\partial_{\bar{z}} u = 0 \qquad z \equiv x + iy \qquad \partial_x u \mp i \partial_y u = 0$$

$$u^{-1} = \sum_{i=1}^{k} \frac{\lambda_i}{z - z_i}$$

$$u \to \infty \quad (|z| \to \infty)$$

$$u \to 0 \quad (z \to z_i)$$





$$T_L = 2\pi k \qquad k \in \pi_2(S^2) = \mathbf{Z}$$

2D Skyrmion

Cond-mat examples: Ferromagnet, quantum Hall systems

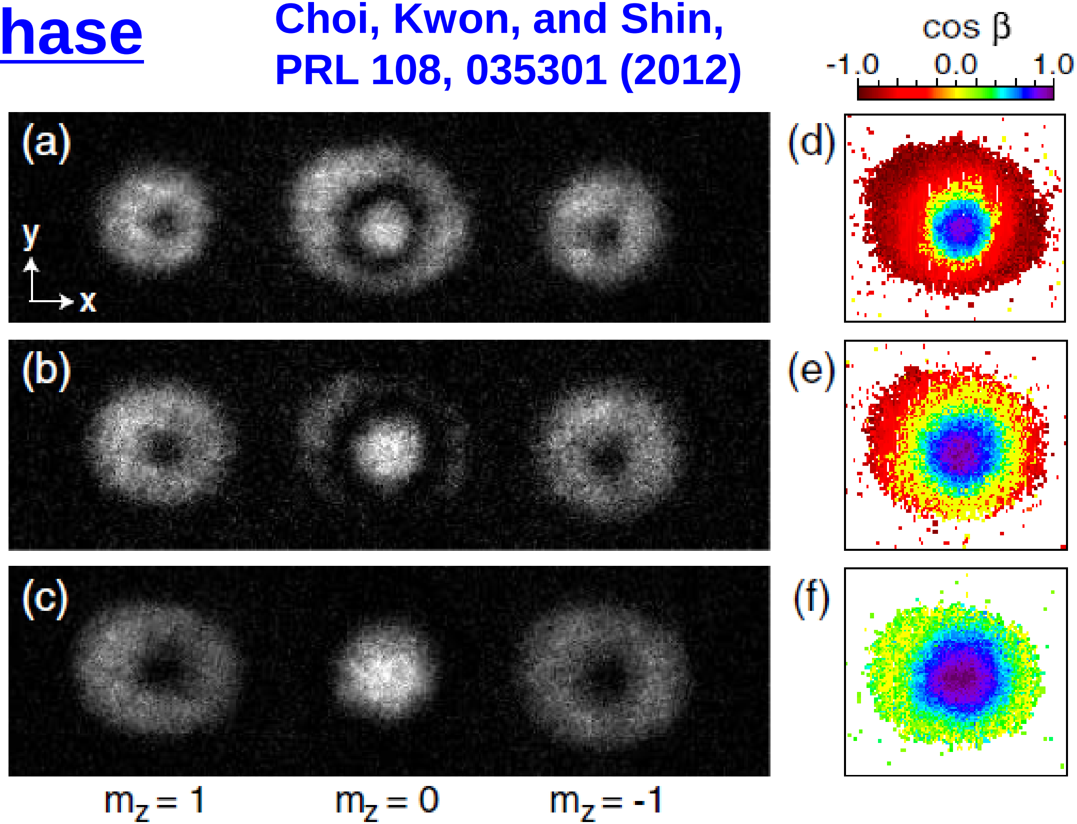
Spin 1 BEC, Polar phase

Choi, Kwon, and Shin, PRL 108, 035301 (2012)

$$\frac{G}{H_P} \cong \frac{U(1)_\Phi \times SO(3)_F}{(\mathbf{Z}_2)_{\Phi+F_x} \times U(1)_{F_z}}$$

$$\cong \frac{S^1{}_\Phi \times S^2{}_F}{(\mathbf{Z}_2)_{\Phi+F}}$$

$$\pi_2\left(\frac{G}{H_P}\right) \cong \mathbf{Z}$$



3D Skyrmion

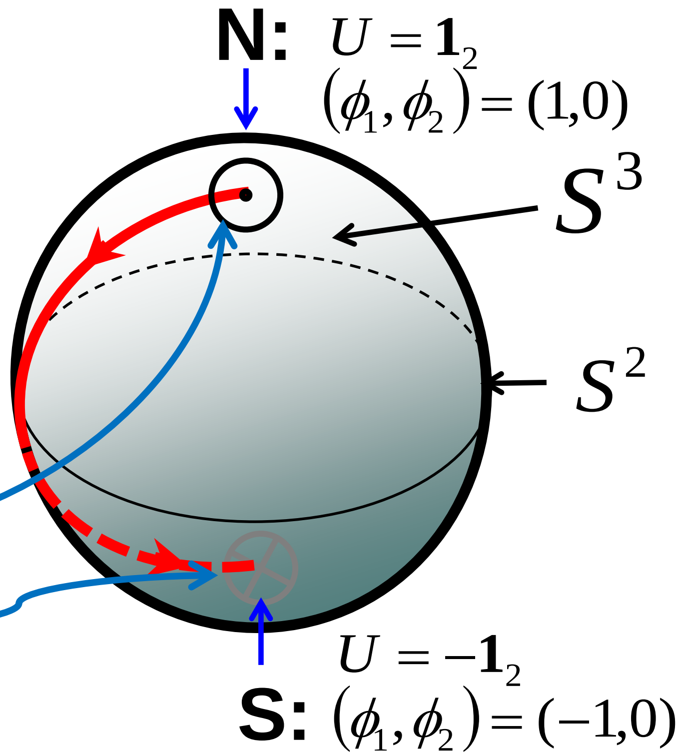
O(4) sigma model~Skyrme model

$$\begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \in \mathbf{C}^2 \qquad |\phi_1|^2 + |\phi_2|^2 = 1 \quad S^3$$

$$\pi_3(S^3) = \mathbf{Z}$$

$$S^3 \cong \mathrm{SU}(2) \qquad U^\dagger U = 1, \quad \det U = |\phi_1|^2 + |\phi_2|^2 = 1$$

$$U \equiv \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \in SU(2)$$



Skyrmion ansatz

$$U(x) = \exp\left(i \frac{f(r)\mathbf{r}\cdot\boldsymbol{\sigma}}{r} \right)$$

$$\to +\mathbf{1}_2, \quad f(r) \to 0 \ (r \to \infty)$$

$$\to -\mathbf{1}_2, \quad f(r) \to 1 \ (r \to 0)$$

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

§ 4-1 Brane annihilation (4p+22p)

§ 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

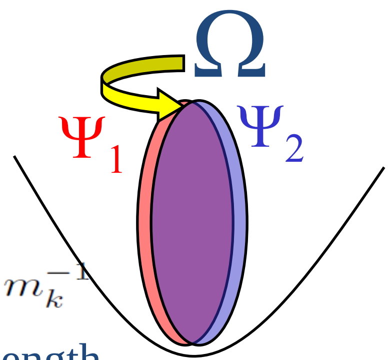
2 component BEC/superfluid

Gross-Pitaevskii energy functional in rotating frame

$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + (V_{\mathrm{ext}} - \mu_i)|\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 \right\}$$

$$i\hbar \frac{\partial \psi_i}{\partial t} = \left[-\frac{\hbar^2}{2m_i}\nabla^2 + V_{\mathrm{ext}} - \mu_i + g_{ii}|\psi_i|^2 + g_{ij}|\psi_j|^2 - \mathbf{\Omega}\cdot\mathbf{L} \right]\psi_i$$

Trapping potential



Atomic interaction $g_{jk} = 2\pi\hbar^2 a_{jk}/m_{jk} \quad m_{jk}^{-1} = m_j^{-1} + m_k^{-1}$

a_{ij}: s-wave scattering length

$$g_{11} = g_{22} \equiv g$$

G.Modugno et al., Phys. Rev. Lett. 89, 190404 (2002) ^{41}K - ^{87}Rb

S. B. Papp et al., Phys. Rev. Lett. 101, 040402 (2008) ^{85}Rb - ^{87}Rb

T. Fukuhara et al., Phys. Rev. A. 79, 021601 (2009) ^{174}Yb - ^{176}Yb

Sigma model representation

K.Kasamatsu., M.Tsubota, M. Ueda, Phys. Rev. A 71, 043611 (2005)

$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \sqrt{n_{\mathrm{T}}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad |\phi_1|^2 + |\phi_2|^2 = 1 \quad \boldsymbol{S^3}$$

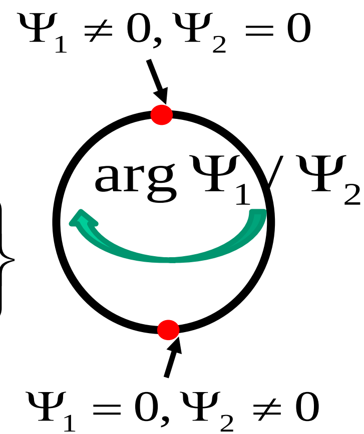
pseudo-spin: $\mathbf{S} = \phi^{\dagger} \boldsymbol{\sigma} \phi = \left(S_x, S_y, S_z\right)^T \qquad \mathbf{S}^2 = 1 \quad \boldsymbol{S^2}$

$\boldsymbol{\sigma}$: Pauli matrix

$$E = \int d\mathbf{r} \left\{ \frac{\hbar^2}{2m} \left[\left(\nabla \sqrt{n_{\mathrm{T}}} \right)^2 + \frac{n_{\mathrm{T}}}{4} \sum_{\alpha} (\nabla S_\alpha)^2 \right] + V_j n_{\mathrm{T}} + \frac{m n_{\mathrm{T}}}{2} (\mathbf{v}_{\mathrm{eff}} - \square \boldsymbol{\Omega} \times \mathbf{r})^2 + c_0 + c_1 S_z + c_2 S_z^2 \right\}$$

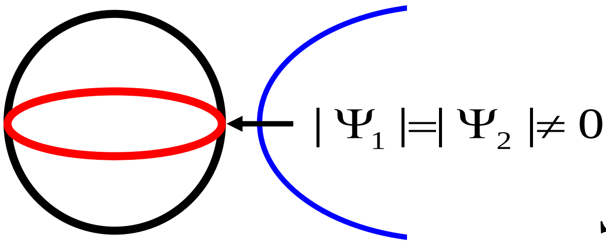
$$c_0 = \frac{n_{\mathrm{T}}}{8} [n_{\mathrm{T}} (g_{11} + g_{22} + 2 g_{12}) - 4(\mu_1 + \mu_2)],$$

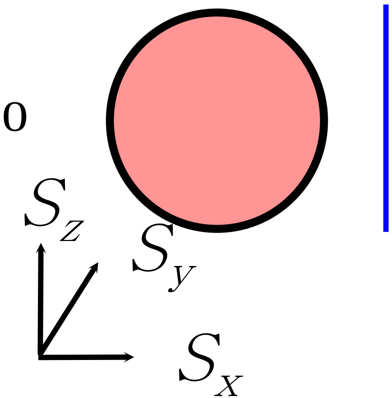
$$c_1 = \frac{n_{\mathrm{T}}}{4} [n_{\mathrm{T}} (g_{11} - g_{22}) - 2(\mu_1 - \mu_2)], \qquad c_2 = \frac{n_{\mathrm{T}}^2}{8} (g_{11} + g_{22} - 2 g_{12})$$

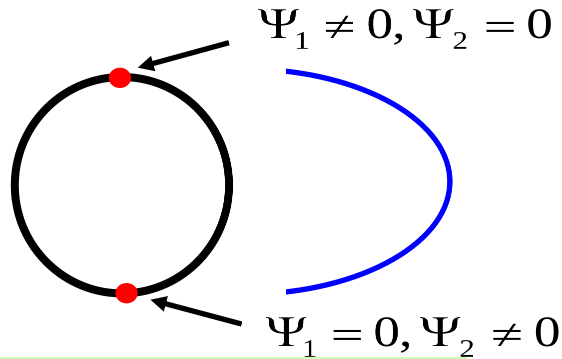


phase structure

$g > g_{12}$	$g = g_{12}$	$g < g_{12}$
Anti-ferromagnetic	**SU(2) symmetric**	**Ferromagnetic**

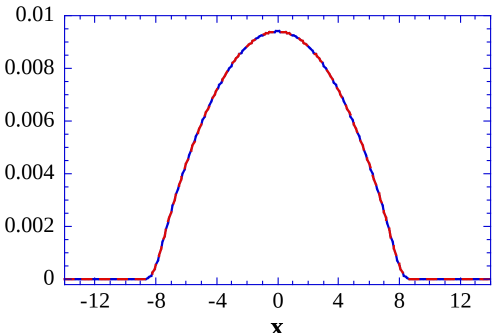


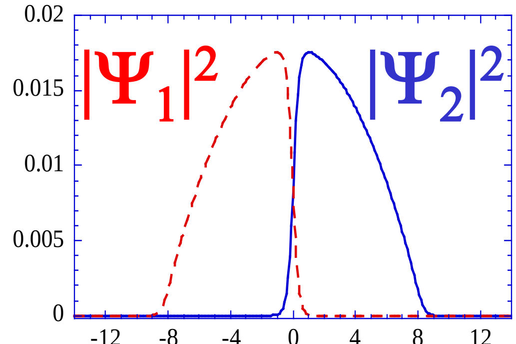




2 comp coexist

2 comp are separated

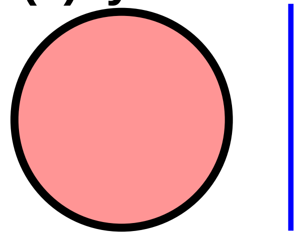




g=g_{12}
SU(2) symmetric

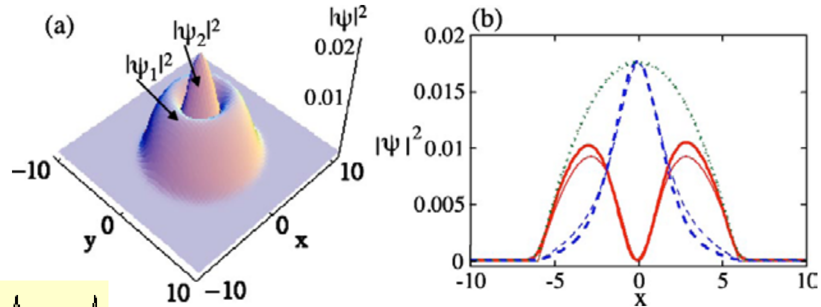
Massless O(3) model

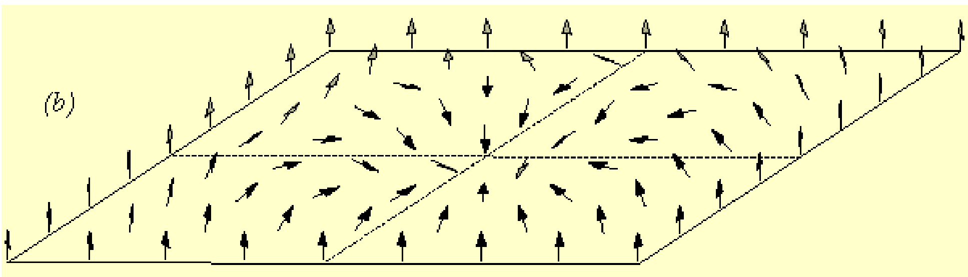
SU(2)symmetric



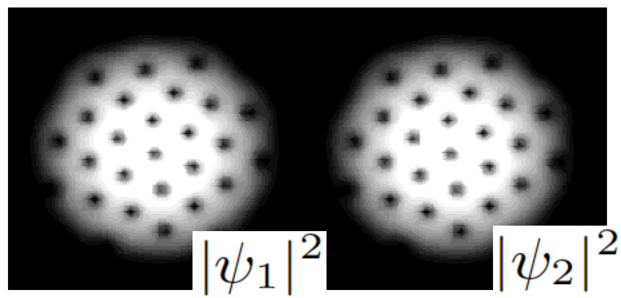
Coreless vortex
= lump, 2D Skyrmion

$$\pi_2(S^2) = \mathbf{Z}$$





Kasamatsu, Tsubota, Ueda



Integer vortex

1 U(1) winding

$$(\Psi_1, \Psi_2) = (f(r)e^{i\theta}, f(r)e^{i\theta}) \sim e^{i\theta}(1,1)$$

$g_{12}<0$ **attraction**

singular vortex(~1comp)

$g_{12}>0$ **repulsion -> splitting**

Vortex molecule

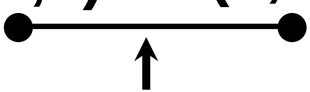
Repulsion balanced with

internal coherent coupling

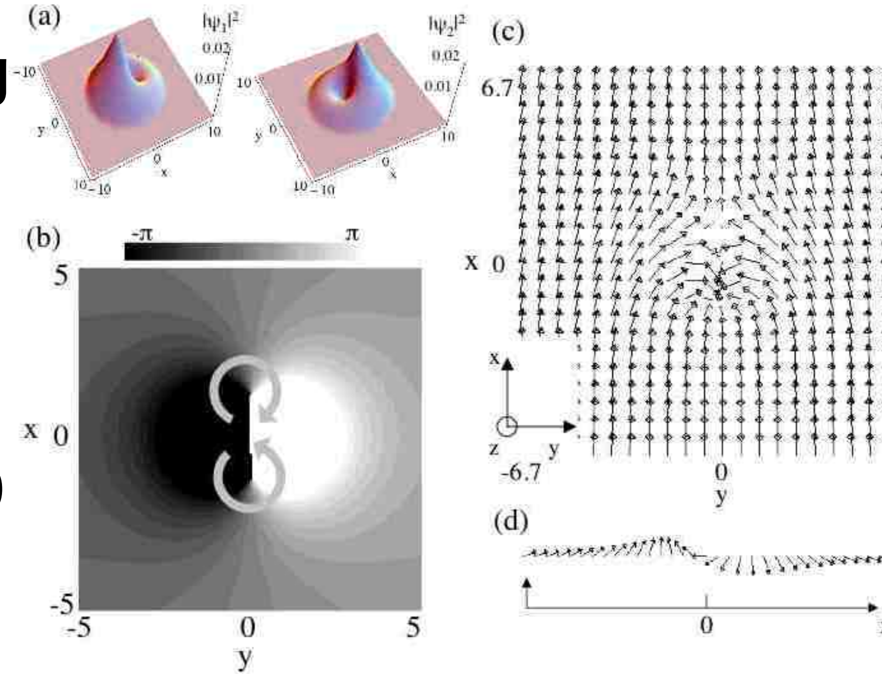
(Rabi frequency)

$$\Delta E = -\hbar(\Psi_2{}^{*}\Psi_1 e^{-i\Delta t} + \mathrm{c.c.})$$

(1,0) **(0,1)**



SineGordon kink



Son-Stephanov('02) **Kasamatsu-Tsubota-Ueda('05)**

3D Skyrmion = vorton in two comonent BECs

$$\pi_3(S^3) = \mathbf{Z}$$

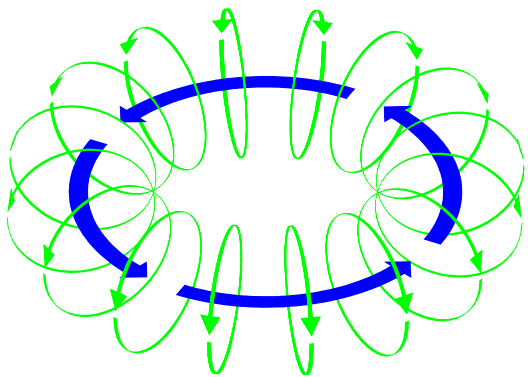
Khawaja & Stoof, *Nature* ('01)
Ruostekoski & Anglin ('01)
Battye, Cooper & Sutcliffe ('02)
Herbut & Oshikawa ('06)

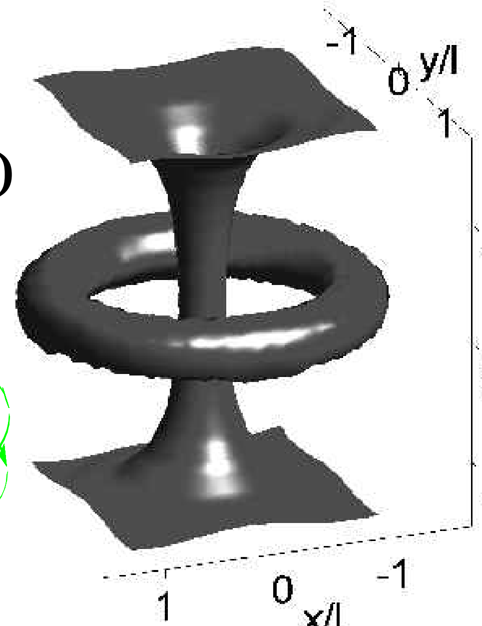
$$\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \sqrt{n_{\mathrm{T}}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad |\phi_1|^2 + |\phi_2|^2 = 1$$

$$S^3 \cong \mathrm{SU}(2)$$

$$U \equiv \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} = \exp\left(i \frac{f(r)\mathbf{r}\cdot\boldsymbol{\sigma}}{r} \right) \in SU(2)$$

$$U^\dagger U = 1,$$
$$\det U = |\phi_1|^2 + |\phi_2|^2 = 1$$



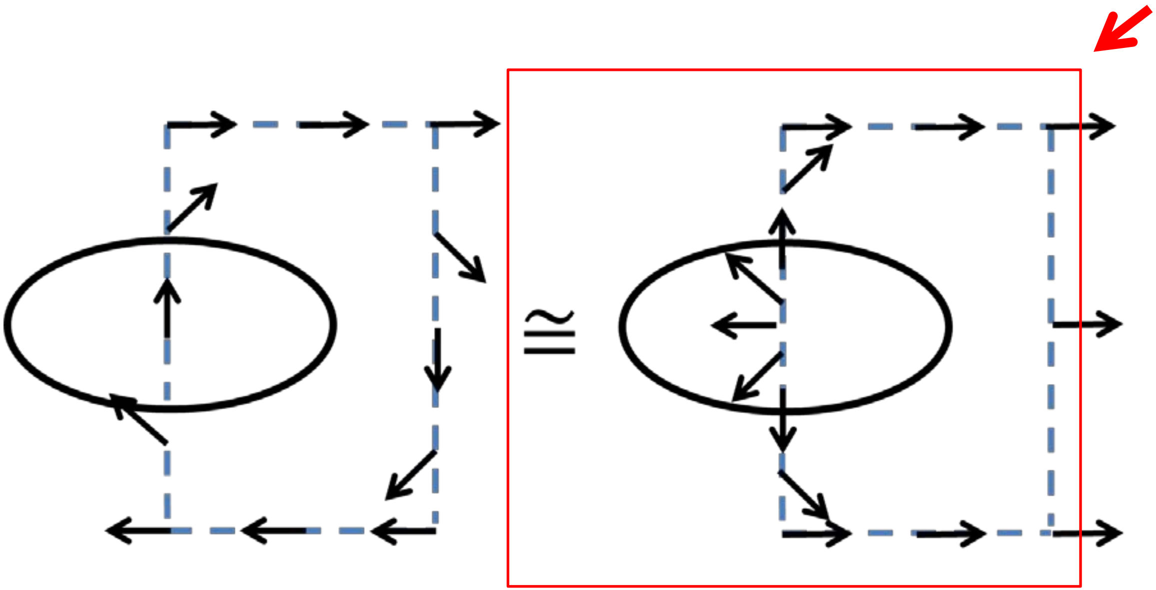


Topological equivalence to 3D skyrmion

Phase of Ψ_1

$$\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}=\begin{pmatrix}1\\ 0\end{pmatrix}$$

@boundary



Vorton **3D skyrmion**

3 component BEC/superfluid

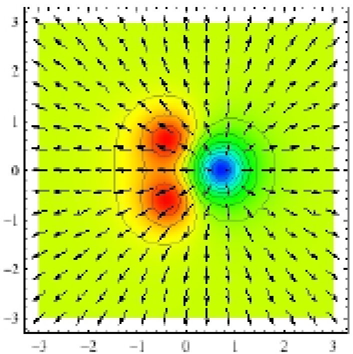
Eto-MN,
Phys.Rev. A85 (2012) 053645

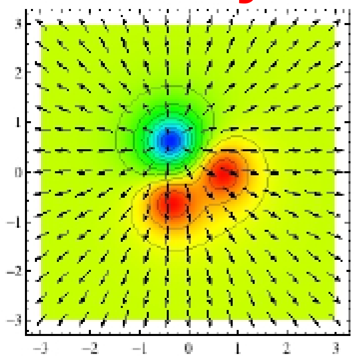
Gross-Pitaevskii energy functional

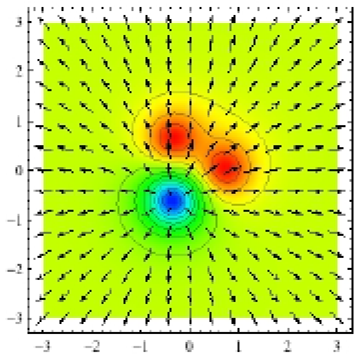
$$E[\psi] = \int d^3\mathbf{r} \left\{ \sum_i \left(\frac{\hbar^2}{2m_i} |\nabla \psi_i|^2 + (V_{\rm ext} - \mu_i)|\psi_i|^2 \right) + \sum_{i,j} \frac{g_{ij}}{2} |\psi_i|^2 |\psi_j|^2 - \omega_{ij} \psi_i^* \psi_j \right\}$$

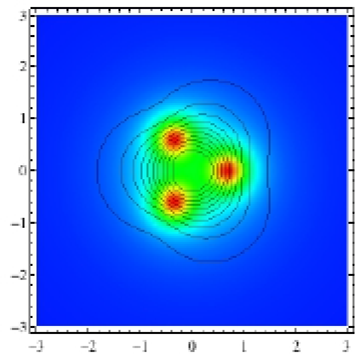
internal coherent coupling (Rabi frequency)

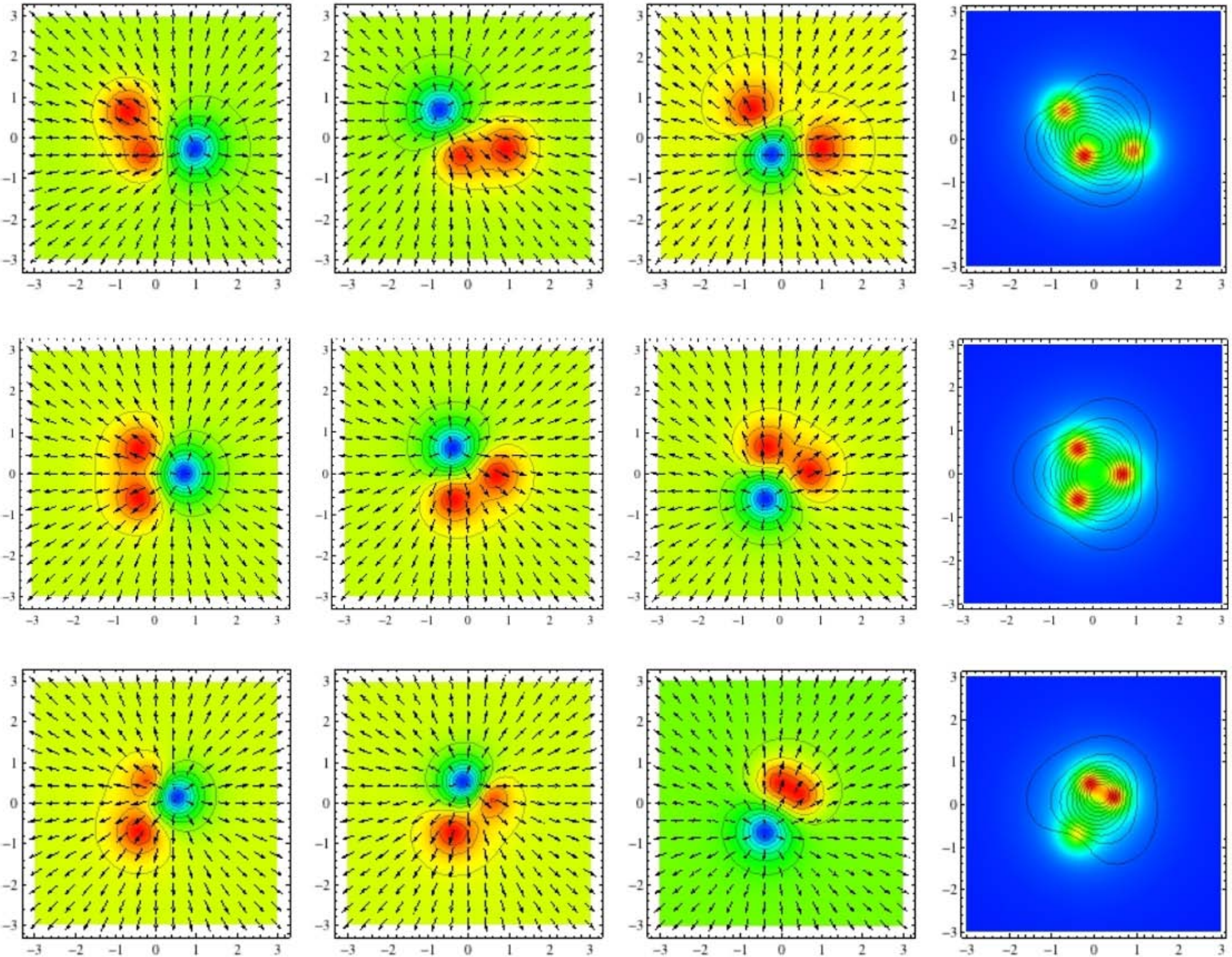
Vortex trimer = CP^2 Skyrmion


(1,0,0)


(0,1,0)


(0,0,1)


enegy density



$$(\omega_{12}, \omega_{23}, \omega_{31}) = (0.01, 0.05, 0.05)$$

asymmetric

$$(\omega_{12}, \omega_{23}, \omega_{31}) = (0.05, 0.05, 0.05)$$

symmetric

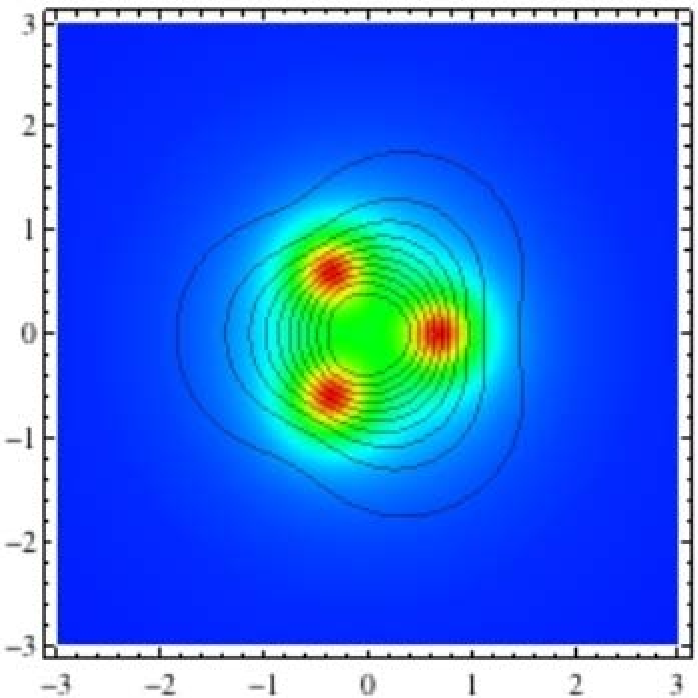
$$(\omega_{12}, \omega_{23}, \omega_{31}) = (0.2, 0.05, 0.05)$$

asymmetric

BEC

Vortex trimer

Y-junction of domain walls

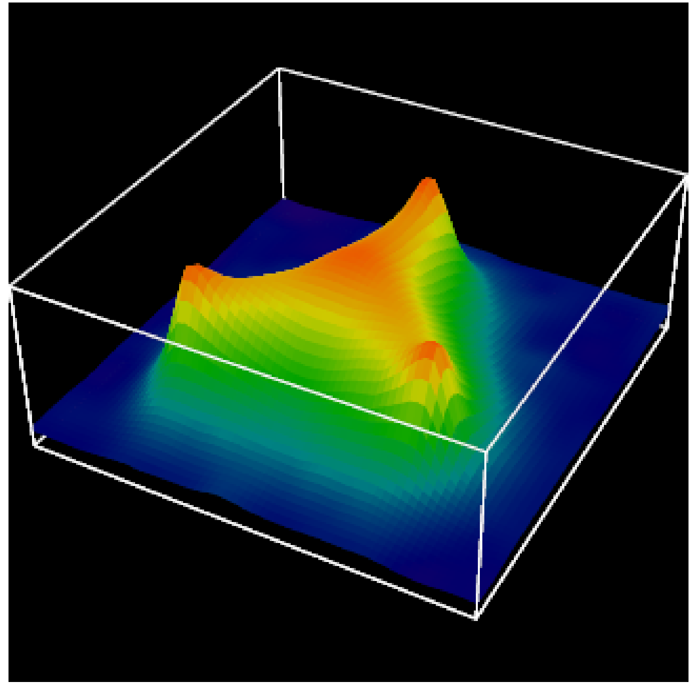


Eto-MN, PRA85 (2012) 053645

Baryon = q-q-q

QCD

Y-junction of fluxes
(not Δ)



Ichie-Suganuma *et.al* (‘03)

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

  § 4-1 Brane annihilation (4p+22p)

  § 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

§ 4-1 Brane annihilation

Creating vortons and three-dimensional skyrmions from domain wall annihilation with stretched vortices in Bose-Einstein condensates

Phys. Rev. A85 (2012) 053639

e-Print: **arXiv:1203.4896 [cond-mat.quant-gas]**

Hiromitsu Takeuchi **(Hiroshima U.)**

Kenichi Kasamatsu **(Kinki U.)**, **Makoto Tsubota** **(Osaka City U.)**

Related papers:

①**Tachyon Condensation in Bose-Einstein Condensates**

e-Print: **arXiv:1205.2330 [cond-mat.quant-gas]**

②**Analogues of D-branes in Bose-Einstein condensates**

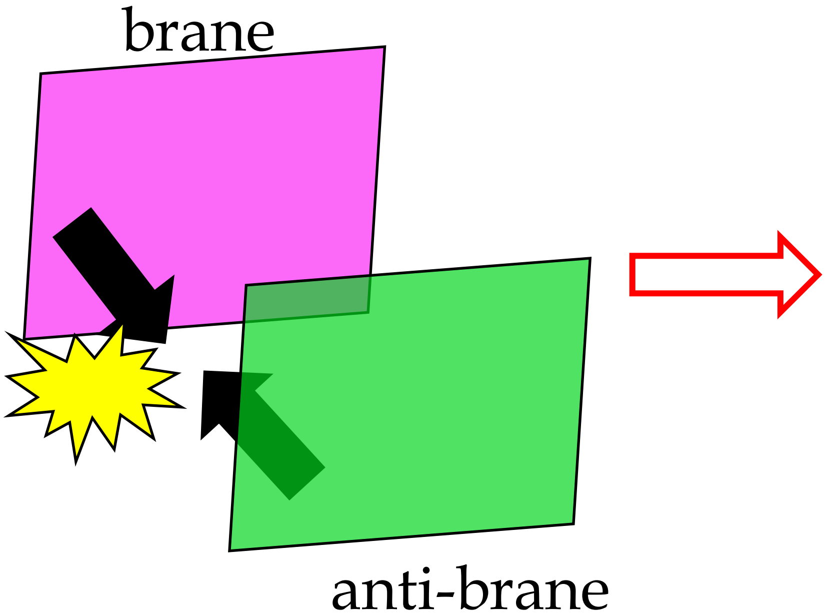
JHEP 1011 (2010) 068

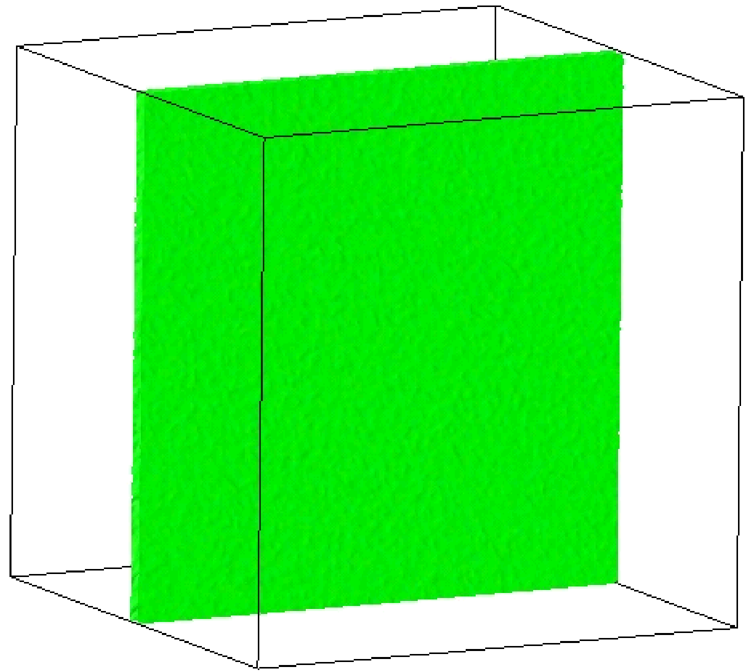
e-Print: **arXiv:1002.4265 [cond-mat.quant-gas]**

Brane-anti-brane annihilation in BEC

closed string production by brane pair annihilation

Simulation by Takeuchi





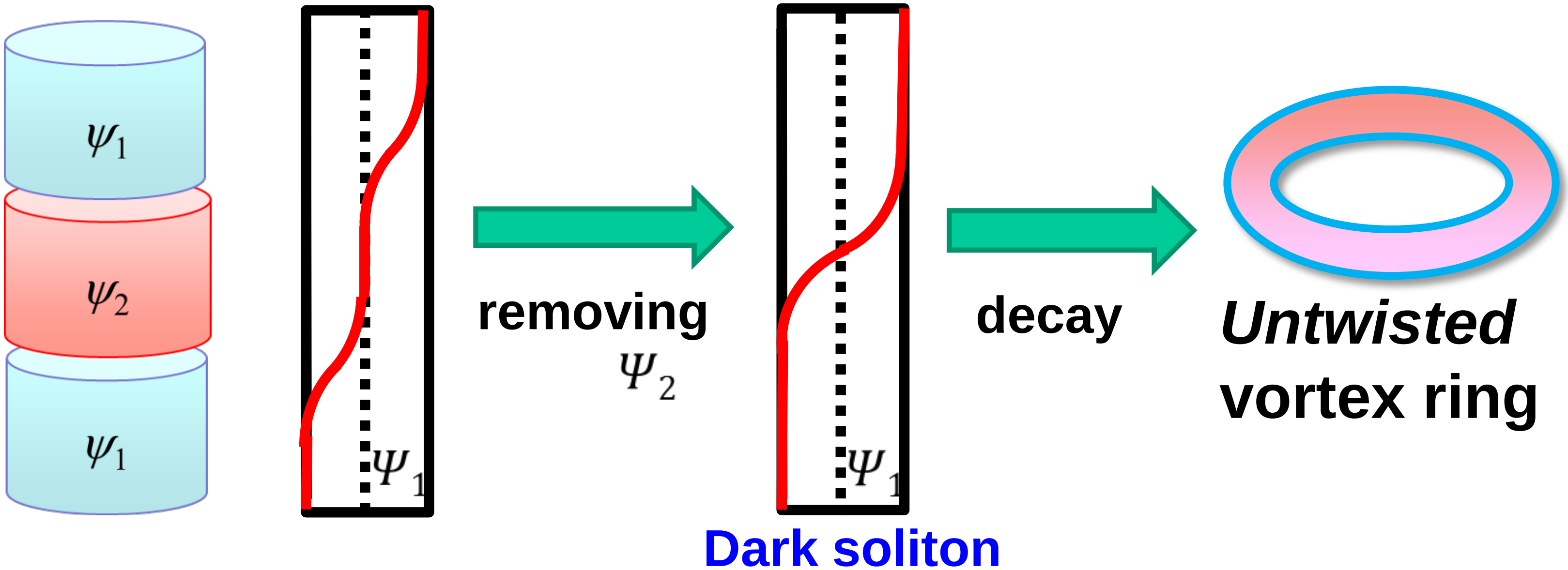
2nd component inside vortex $-\pi$ [color bar] π

Experiments

Watching Dark Solitons Decay into Vortex Rings in a Bose-Einstein Condensate

B. P. Anderson *et.al.*, Phys. Rev. Lett. 86, 2926–2929 (2001)

(JILA, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado)



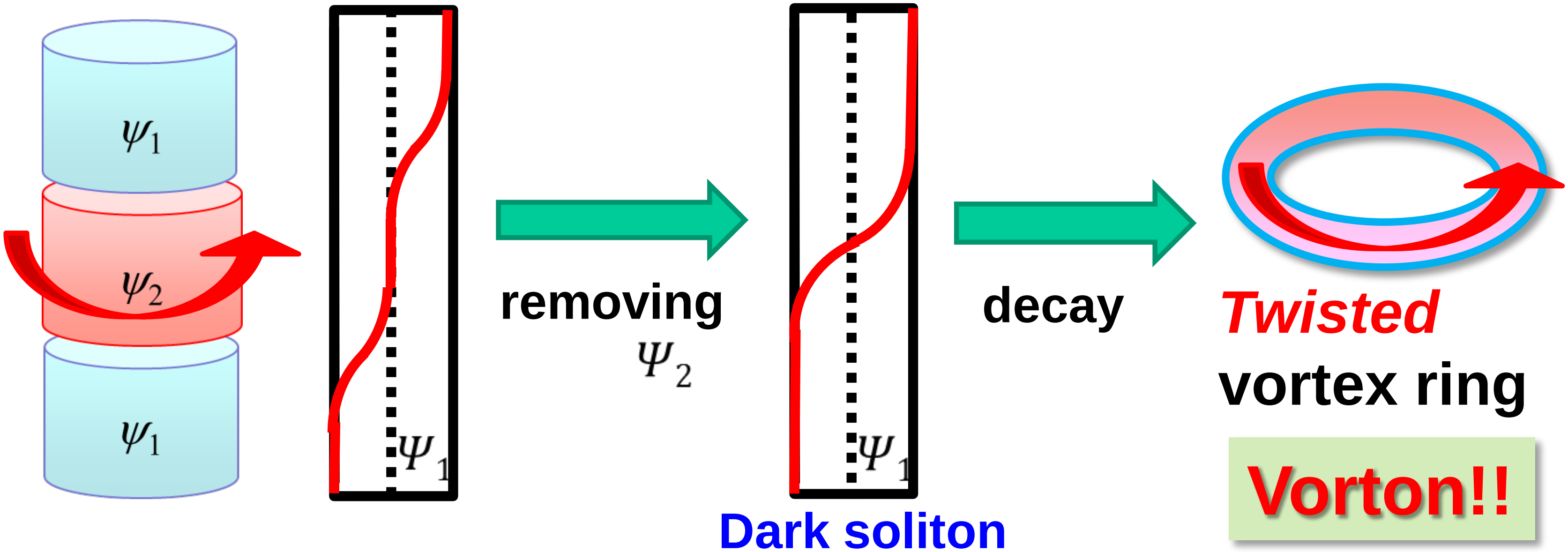
Experiments

Our proposal

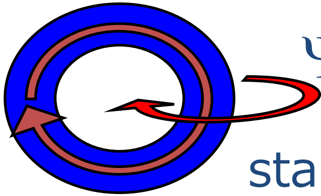
Watching Dark Solitons Decay into Vortex Rings in a Bose-Einstein Condensate

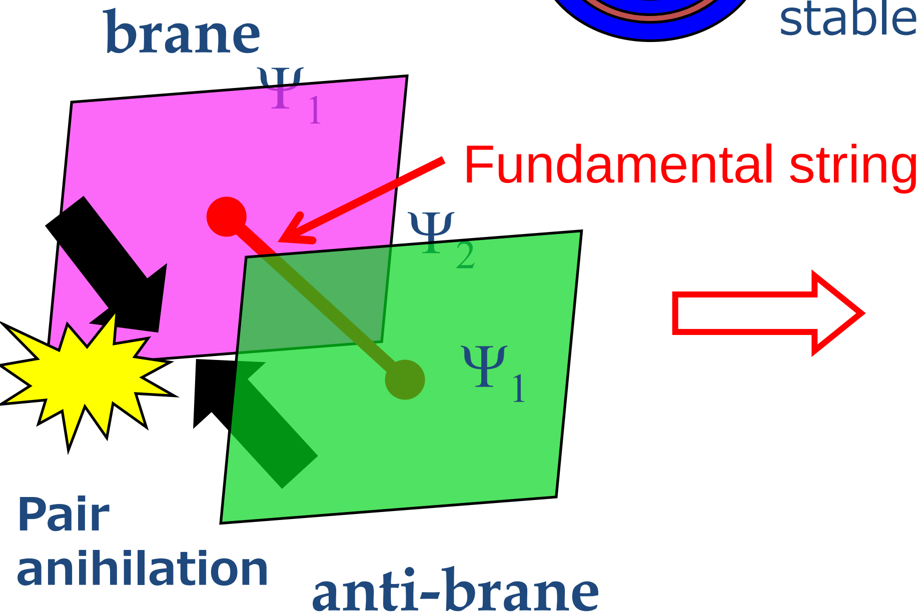
B. P. Anderson *et.al.*, Phys. Rev. Lett. 86, 2926–2929 (2001)

(**JILA,** National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado)

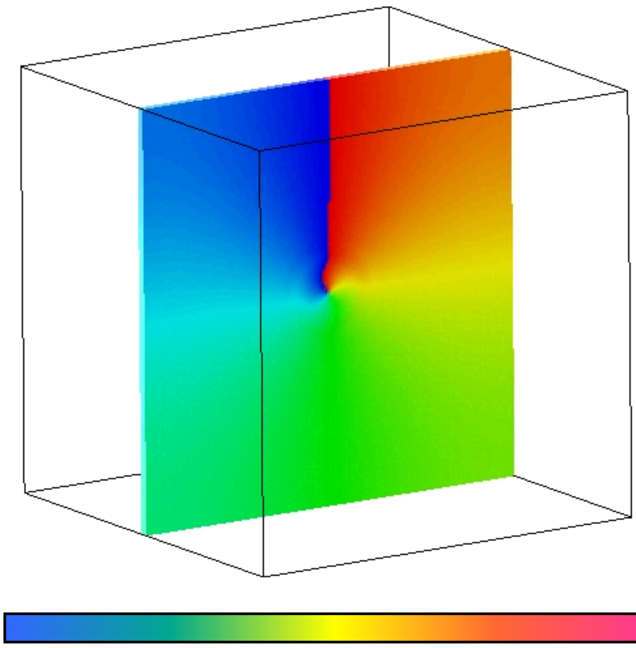


Brane annihilation with stretched string





Simulation by Takeuchi



$g < g_{12}$

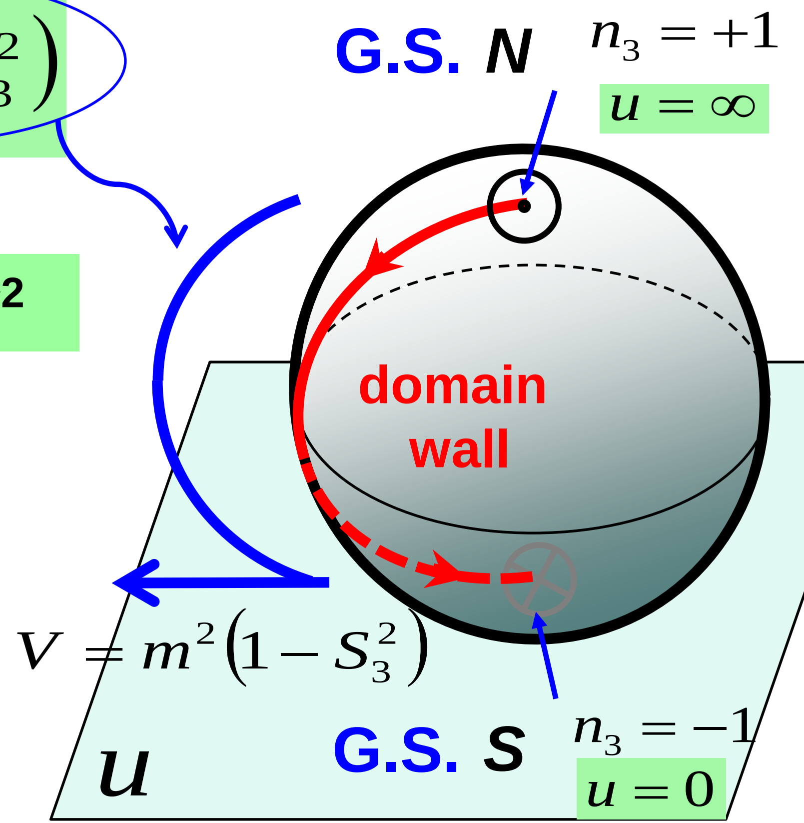
ferromagnetic

Massive O(3) sigma model

$$E = \frac{1}{2}(\nabla S)^2 + m^2\left(1 - S_3^2\right)$$

$S(x) = (S_1, S_2, S_3)$ $S^2 = 1$

Target space = S^2



equivalent to

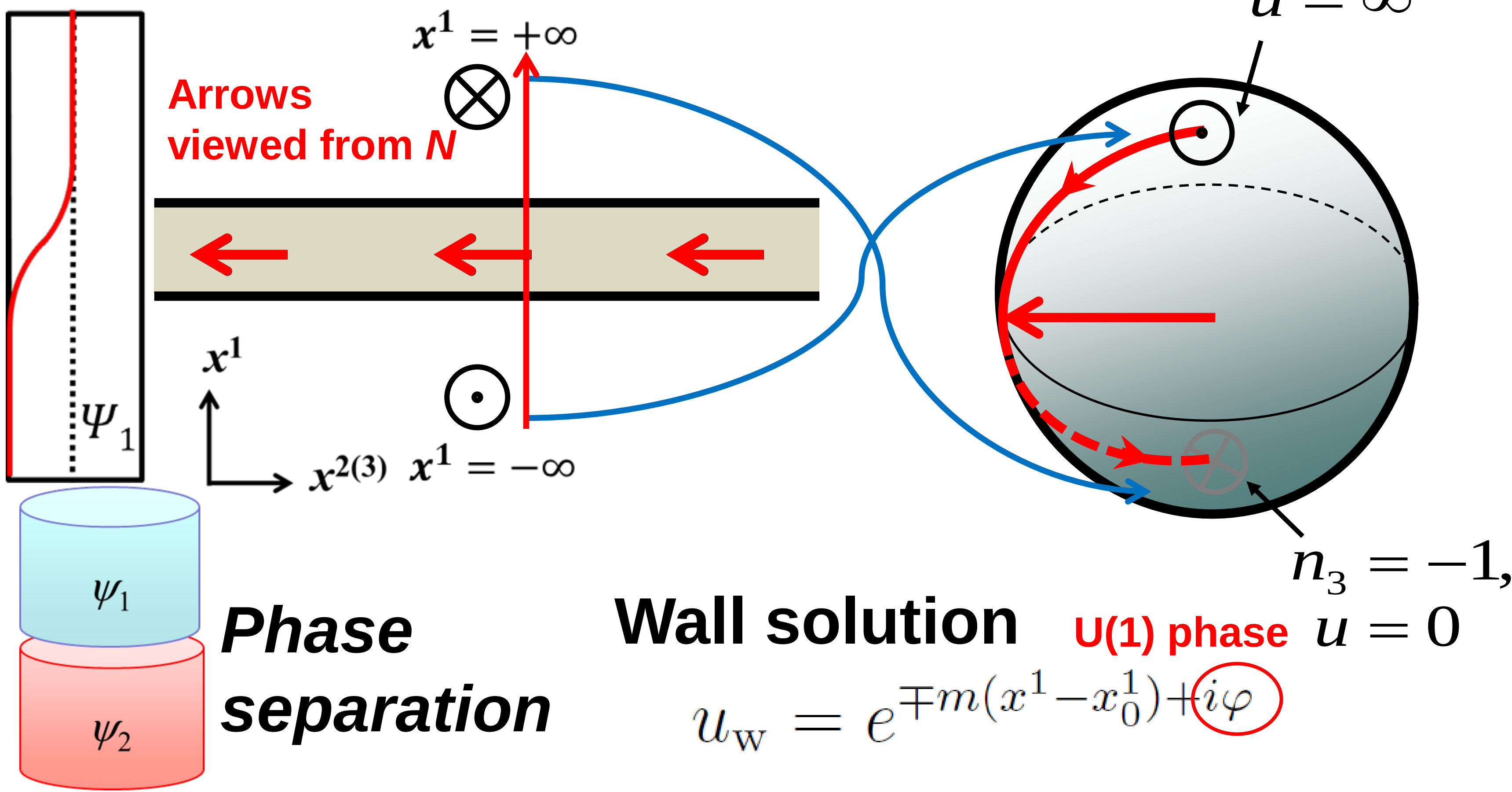
CP[1] model

$$E = \int d\mathbf{r} \frac{\sum_\alpha |\partial_\alpha u|^2 + m^2 |u|^2}{\left(1 + |u|^2\right)^2}$$

Stereographic coordinate *u*

$$u = \frac{S_1 - iS_2}{1 - S_3}$$

Single domain wall



Bogomol'nyi completion for domain wall

$$E = \int dx^1 \frac{\sum_\alpha |\partial_\alpha u|^2 + m^2|u|^2}{\left(1+|u|^2\right)^2}$$

$$= \int dx^1 \left[\frac{|\partial_1 u \mp 2mu|^2}{\left(1+|u|^2\right)^2} \pm \frac{2m\left(u^* \partial_1 u + u \partial_1 u^*\right)}{\left(1+|u|^2\right)^2} \right]$$

$$\geq |T_W|$$

Topological charge

$$T_W = \pm \int dx^1 \frac{2m\left(u^* \partial_z u + u \partial_z u^*\right)}{\left(1+|u|^2\right)^2}$$

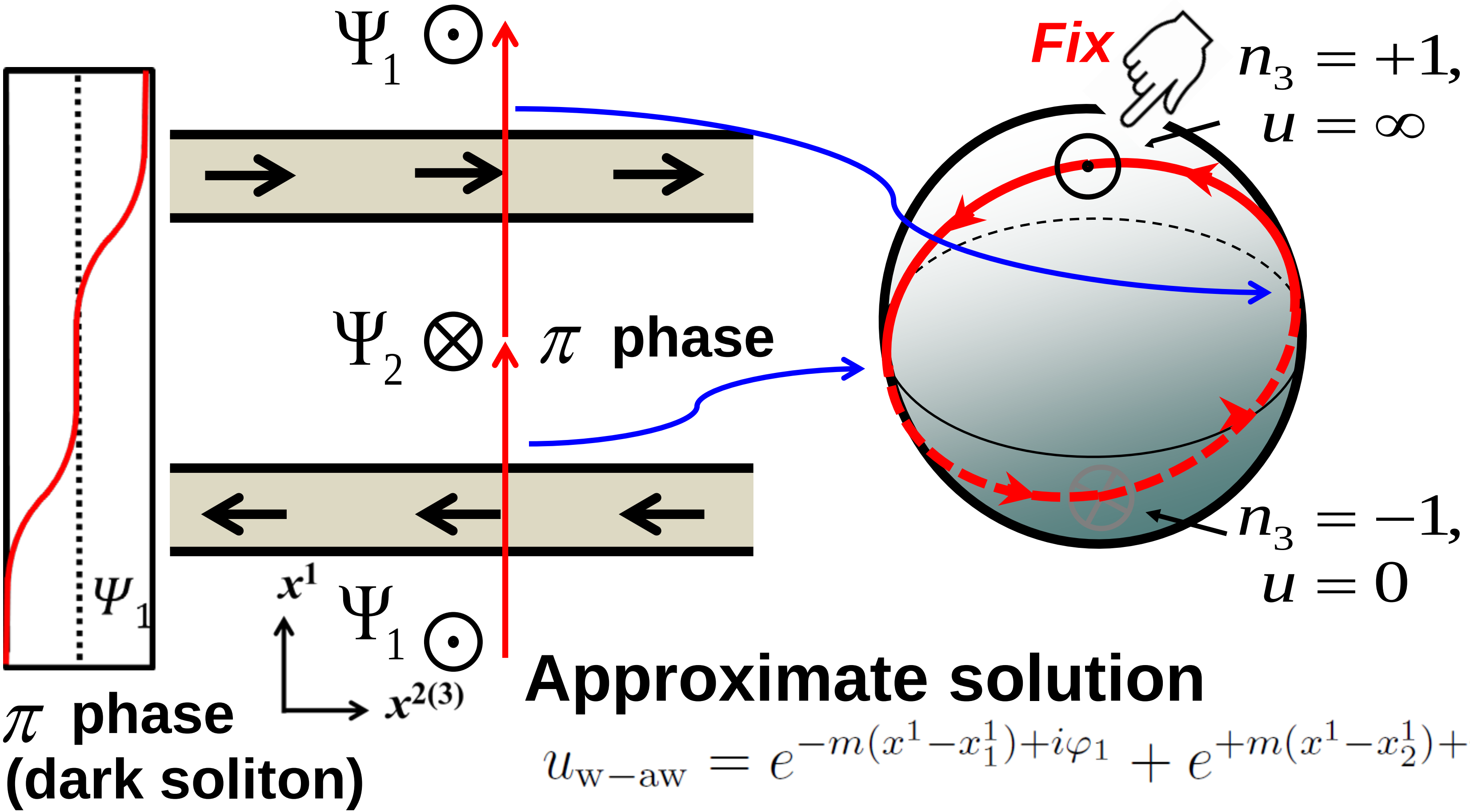
$$= \pm m \int dx^1 \partial_1 \left(\frac{1-|u|^2}{1+|u|^2} \right) = \pm m \left[\frac{1-|u|^2}{1+|u|^2} \right]_{x^1=-\infty}^{x^1=+\infty}$$

BPS equation

$$\partial_1 u \mp mu = 0$$

$$\rightarrow u_w = e^{\pm m x^1 + i\varphi}$$

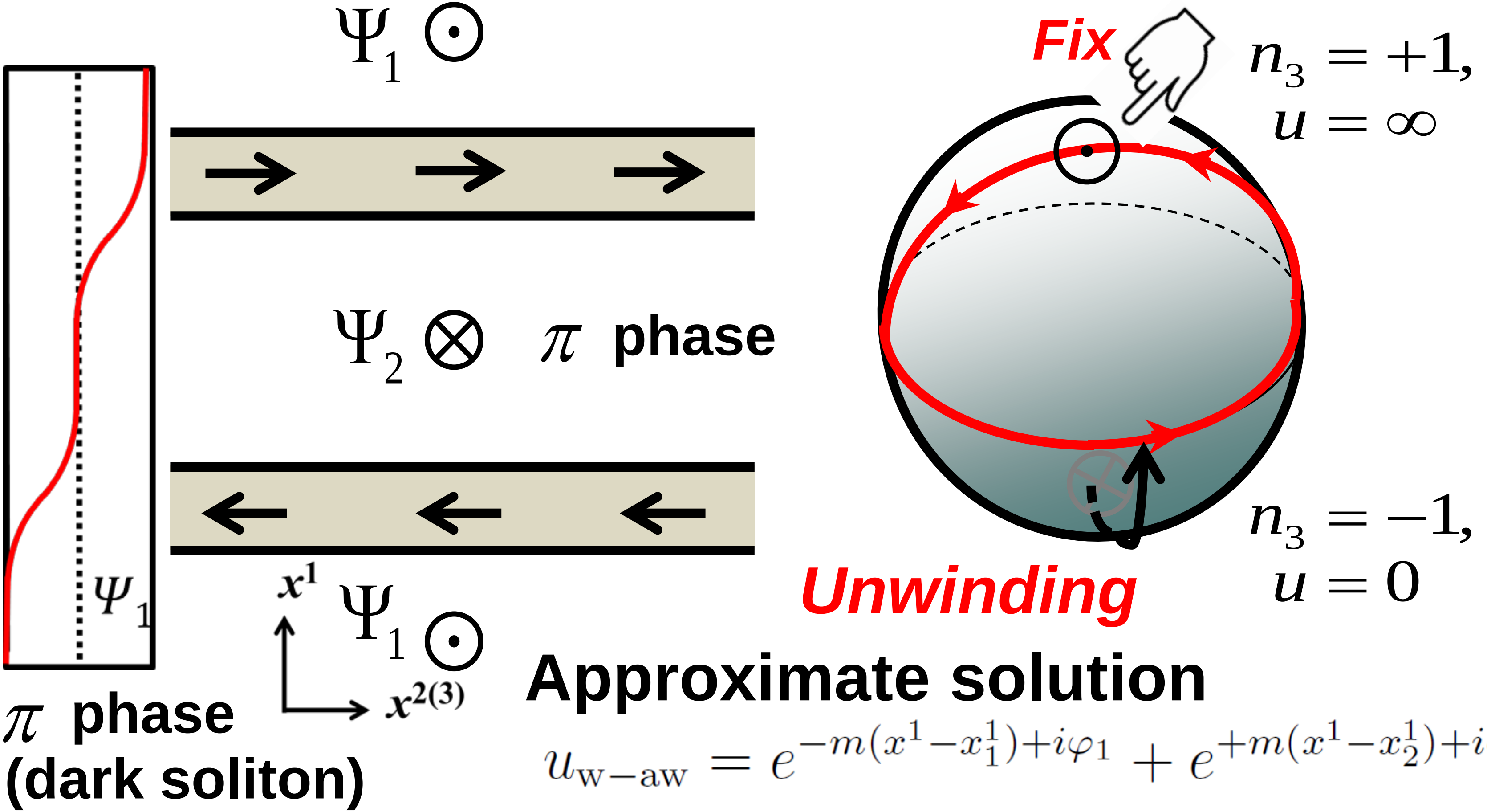
A pair of a domain wall and an anti-domain wall



Approximate solution

$$u_{\mathrm{w-aw}} = e^{-m(x^1 - x^1_1) + i\varphi_1} + e^{+m(x^1 - x^1_2) + i\varphi_2}$$

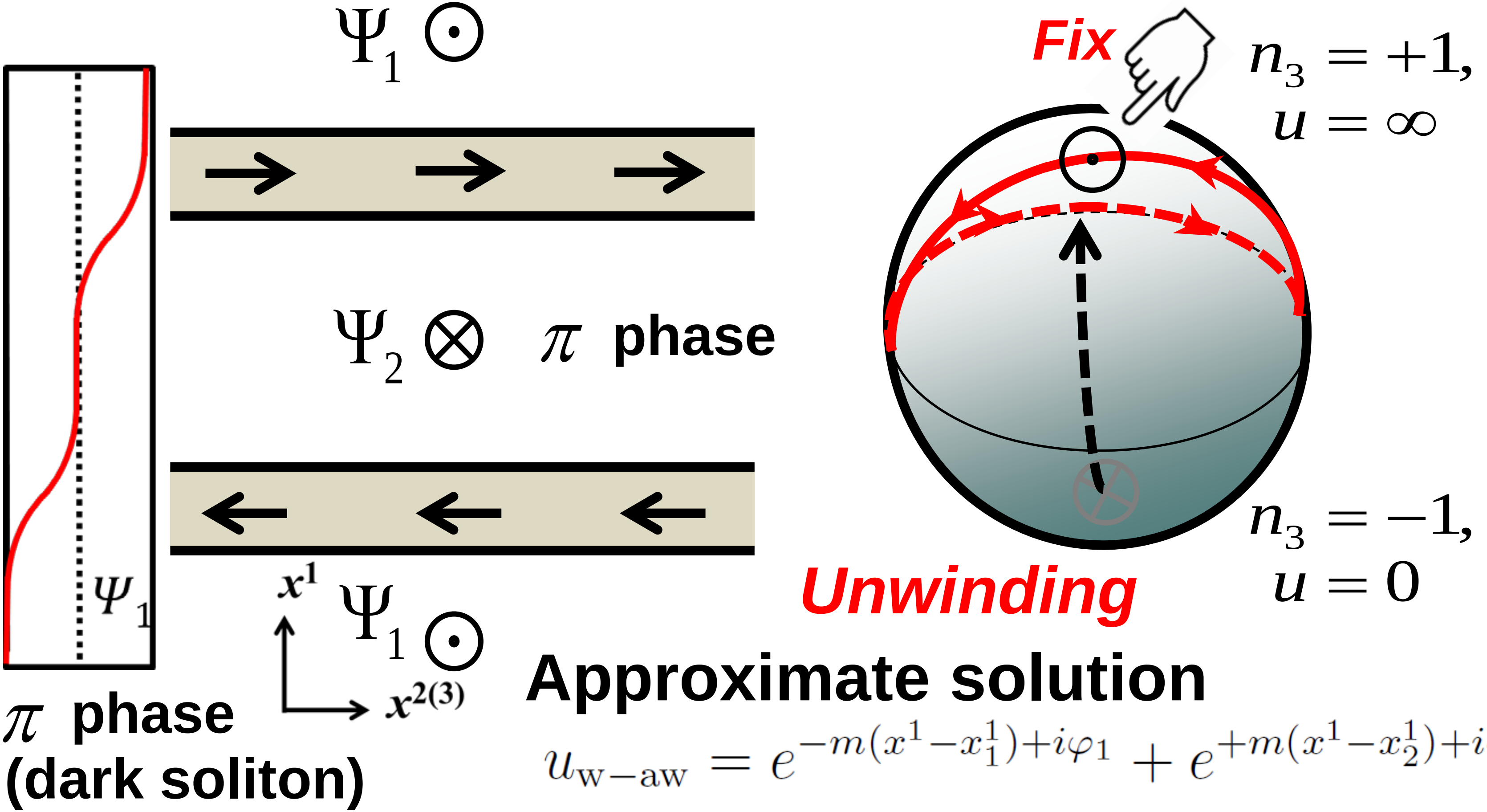
A pair of a domain wall and an anti-domain wall



Approximate solution

$$u_{\mathrm{w-aw}} = e^{-m(x^1 - x^1_1) + i\varphi_1} + e^{+m(x^1 - x^1_2) + i\varphi_2}$$

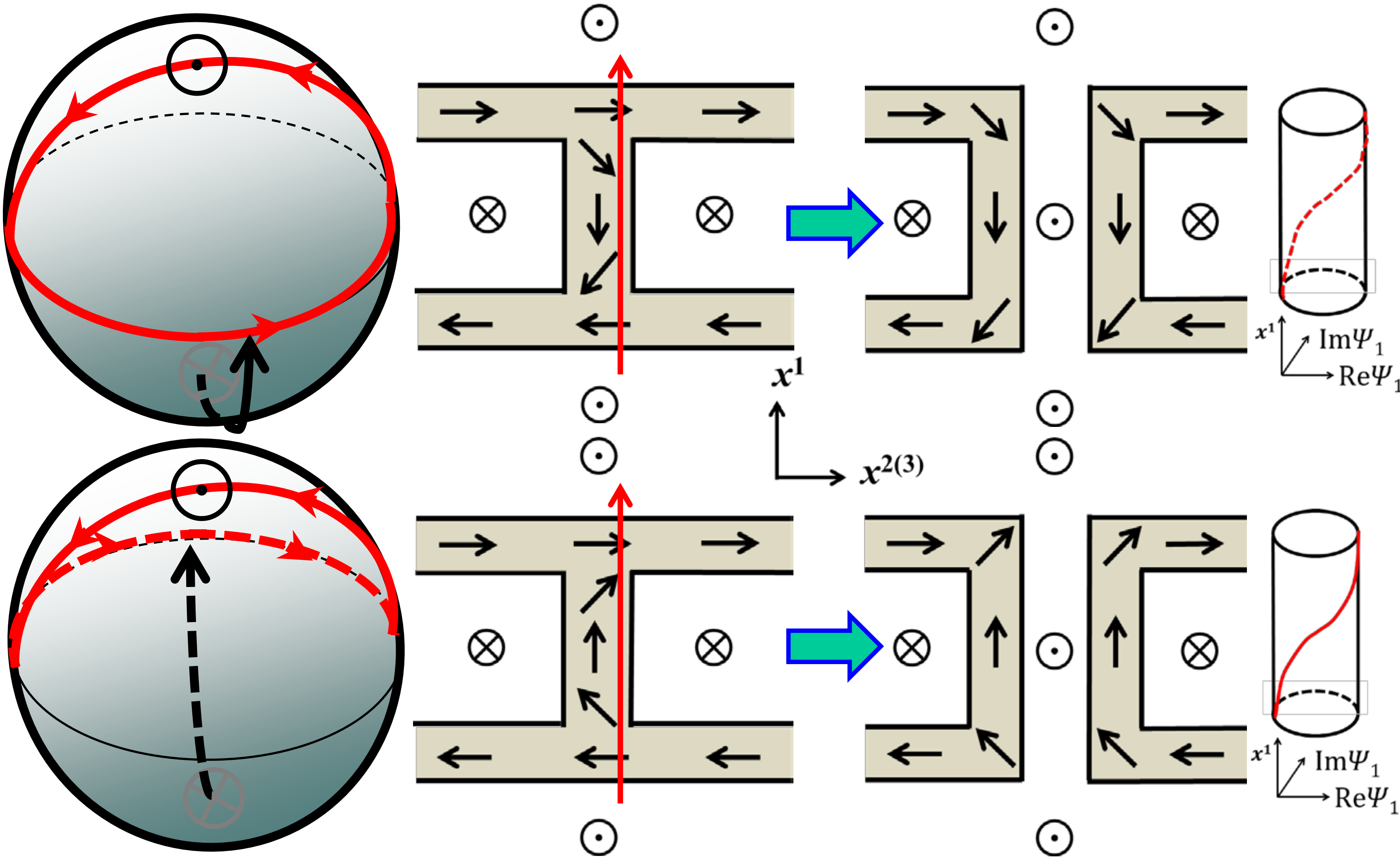
A pair of a domain wall and an anti-domain wall



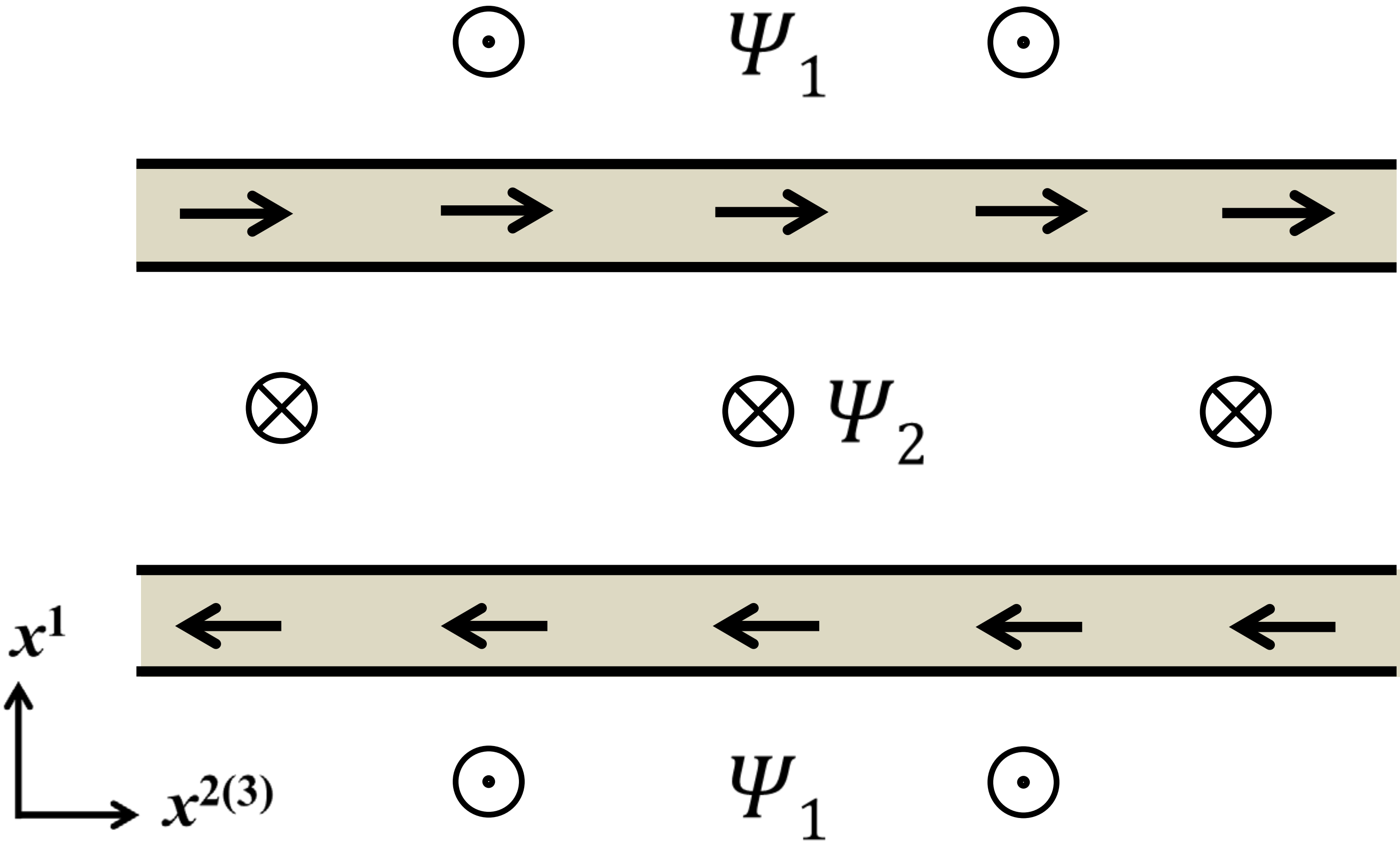
π phase
(dark soliton)

Approximate solution

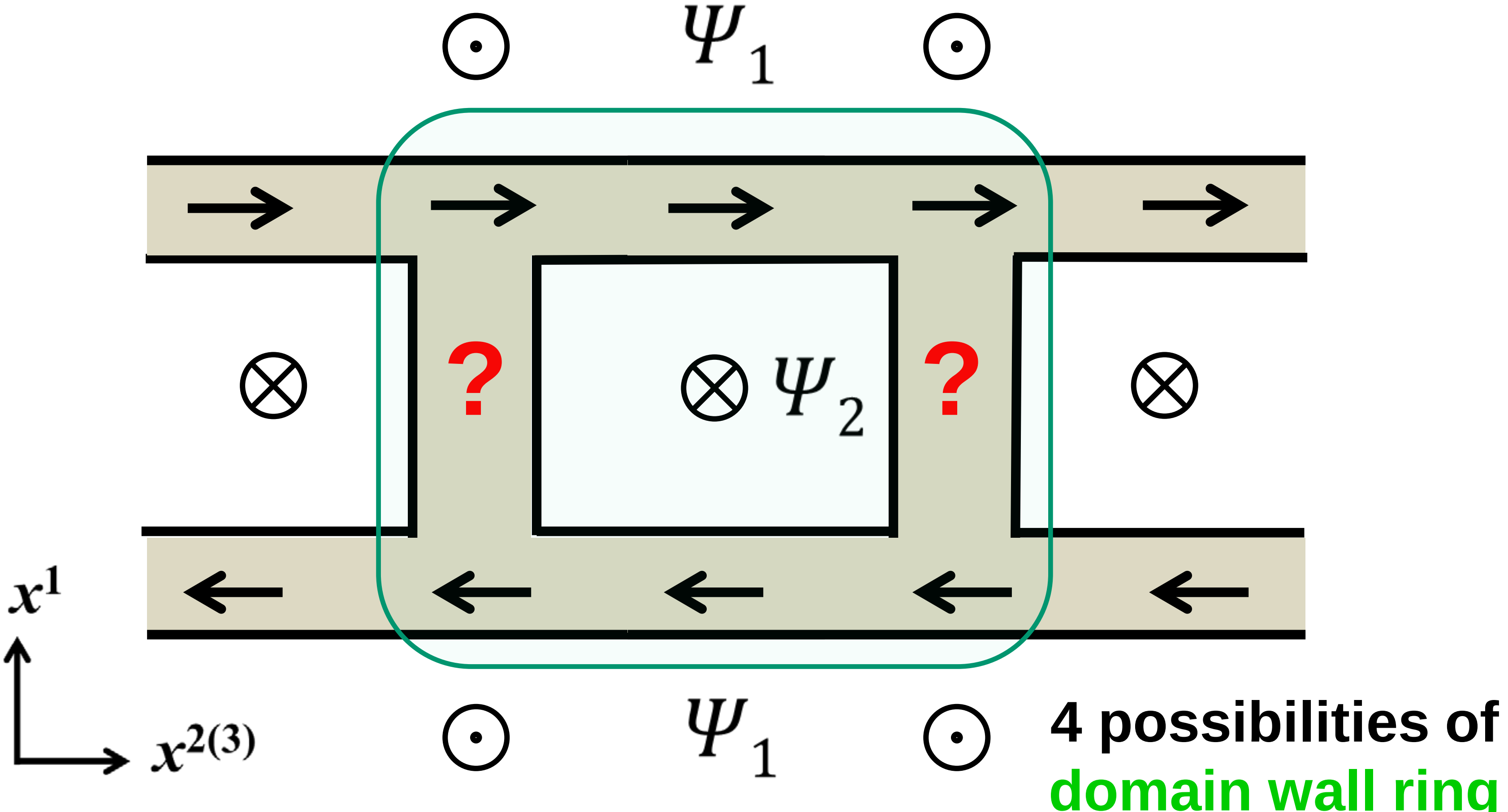
$$u_{\mathrm{w-aw}} = e^{-m(x^1 - x^1_1) + i\varphi_1} + e^{+m(x^1 - x^1_2) + i\varphi_2}$$



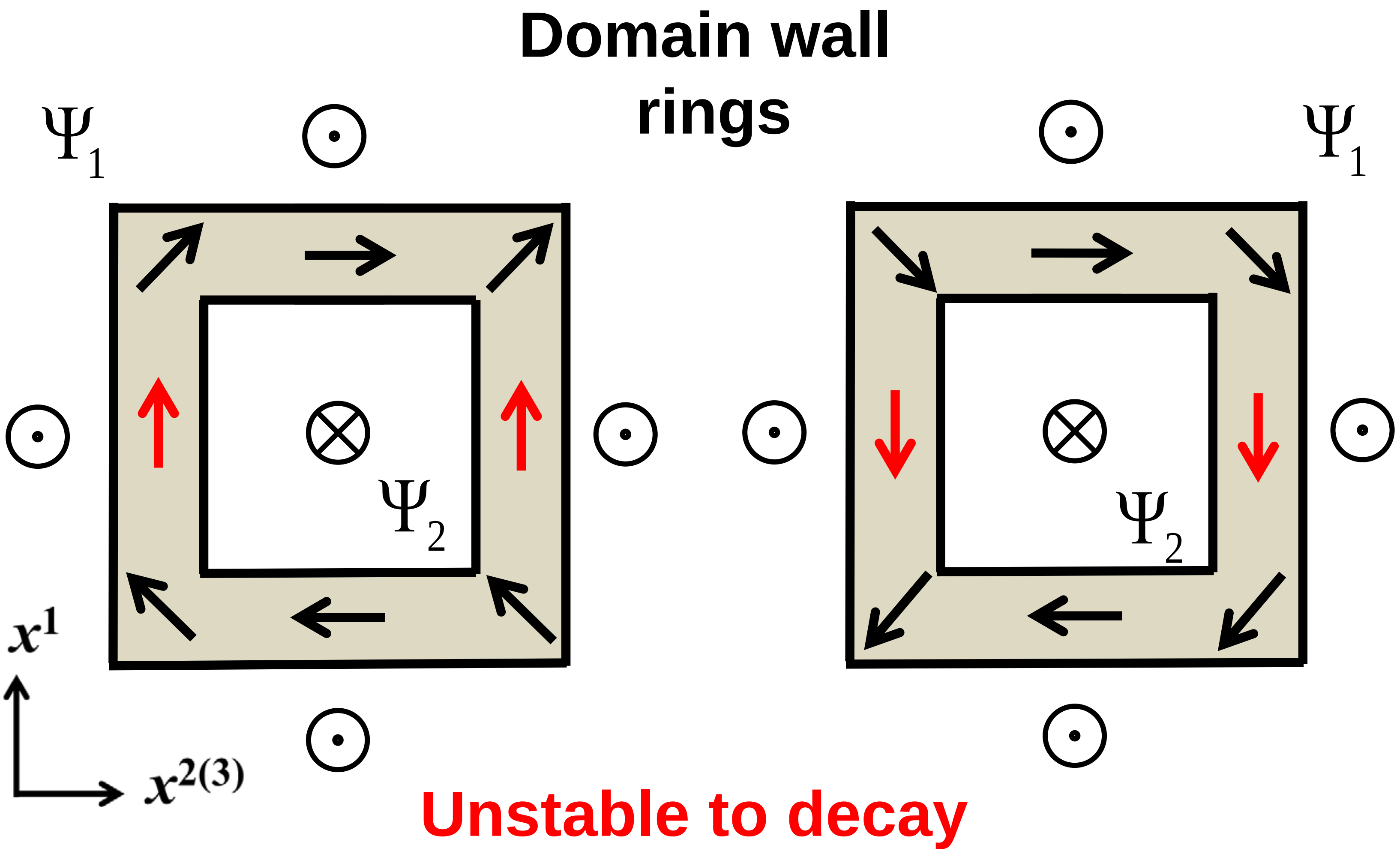
x^1
$x^{2(3)}$
x^1
ImΨ_1
ReΨ_1
x^1
ImΨ_1
ReΨ_1

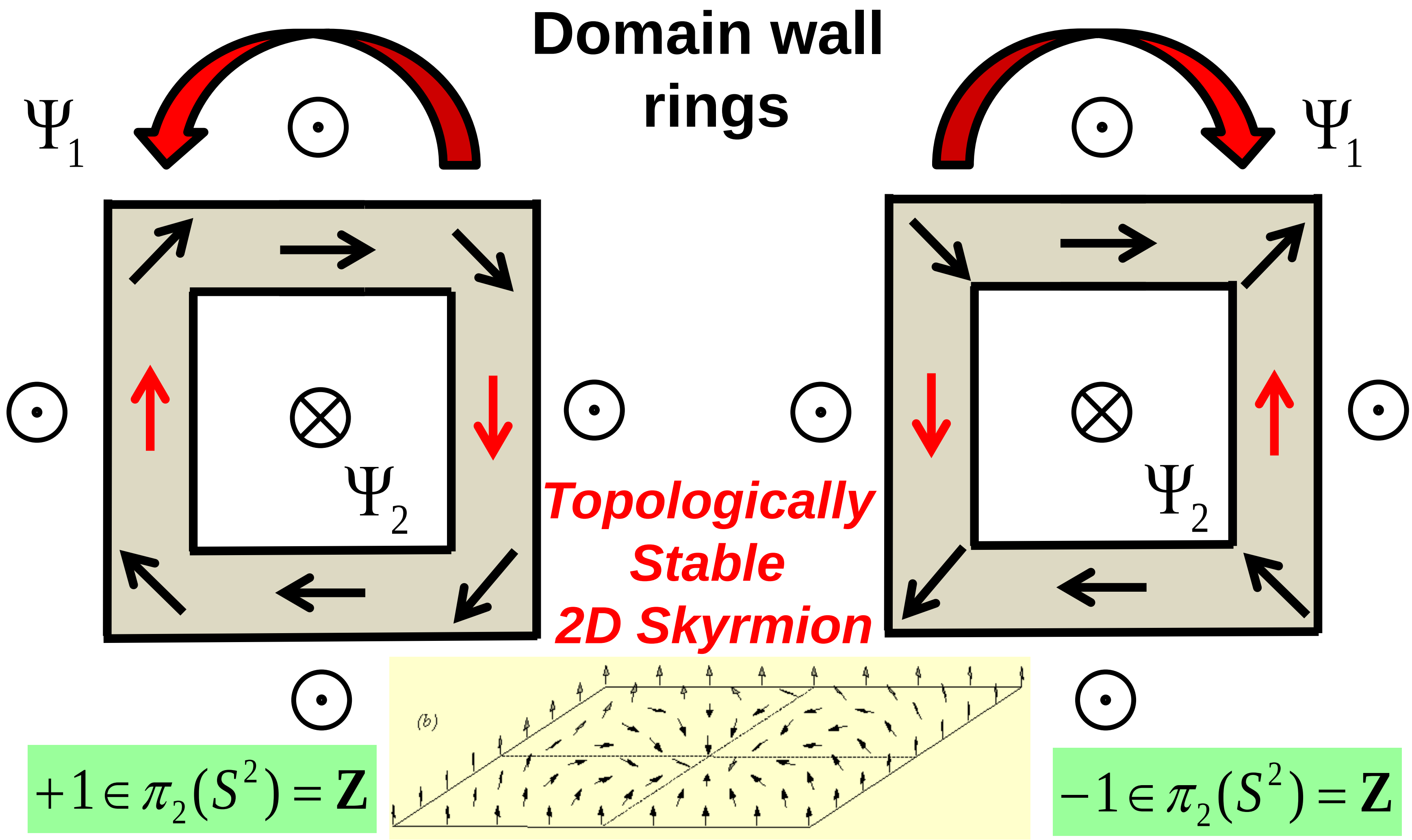


Ψ1
Ψ2
Ψ1
x1
x2(3)



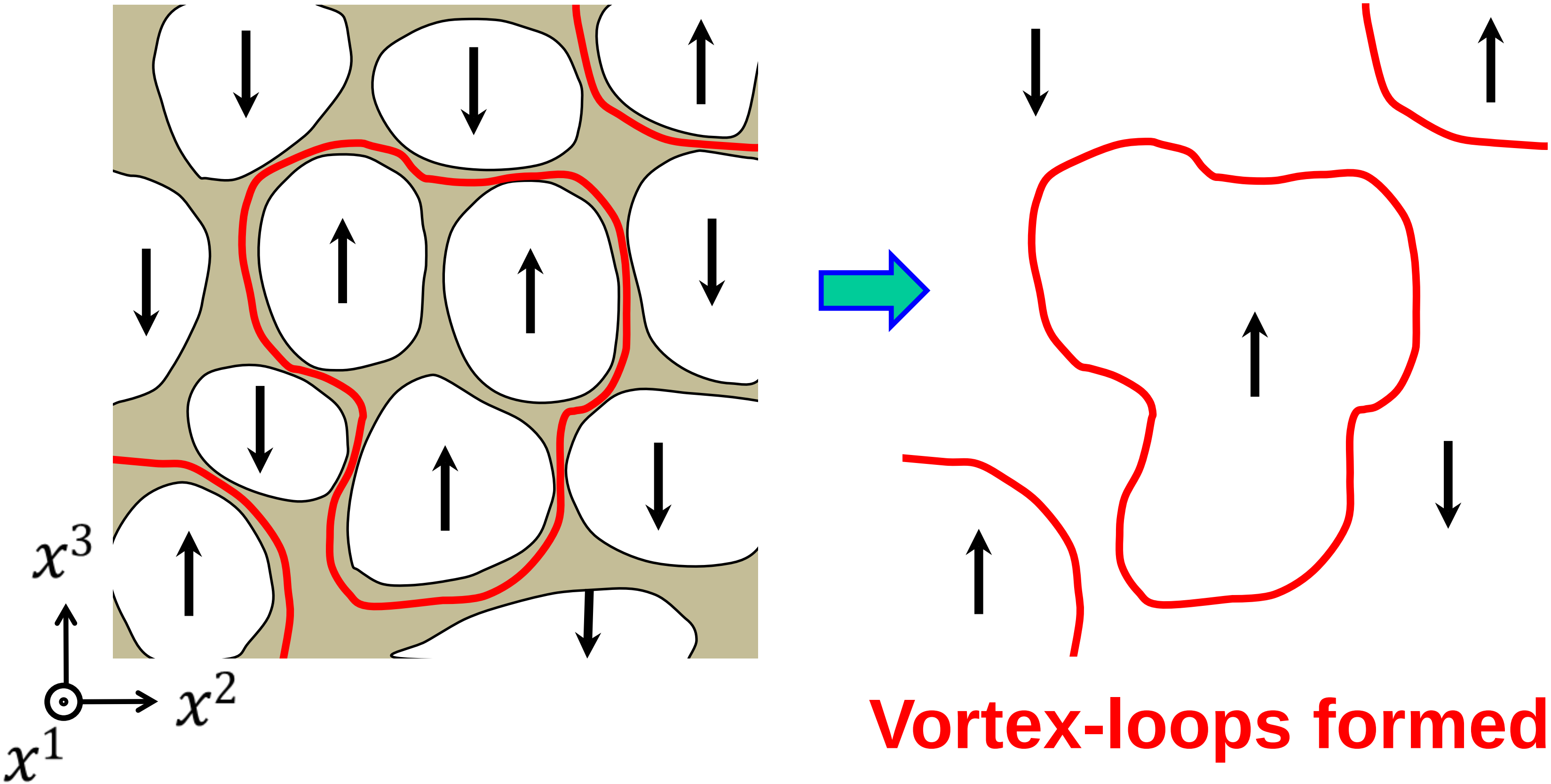
Ψ1
Ψ2
?
?
Ψ1
x1
x2(3)
4 possibilities of
domain wall ring


Domain wall
rings
Ψ1
Ψ2
Ψ1
Ψ2
x1
x2(3)
Unstable to decay

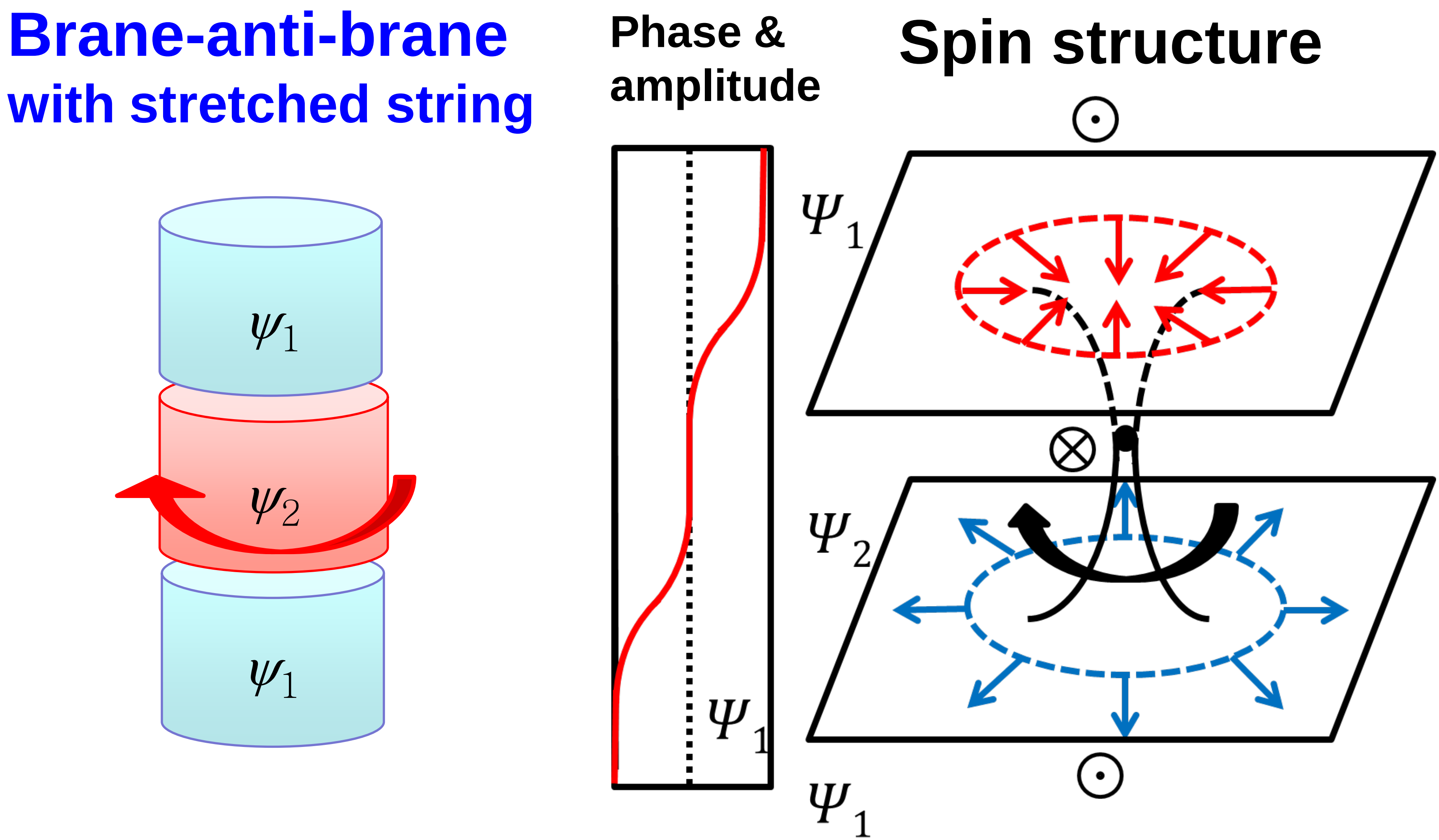


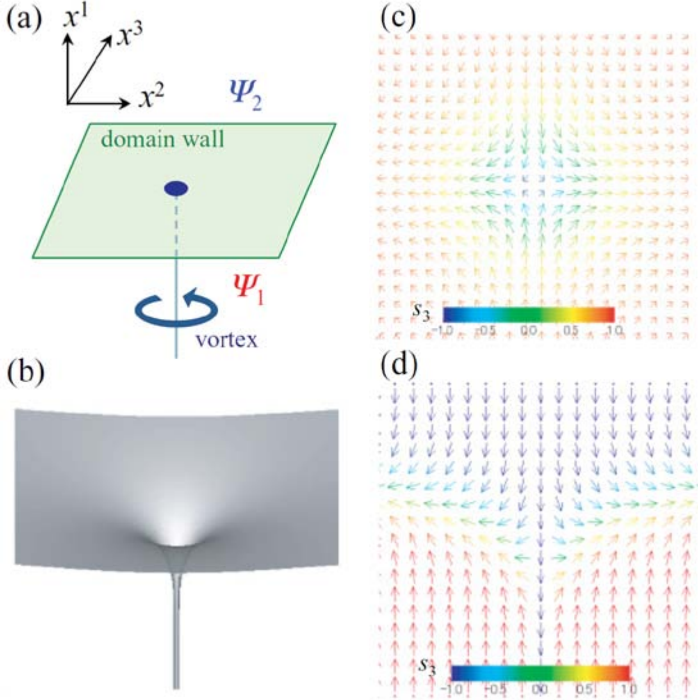
Domain wall rings
Ψ1
Ψ2
Ψ1
Ψ2
Topologically Stable 2D Skyrmion
(b)
+1 ∈ π2(S2) = Z
−1 ∈ π2(S2) = Z

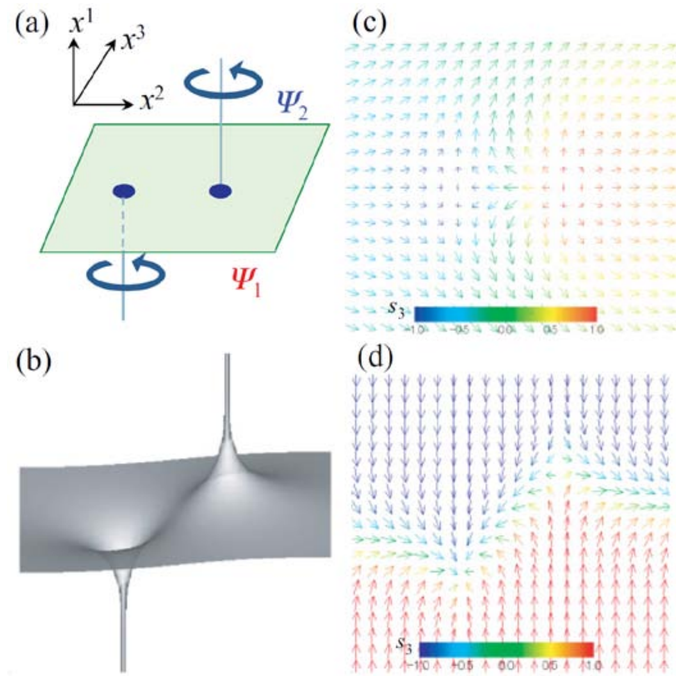
Wall annihilations in 3 dimensions



Vortex-loops formed


Brane-anti-brane
with stretched string
ψ_1
ψ_2
ψ_1
Phase &
amplitude
Ψ_1
Spin structure
Ψ_1
Ψ_2
Ψ_1





Exact analytic solutions

$$u(x^1, z) = u_{\rm W}(x^1) u_{\rm V}(z),$$

$$u_{\rm W}(x^1) = e^{\mp M(x^1 - x^1_0) - i\phi_0}, \quad u_{\rm V}(z) = \frac{\prod_{j=1}^{N_{v_1}} \left(z - z_j^{(1)}\right)}{\prod_{j=1}^{N_{v_2}} \left(z - z_j^{(2)}\right)}$$

All exact(analytic) solutions of ¼ BPS wall-vortex states

Y.Isozumi, MN, K.Ohashi, N.Sakai
Phys.Rev. D71 (2005) 065018

Bogomol'nyi-Prasad-Sommerfield (BPS) bound for vortex-domain wall

$$E = \int d\mathbf{r} \frac{\sum_\alpha |\partial_\alpha u|^2 + M^2 |u|^2}{\left(1+|u|^2\right)^2}$$

$$= \int d\mathbf{r} \left[\frac{|\partial_x u \mp i \partial_y u|^2}{\left(1+|u|^2\right)^2} \pm \frac{i\left(\partial_x u^* \partial_y u - \partial_y u^* \partial_x u\right)}{\left(1+|u|^2\right)^2} \right.$$

$$\left. + \frac{|\partial_z u \mp 2Mu|^2}{\left(1+|u|^2\right)^2} \pm \frac{2M\left(u^* \partial_z u + u \partial_z u^*\right)}{\left(1+|u|^2\right)^2} \right]$$

$$\geq |T_W| + |T_V|$$

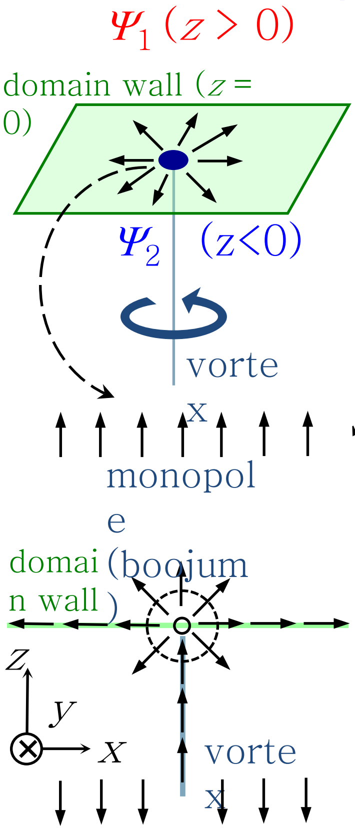
$T_V = 2\pi N_V$
vortex
(2d Skyrmion)
charge

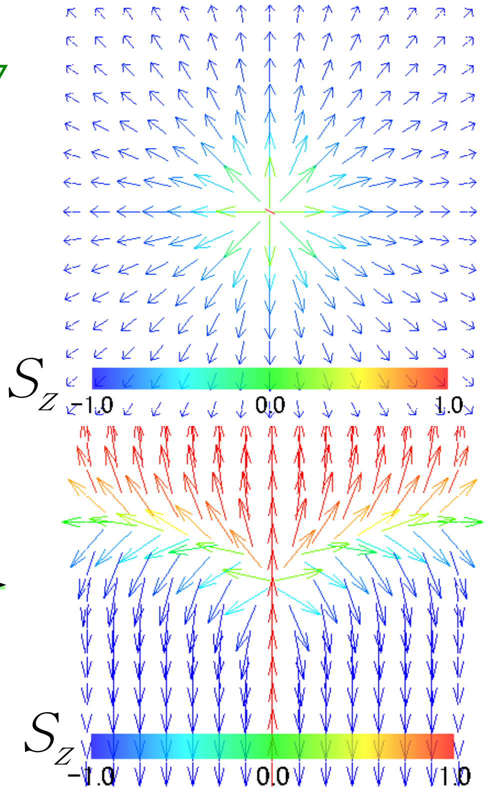
$T_W = \pm M, 0$
domain wall
charge

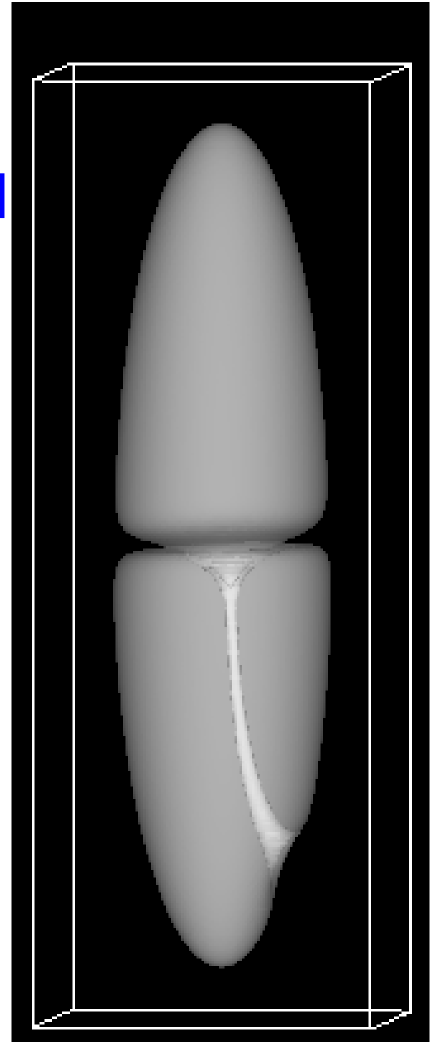
D-brane in a laboratory

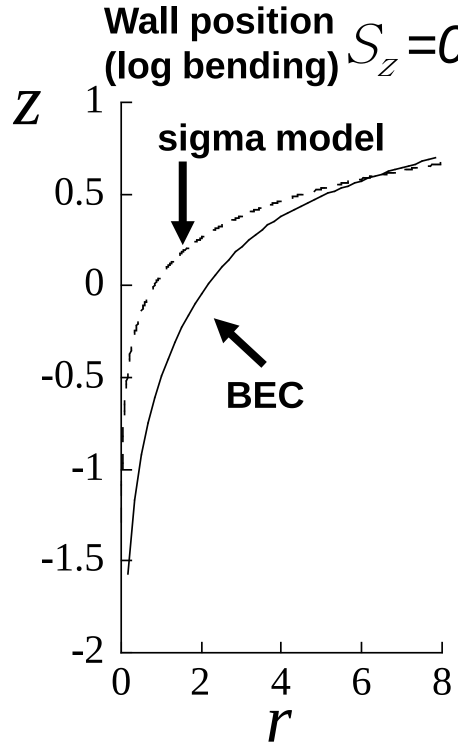
Kasamatsu-Takeuchi-MN-Tsubota

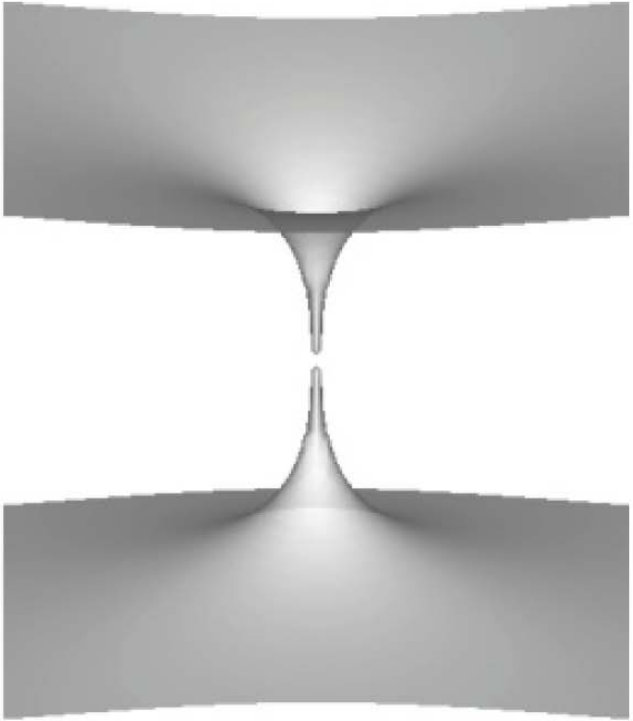
JHEP 1011:068,2010[arXiv:1002.4265]



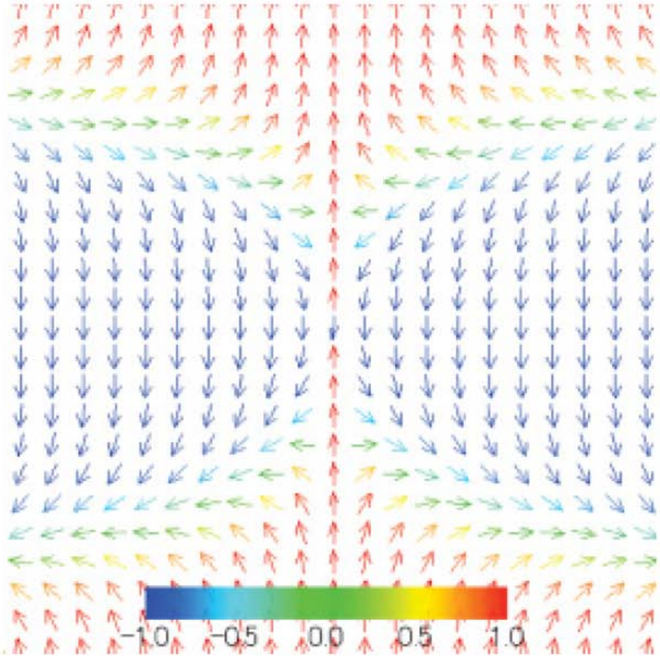








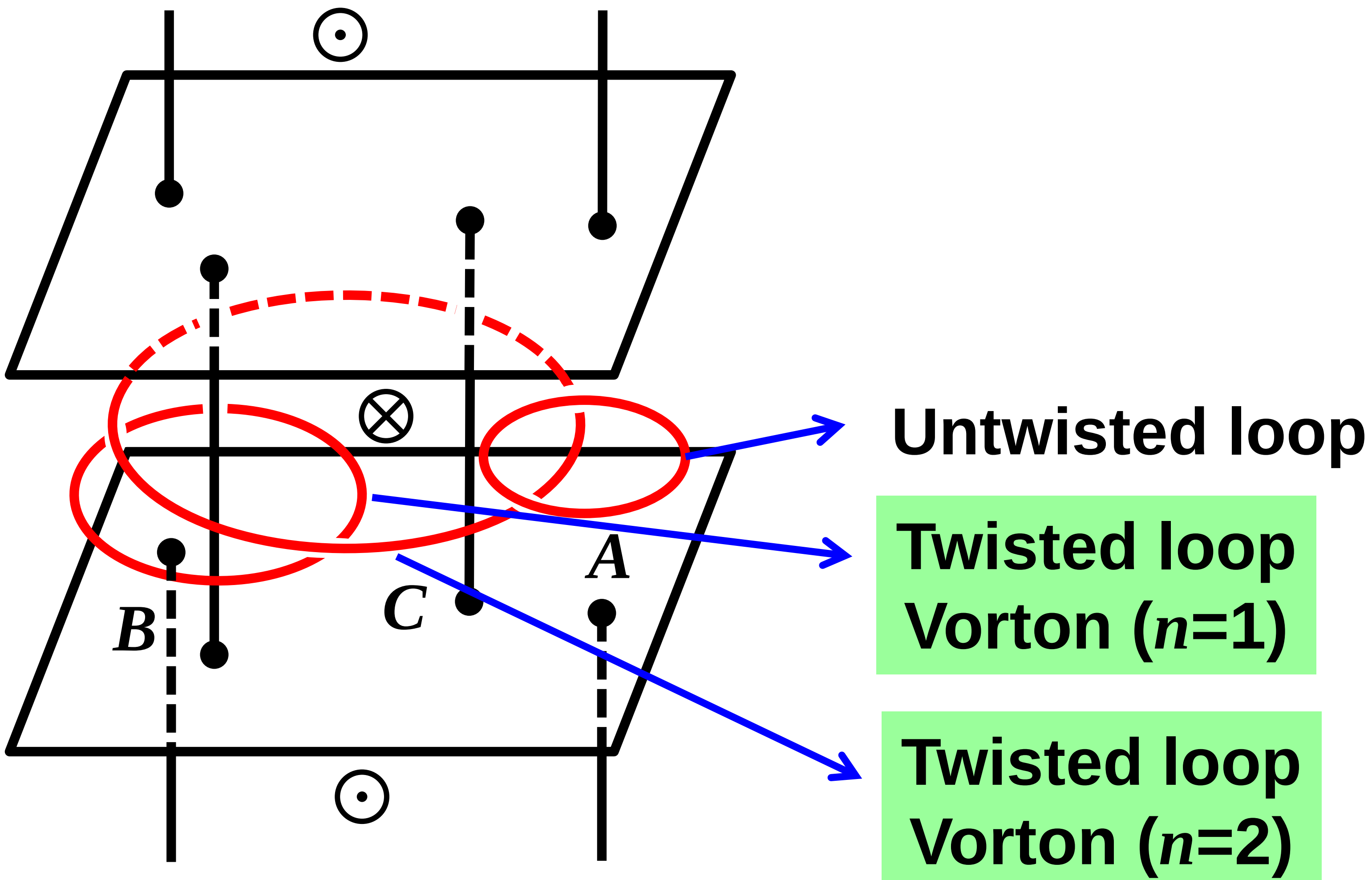
(b)



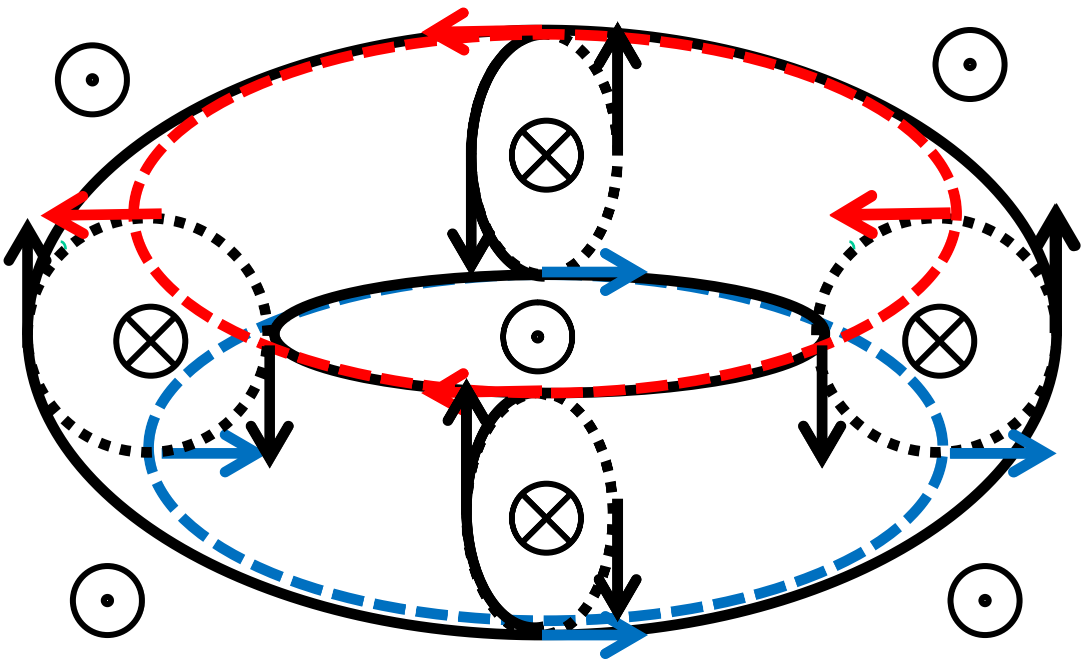
(c)

$$u(x^1,z) = u_{\mathrm{W}}(x^1)u_{\mathrm{V}}(z),$$ **Analytic (approximate) solution**

$$u_{\mathrm{W}}(x^1) = e^{-M(x^1-x^1_1)-i\phi_1} + e^{M(x^1-x^1_2)-i\phi_2}, \; u_{\mathrm{V}}(z) = 1/z$$

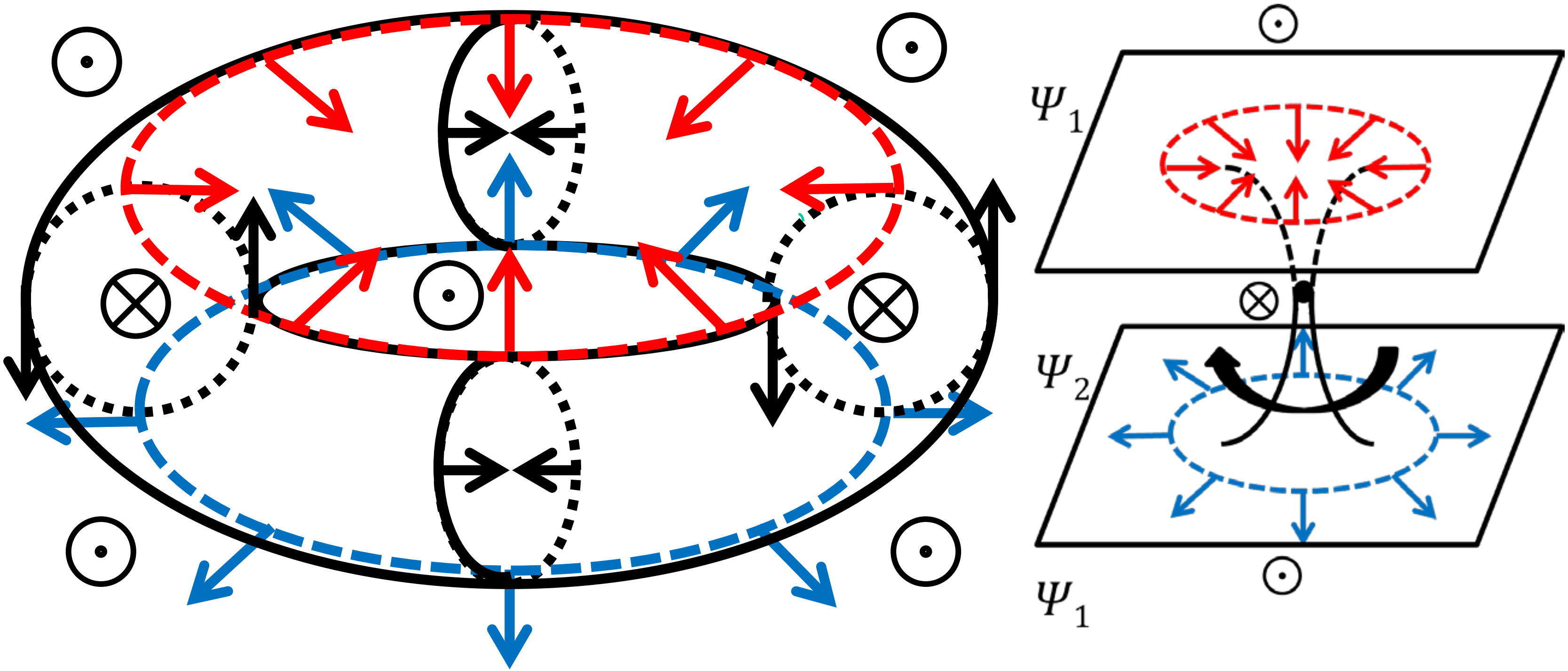


Untwisted loop
Twisted loop
Vorton (n=1)
Twisted loop
Vorton (n=2)
A
B
C

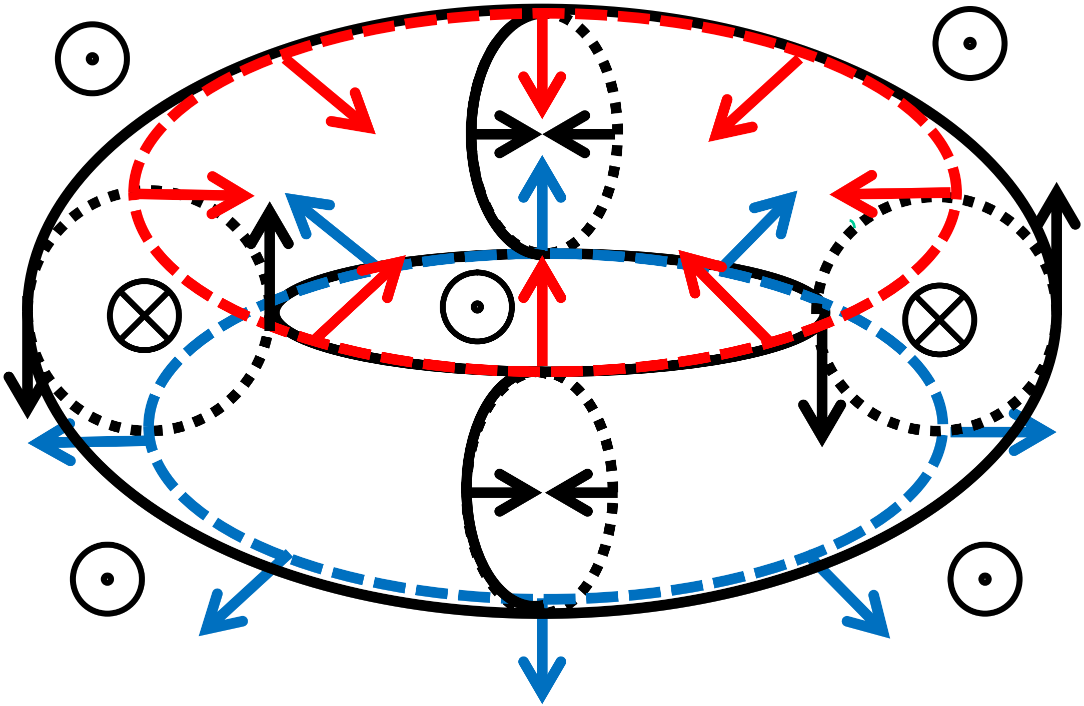


Untwisted loop

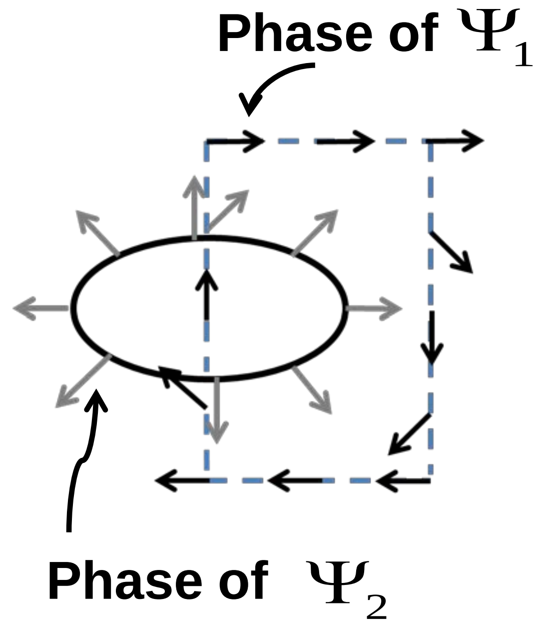
Unstable to decay



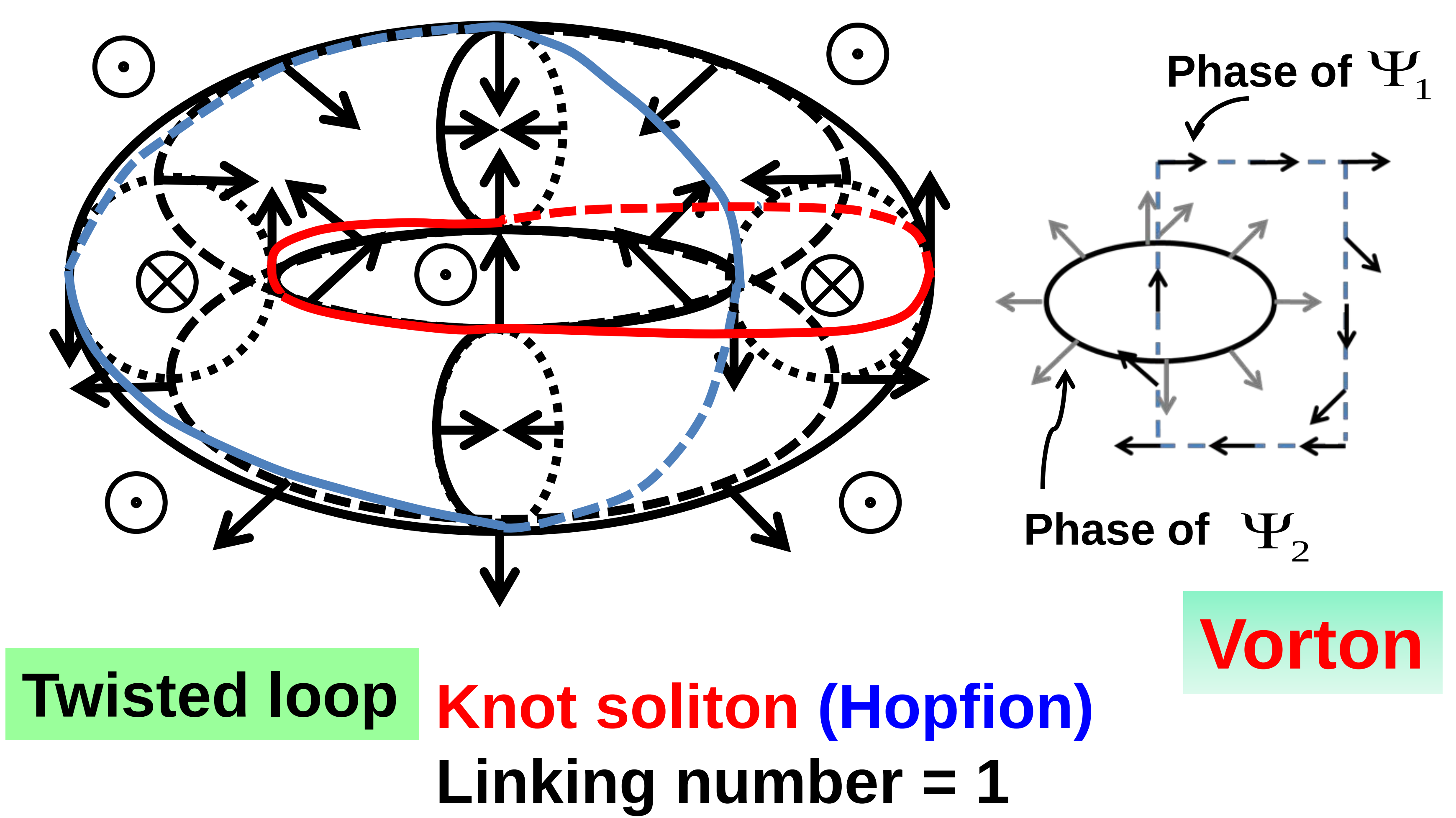
Twisted loop



Twisted loop



Vorton


Phase of Ψ_1
Phase of Ψ_2
Vorton
Twisted loop
Knot soliton (Hopfion)
Linking number = 1

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

§ 4-1 Brane annihilation (4p+22p)

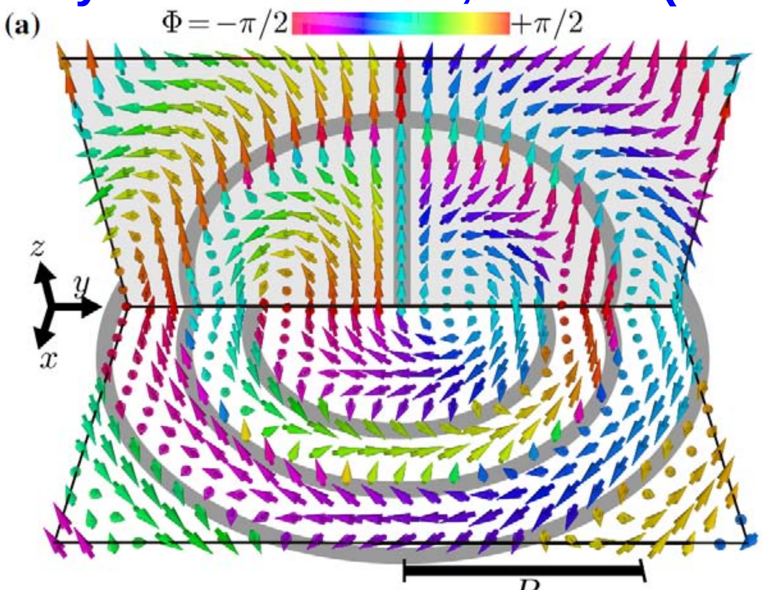
§ 4-2 Non-Abelian gauge field (7p)

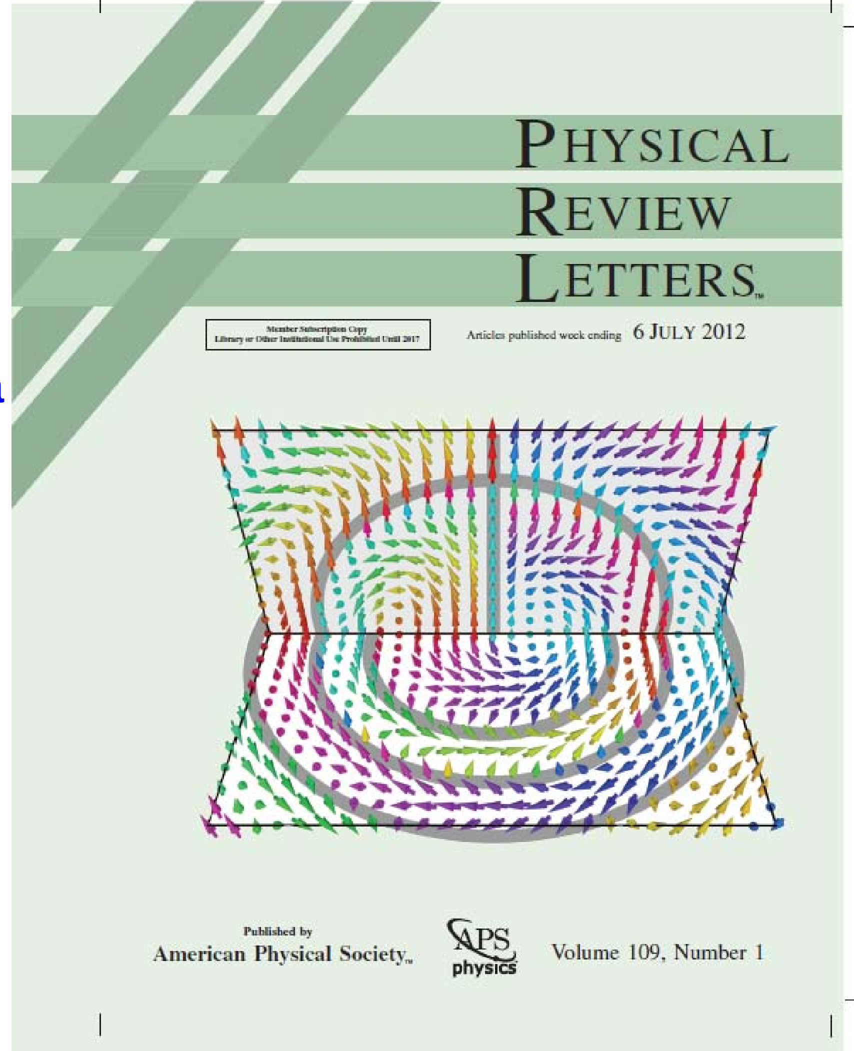
§ 5 Conclusion (1p)

§ 4-2 Non-Abelian gauge field

Artificial "SU(2) gauge field" stabilizes 3D Skyrmion

Kawakami, Mizushima, MN & Machida
Phys. Rev. Lett. 109, 015301 (2012)





Non-Abelian gauge fields

VOLUME 52, NUMBER 24 PHYSICAL REVIEW LETTERS 11 JUNE 1984

Appearance of Gauge Structure in Simple Dynamical Systems

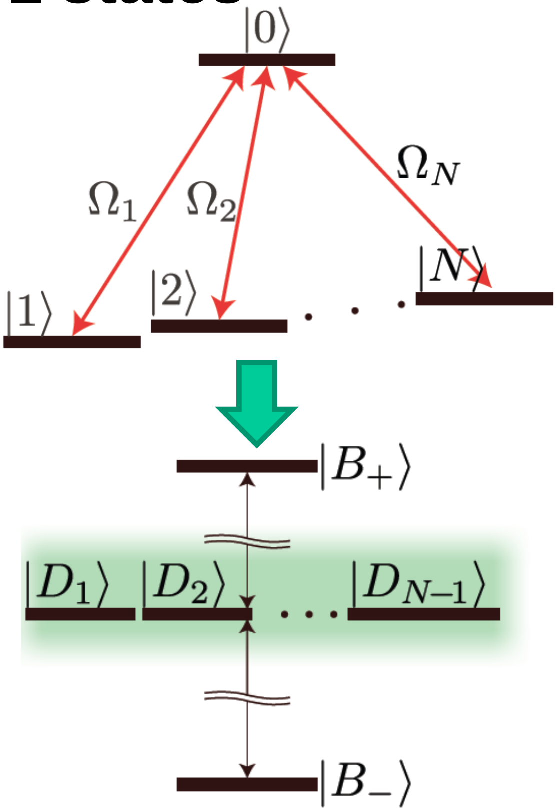
Frank Wilczek and A. Zee[(a)]
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 9 April 1984)

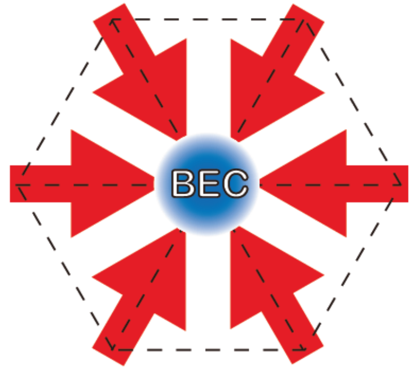
Generalizing a construction of Berry and Simon, we show that non-Abelian gauge fields arise in the adiabatic development of simple quantum mechanical systems. Characteristics of the gauge fields are related to energy splittings, which may be observable in real systems. Similar phenomena are found for suitable classical systems.

Non-Abelian gauge fields is induced on degenerate states by Berry phase.

N+1 states

Juzeliūnas, Ruseckas & Dalibard
Phys. Rev. A 81, 053403 (2010)





$$|\Psi(\boldsymbol{r})\rangle = \sum_{i=1}^{N-1} \psi_i(\boldsymbol{r})\,|D_i(\boldsymbol{r})\rangle$$

N-1 dark states $\{|D_A\rangle\}$
+ 2 bright states

(N-1)x(N-1) gauge fields

SU(2) gauge fields

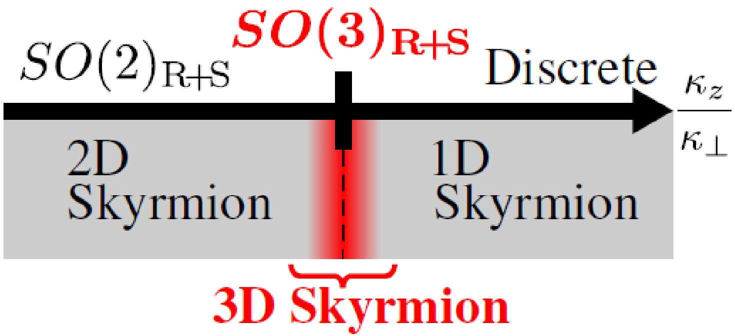
$$\mathbf{A}_i = \sum_{a,i} A_i^a \sigma_a$$

Gauge fields	Formulae	Generating schemes
Rashba +Dresselhause (1D)	$A \propto \hat{x}\sigma_x$	Lin, Garcia, Spielman, Nature **471**, 83 (2011)
Rashba (2D)	$A \propto \hat{x}\sigma_x + \hat{y}\sigma_y$	Juzeliunas, Ruseckas, Dalibard PRA **81**, 053403 (2010) Campbell, Juzeliunas, Spielman PRA **84**, 025602 (2011) etc…
3D-Rashba	$A \propto \hat{x}\sigma_x + \hat{y}\sigma_y + \hat{z}\sigma_y$	Anderson, Juzeliunas, Spielman, Galitski, arXiv:1112.6022
Non-Abelian monopole	$A = -\frac{\cos\theta}{r\sin\theta} e_\phi \sigma_x + \cdots$	Ruseckas, Juzeliunas, *et al.*, PRA **95**, 010404 (2005)

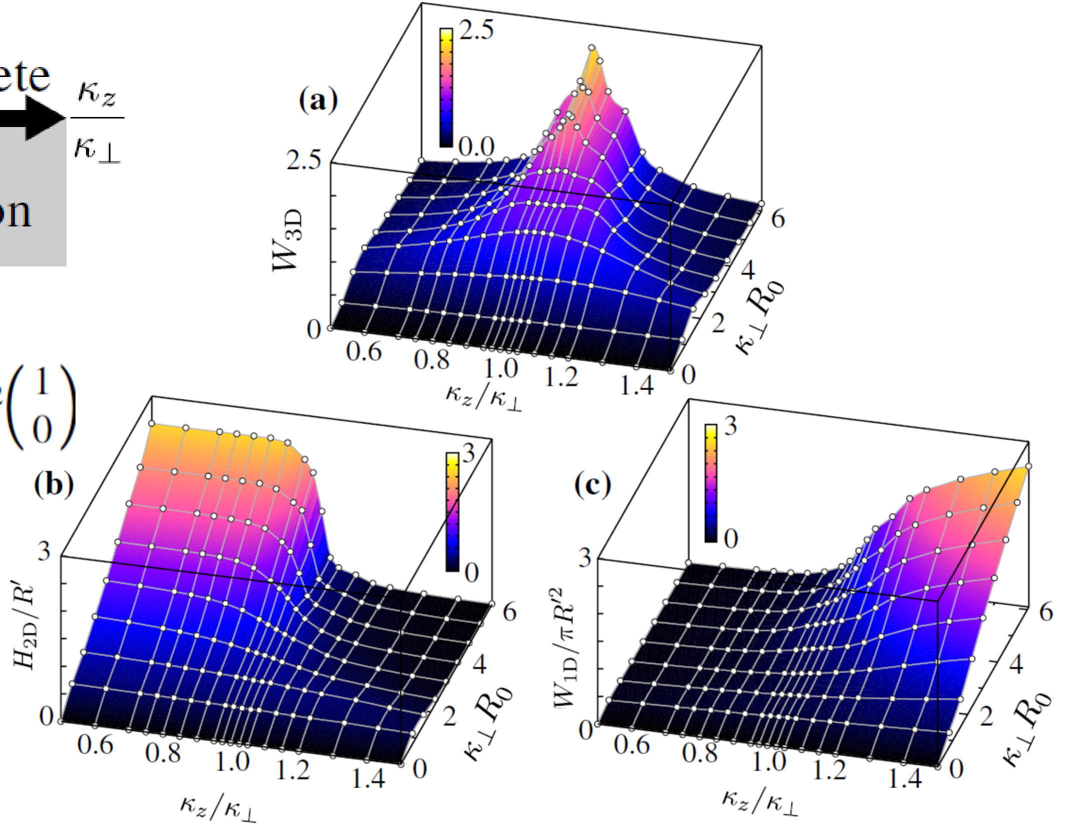
We use $\mathbf{A}_i = \sum_{a,i} A_i^a \sigma_a = \kappa_\perp (\hat{\mathbf{x}}\sigma_x + \hat{\mathbf{y}}\sigma_x) + \kappa_z \hat{\mathbf{z}}\sigma_x$

Crossover of Skyrmions

$$W_{3D} = \frac{1}{8\pi^2} \int d^3\boldsymbol{r}\, \epsilon_{ijk} \sin\theta_S (\partial_i \theta_S)(\partial_j \phi_S)(\partial_k \Phi)$$

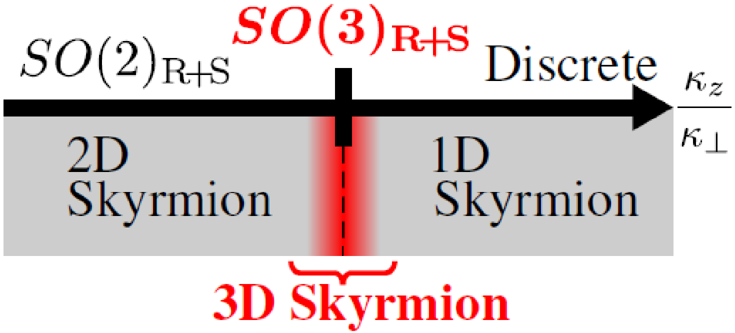


$$\begin{pmatrix} \Psi_\uparrow(\boldsymbol{r}) \\ \Psi_\downarrow(\boldsymbol{r}) \end{pmatrix} = \sqrt{n(\boldsymbol{r})} e^{i\Phi} e^{-i\sigma_z \phi_s/2} e^{-i\sigma_y \theta_s/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



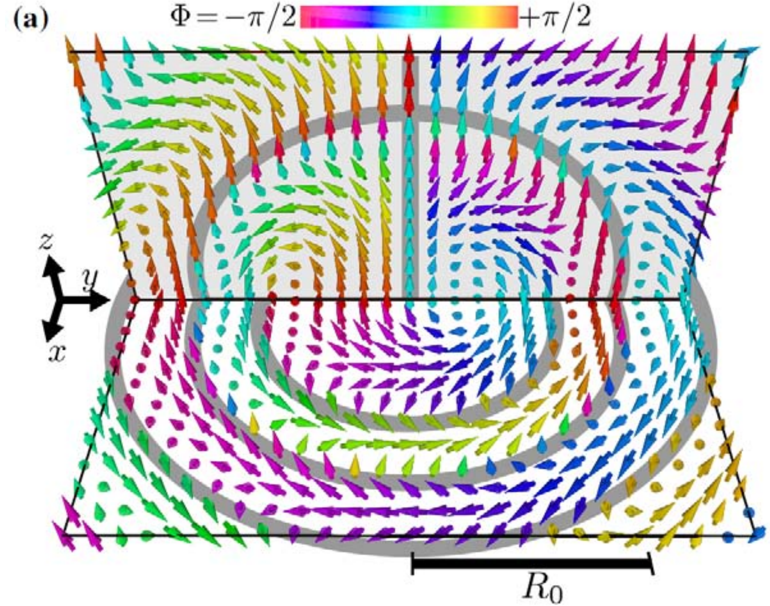
$$W_{2D} = \frac{1}{4\pi} \int d^3\boldsymbol{r}\, \epsilon_{ij} \sin\theta_S (\partial_i \theta_S)(\partial_j \phi_S) \quad W_{1D} = \frac{1}{2\pi} \int dz \frac{\Psi_\mu^* \partial_z (\sigma_z)_{\mu\nu} \Psi_\nu}{\Psi_\eta^* \Psi_\eta} + \text{c.c.}$$

Crossover of Skyrmions

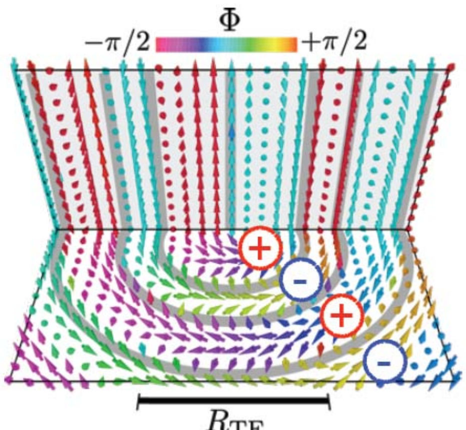


$$\begin{pmatrix}\Psi_\uparrow(\boldsymbol{r})\\ \Psi_\downarrow(\boldsymbol{r})\end{pmatrix} = \sqrt{n(\boldsymbol{r})}\, e^{i\Phi} e^{-i\sigma_z\phi_s/2} e^{-i\sigma_y\theta_s/2}\begin{pmatrix}1\\0\end{pmatrix}$$

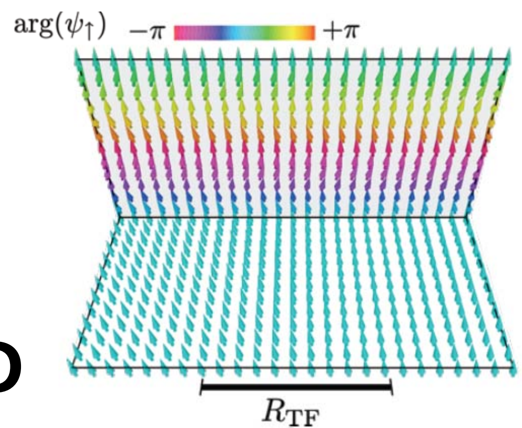
3D



2D

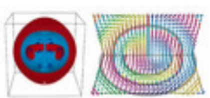


1D



岡山大など、概念上の素粒子「スカーミオン」を安定に作り出すことを提唱

マイナビニュース 7月20日(金)16時10分配信



拡大写真
（写真：マイナビニュース）

岡山大学と慶應義塾大学(慶応大)は7月19日、陽子や中性子のような「核子」と呼ばれる粒子を理解するために導入された数学的概念であり、未だにその性質に謎が多く、素粒子理論に不可欠な「トポロジカル構造」である素粒子「スカーミオン」の理解に不可欠な構造を、現実に数ナノケルビン程度まで冷却された原子気体において安定に作り出すことを世界で初めて提唱したことを発表した。

成果は、岡山大大学院 自然科学研究科 先端基礎科学専攻の川上拓人大学院生(物性理論)、同水島健助教、同町田一成特命教授、慶応大 自然科学研究教育センターの新田宗土准教授(素粒子論)らの研究グループによるもの。研究の詳細な内容は、7月2日付けで米国物理学会速報誌「Physical Review Letters」オンライン版に掲載された。また、「Physical Review

Plan of my talk

§ 1 Introduction(BEC and Vortices) (13p)

§ 2 Skyrmions (7p)

§ 3 Multi-component BECs (7p+3p)

§ 4 3D Skyrmions in BECs

§ 4-1 Brane annihilation (4p+22p)

§ 4-2 Non-Abelian gauge field (7p)

§ 5 Conclusion (1p)

§ 5 Conclusion

- 位相的励起、特に渦やスカーミオンは、物性物理で広く現れ、系の相やダイナミクスを支配する重要な自由度である。
- 位相的励起を観測することで、系の自由度、対称性、超流動性、超伝導性などがわかる(こともある)。
- 基礎物理（素粒子物理、ハドロン物理(QCD)、宇宙論）でも現れ重要。

渦やスカーミオンの物理学の構築に向けて
両分野の交流が不可欠