Effect of vertex correction in QED3

S-D equation

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Thermal quantum field theory & its application

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Yukawa Hall

@outline

QED3 is super renormalizable

[e^2]= M Infrared divergence is severe

Schwinger-Dyson equation

for chiral sym & mass generattion

bare vertex violates Ward-Takahashi identity

we need vertex correction

our results agree with Fisher,Alkofer,Maris(04)

@results

Chern-Simon QED has largest vertex correction

vev ~ln(Λ),gauge invariant

former analysis of Chern-Simon QED

Ladder S-D equation

Bashir,Raya

Matsuyama,Nagahiro

Hoshino,Inagaki,Sakamoto

(not published)

1 vertex correction by Ward-Takahashi identity

Fundamental Green function

$$S_F(x-y),$$
$$D_{F\mu\nu}(x-y),$$
$$\Gamma_\mu(x,y;z)$$

are not independent.Ward-Takahashi-identity by gauge invariance

$$(p-q)^\mu \Gamma_\mu(p,q) = S_F^{-1}(q) - S_F^{-1}(p)$$

S-D

$$
\begin{aligned}
S_F^{-1}(p) &= S_F^{(0)-1} - ie^2 \int \frac{d^3k}{(2\pi)^3} \Gamma_\mu(p,k) S_F(k) \gamma_\nu D_F^{\mu\nu}(p-k) \\
&= A(p) p \cdot \gamma - B(p)
\end{aligned}
$$

S-D is an quantum equation of motion

Gauge-Transform

Part I

$$
\begin{aligned}
\psi &- > \mathrm{exp}(ie\theta(x))\psi \\
\overline{\psi} &- > \overline{\psi}\,\mathrm{exp}(-ie\theta(x)) \\
A_\mu &- > A_\mu + \partial_\mu \theta(x)
\end{aligned}
$$

$$
\frac{\delta L}{\delta \theta} = \partial_\mu \frac{\delta L}{\delta(\partial_\mu \theta)} = 0
$$

$$J_\mu = \frac{\delta L}{\delta(\partial_\mu \theta)}$$

$$\partial_\mu J_\mu = 0$$

using

$$\delta(x_0 - y_0)[J_0(x), \phi(y)] = \delta\phi(y)\delta^{(n-1)}(x-y)$$

$$\partial^z_\mu T(\psi(x)\overline{\psi}(y)J_\mu(z))$$
$$= eT(\psi(x)\overline{\psi}(y))[\delta(y-z) - \delta(z-x)]$$

for chiral symmetry

$$\partial^z_\mu T(\psi(x)\overline{\psi}(y)J_{5\mu}(z))$$
$$= T(\psi(x)\overline{\psi}(y))[\gamma_5\delta(y-z) + \gamma_5\delta(z-x)]$$

$$T(\psi(x)\overline{\psi}(y)A_\mu(z))$$
$$= -e\int dx'dy'dz'S'_F(x-x')\Gamma_\mu(x',y';z')S'_F(y'-y)$$
$$* D_{\mu\nu}(z'-z)$$

using

$$\Box_x \partial^x_\mu \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle = i\partial_\nu \delta^{(n)}(x-y)$$

we have

Part II

$$\Box_z \partial^z_\mu \left\langle 0|T(\psi(x)\overline{\psi}(y)A_\mu(z))|0\right\rangle$$
$$= -eS'_F(x-y)[\delta^{(n)}(y-z) - \delta^{(n)}(x-z)]$$

$$- ie\int d^n x' d^n y' S'_F(x-x')\partial_\nu \Gamma_\nu(x',y';z)S'_F(y'-y)$$
$$= -eS'_F(x-y)[\delta^{(n)}(y-z) - \delta^{(n)}(x-z)]$$

1.0.1

Ball-Chiu Ansatz(1980),Bashir,Pennington(1994)

Part III

$$S_F^{-1}(p) = A(p)\gamma \cdot p - B(p)$$
$$\Gamma_\mu(p,q) = \Gamma_\mu^L(p,q) + \Gamma_\mu^T(p,q)$$
$$(p-q)^\mu \Gamma_\mu^L(p,q) = S_F^{-1}(q) - S_F^{-1}(p)$$
$$(p-q)^\mu \Gamma_\mu^T(p,q) = 0$$

Assume

$$\Gamma_\mu^L(p,q) = a(p,q)\gamma_\mu + b(p,q)(p+q)\cdot\gamma(p+q)_\mu - c(p,q)(p+q)_\mu$$

solution

$$\Gamma^L_\mu(p,q) = \frac{A(p)+A(q)}{2}\gamma_\mu + \frac{A(p)-A(q)}{2(p^2-q^2)}(p+q)\cdot\gamma(p+q)_\mu$$
$$-\frac{B(p)-B(q)}{p^2-q^2}(p+q)_\mu$$

Differential form

$$\Gamma_\mu(p,q) = A(q)\gamma_\mu + 2\frac{dA(q)}{dq^2}q\cdot\gamma q_\mu - 2\frac{dB(q)}{dq^2}q_\mu$$

1.1 $\langle\overline{\psi}\psi\rangle$ gauge invariant ? OK within error

$$\langle\overline{\psi}\psi\rangle = -trS_F(x) = -(2,4)\int\frac{d^nk}{(2\pi)^n}\frac{B(k)}{A^2(k)k^2+B^2(k)}$$

1.2 role of $\Delta B = B(p) - B(q)$ term

1.2.1 neglect $\Delta B = B(p) - B(q)$ in S-D equation->A(p)=1,B(p)$\neq 0 (ladder)$

1.3 large effect

$\left\langle \overline{\psi}\psi \right\rangle$ becomes large by 50% from ΔB

2 Chern-Simon QED

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{\mu}{4}\epsilon_{\mu\nu\rho}F_{\mu\nu}A_\rho + ...$$

$F \times A$:Parity violating

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - p_\mu p_\nu/p^2 - i\mu\epsilon_{\mu\nu\rho}p_\rho/p^2}{p^2 - \mu^2 + i\epsilon} + \xi\frac{p_\mu p_\nu}{p^4}$$

Ladder S-D Landau gauge

$$B(p) = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dqq^2}{q^2A(q)^2 + B(q)^2}[2(B(q)I_0(p,q) - \mu A(q)I_2(p,q)_-)].$$

$$p^2(A(p) - 1) = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dqq^2}{q^2A(q)^2 + B(q)^2}[2(A(q)I_3(p,q) - \mu B(q)I_2(p,q)_+)]$$

$$I_0(p,q) = \frac{-1}{2pq}\ln(\frac{(p-q)^2 + \mu^2}{(p+q)^2 + \mu^2}),$$

$$I_2(p,q)_\pm = \frac{-1}{4pq}\ln(\frac{(p-q)^2 + \mu^2}{(p+q)^2 + \mu^2}) \pm \frac{p^2 - q^2}{4\mu^2 pq}\ln(\frac{1 + \mu^2/(p-q)^2}{1 + \mu^2/(p+q)^2})$$

$$I_3(p,q) = \frac{(p^2-q^2)^2}{8\mu^2 pq}\ln\left(\frac{1+\mu^2/(p-q)^2}{1+\mu^2/(p+q)^2}\right) - \frac{1}{2} - \frac{\mu^2}{8pq}\ln\left(\frac{(p-q)^2+\mu^2}{(p+q)^2+\mu^2}\right).$$

$$B \to \frac{a}{p} + \frac{b}{p^2}$$

at large p but B has node

$$B(p) - \mu A(p) \to 0$$

for large p.Complication

1 vertex correction by Ward-Takahashi identity

Fundamental Green function

$$S_F(x-y),$$
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are not independent.Ward-Takahashi-identity by gauge invariance

$$(p-q)^\mu \Gamma_\mu(p,q) = S_F^{-1}(q) - S_F^{-1}(p)$$

S-D

$$S_F^{-1}(p) = S_F^{(0)-1} - ie^2 \int \frac{d^3k}{(2\pi)^3} \Gamma_\mu(p,k) S_F(k) \gamma_\nu D_F^{\mu\nu}(p-k)$$
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Part I

$$\psi -> \exp(ie\theta(x))\psi$$
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$$A_\mu -> A_\mu + \partial_\mu \theta(x)$$

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using

$$\delta(x_0 - y_0)[J_0(x), \phi(y)] = \delta\phi(y)\delta^{(n-1)}(x-y)$$

$$\partial_\mu^z T(\psi(x)\overline{\psi}(y)J_\mu(z))$$
$$= eT(\psi(x)\overline{\psi}(y))[\delta(y-z) - \delta(z-x)]$$

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$$T(\psi(x)\overline{\psi}(y)A_\mu(z))$$
$$= -e\int dx'dy'dz'S_F^{'}(x-x')\Gamma_\mu(x',y';z')S_F^{'}(y'-y)$$
$$* D_{\mu\nu}(z'-z)$$

using

$$\Box_x A_\mu(x) = -J_\mu(x)$$
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we have

Part II

$$\Box_z \partial_\mu^z \left\langle 0|T(\psi(x)\overline{\psi}(y)A_\mu(z))|0\right\rangle$$
$$= -eS_F'(x-y)[\delta^{(n)}(y-z) - \delta^{(n)}(x-z)]$$

$$- ie \int d^n x' d^n y' S_F'(x-x')\partial_\nu \Gamma_\nu(x',y';z)S_F'(y'-y)$$
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$$\begin{aligned} S_F^{-1}(p) &= A(p)\gamma \cdot p - B(p) \\ \Gamma_\mu(p,q) &= \Gamma_\mu^L(p,q) + \Gamma_\mu^T(p,q) \\ (p-q)^\mu \Gamma_\mu^L(p,q) &= S_F^{-1}(q) - S_F^{-1}(p) \\ (p-q)^\mu \Gamma_\mu^T(p,q) &= 0 \end{aligned}$$

Assume

$$\Gamma_\mu^L(p,q) = a(p,q)\gamma_\mu + b(p,q)(p+q)\cdot\gamma(p+q)_\mu - c(p,q)(p+q)_\mu$$

solution

$$\Gamma_\mu^L(p,q) = \frac{A(p)+A(q)}{2}\gamma_\mu + \frac{A(p)-A(q)}{2(p^2-q^2)}(p+q)\cdot\gamma(p+q)_\mu - \frac{B(p)-B(q)}{p^2-q^2}(p+q)_\mu$$

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$$B \to \frac{a}{p} + \frac{b}{p^2}$$

at large p but B has node

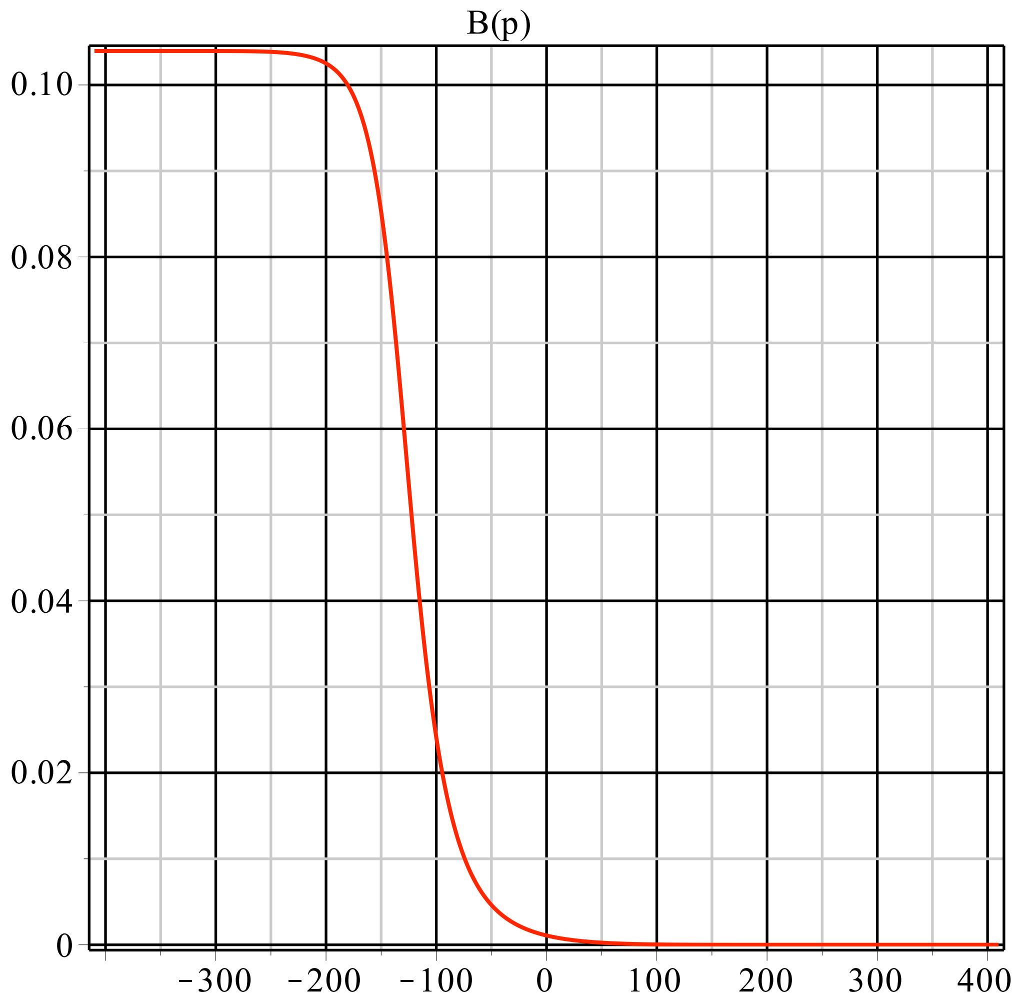
$$B(p) - \mu A(p) \to 0$$

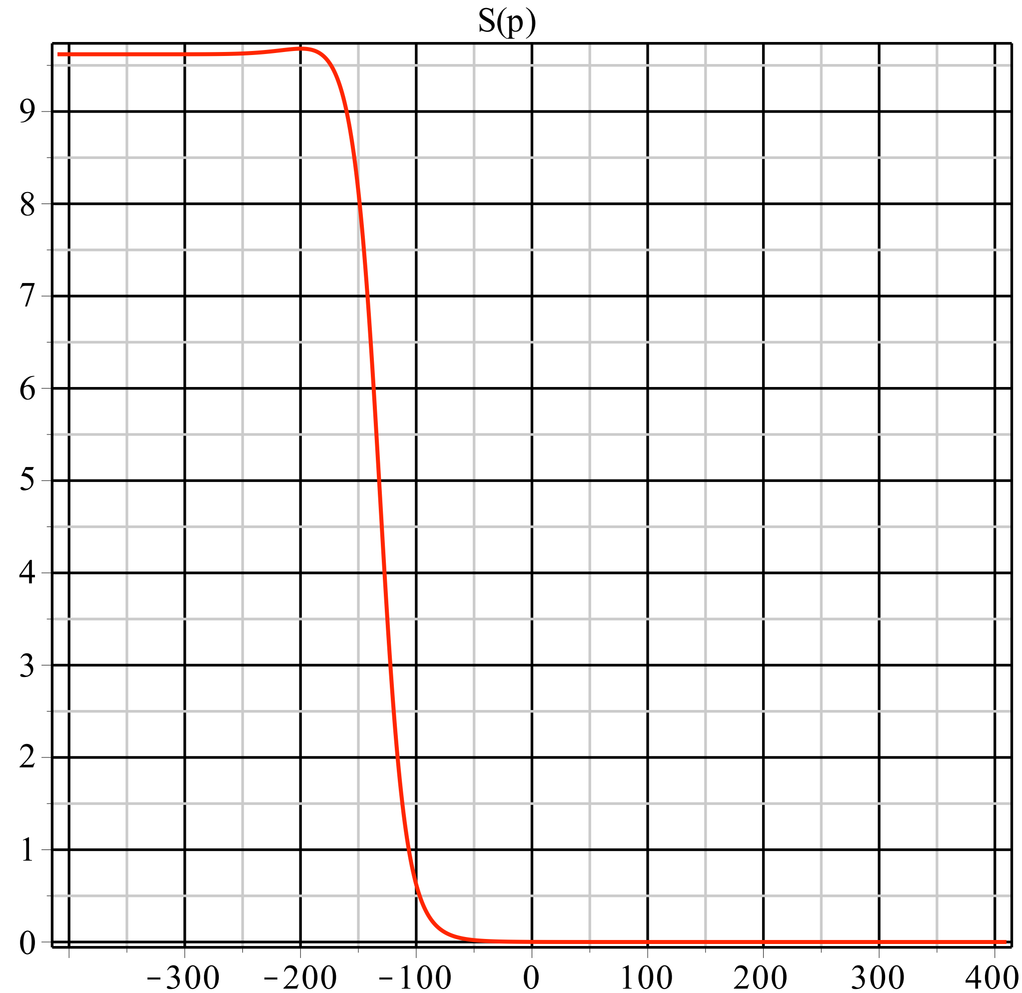
for large p.Complication

Numerical results of Schwinger-Dyson with veretex correction

Laddar Landau gauge

A=1





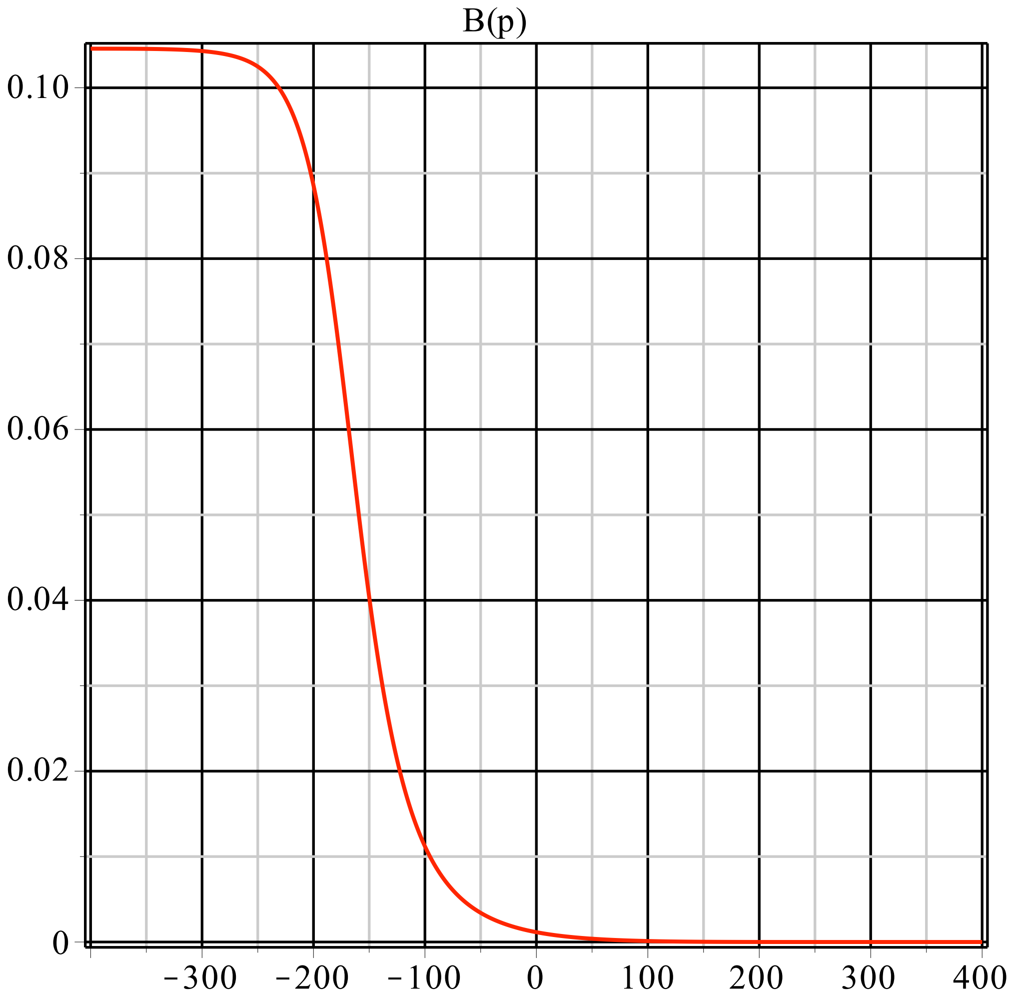
> $p := 10^{-4} \sim 10^{5}$; *double exponentiation formula used*;

>

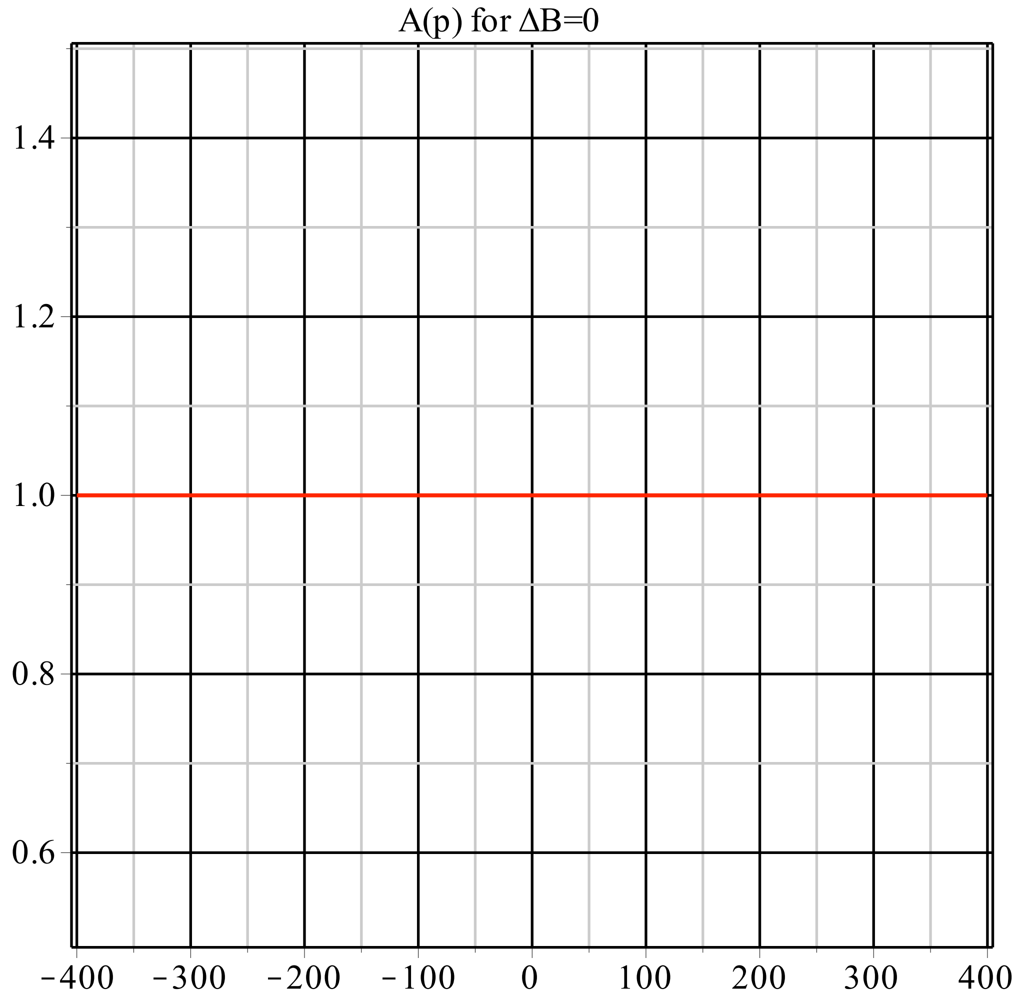
> $vev := 2.2 \cdot 10^{-3} \left(e^{4}\right)$

$$vev := 0.002200000000\ e^{4} \quad (1)$$

Veretx corection



> **if** $\Delta B = 0$

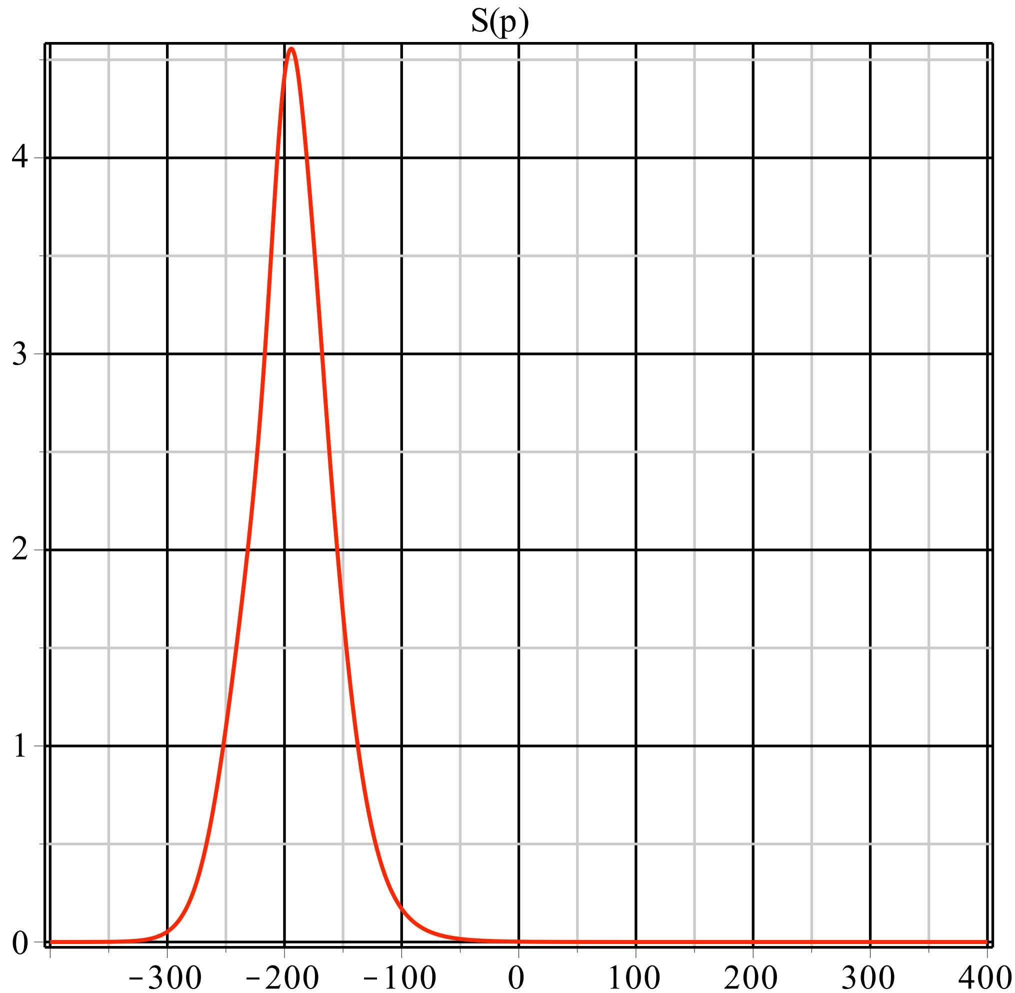


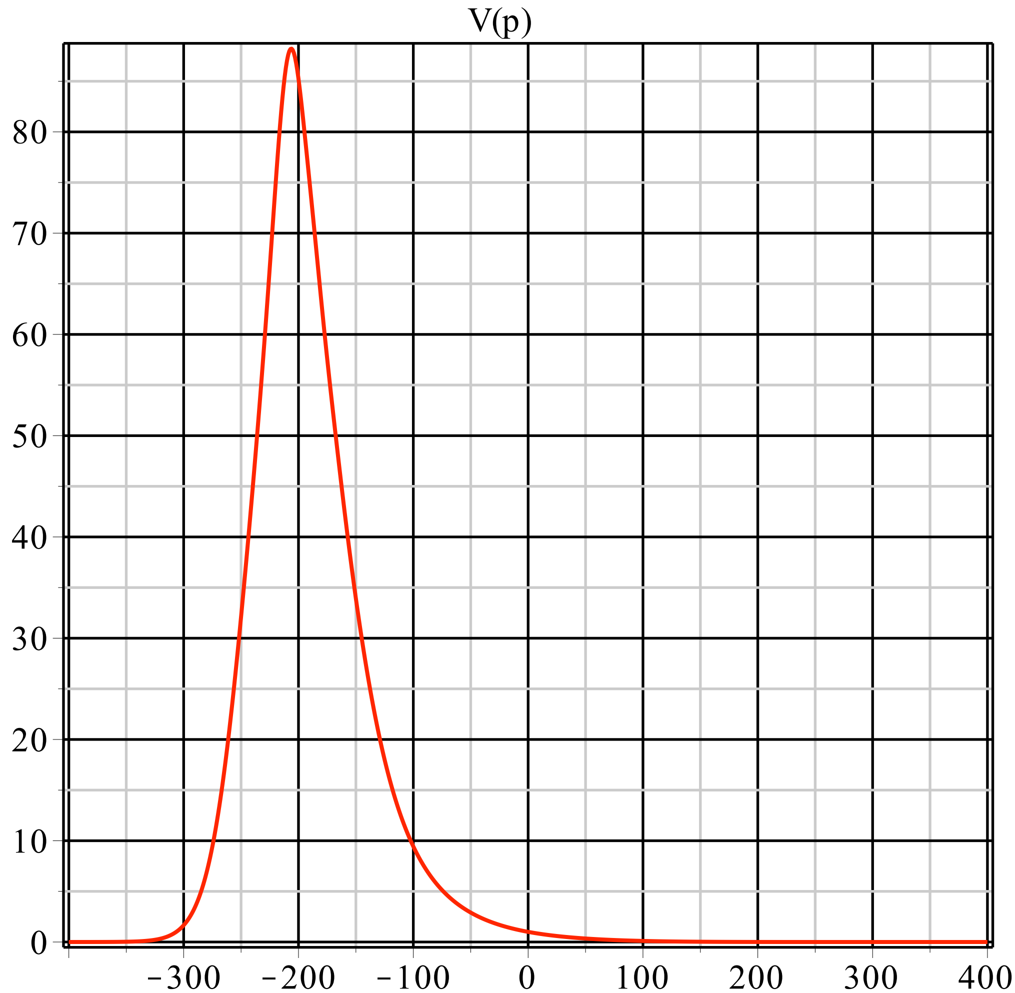
B(p) is proportional to 1/p^2

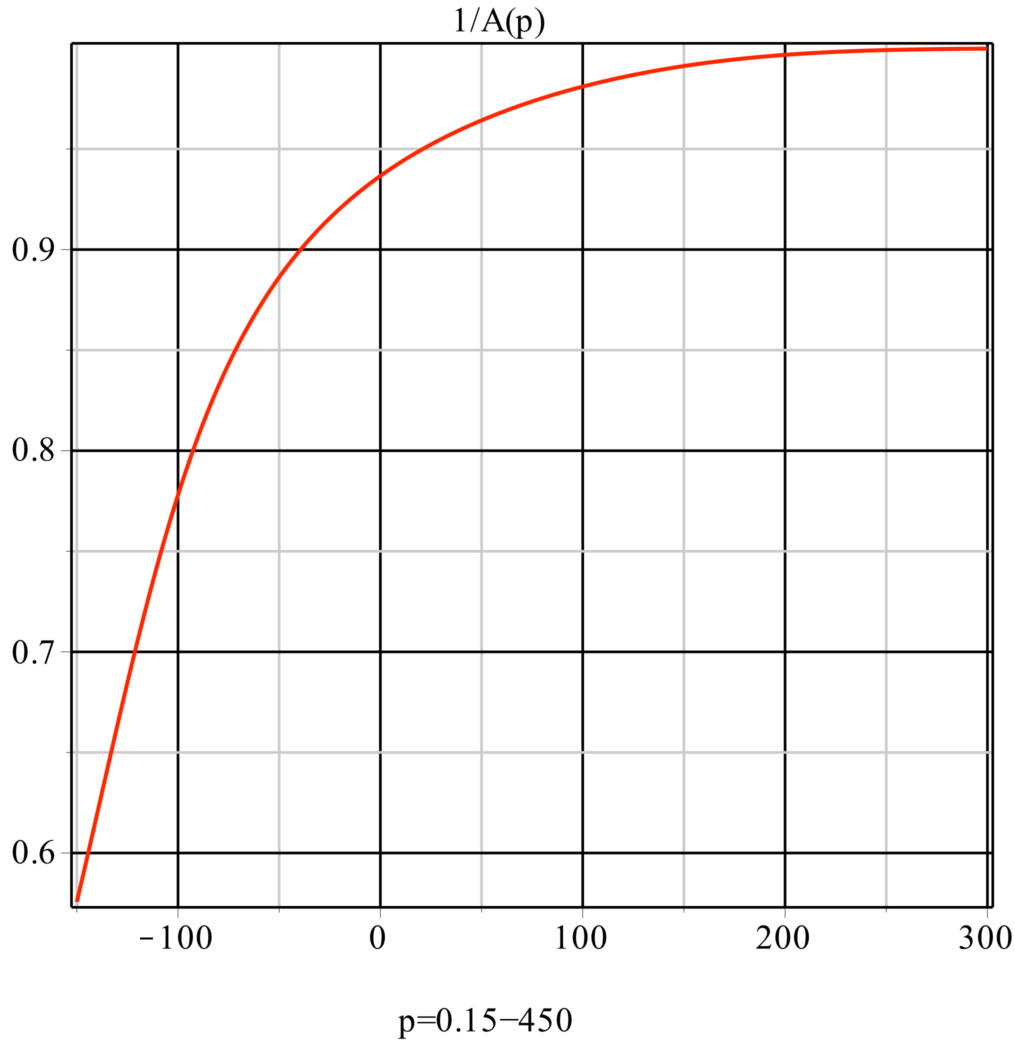
> $\frac{A(p)-A(q)}{p^2-q^2}, \frac{B(p)-B(q)}{p^2-q^2} \rightarrow$ *infrared divergent*

> $S(p) := \frac{B(p)}{A(p)^2 p^2 + B(p)^2}, V(p) := \frac{A(p)}{A(p)^2 p^2 + B(p)^2}$ *are finite*

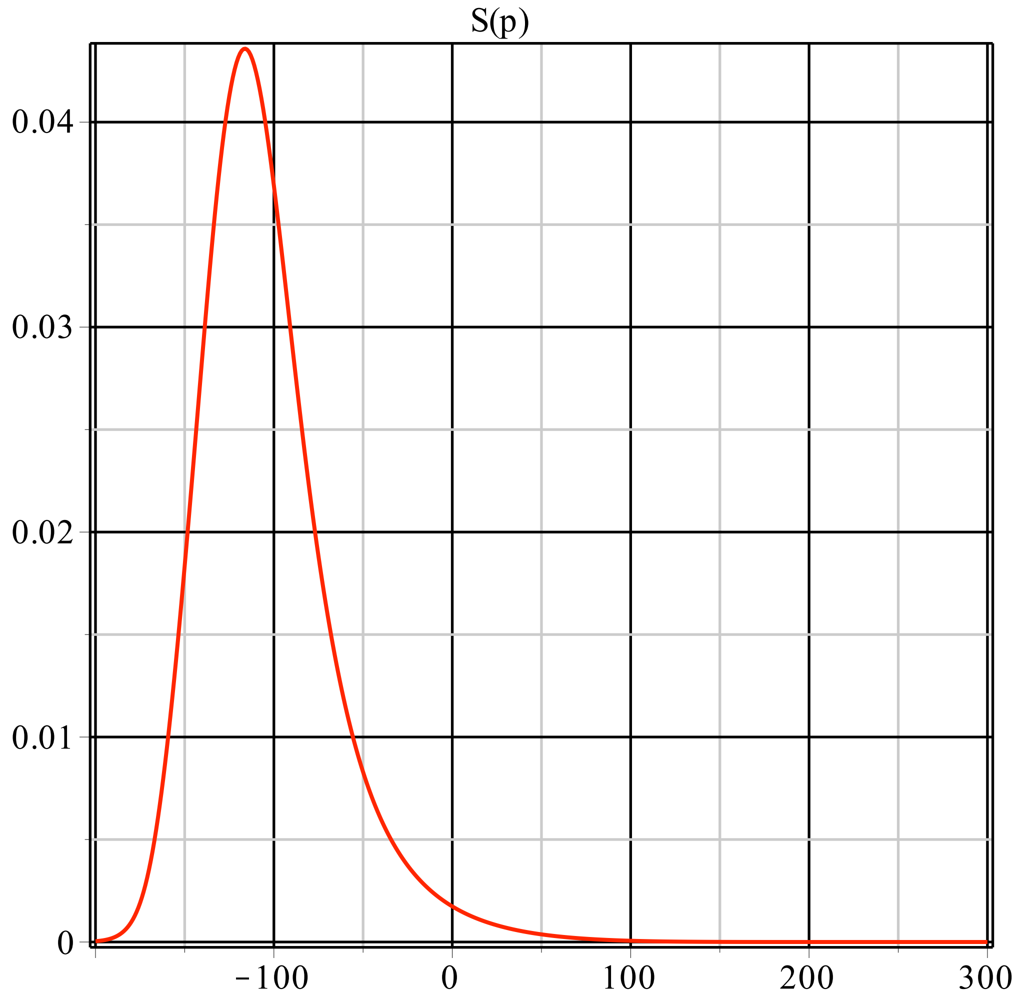
> **Landau gauge**


S(p)
0
1
2
3
4
-300
-200
-100
0
100
200
300
400

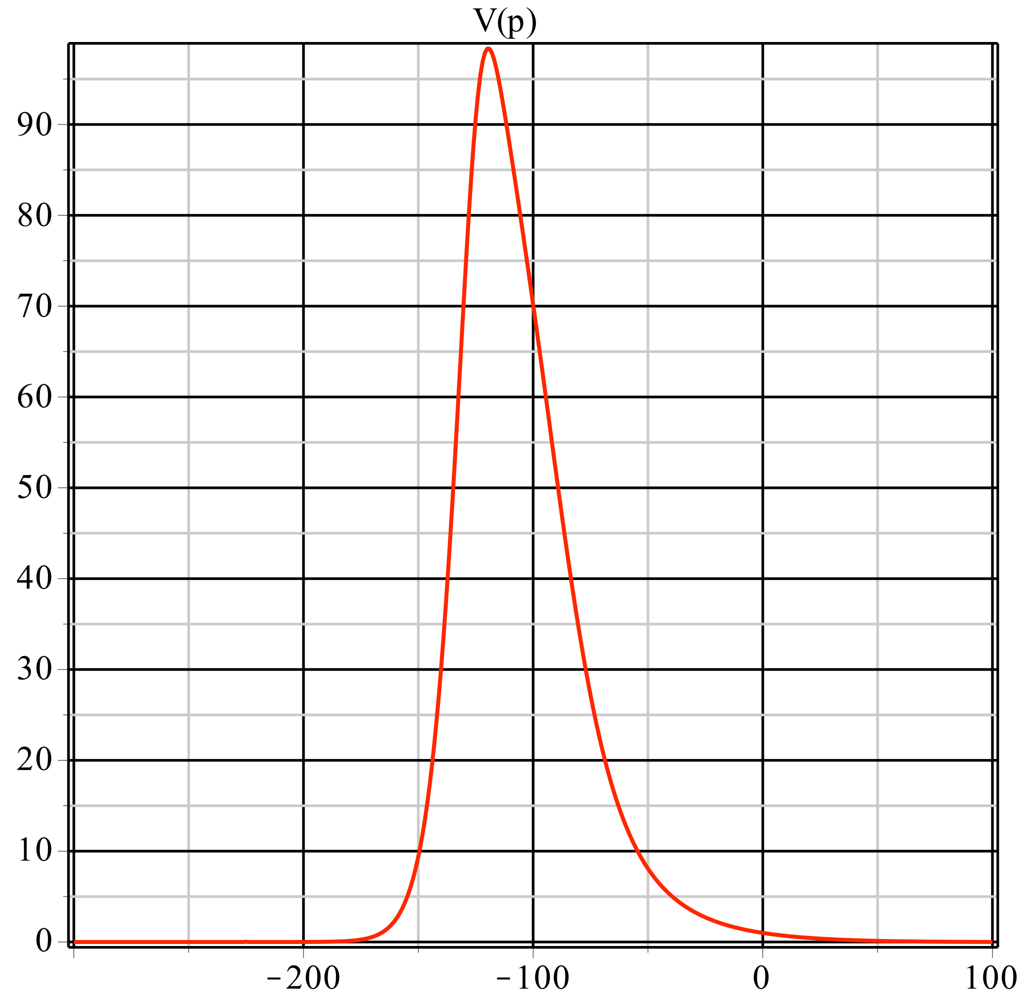

V(p)
80
70
60
50
40
30
20
10
0
-300
-200
-100
0
100
200
300
400



> Feynman gauge



S(p)
0.04
0.03
0.02
0.01
0
-100
0
100
200
300

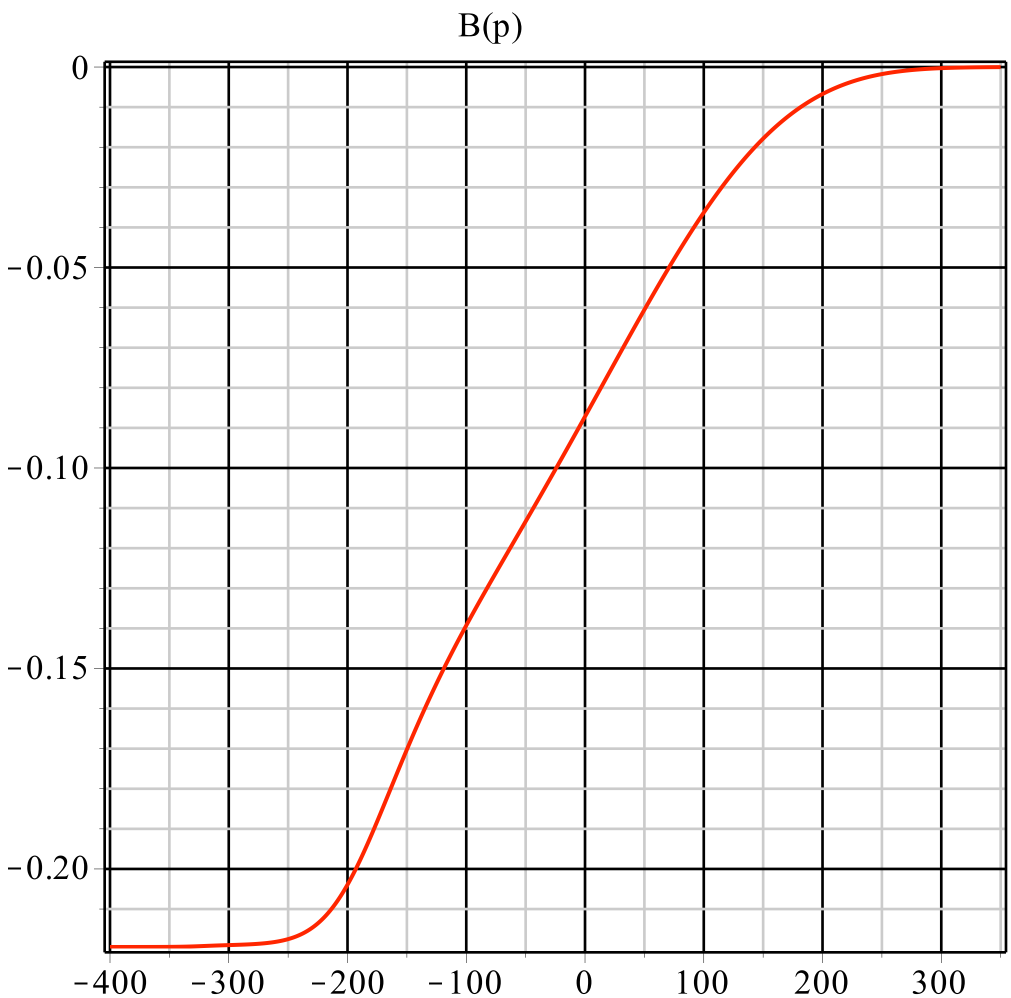


looks like Wave packet

> $vev := 3.2 \sim 3.5 \cdot 10^{-3}\left(e^{4}\right)$ *looks gauge invariant within* **error** $d = 0 \sim 1$

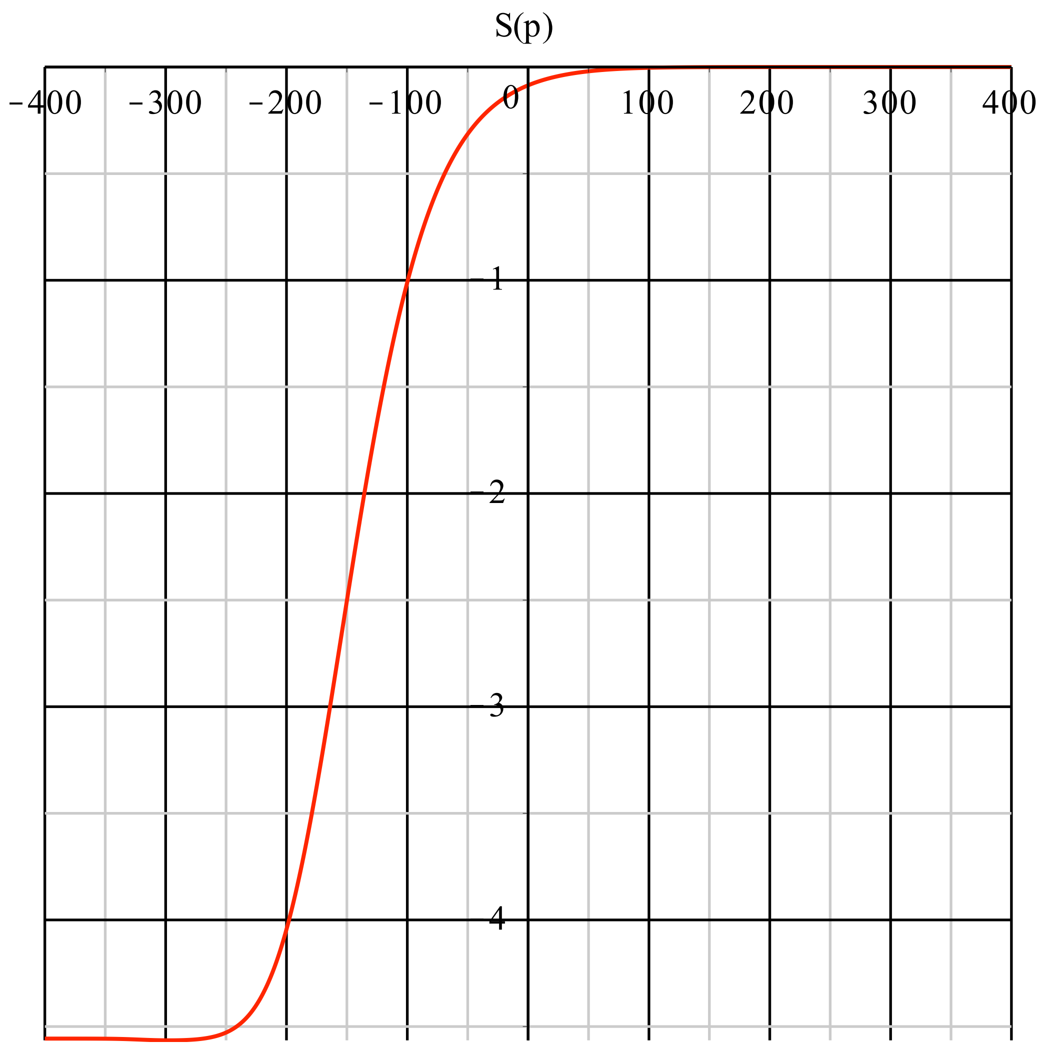
Chern-Simon QED

Largest change between laddar and vertex correction



B(p)
0
−0.05
−0.10
−0.15
−0.20
−400
−300
−200
−100
0
100
200
300

>



B(p) is proportional to 1/p,ΔA,ΔB->infrared divergent

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> landau gauge
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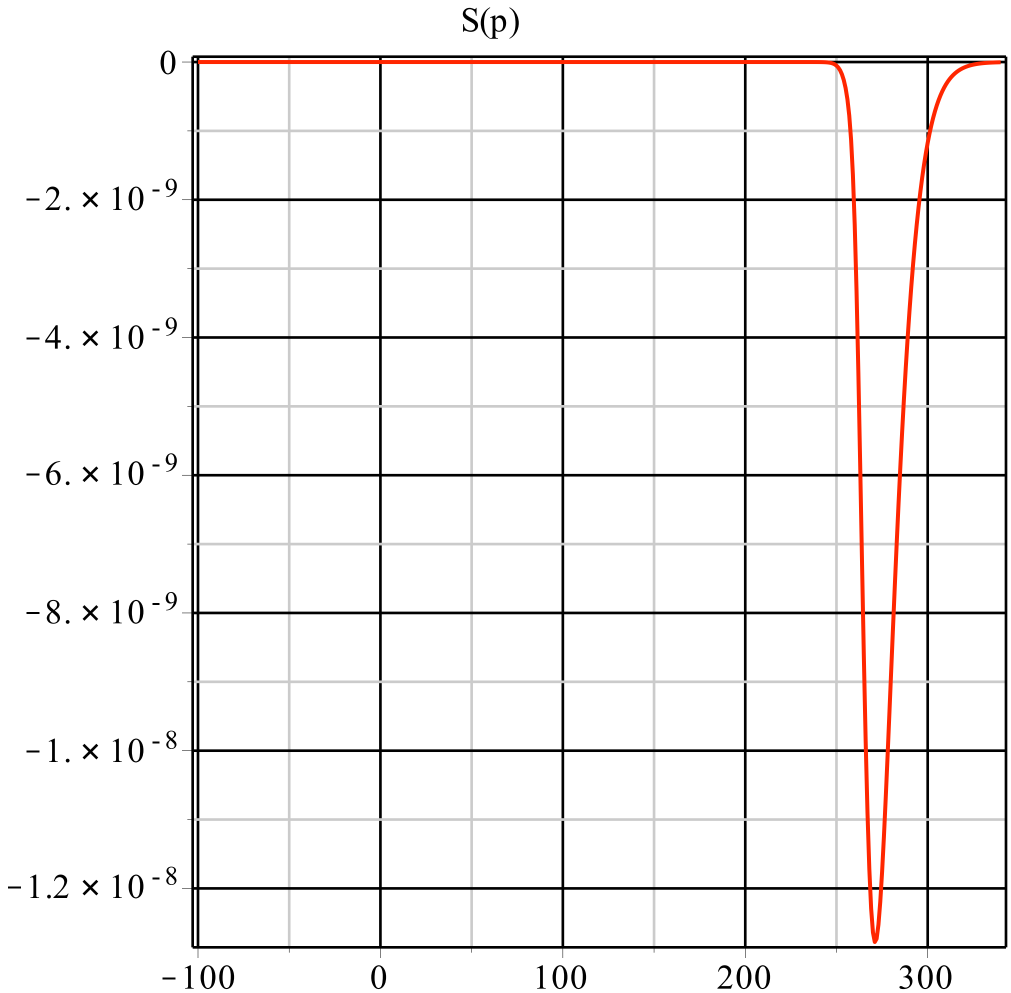
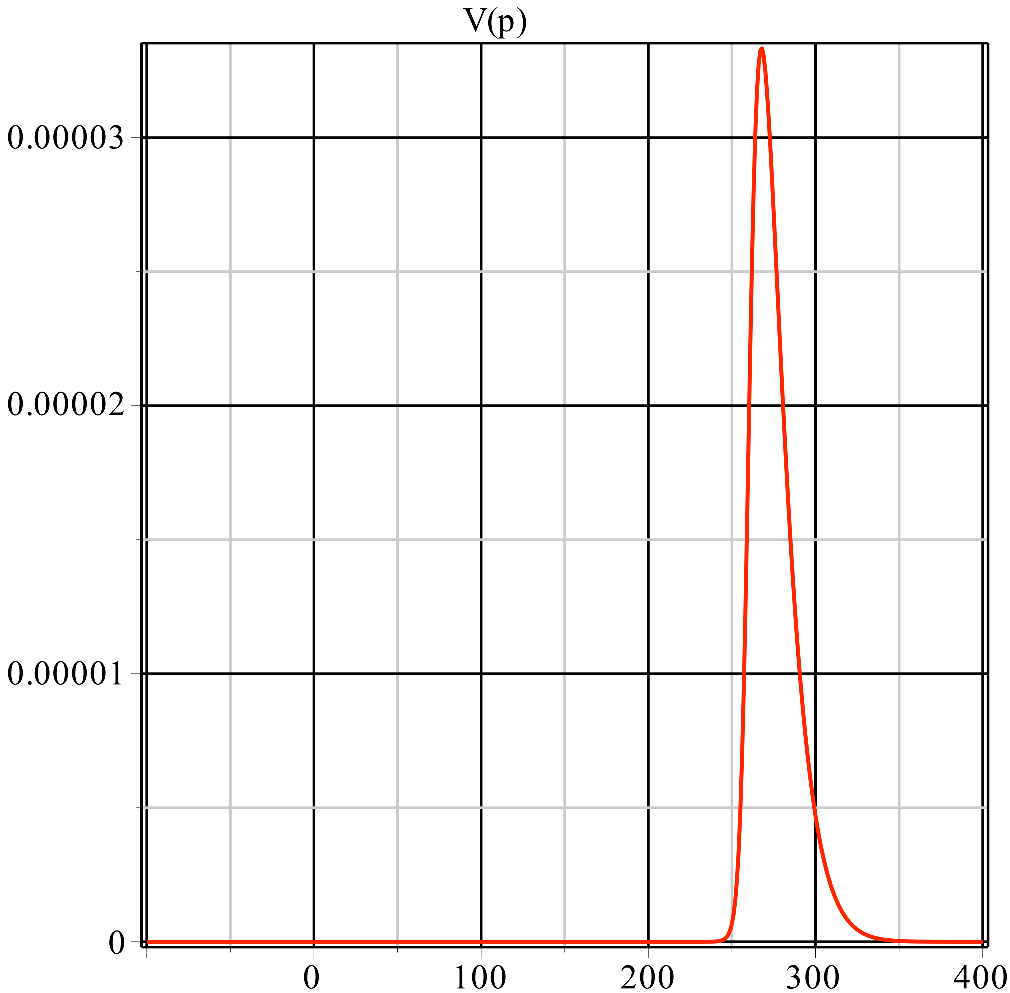


S(p)
0
−2. × 10^-9
−4. × 10^-9
−6. × 10^-9
−8. × 10^-9
−1. × 10^-8
−1.2 × 10^-8
−100
0
100
200
300



> *vev is gauge invariant but* $\log(\Lambda)$ *dependence*

Laddar approximation is not good!!

>