熱場2012
(~_~ ;)
12/8/24

Lattice QCD at finite T and μ

– updates from Lattice 2012 –

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Lattice 2012
June 24-29, 2012 @ Cairns, Australia

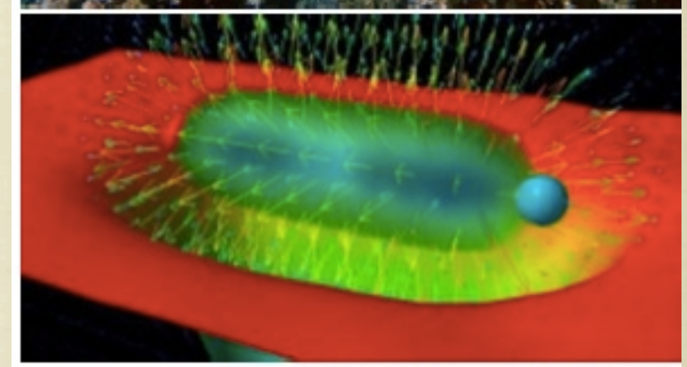
Plenaries:

High T QCD	M. P. Lombardo
Complex Langevin	G. Aarts

Parallels (Non-zero T and μ):

35 talks (+α in other sessions/posters)

~ 15% of whole presentations

Updates in

- phase structure
- EOS
- finite μ

- other developments

Not covered: models, conformals, ...

apologize ^^;

phase structure

PROSPECTIVE PHASE STRUCTURE AT $\mu = 0$

GL effective models + lattice results

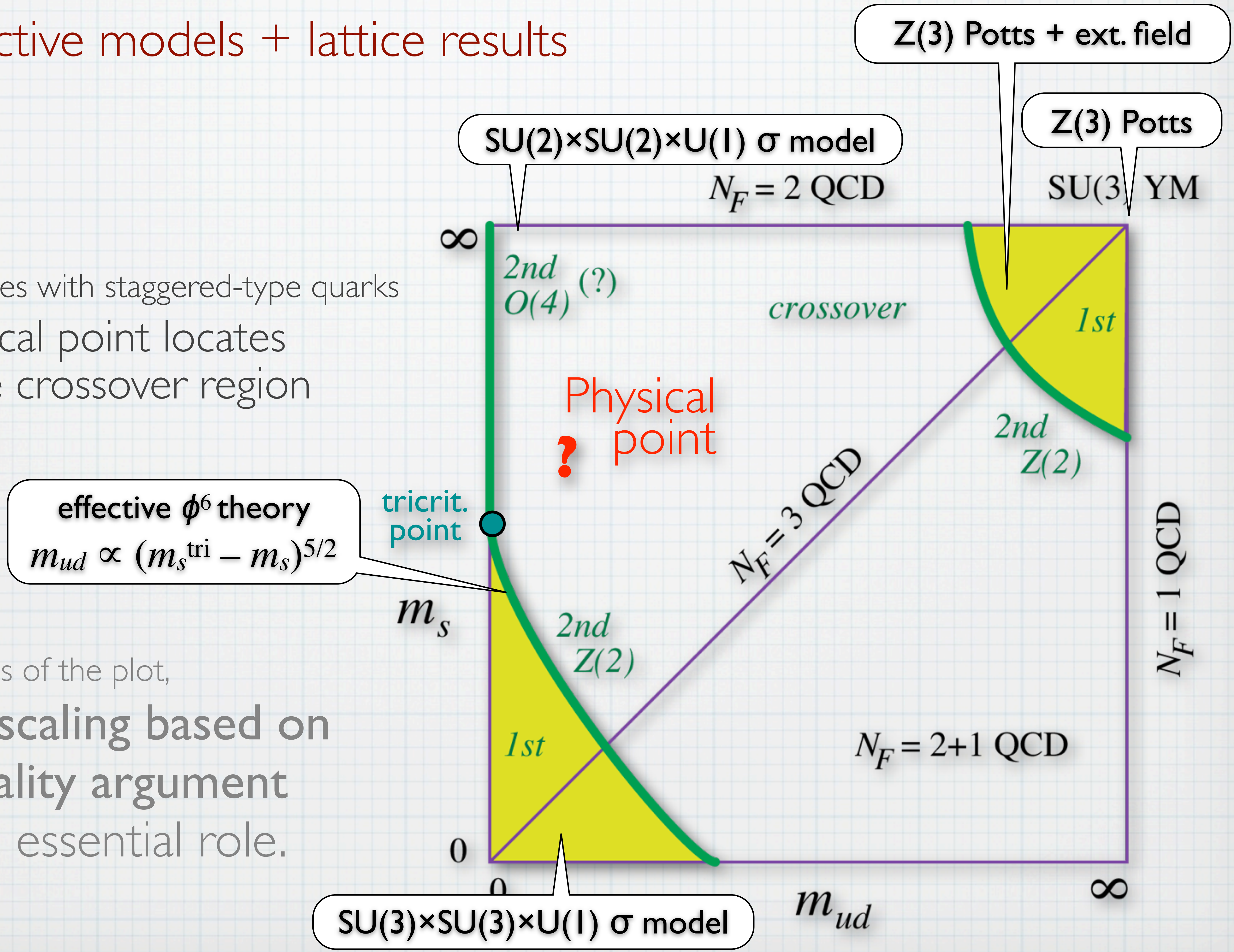
Lattice studies with staggered-type quarks

=> Physical point locates in the crossover region

To fix details of the plot,

critical scaling based on universality argument

plays an essential role.

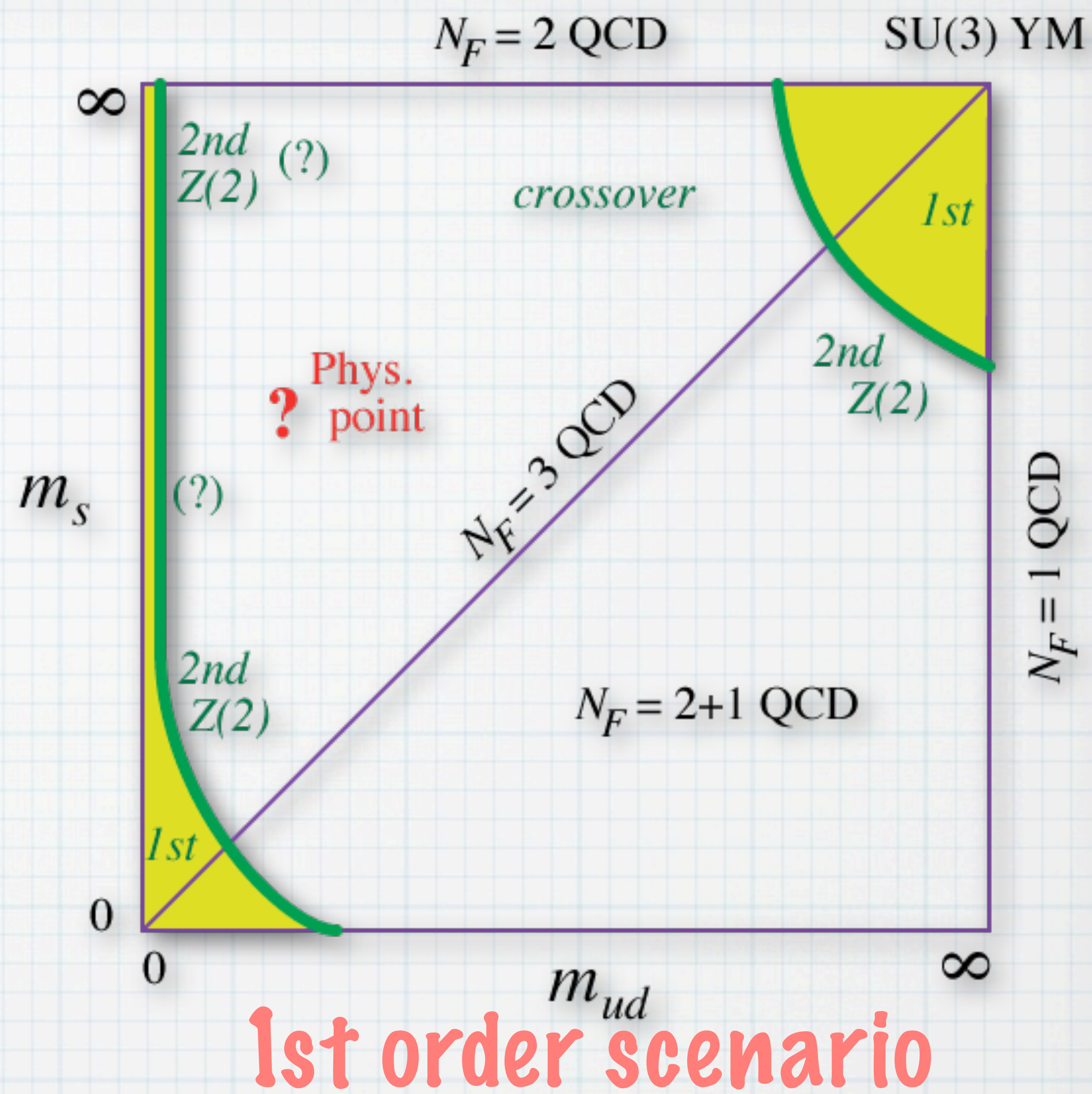


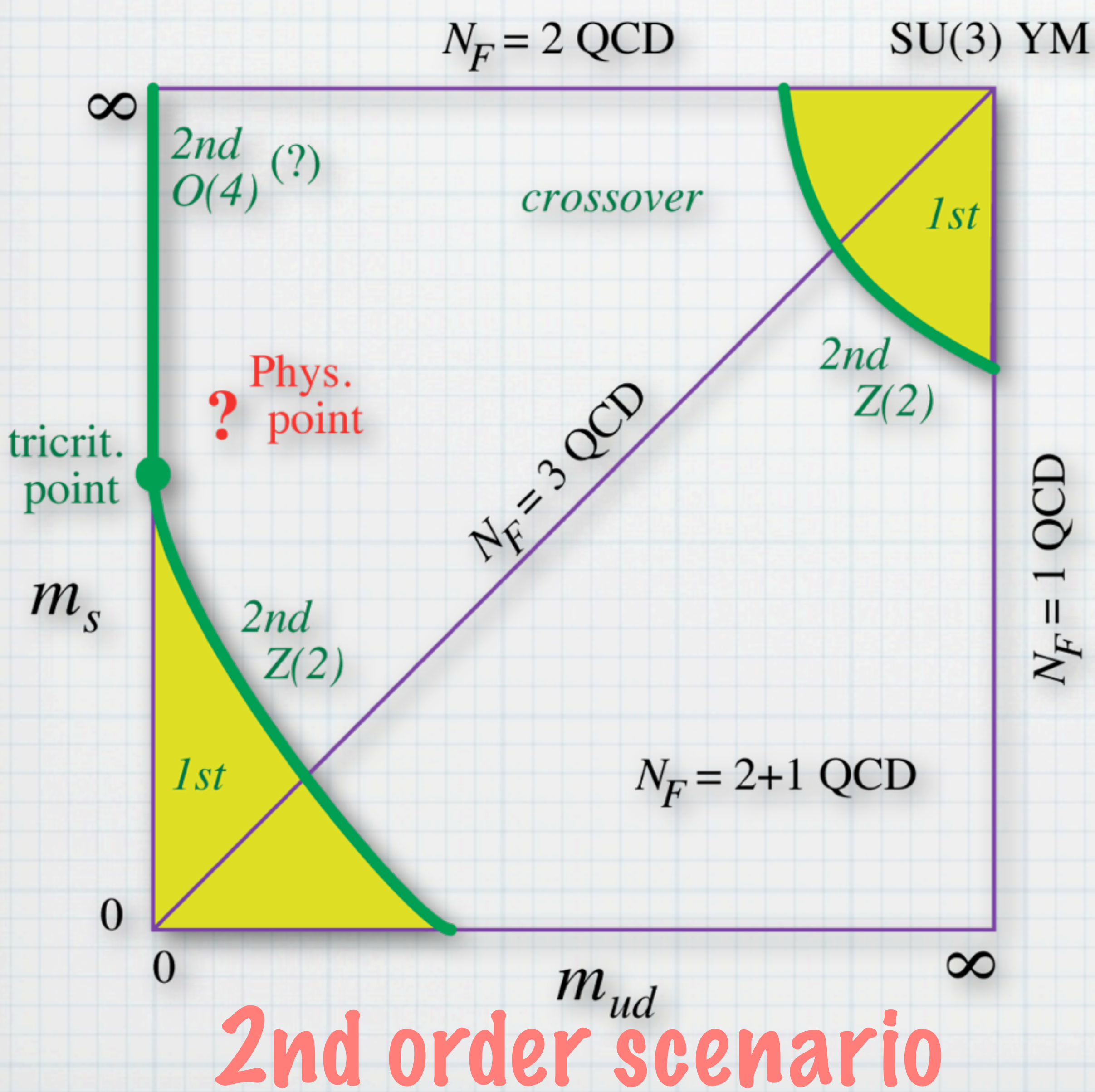
U(1)A plays a role here:

Explicitly broken by anomaly at all *T*.

Anomaly suppressed by Debye-screening of large instantons at $T \sim \infty$.

=> How about around *Tc*?





In case anomaly negligible around *Tc*,

$N_F = 2$: 1st order chiral trans. with Ising crit. end point

though 2nd order not excluded

$N_F = 3$: smaller 1st order region

<= anomaly was a source of the M^3 term

Studies on the lattice

Computationally less expensive staggered-type quarks have been leading the lattice studies.

	SU(N)XSU(N)	$U_A(1)$
Staggered	Remnant U(1)	Broken
Wilson	Broken	Broken
Domain Wall	Exact (for $L \to \infty$)	Exact (for $L \to \infty$)
Overlap	Exact	Exact

Lombardo @ Lat12

Staggered:

It turned out from studies of T>0 QCD around '10,

a good control of taste violation essential to extract physical predictions from staggered-type quarks. => improvements

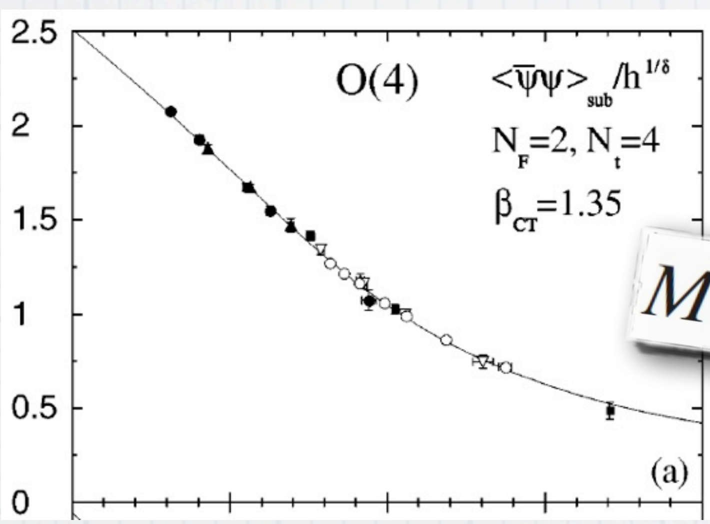
O(4) scaling tests

Wilson-type quarks (N_F=2)

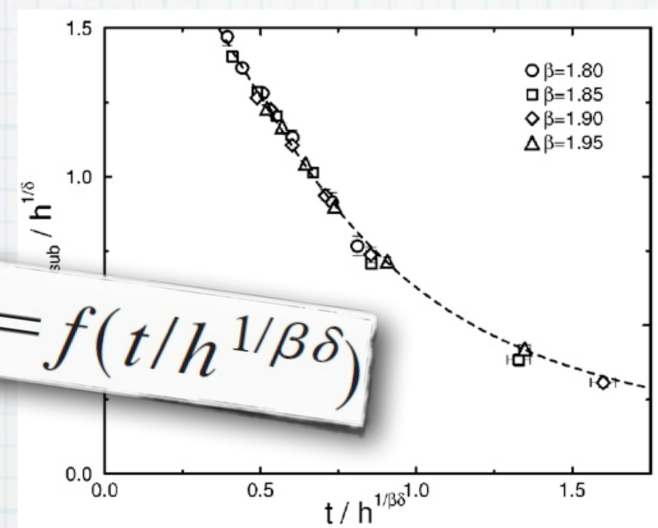
Proper renormalization needed to recover the chiral symmetry in the continuum limit.

$$M \sim \langle\bar{\Psi}\Psi\rangle_{\rm sub} = 2m_q aZ \sum_x \langle\pi(x)\pi(0)\rangle$$ via axial W.I Bochicchio et al.('85)

QCD data vs. O(4) scaling function and exponents



$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$



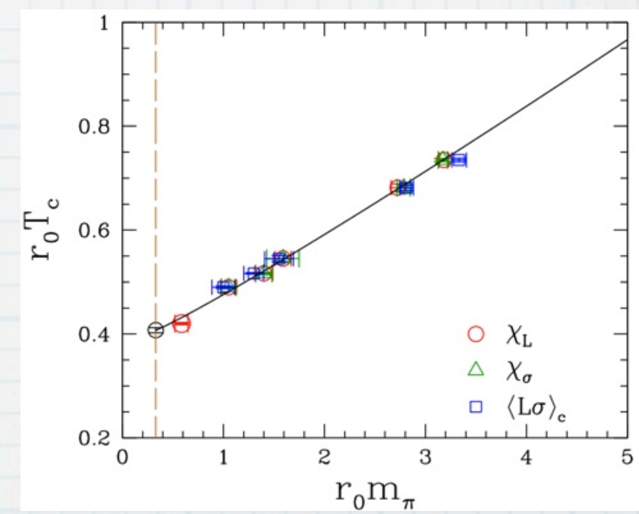
Iwasaki et al. (QCDPAX)
PRL78('97)

- Iwasaki gauge + Wilson
- Nt=4, m_π ~ 600-900 MeV

AliKhan et al. (CP-PACS)
PRD63('01)

- Iwasaki gauge + Clover
- Nt=4, m_π ~ 600-1000 MeV

O(4) scaling fit for *Tc*



Bornyakov et al. (QCDSF)
PRD82('10)

- plaquette gauge + Clover
- Nt = 8,10,12, $m_\pi \approx$ 420-1300 MeV

No indication of 1st order chiral transition.

QCD data well described by the O(4) scaling function with O(4) exponents.

➡ **Consistent with the O(4) scaling**, though quarks are heavy.

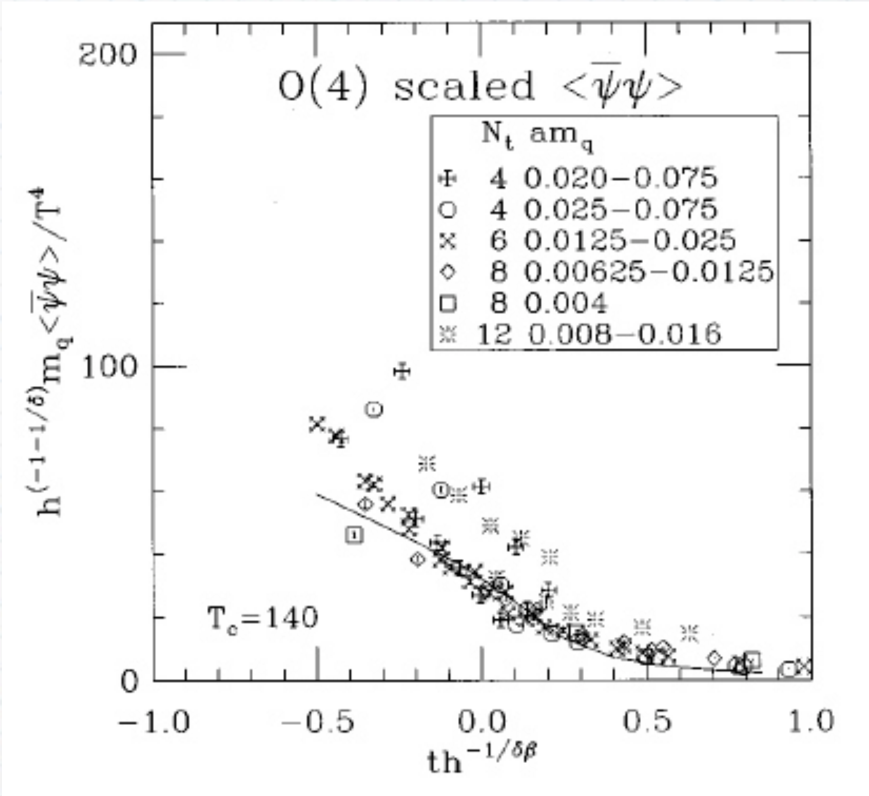
Unimproved staggered quarks (N_F=2)

★ Investigations with unimproved actions:

puzzling

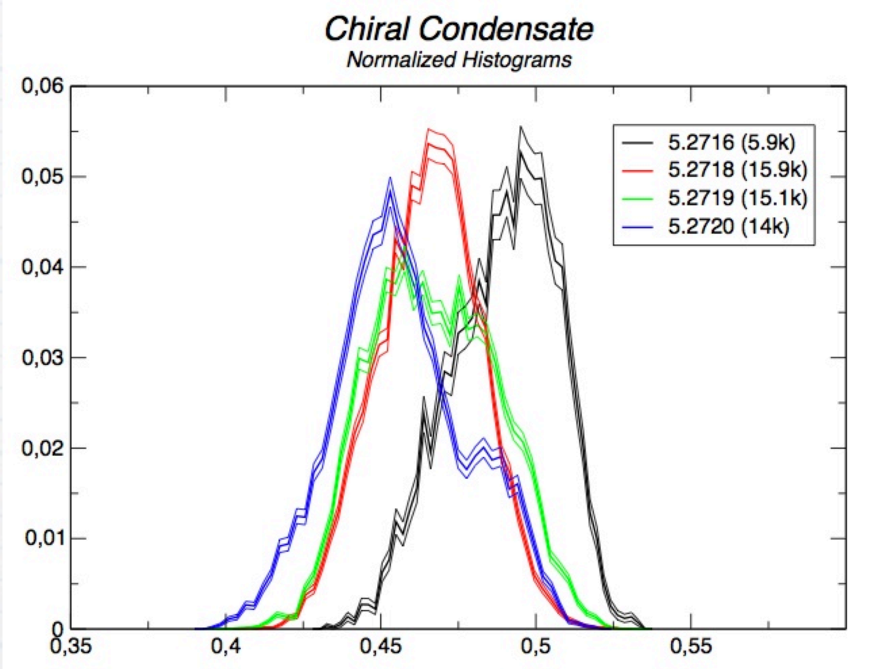
=> Transition looks continuous,
but neither O(2) nor O(4)

Bielefeld ('94): $m_q a$=0.02-0.075 , Nt=4-8
MILC ('94-96) :$m_q a$=0.008-0.075 , Nt=4-12 =>
JLQCD ('98): $m_q a$=0.01-0.075, Nt=4



=> 1st order?

D'Elia et al. PRD 72('05)
Cossau et al. Lat08: $m_q a$=0.01335-, Nt=4 =>



Staggered:

It turned out from studies of T>0 QCD around '10,

a good control of taste violation essential to extract physical predictions from staggered-type quarks. => improvements

Improved staggered quarks (N_F=2+1)

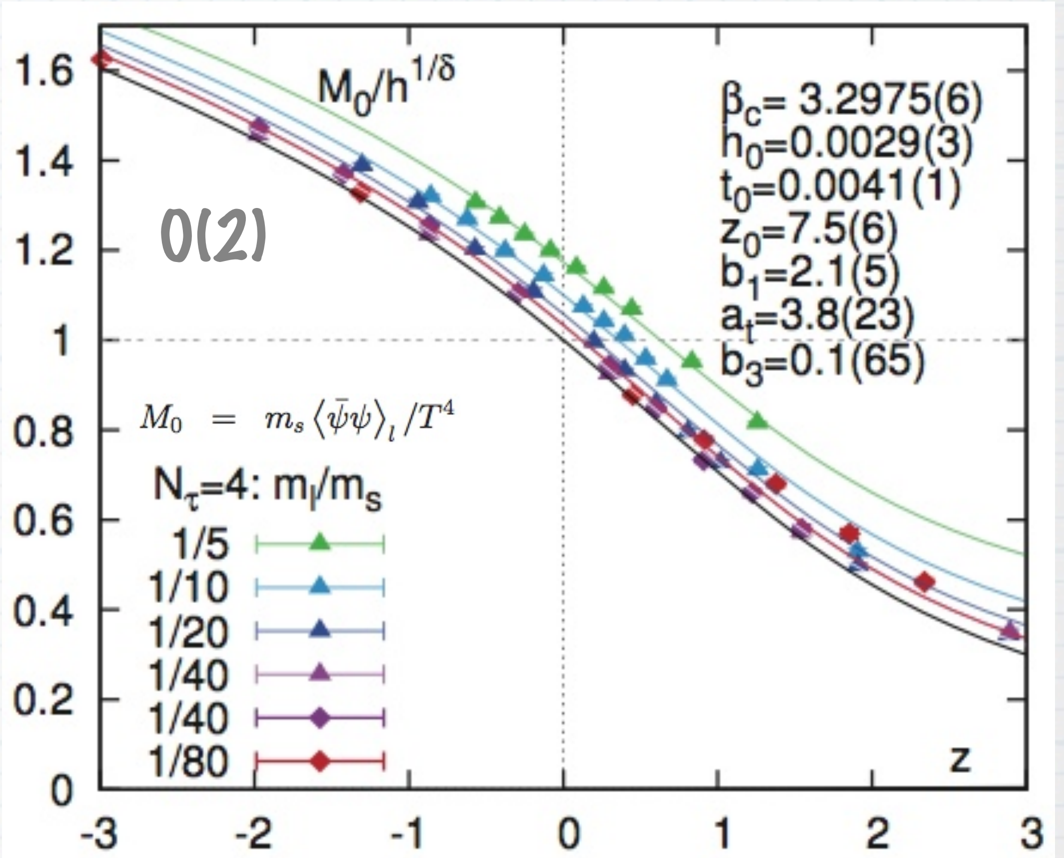
Ejiri et al. (BNL-Bi) PRD80('09) (N_t = 4); Lat10 (N_t = 8)

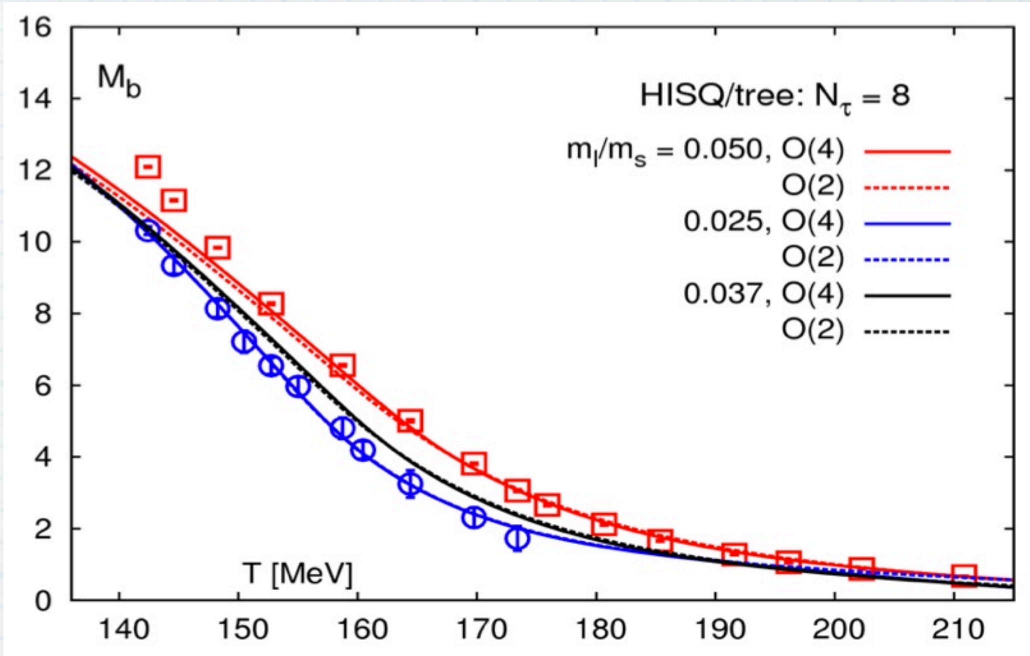
- p4, Nt=4, 8
  m_s≈physical, m_l/m_s= 1/80 – 1/20
  (m_π^{pNG} ≈ 75 – 150 MeV)

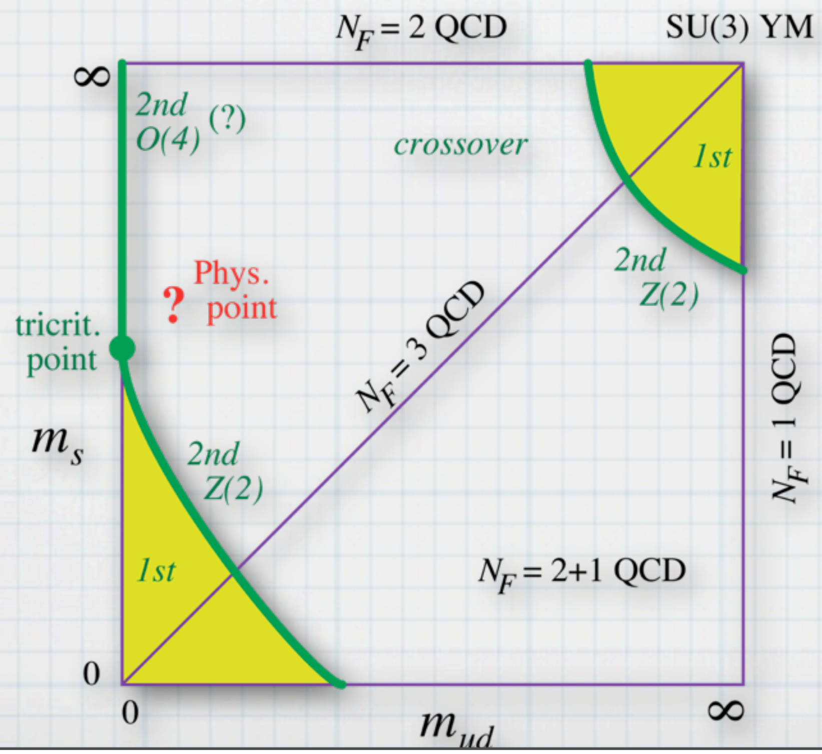
HotQCD @ Lat11

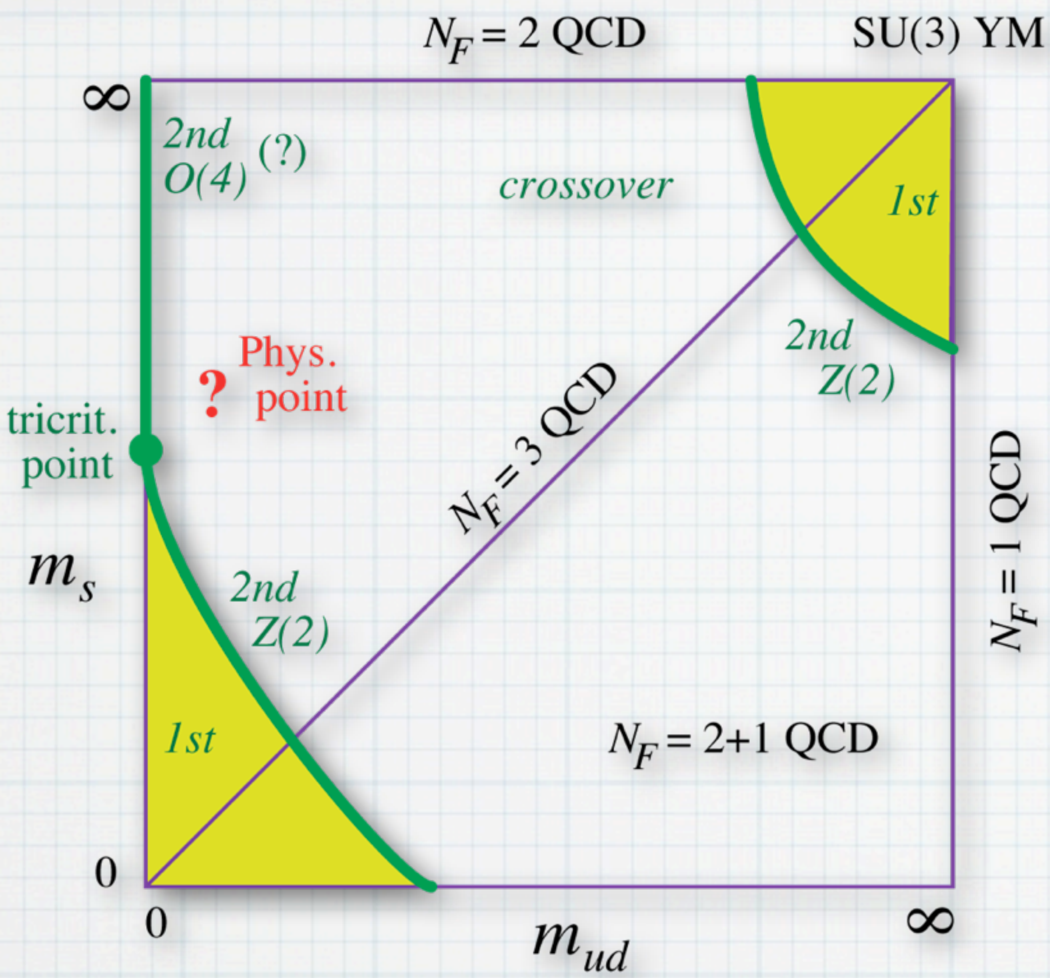
- HISQ, Nt=8
- m_s ≈ physical, m_l/m_s = 1/27 – 1/20

➡ Consistent with O(2) [O(N)]

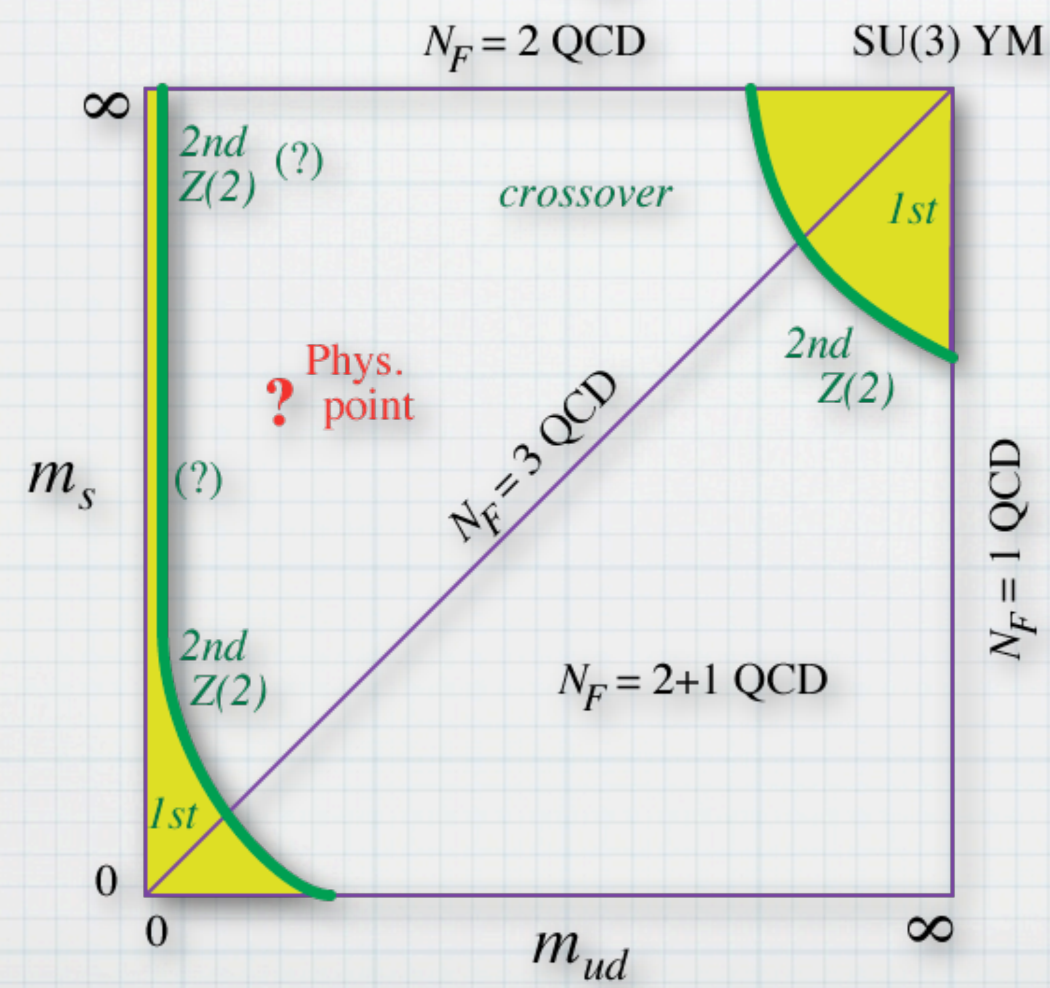




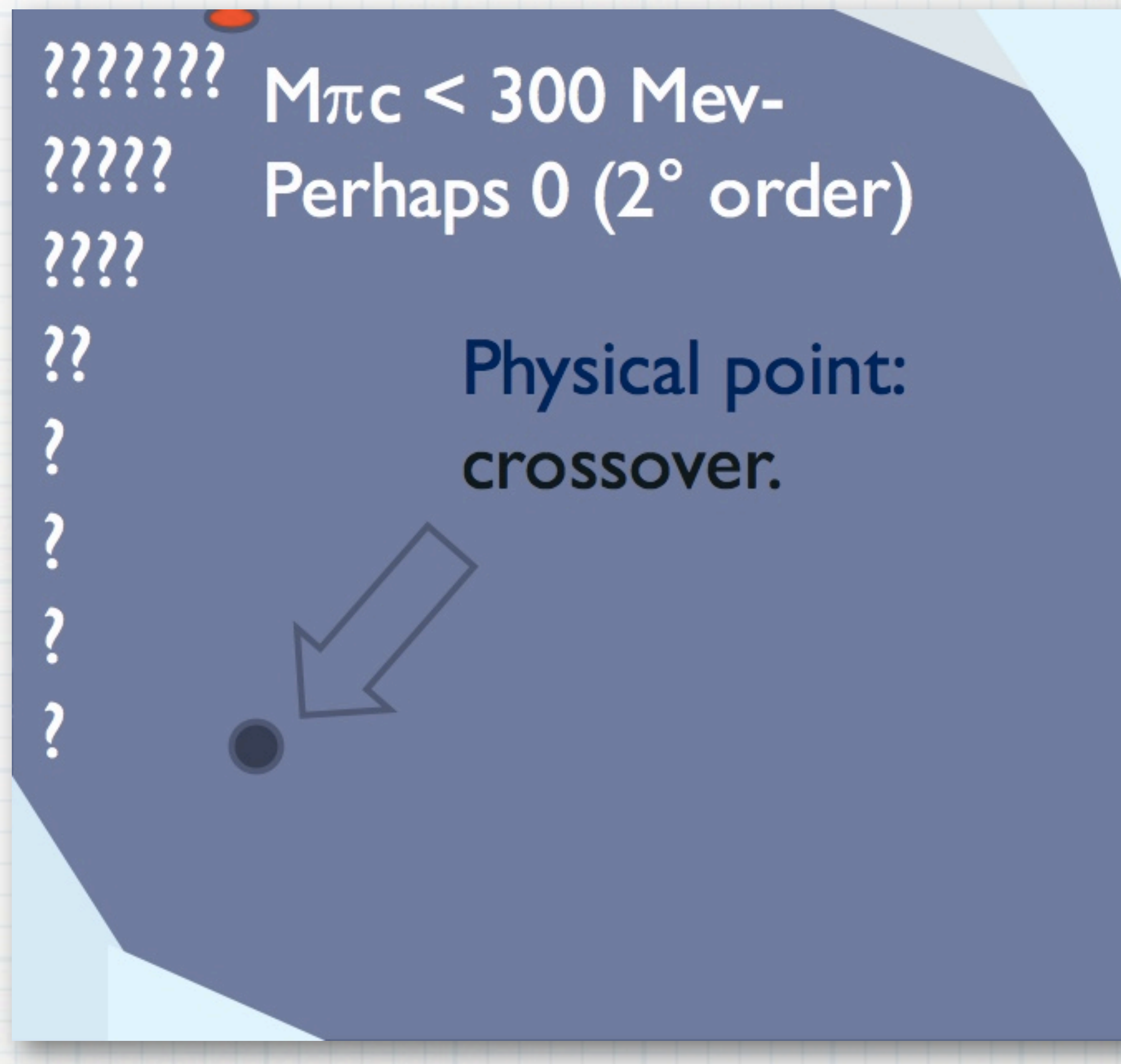




$N_F = 2$ QCD
SU(3) YM
∞
2nd O(4) (?)
crossover
1st
Phys. point
?
tricrit. point
2nd Z(2)
$N_F = 3$ QCD
$N_F = 1$ QCD
m_s
2nd Z(2)
1st
$N_F = 2+1$ QCD
0
0
m_{ud}
∞



$N_F = 2$ QCD
SU(3) YM
∞
2nd Z(2) (?)
crossover
1st
2nd Z(2)
Phys. point
?
m_s
(?)
$N_F = 3$ QCD
$N_F = 1$ QCD
2nd Z(2)
$N_F = 2+1$ QCD
1st
0
0
m_{ud}
∞



???????
?????
????
??
?
?
?
?
Mπc < 300 Mev-
Perhaps 0 (2° order)
Physical point:
crossover.

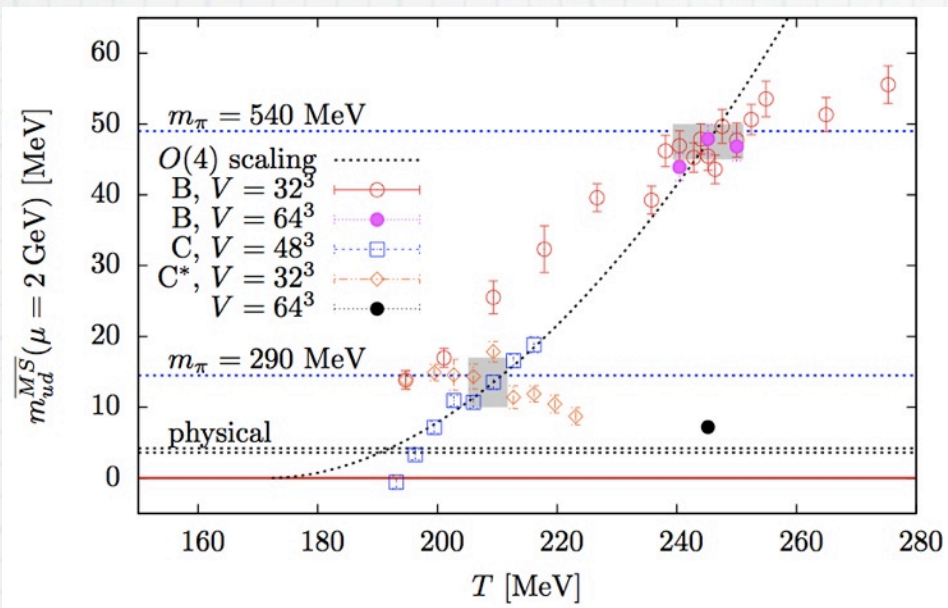
M. P. Lombardo

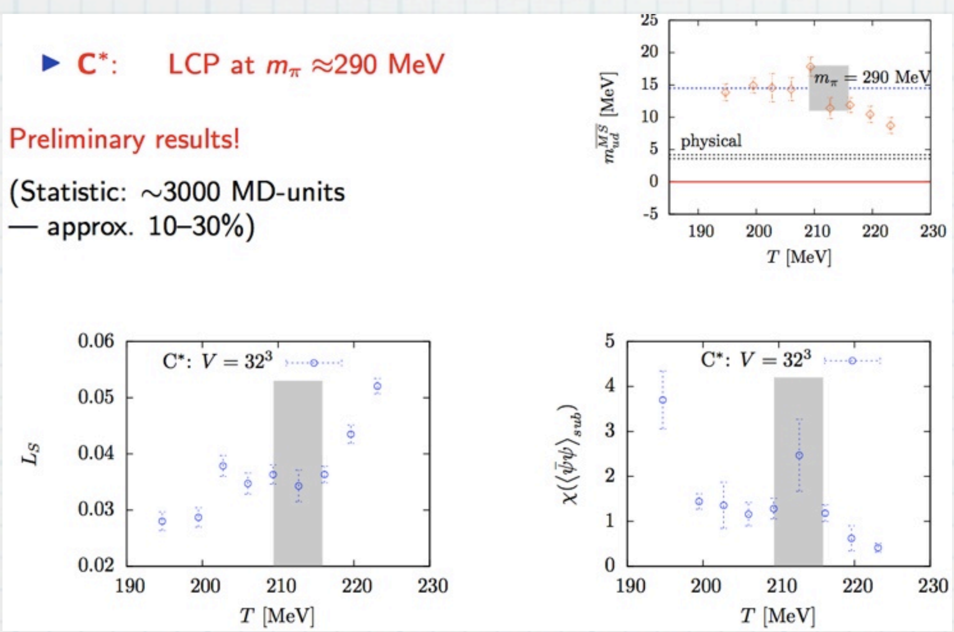
Wilson-type quarks updates (N_F=2)

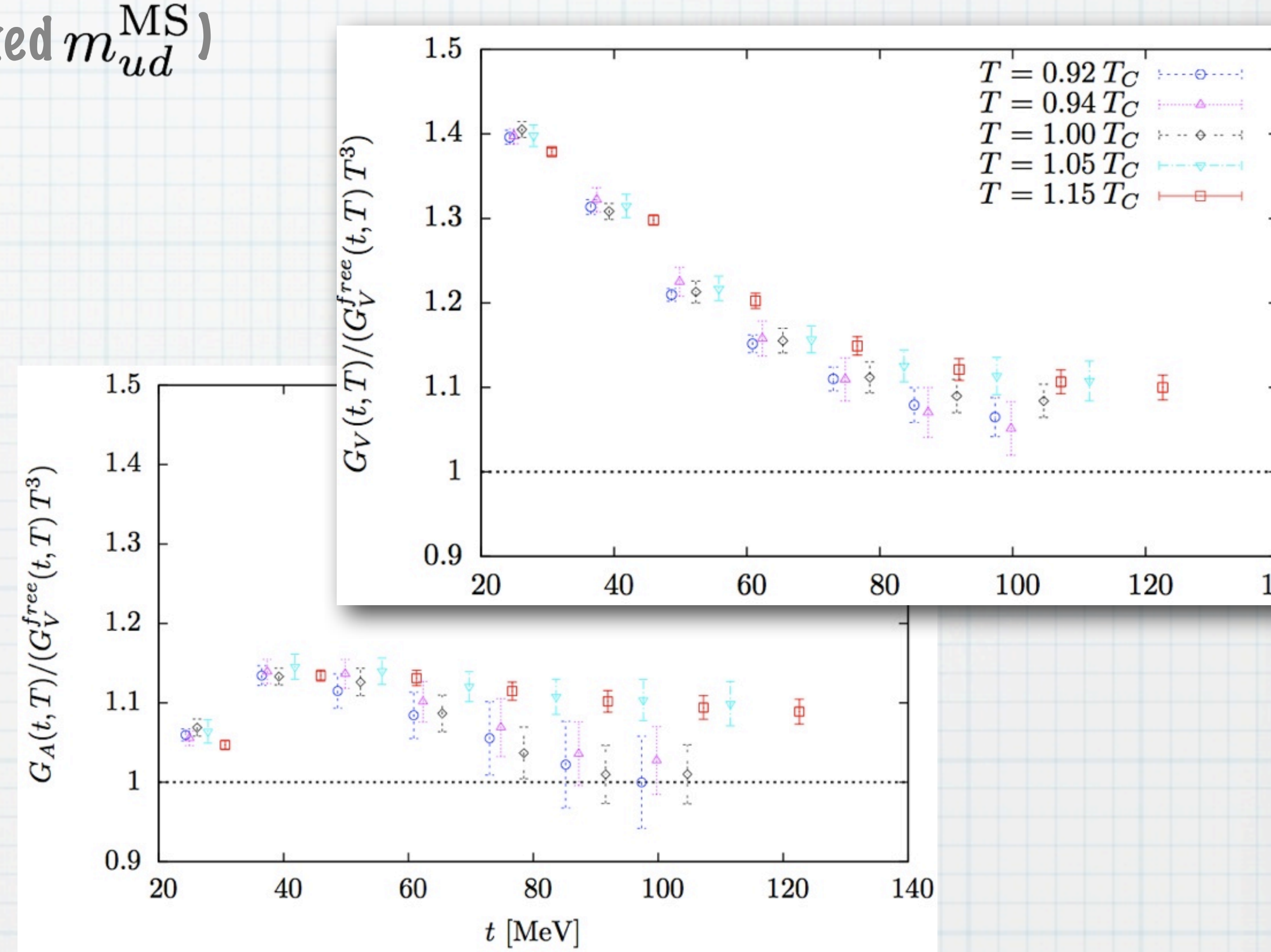
- **Brandt (Mainz) N_F=2 clover + plaquette gauge**

check of the chiral transition / O(4) scaling on large lattices: $N_t = 16,\ V = 32^3\text{-}64^3$

update from Lattice 2010: keep LCP (fixed $m_{ud}^{\overline{\rm MS}}$)







G_V and G_A move closer together for large t above T_C.

⇒ Chiral symmetry restoration!

Scaling not clear yet.

Wilson-type quarks updates (N_F=2)

* Burger (tmfT) N_F=2 twisted mass + tree-level Symanzik gauge

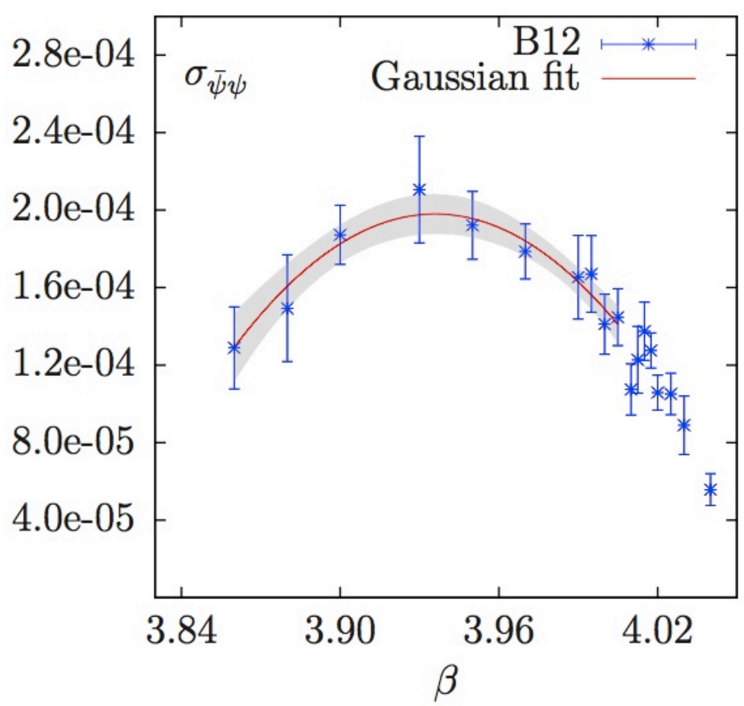
  $N_t = 12, (10), \quad V = 32^3$

  Tc and EOS on LCP for four $m_\pi \approx$ 280-480 MeV

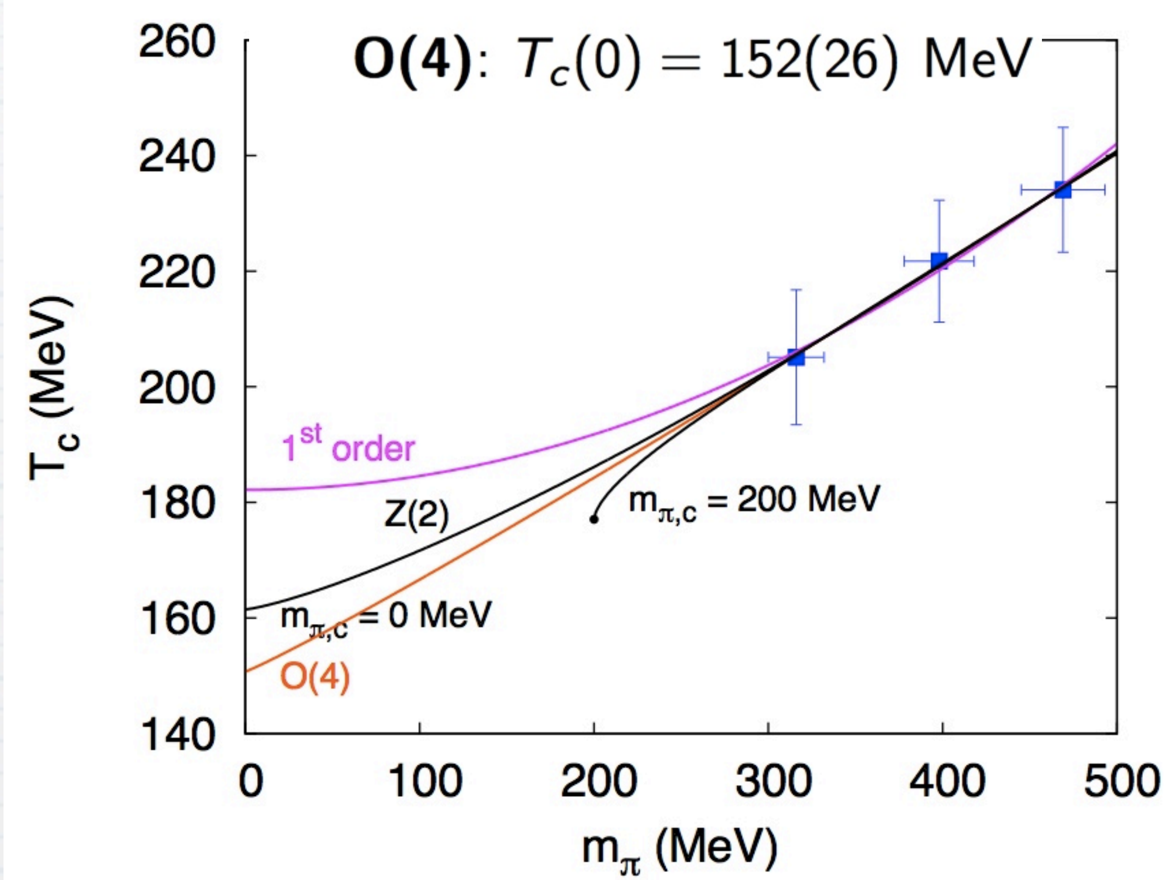
  Note: isospin sym. broken by twisting. O(4) only in the cont. lim.

$$\sigma_{\langle\bar\psi\psi\rangle} = V/T\left(\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2\right)$$

$m_\pi \approx 400$ MeV:



$$T_c(m_\pi) = T_c(0) + A m_\pi^{2/(\beta\delta)}$$



Scaling not clear yet.

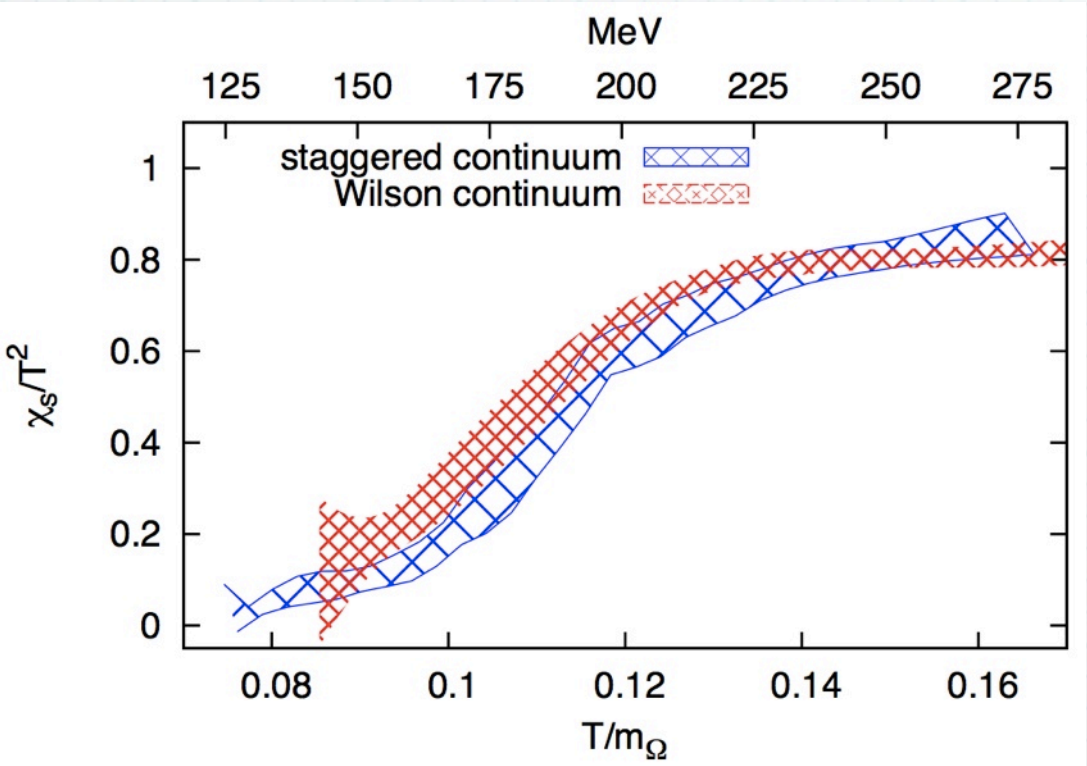
Wilson-type quarks updates (N_F=2+1)

- Nógrádi (Budapest-Wuppertal)
  N_F=2+1 stout-smeared clover + tree-level Symanzik gauge

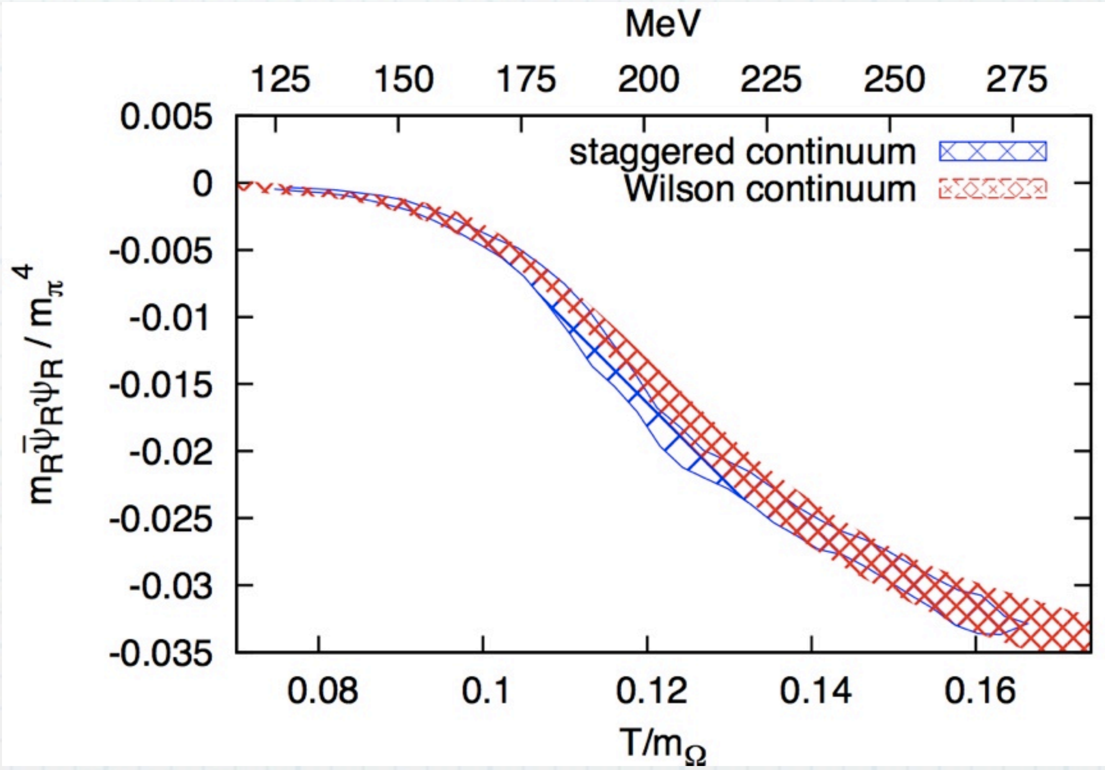
$N_t = 6\text{-}28, \quad V = 32^3\text{-}64^3$ on LCP for $m_\pi \approx 545$ MeV

Comparison with staggered. Update from Lat 11: Z_A.

quark number susceptibility



$m_R\bar{\psi}_R\psi_R(T) = 2N_f m^2_{PCAC} Z_A^2 (PP(T) - PP(0))$



- Agreement between continuum staggered and continuum Wilson results

Scaling not tested.

Wilson-type quarks updates (N_F=2+1)

* Umeda (WHOT-QCD) N_F = 2+1 NP-clover + Iwasaki gauge

Fixed-scale approach using CP-PACS+JLQCD T=0 configuration

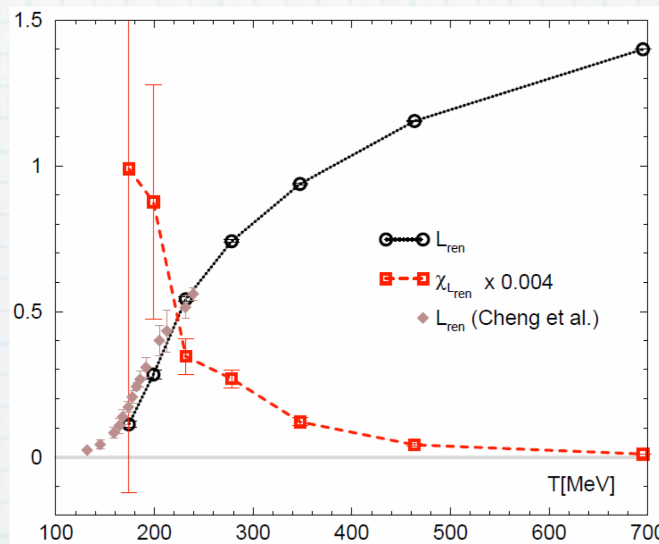
$m_\pi \approx 636$MeV, $a \approx 0.07$fm, $28^3 \times 56$ ($L \approx 2$fm)

N_t = 4-16 $V = 32^3$

- Final results for EOS published in PRD (2012) => later

- Renormalized Polyakov loop

$$L_{\text{ren}} = \exp\left(\frac{c_{\text{m}} N_t}{2}\right)\langle L\rangle$$



consistent with p4 results

(Nt = 8, m_{ud}/m_s = 0.05)

(rescaled with r_0 = 0.5 fm)

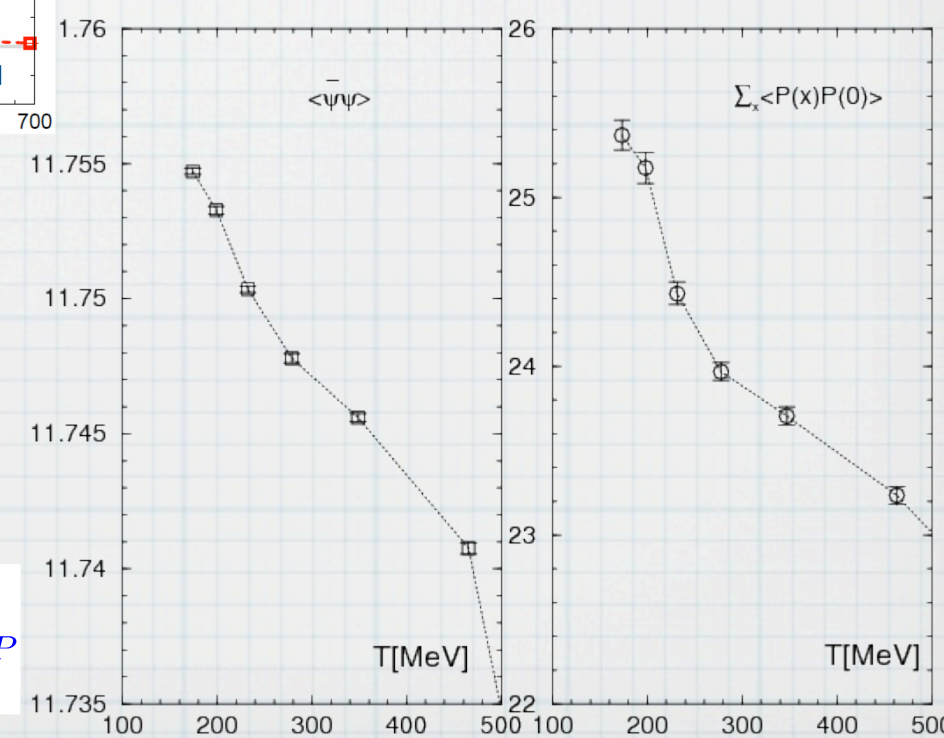
- Chiral condensate: 2 definitions

Additive & multiplicative renormalizations required.

$$\langle\bar{\psi}\psi\rangle(T) = \frac{T}{V}\langle TrD^{-1}\rangle = \frac{\sigma_R(T)}{Z_{\bar{\psi}\psi}} + c_{\bar{\psi}\psi}$$

But, they are all constants in the fixed-scale approach!

$$\left\langle\sum_x P(x)P(0)\right\rangle(T) = \frac{\sigma_R(T)}{Z_{PP}} + c_{PP}$$



Fate of $U(1)_A$ at $T \approx T_c$

$U(1)_A$ **explicitly broken** at all T, but will restore at $T=\infty$.

Is $U(1)_A$ "effectively" restored at T_c ??

<= e.g. by formation of instanton-antiinstanton molecules

If so, the 1st order scenario becomes preferable,

though 2nd order transition not excluded.

$N_F=2$:

$$\vec{\pi} = \bar{q}\,\gamma_5\,\frac{\vec{\tau}}{2}\,q \quad \longleftrightarrow \quad \sigma = \bar{q}\,q$$

$SU(2)_L \times SU(2)_R$

$U(1)_A$

$$\vec{\delta} = \bar{q}\,\frac{\vec{\tau}}{2}\,q \quad \longleftrightarrow \quad \eta = \bar{q}\,\gamma_5\,q$$

^ disconnected diagrams required

If $U(1)_A$ "restored" => π-δ, σ-η degeneracy

Fate of $U(1)_A$ at T>Tc

If $U(1)_A$ "restored" => π-δ degeneracy

=> $\chi_\pi = \chi_\delta$ (note: =>'s are not <=>)

where $\chi_\pi = \frac{T}{V}\langle \mathrm{Tr} M^{-1}\gamma_5 M^{-1}\gamma_5 \rangle$

$\chi_\delta = \frac{T}{V}\langle \mathrm{Tr} M^{-1} M^{-1} \rangle$

Banks-Casher: $-\langle \bar{q}q \rangle \xrightarrow{V\to\infty} \int_0^\infty d\lambda \, \frac{2m_q \, \rho(\lambda)}{\lambda^2 + m_q^2} \xrightarrow{m_q\to 0} \pi\rho(0)$

$SU(N_F)_A$ restoration <=> ρ(0)=0 in the massless limit.

$$\chi_\pi - \chi_\delta \xrightarrow{V\to\infty} \int_0^\infty d\lambda \, \frac{4m_q^2 \, \rho(\lambda)}{(\lambda^2 + m_q^2)^2} \xrightarrow{m_q\to 0} ??$$

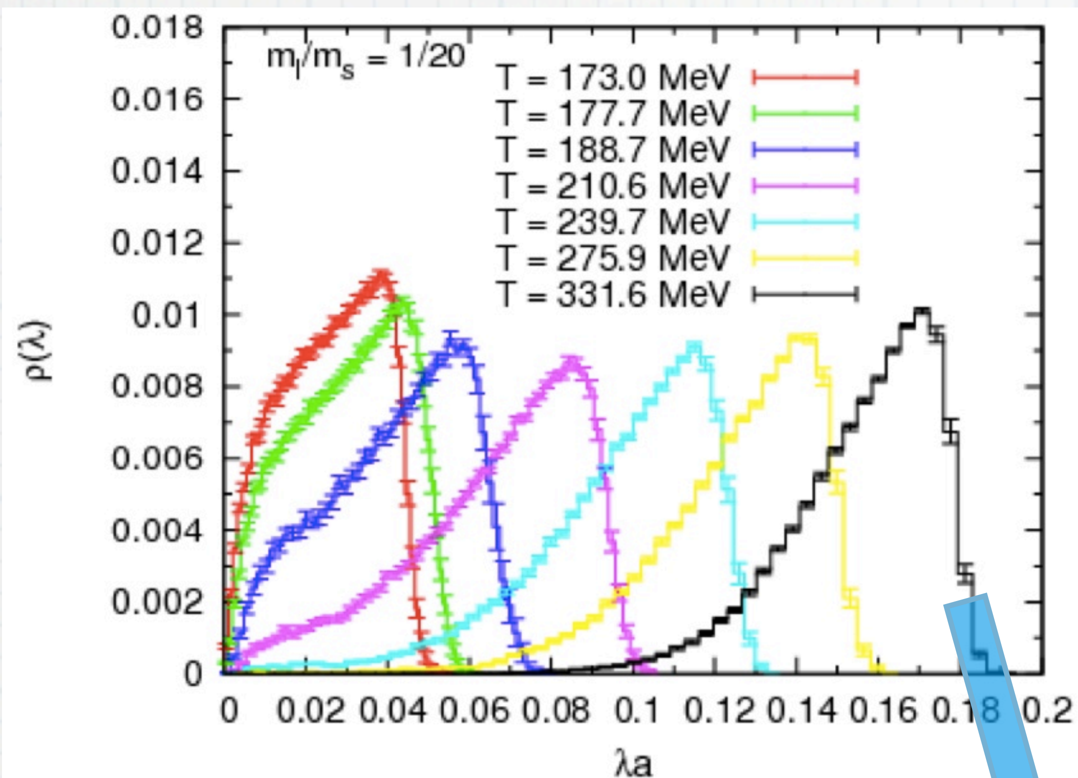
$$\rho(\lambda) \sim m^a \lambda^b \quad (a+b>0) \implies \chi_\pi - \chi_\delta \neq 0 \ \text{if} \ a+b \leq 1$$

at $T > Tc$

Bazavov et al., arXiv:1205.3535

Fate of $U(1)_A$ at T>Tc

* Ohno (HotQCD) N_F=2+1 HISQ Nt=8, V=32^3-48^3

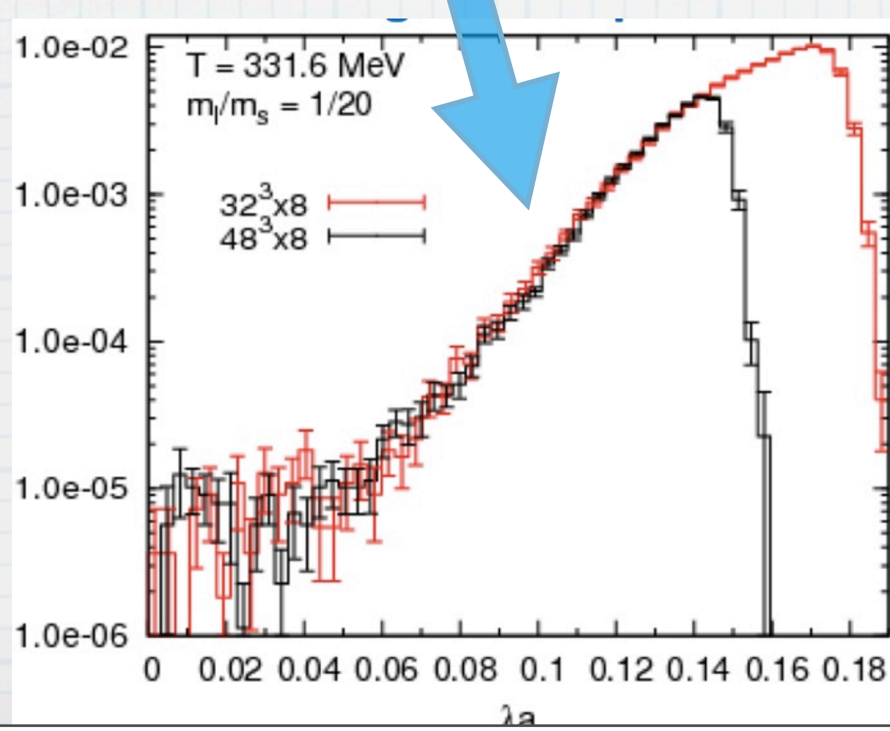


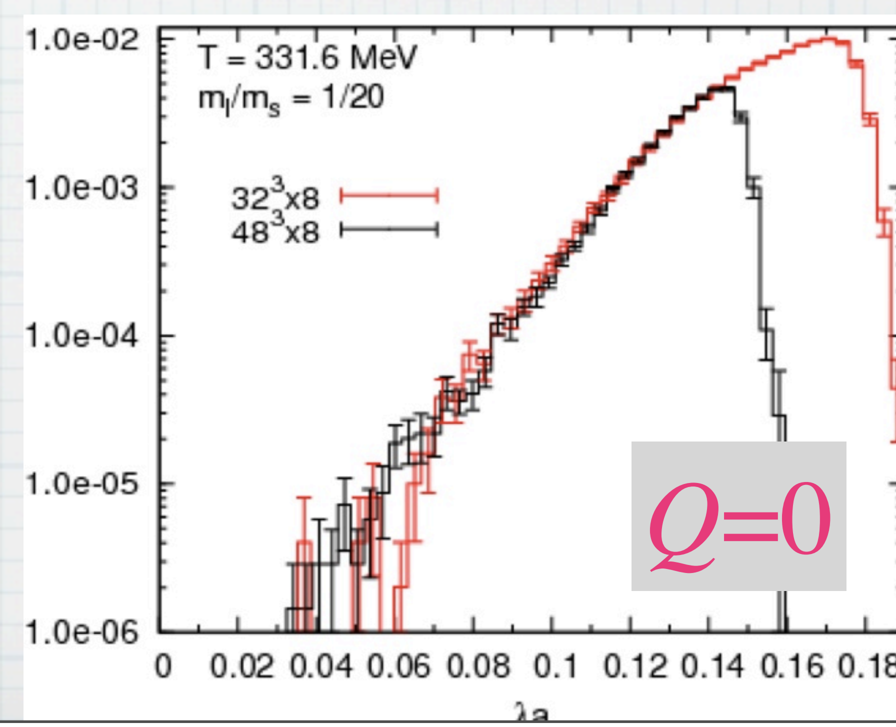
V-indep. tails remain => rho(0)≠ 0

But they are $Q\neq0$ contributions.
=> large statistics needed to conclude at large *V*.

i.e. not clear

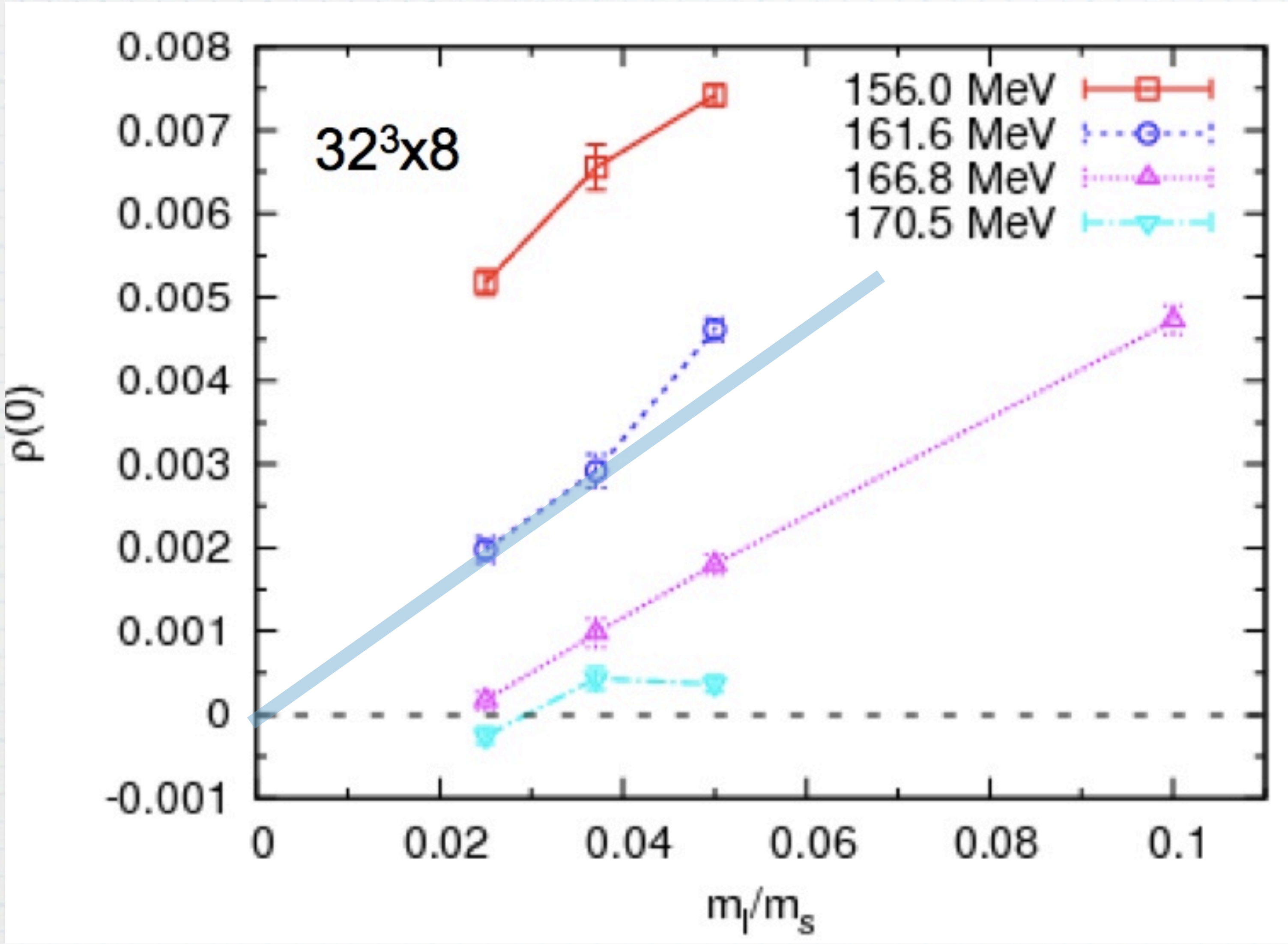
log-scale





Fate of $U(1)_A$ at T>Tc

- Ohno (HotQCD) N_F=2+1 HISQ Nt=8, V=32^3-48^3



c.f. T_{pc} estimated from the chiral susceptibility:
157(3) MeV (m_l/m_s=1/40)
162.9(1.8) MeV (m_l/m_s=1/20)

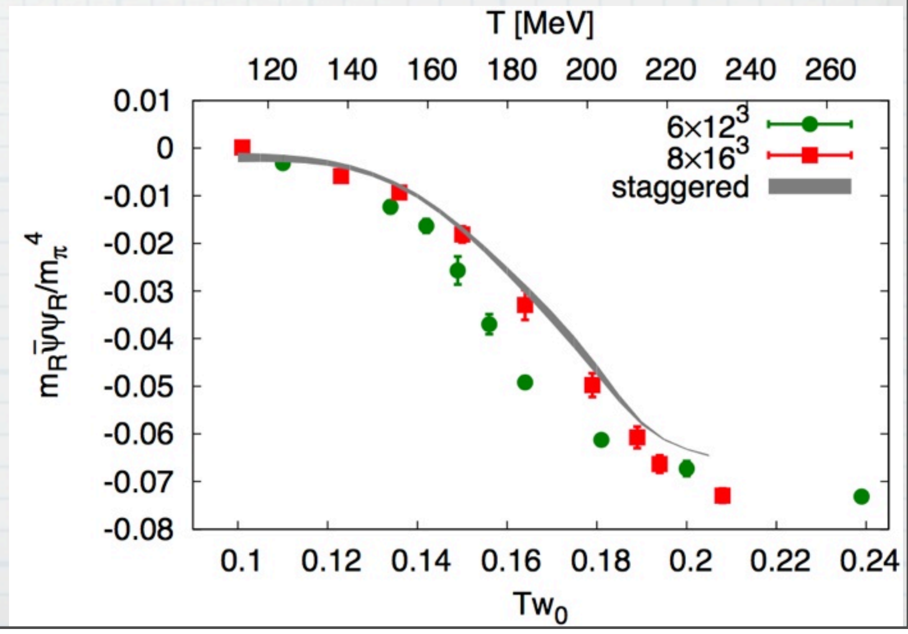
Assuming ρ(0) = 0 in the chiral limit, ρ(0) in the small quark mass region seems to linearly approach the origin up to T = 161.6 MeV.

This suggests that $U_A(1)$ symmetry remains broken in the chiral limit just above T_c.

Fate of $U(1)_A$ at T>Tc

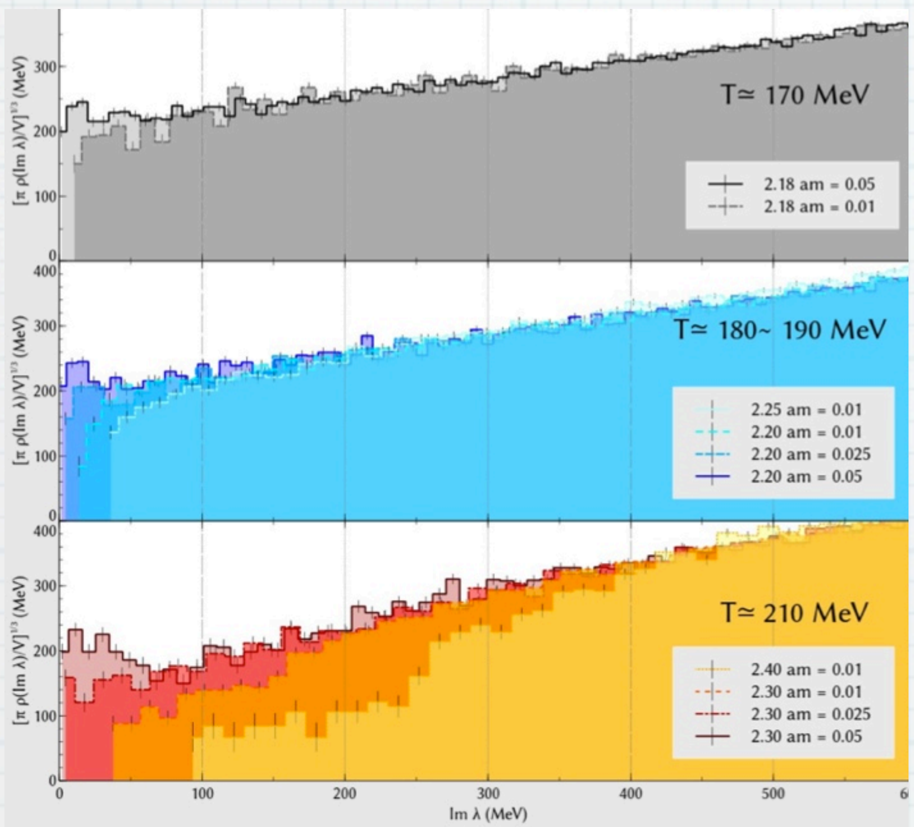
	SU(N)XSU(N)	$U_A(1)$
Staggered	Remnant U(1)	Broken
Wilson	Broken	Broken
Domain Wall	Exact (for L $\rightarrow \infty$)	Exact (for L $\rightarrow \infty$)
Overlap	Exact	Exact

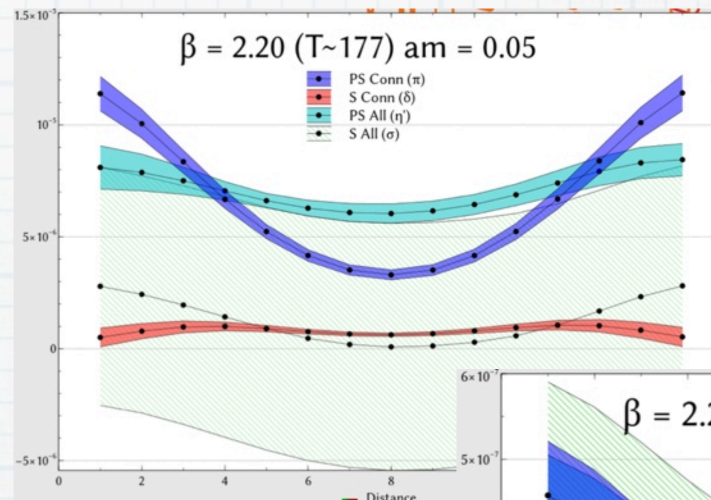
* **Cossu (JLQCD)** N_F=2 overlap + fixed-topology Iwasaki gauge
* **Lin (HotQCD)** N_F=2+1 DW + Iwasaki
* **Krieg (Budapest-Wuppertal)**
  N_F = 2+1 overlap + fixed-topology Symanzik
  m_π=350MeV, 12^3x6, 16^3x8
  => good agreement with stag.

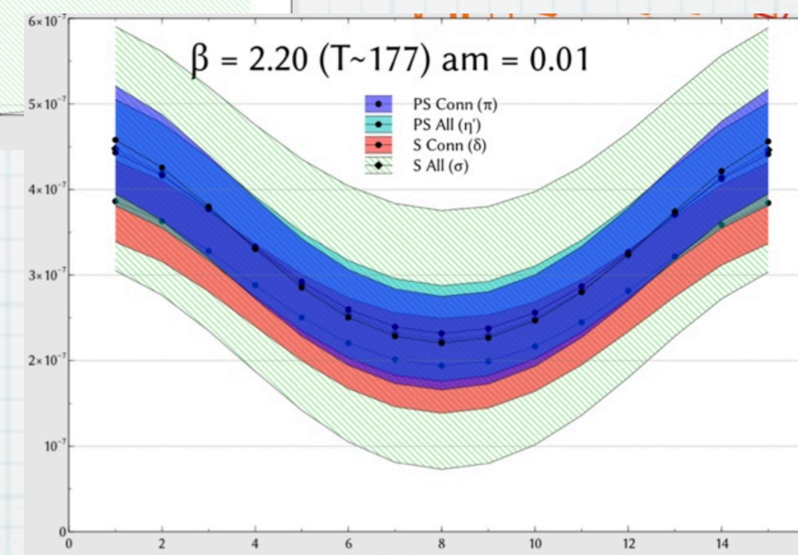


Fate of $U(1)_A$ at T>Tc

* **Cossu (JLQCD)** N_F=2 overlap + fixed-topology Iwasaki gauge
  Nt=8, V=16^3, m_π≈290MeV







- Full QCD spectrum shows <u>a gap at high temperature</u> even at pion masses ~250 MeV
- Correlators <u>show degeneracy of all channels</u> when mass is decreased
- Results support effective restoration of $U(1)_A$ symmetry

at these T's. / How about at Tc ?? / V-dep. should be checked.

Fate of $U(1)_A$ at T>Tc

* **Lin (HotQCD)** N_F=2+1 DW + Iwasaki gauge
  also arXiv:1205.3535
  Nt=8, V=16^3-32^3, m_π=200MeV DSDR (or Ls=96) to reduce m_{res}
  DSDR allows topological tunnelings
  Lowest 100 eigenvalues:

V=64^3 in progress

FIGURE: 159MeV

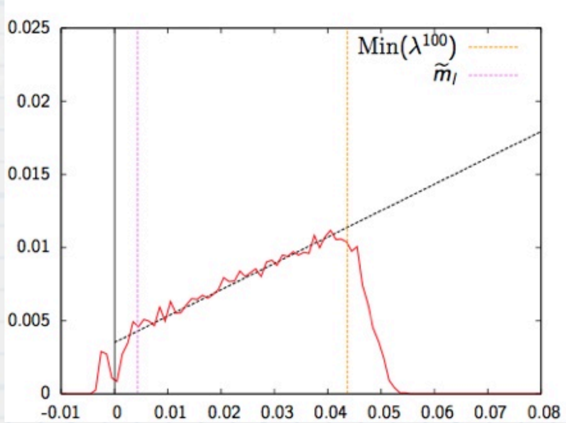
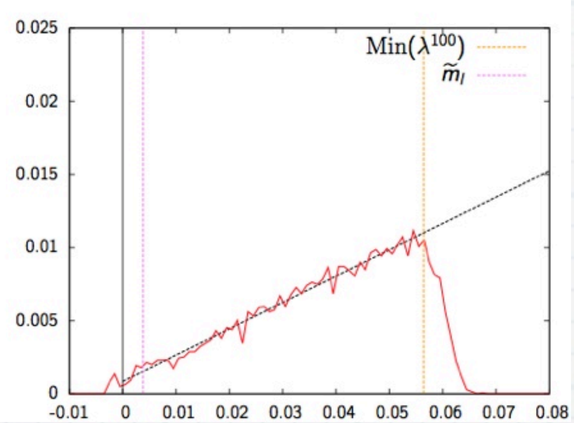


FIGURE: 168MeV



T =177 MeV

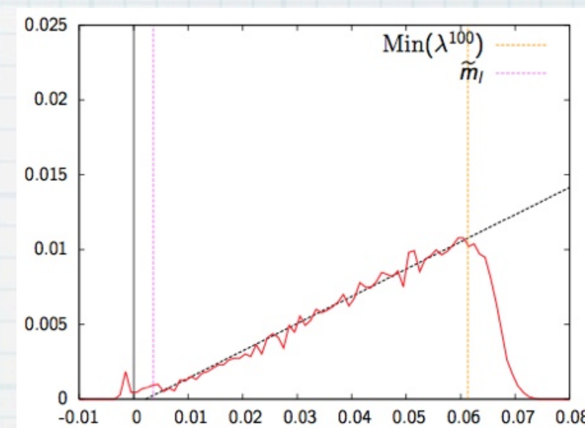


FIGURE: 186MeV

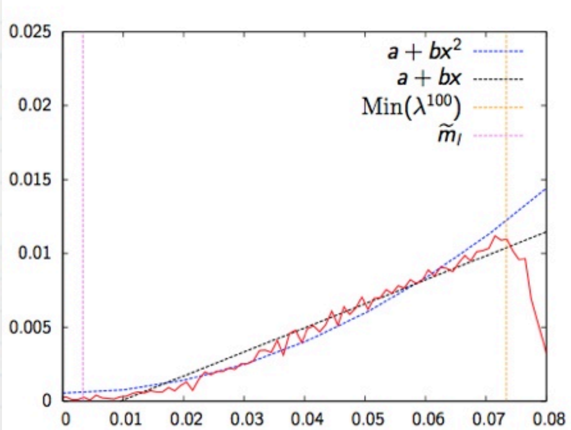
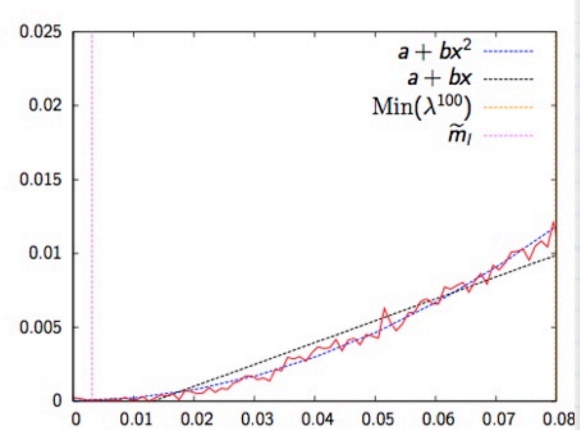


FIGURE: 195MeV



Intercept ~ 0
Linear slope visible.

$$\frac{\chi_\pi - \chi_\delta}{T^2} = 48.72+9.70$$

Consistent with their correlation functions.

Fate of $U(1)_A$ at T>Tc

* S. Aoki
  N_F=2 Chiral WT of Gisparg-Wilson fermions

$$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{k=0}^{\infty} \rho_k^A \frac{|\lambda|^k}{k!}$$

$$D(A)\phi_n^A = \lambda_n^A \phi_n^A$$

$\langle \rho_0^A \rangle_m = O(m^4)$ $\langle \rho_1^A \rangle_m = O(m^2)$ $\langle \rho_2^A \rangle_m = O(m^2)$

$$\lim_{m\to 0} \chi^{\pi-\eta} = \lim_{m\to 0} \lim_{V\to\infty} \frac{N_f^2}{m^2 V^2} \langle Q(A)^2 \rangle_m = 0$$

at all *T*'s above *Tc*.

More generally, for $\mathcal{O} = \mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$

$$\lim_{m\to 0} \lim_{V\to\infty} \frac{1}{V^k} \langle \delta^0 \mathcal{O} \rangle_m = 0$$

δ^0 : singlet rotation

Breaking of U(1)_A symmetry is absent for these "bulk quantities".

V-dep. important to check in the lattice results.

EOS

EOS

$$\epsilon = -\frac{1}{V}\frac{\partial \ln Z}{\partial T^{-1}}, \quad p = T\frac{\partial \ln Z}{\partial V}$$

with $Z = \mathrm{Tr} e^{-H/T} = \int_{b.c.} \mathcal{D}\phi\, e^{-S}$

Trace anomaly

$$\frac{\epsilon - 3p}{T^4} = N_t^4 \left\{ \left\langle a\frac{dS}{da} \right\rangle - \left\langle a\frac{dS}{da} \right\rangle_{T=0} \right\} = \frac{N_t^3}{N_s^3} \sum_i a\frac{db_i}{da} \left\{ \left\langle \frac{\partial S}{\partial b_i} \right\rangle - \left\langle \frac{\partial S}{\partial b_i} \right\rangle_{T=0} \right\}$$

lattice beta functions along LCP

$b = (\beta, \kappa_{ud}, \kappa_s, \cdots) \equiv (b_1, b_2, \cdots)$

measured by the simulation.
T=0 subtraction for ren.

Integral method for p (fixed-Nt approach)

Differentiate and integrate a thermodyn. relation $p = (T/V)\ln Z$

$$p = \frac{T}{V}\int_{b_0}^{b} db \frac{1}{Z}\frac{\partial Z}{\partial b} = -\frac{T}{V}\int_{b_0}^{b} \sum_i db_i \left\{ \left\langle \frac{\partial S}{\partial b_i} \right\rangle - \left\langle \frac{\partial S}{\partial b_i} \right\rangle_{T=0} \right\}$$

such that $p(b_0) \approx 0$

numerical integration
in the coupling param. space

EOS

$$\epsilon = -\frac{1}{V}\frac{\partial \ln Z}{\partial T^{-1}}, \quad p = T\frac{\partial \ln Z}{\partial V}$$

with $Z = \mathrm{Tr} e^{-H/T} = \int_{b.c.} \mathcal{D}\phi\, e^{-S}$

Trace anomaly

$$\frac{\epsilon - 3p}{T^4} = N_t^4 \left\{ \left\langle a\frac{dS}{da} \right\rangle - \left\langle a\frac{dS}{da} \right\rangle_{T=0} \right\} = \frac{N_t^3}{N_s^3} \sum_i a\frac{db_i}{da} \left\{ \left\langle \frac{\partial S}{\partial b_i} \right\rangle - \left\langle \frac{\partial S}{\partial b_i} \right\rangle_{T=0} \right\}$$

lattice beta functions along LCP

$b = (\beta, \kappa_{ud}, \kappa_s, \cdots) \equiv (b_1, b_2, \cdots)$

measured by the simulation.
T=0 subtraction for ren.

T-integration method for *p* (fixed-scale approach)

$$T = \frac{1}{N_t a}$$

vary *T* by varying *Nt* at fixed *a* (i.e. fixed coupling param's)

$$T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right) = \frac{\epsilon - 3p}{T^4} \quad => \quad \frac{p}{T^4} = \int_{T_0}^{T} dT \frac{\epsilon - 3p}{T^5}$$

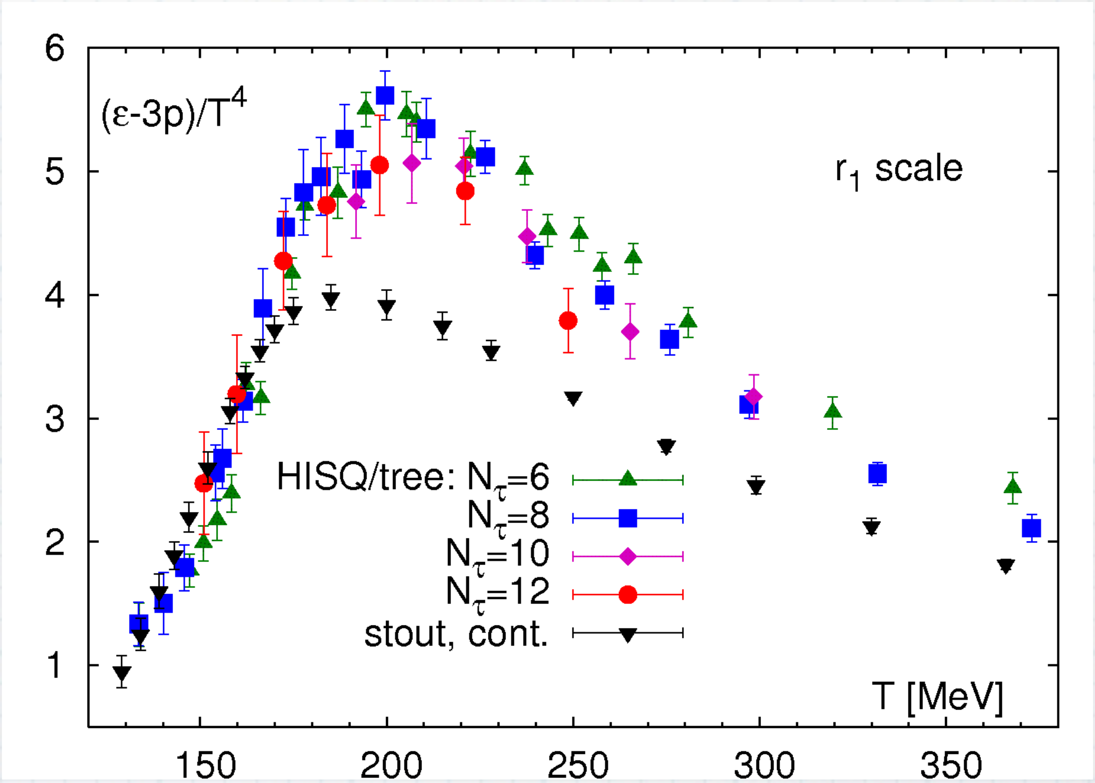
$p(T_0) \approx 0$

Umeda et al., PRD79, 051501 ('09)

EOS updates

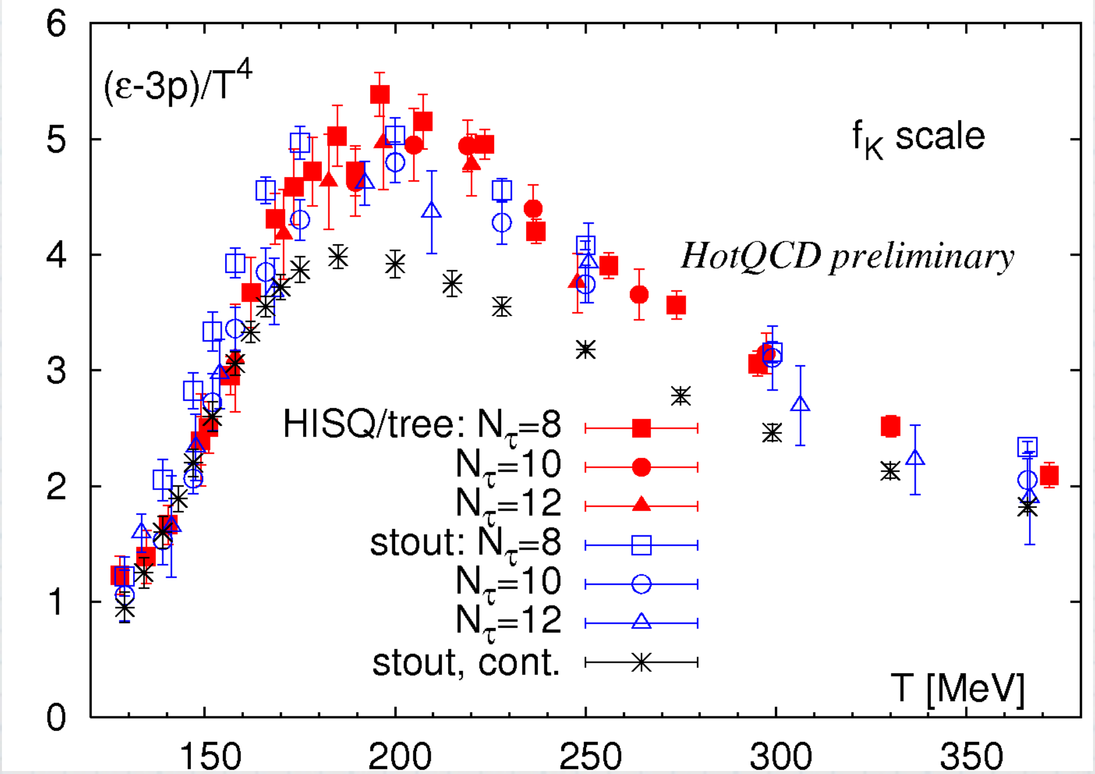
* Petreczky (HotQCD) $N_F = 2+1$ HISQ
  - $m_s \approx$ physical, $m_l/m_s = 1/27 - 1/20$

Discrepancies between HISQ(HotQCD) and stout(Budapest-Wuppertal)



• The differences between HISQ/tree and stout data are statistically not significant for $N_\tau \geq 8$

• The scale setting procedure could make a difference, the use of f_K scale improves the agreement between different actions, though the effect is negligible for HISQ/tree $N_\tau = 10, 12$



EOS updates

* Burger (tmfT) N_F=2 twisted mass + tree-level Symanzik gauge

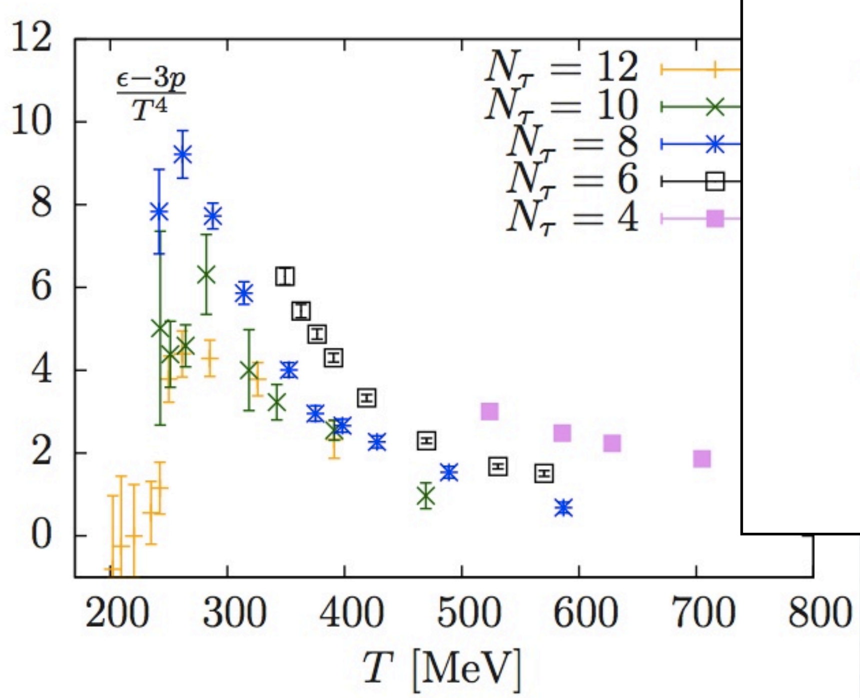
  N_t = 12, (10), $V = 32^3$

  Tc and EOS on LCP for four $m_\pi \approx$ 280-480 MeV

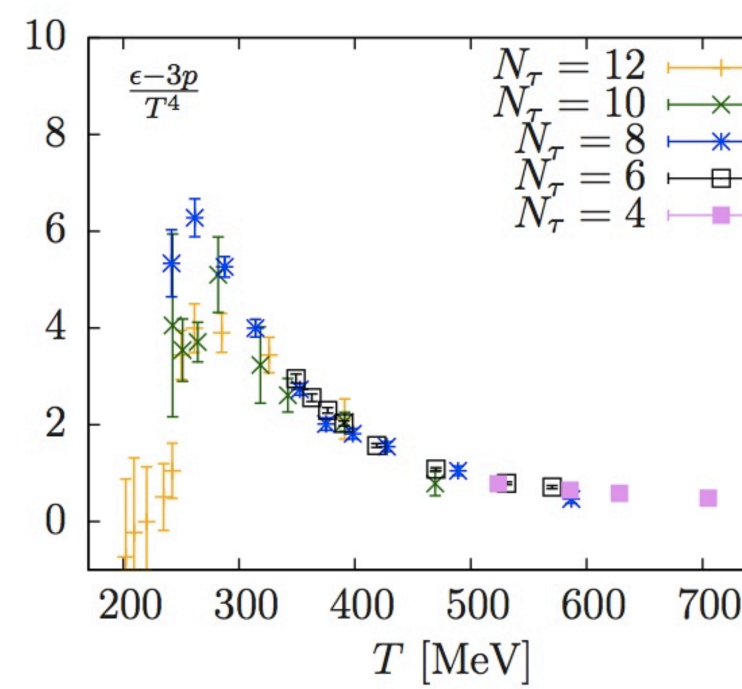
Beta function by *r_chi* on each (approximate) LCP
Tree-level corrections to remove leading tm artifacts / to improve large *Nt* behavior
T-integration to obtain *p*

uncorrected

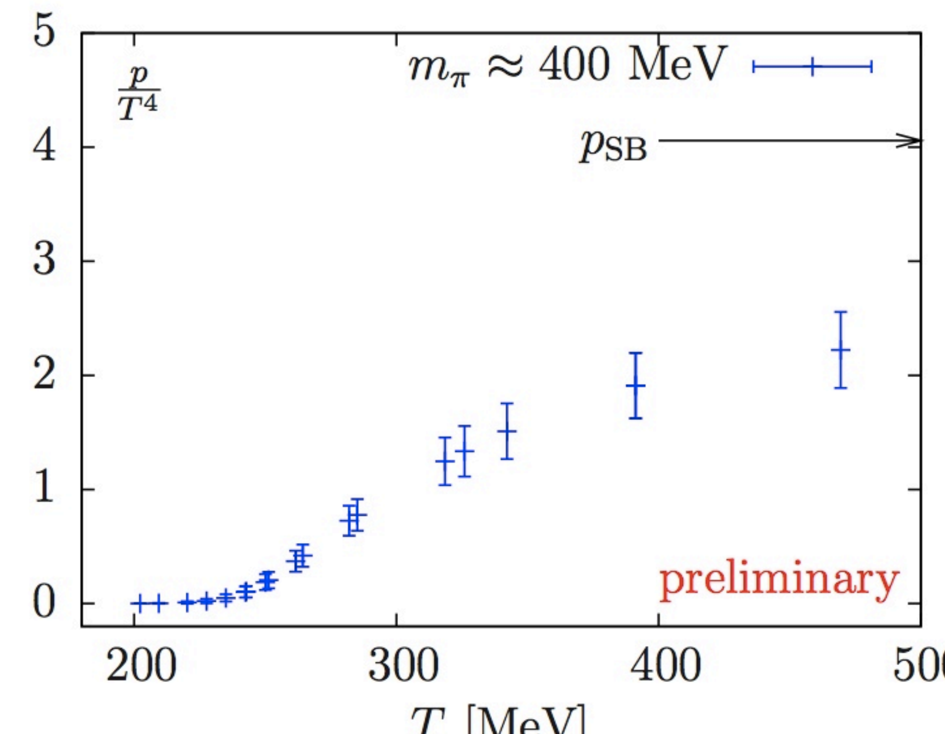
$m_\pi \approx 400$ MeV:



corrected



$m_\pi \approx 400$ MeV:



EOS updates

* **Umeda (WHOT-QCD) $N_F = 2+1$ NP-clover + Iwasaki gauge**

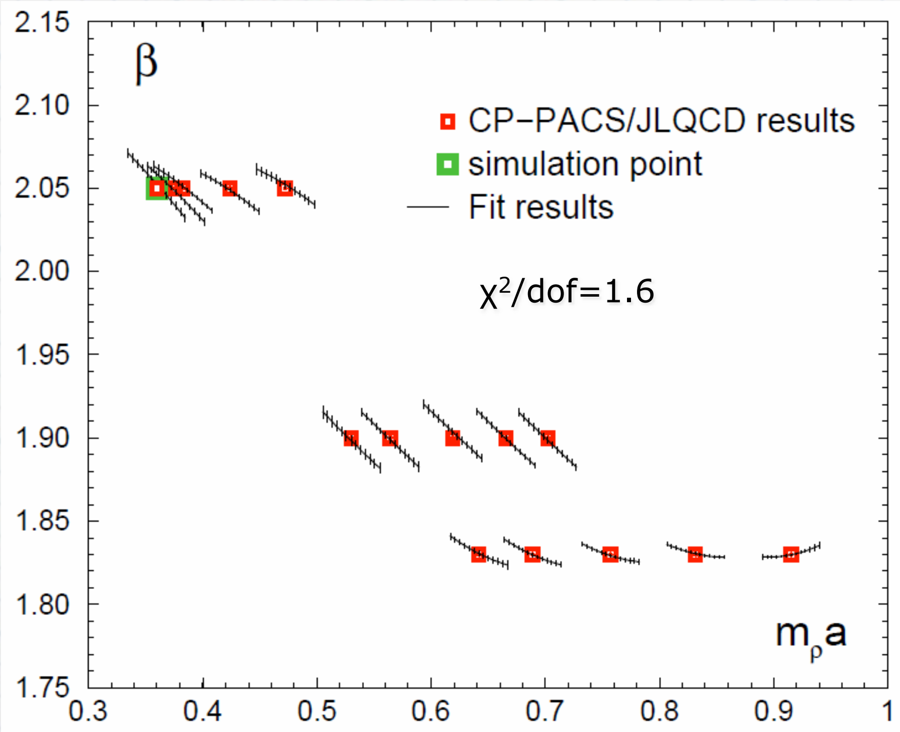
Fixed-scale approach using CP-PACS+JLQCD T=0 configuration

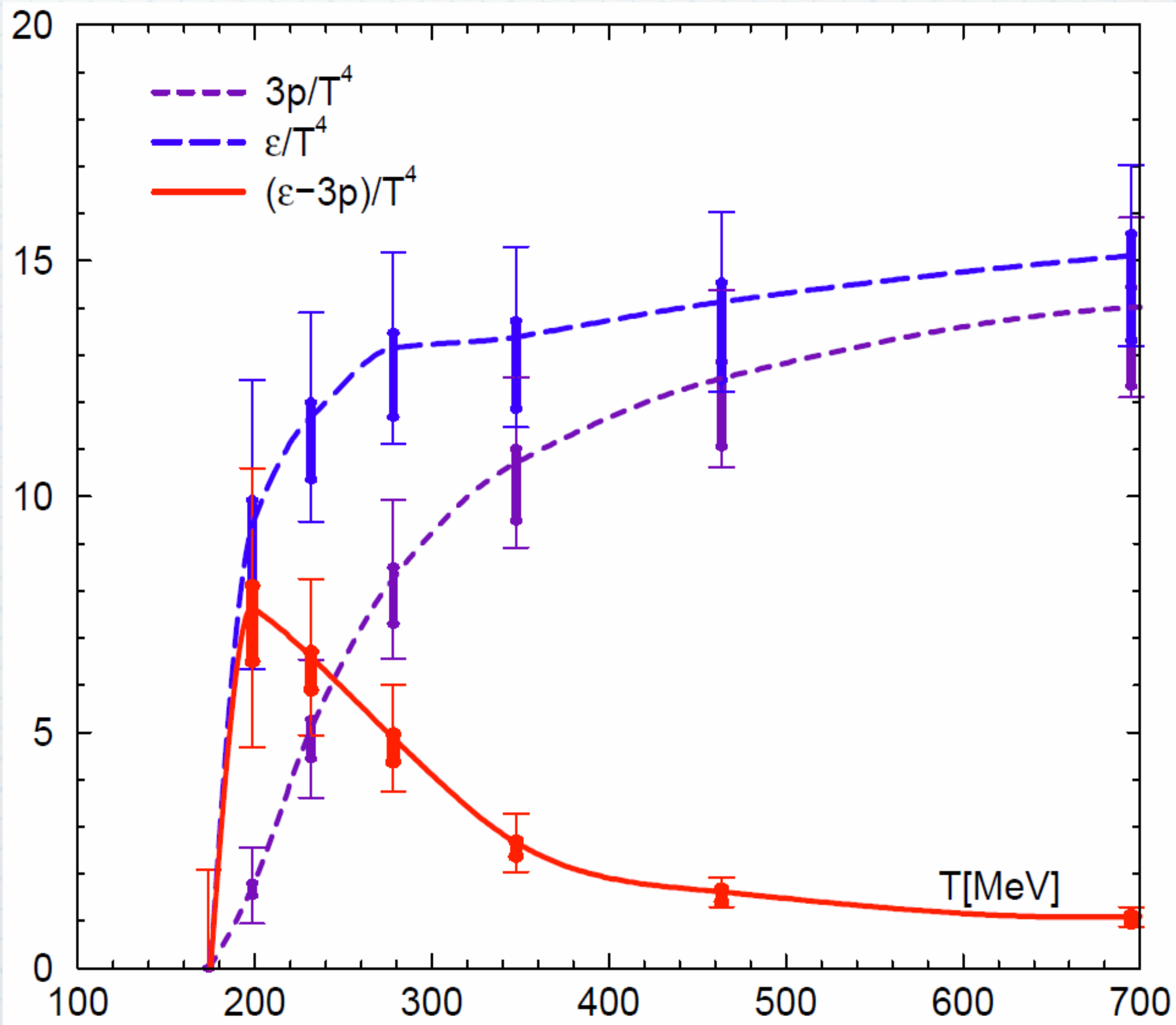
$m_\pi \approx 636$MeV, $a \approx 0.07$fm, $28^3 \times 56$ ($L \approx 2$fm)

$N_t = 4\text{-}16$ $V = 32^3$

Final result published in PRD (2012):

Beta function by direct fit method
T-integration to obtain p



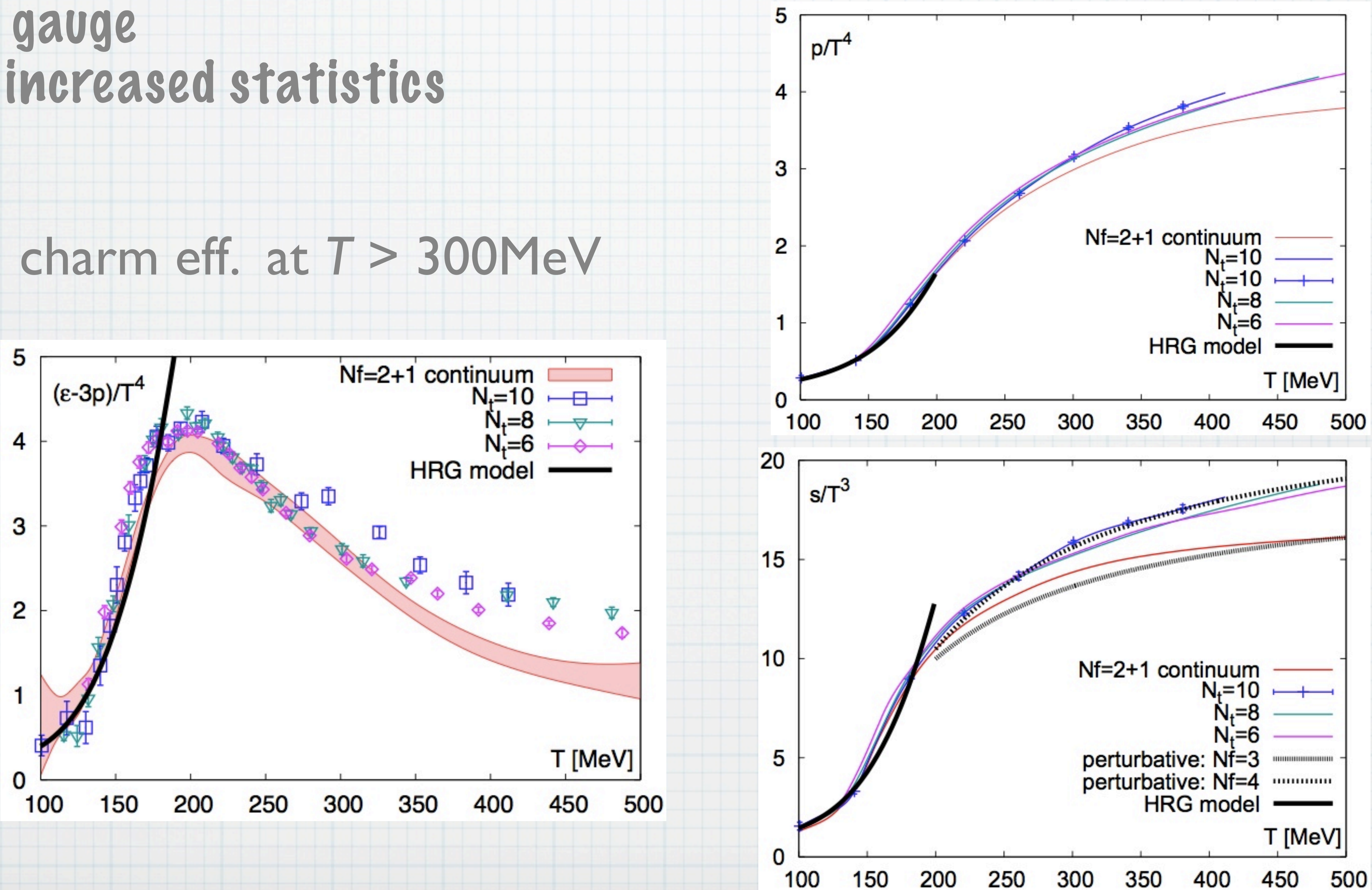


thick error bar = system. error from the beta function

EOS charm effects

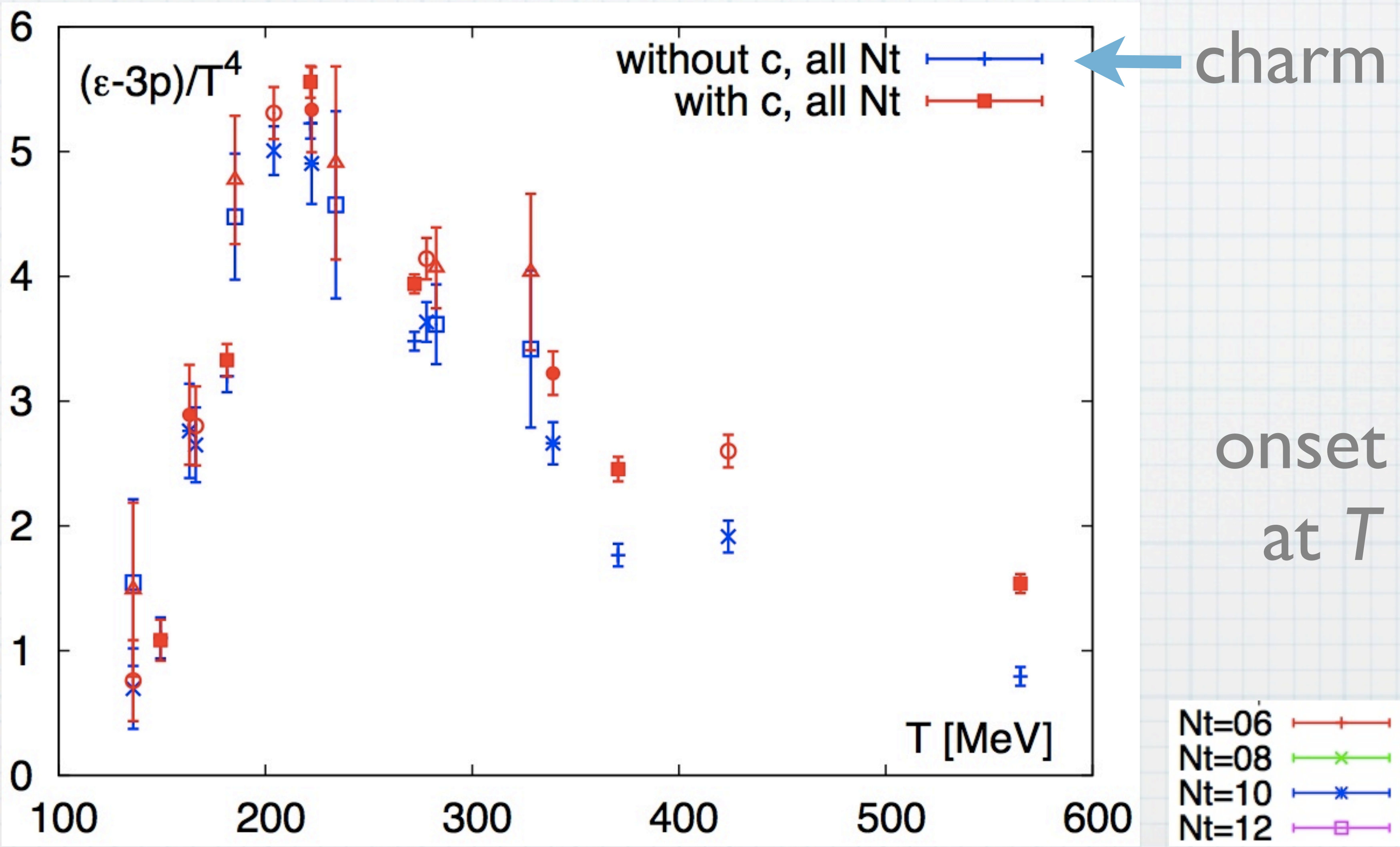
* Krieg (Budapest-Wuppertal) $N_F = 2+1+1$ stout + Symanzik gauge
increased statistics

charm eff. at T > 300MeV



EOS charm effects

* Heller (HotQCD) N_F = 2+1+1 HISQ + tadpole-impr. 1-loop Symanzik gauge
  Naik term to improve the charm dispersion
  LCP at m_{ud}/m_s = 1/5, m_s, m_c ≈ phys., Nt=6-12
  Preliminary: No continuum extrapolations yet.
  Contribution from variation of the charm Naik term not included yet.



charm sea included!

onset of charm eff.
at T ≈ 350MeV

$\mu \neq 0$

Difficulties at $\mu \neq 0$

- **LQCD at $\mu \neq 0$** $\quad U_4 \longrightarrow \begin{cases} U_4 e^{\mu a} & \cdots \quad \text{positive } t \text{ direction} \\ U_4 e^{-\mu a} & \cdots \quad \text{nagative } t \text{ direction} \end{cases}$

  in the temporal hopping term of corresponding quark.

- **Complex phase problem (sign problem)** $[\det M(\mu)]^* = \det M(-\mu^*)$

  => Importance sampling not naively justified

  => Exponential cancellation due to the phase fluctuation of $\det M$

- **Techniques for small μ/T**

✦ Taylor expansion
✦ Reweighting
✦ Canonical
✦ Imaginary μ
✦ Complex Langevin
✦ Direct calculation of many body propagators, etc.

& Combination of them

[+ density of state method / cumulant expansion / ...]

=> only $\mu/T \leq O(1)$ accessible so far

See Nagata (XQCD-J)@Lat12 for a recent attempt towards large μ.

3 and 4 flavors

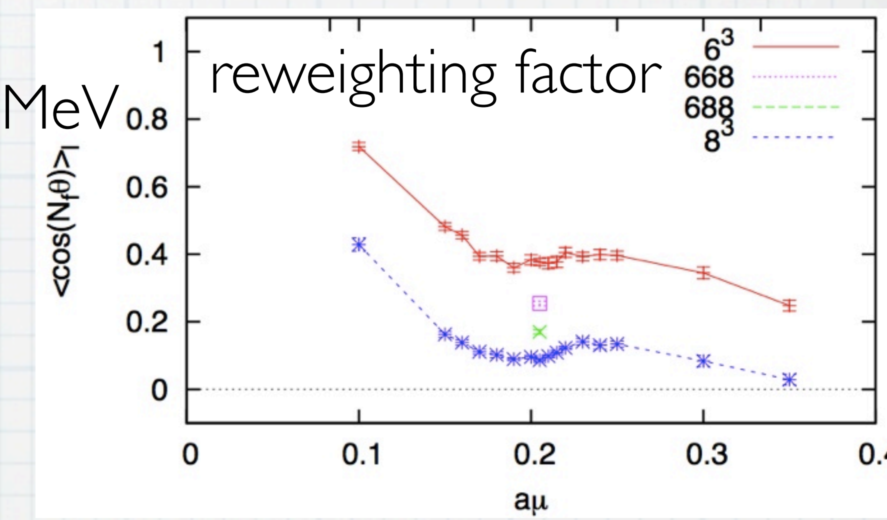
- S. Takeda, Y. Nakamura, Jin (with Kuramashi, Ukawa)

  phase-quenched simulation + phase reweighting

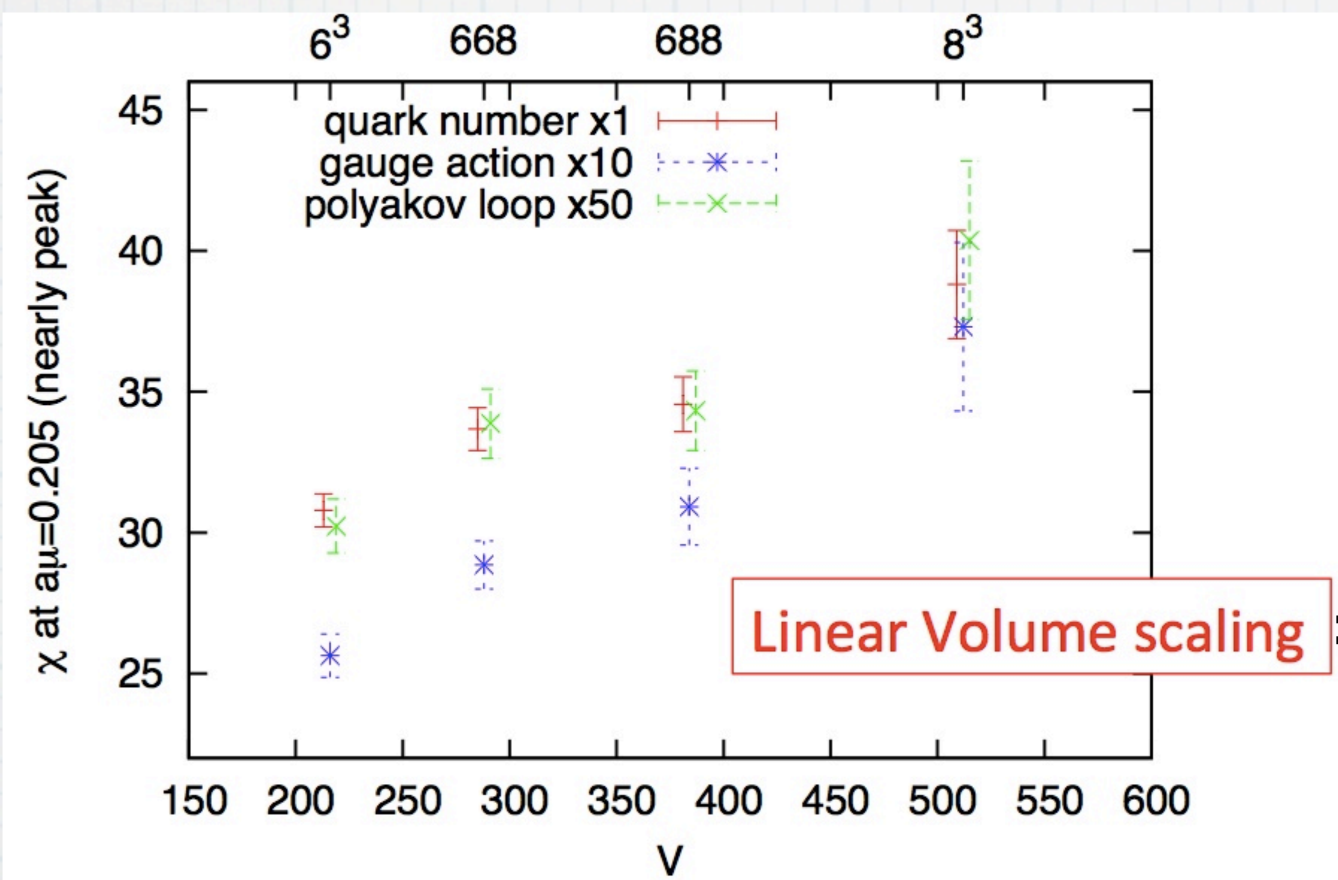
  <= winding number expansion (Danzer-Gattlinger's factorization method)

  canonical ensembles with fixed quark numbers

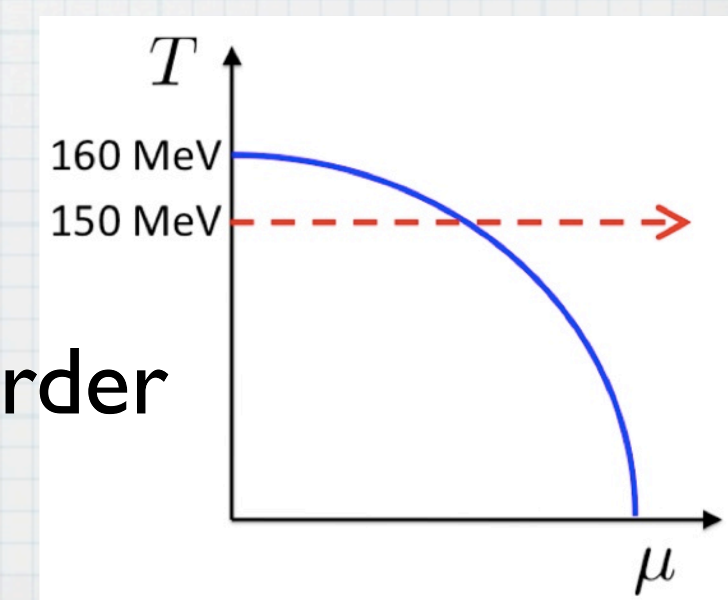
N_F=4: $N_t=4$, $V=6^3$-8^3, clover, $m_\pi \approx 830$ MeV, $T \approx 150$MeV



Peak height of the quark # suscept. at μ≠0

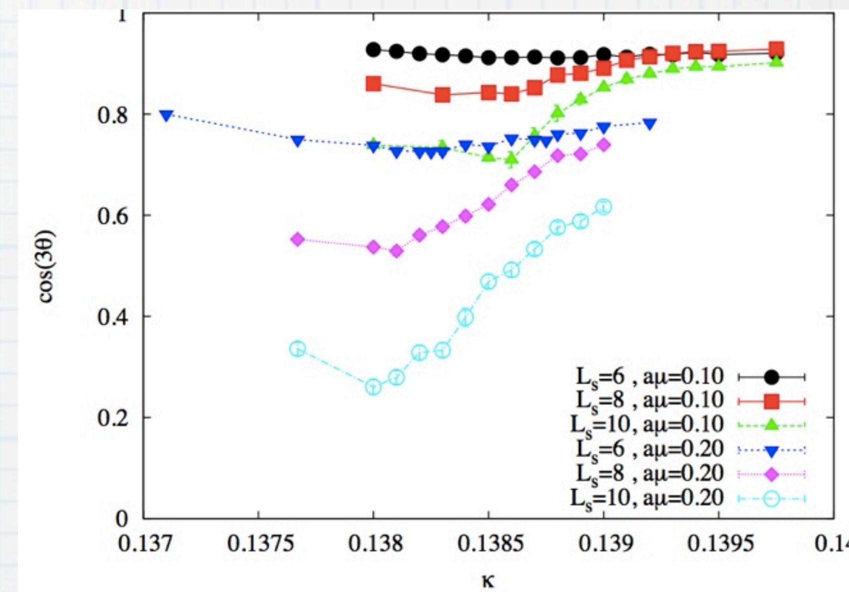


=> 1st order

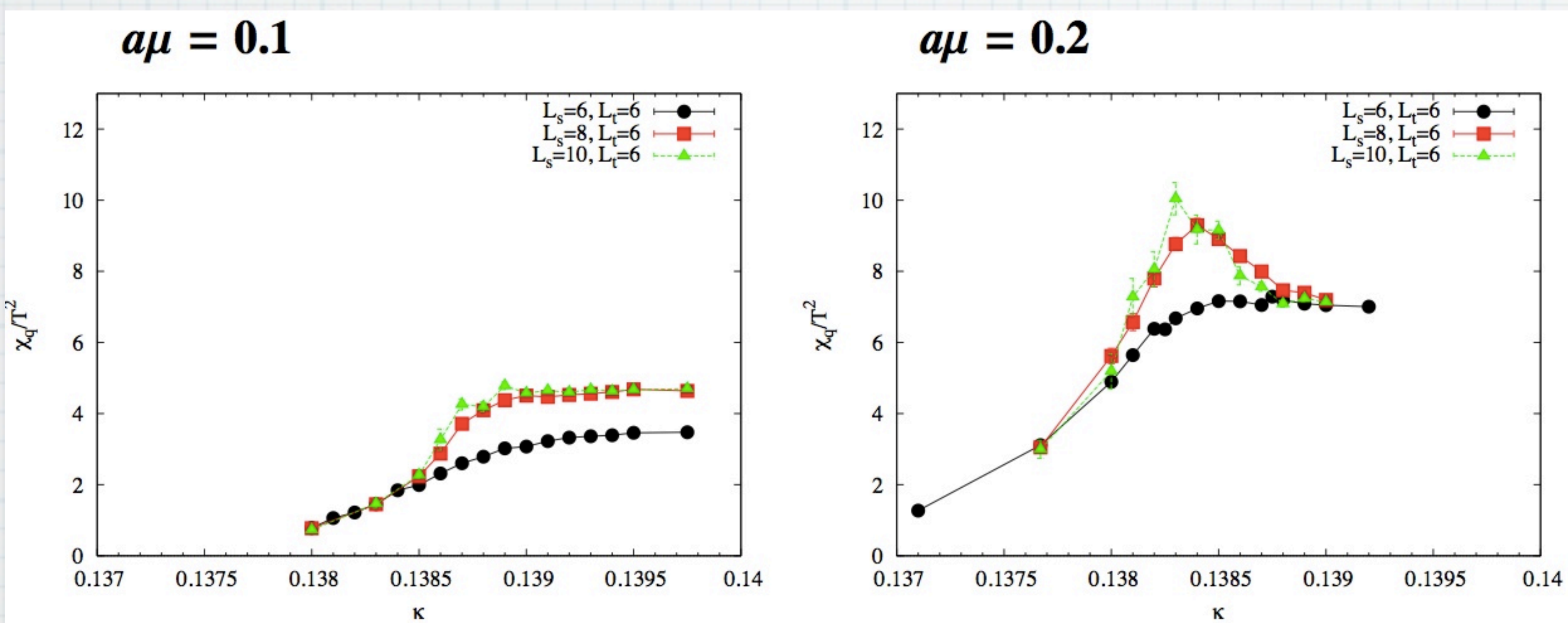


3 and 4 flavors

* **S. Takeda, Y. Nakamura, Jin (with Kuramashi, Ukawa)**
  phase-quenched simulation + phase reweighting
  N_F=3: N_t=6, V=6^3-10^3, clover + Iwasaki, m_π≈400-1200MeV, T≈210MeV



reweighting factor



in finite size study up to $L_s = 10$, no clear sign of 1^{st} or 2^{nd} order phase transition for $T \sim 200$ MeV $m_\pi \sim 900$ MeV, more statistics/larger volume?

Histogram method

* Nakagawa, Ejiri (WHOT-QCD)

$$\frac{\mathcal{Z}(\beta,\mu)}{\mathcal{Z}(\beta,0)} = \frac{1}{\mathcal{Z}(\beta,0)} \int \mathcal{D}U e^{i\theta(\mu)} \left|\det M(\mu)\right|^{N_\mathrm{f}} e^{6\beta N_\mathrm{site} \hat{P}}$$

$$= \int dP dF w(P,F;\beta,\mu) \left\langle e^{i\theta(\mu)} \right\rangle (P,F;\beta,\mu)$$

$$P = -\frac{S_g}{6\beta N_\mathrm{site}}$$

$$F(\mu) = N_\mathrm{f} \ln \left| \frac{\det M(\mu)}{\det M(0)} \right|$$

Phase-quenched distribution function for P and F

$$w(P',F';\beta,\mu) = \frac{1}{\mathcal{Z}(\beta,0)} \int \mathcal{D}U \delta(\hat{P}-P')\delta(\hat{F}-F') \underbrace{\left|\det M(\mu)\right|^{N_f} e^{6\beta N_\mathrm{site}\hat{P}}}_{\text{phase quenched measure}}$$

Phase-reweighting factor in terms of phase-quenched expectation values

$$\left\langle e^{i\theta(\mu)} \right\rangle (P',F';\beta,\mu) = \frac{\left\langle\left\langle e^{i\theta(\mu)} \delta(\hat{P}-P')\delta(\hat{F}-F') \right\rangle\right\rangle_{(\beta,\mu)}}{\left\langle\left\langle \delta(\hat{P}-P')\delta(\hat{F}-F') \right\rangle\right\rangle_{(\beta,\mu)}}$$

★ Phase-reweighting factor $\left\langle e^{i\theta(\mu)} \right\rangle (P', F'; \beta, \mu)$

Potential source of the sign problem.

=> **Cumulant expansion method** Ejiri PRD77('08);WHOT PRD82('10)

$$\left\langle e^{i\theta(\mu)} \right\rangle_{(P,F,\mu)} = \exp\left[i\langle\theta\rangle_c - \frac{1}{2}\left\langle\theta^2\right\rangle_c - \frac{i}{3!}\left\langle\theta^3\right\rangle_c + \frac{1}{4!}\left\langle\theta^4\right\rangle_c + \cdots \right]$$

Odd terms = 0 due to the time-reversal sym.: μ ↔ -μ.

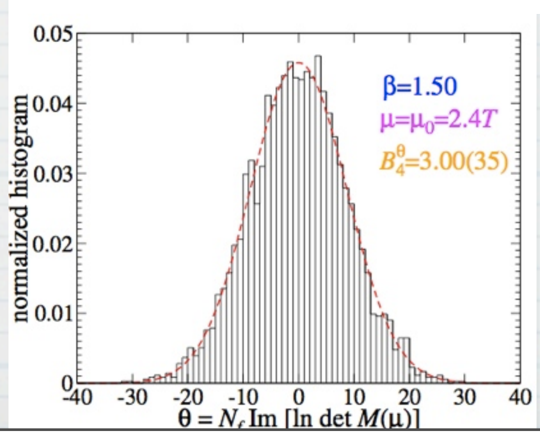
=> The phase factor is real & positive.

No sign problem if the cumulant expansion converges

=> Look for a **definition** of θ that distributes ≈Gaussian.

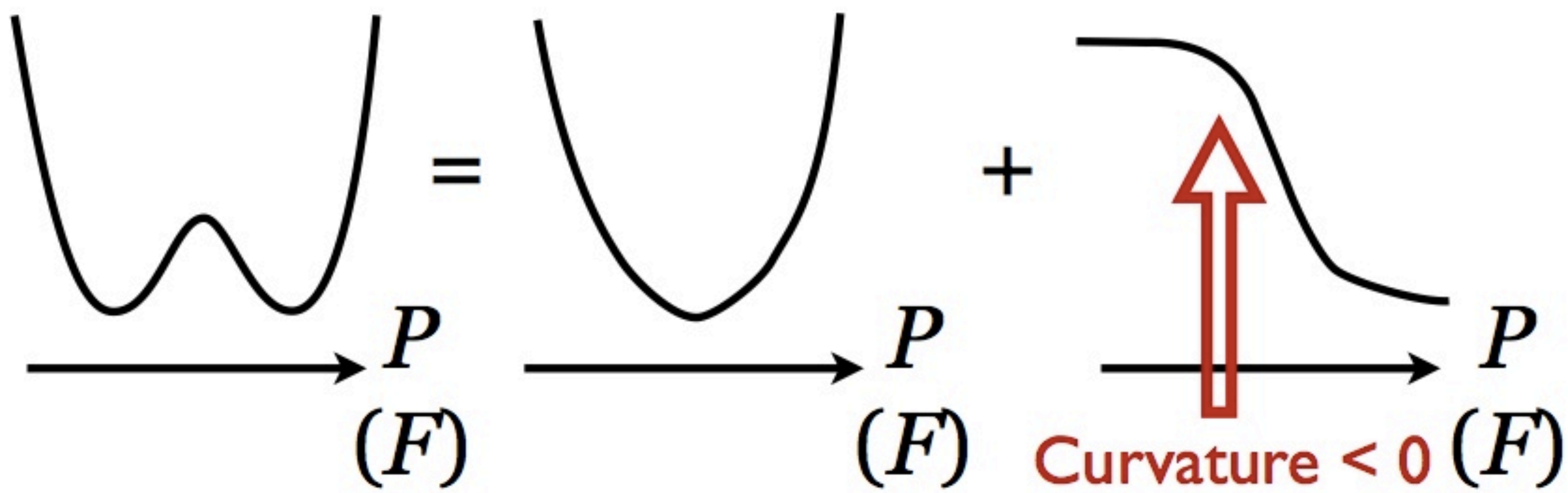
Our proposal:

$$\theta(\mu) = N_f \mathfrak{Im}\left[\ln\det M(\mu)\right] = N_f \int_0^{\mu/T} \mathfrak{Im}\left[\frac{\partial(\ln\det M(\bar\mu))}{\partial(\bar\mu/T)}\right] d\left(\frac{\bar\mu}{T}\right)$$



★ Effective potential $\frac{\mathcal{Z}(\beta,\mu)}{\mathcal{Z}(\beta,0)} = \int dPdFe^{-V(P,F;\beta,\mu)}$

$$V(P,F;\beta,\mu) = -\ln w(P,F;\beta,\mu_0) + \frac{1}{2}\left\langle\theta^2\right\rangle_c (P,F;\beta,\mu,\mu_0)$$



$$\begin{aligned}
w(F;\beta,\mu) &= R(P,F;\beta,\mu,\mu_0)w(P,F;\beta,\mu_0) \\
R(F;\beta,\mu,\mu_0) &= \frac{\left\langle\left\langle \left|\frac{\det M(\mu)}{\det M(\mu_0)}\right|^{N_f} \delta(\hat{P}-P')\delta(\hat{F}-F')\right\rangle\right\rangle_{(\beta,\mu_0)}}{\left\langle\left\langle \delta(\hat{P}-P')\delta(\hat{F}-F')\right\rangle\right\rangle_{(\beta,\mu_0)}} \\
\left\langle e^{i\theta(\mu)}\right\rangle (P',F';\mu,\mu_0) &= \frac{\left\langle\left\langle e^{i\theta(\mu)} \left|\frac{\det M(\mu)}{\det M(\mu_0)}\right|^{N_f} \delta(\hat{P}-P')\delta(\hat{F}-F')\right\rangle\right\rangle_{(\beta,\mu_0)}}{\left\langle\left\langle \left|\frac{\det M(\mu)}{\det M(\mu_0)}\right|^{N_f} \delta(\hat{P}-P')\delta(\hat{F}-F')\right\rangle\right\rangle_{(\beta,\mu_0)}}
\end{aligned}$$

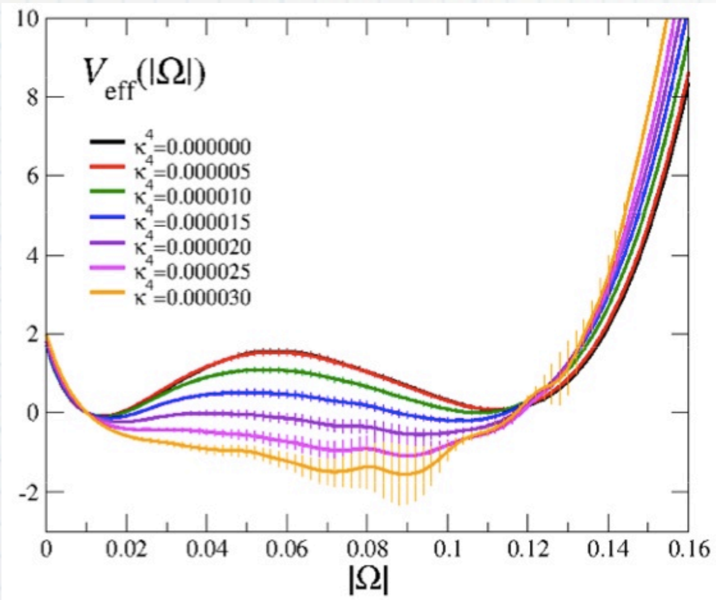
Shifted $\mu \to \mu_0$ to reduce the overlap problem.

★ **Test in the heavy quark region:** unimproved Wilson + plaquette gauge

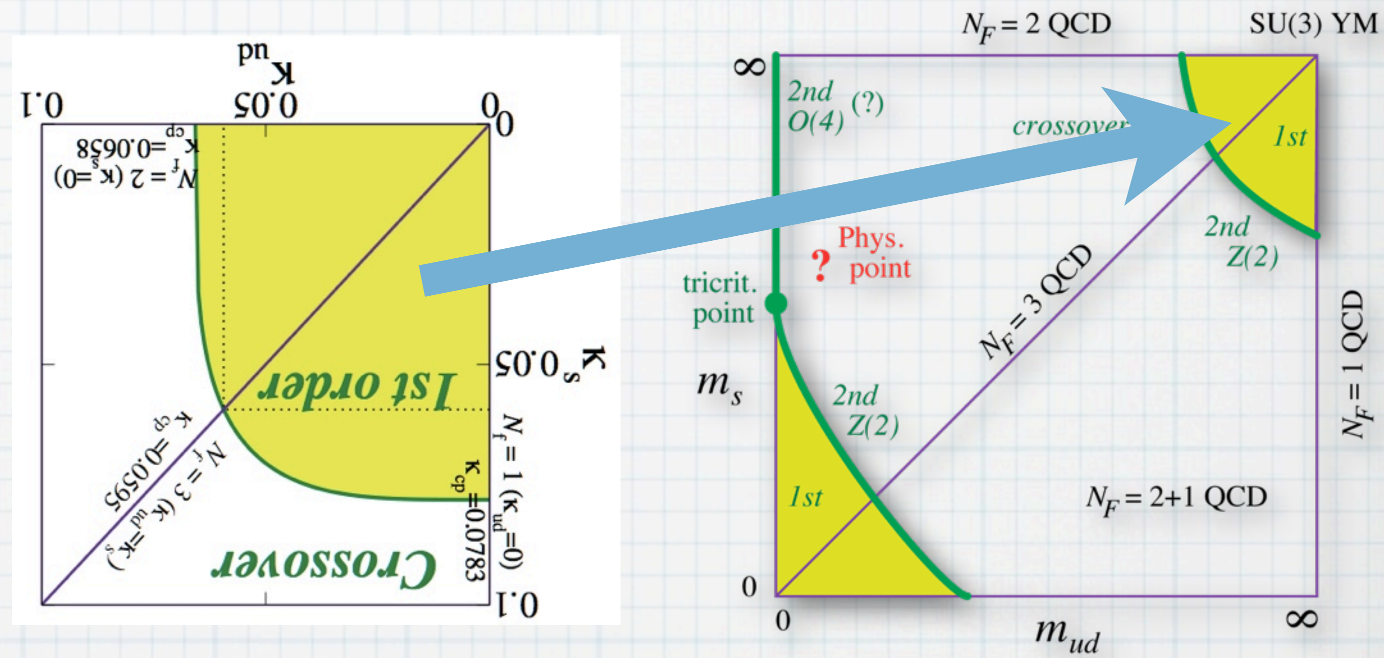
Useful to consider Ω_R instead of *F*. <= hopping param. expansion

μ => Ω_I (imag. part Polyakov)

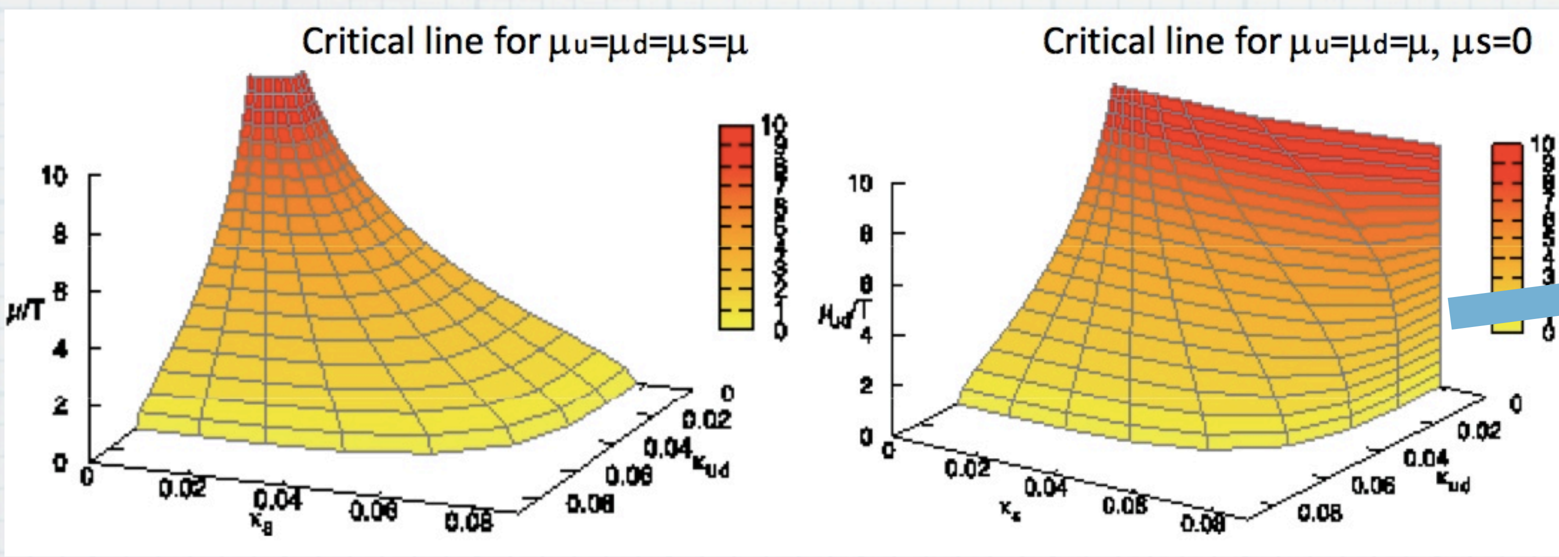
μ=0 case: 1st order at heavy (small κ) ⇒ crossover at light (large κ)

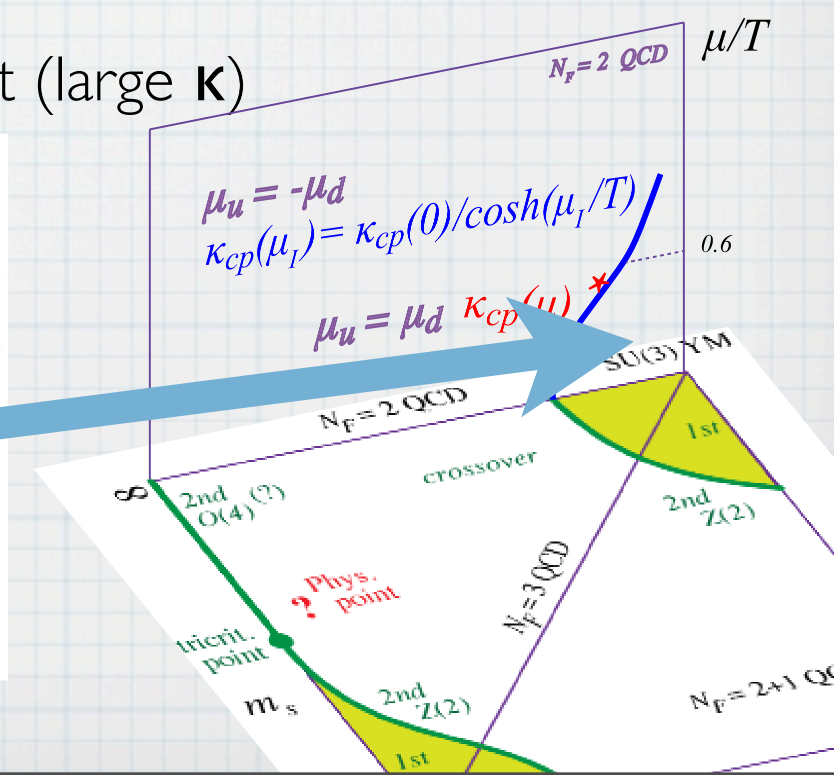


N_F=2+1 =>



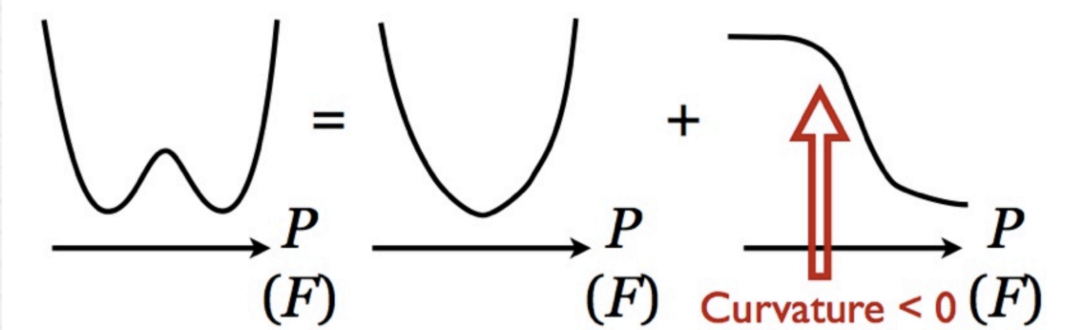
μ≠0 case: 1st order at heavy (small κ) ⇒ crossover at light (large κ)

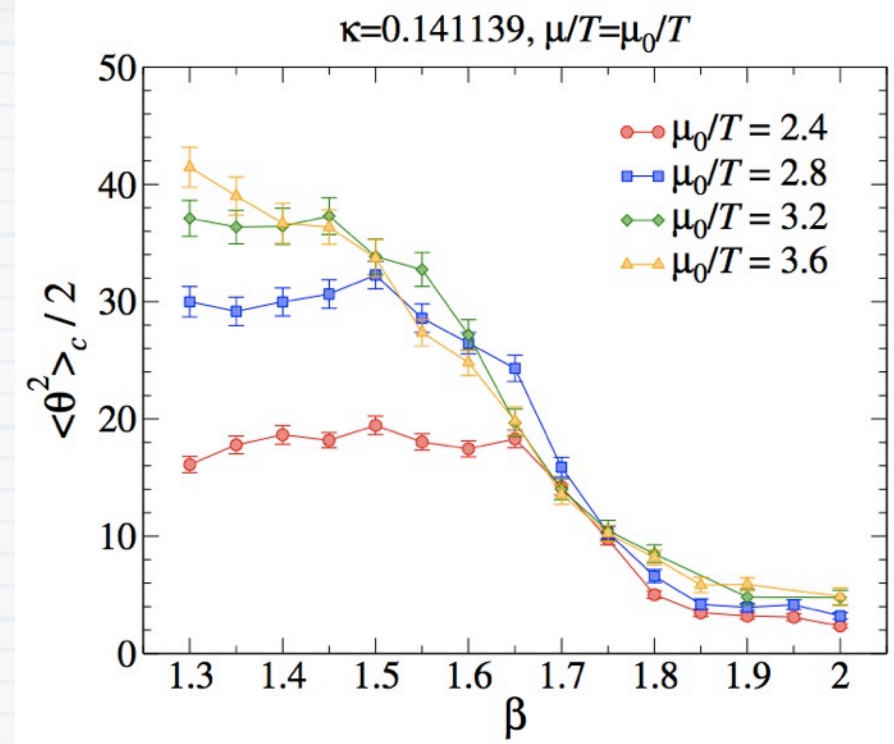


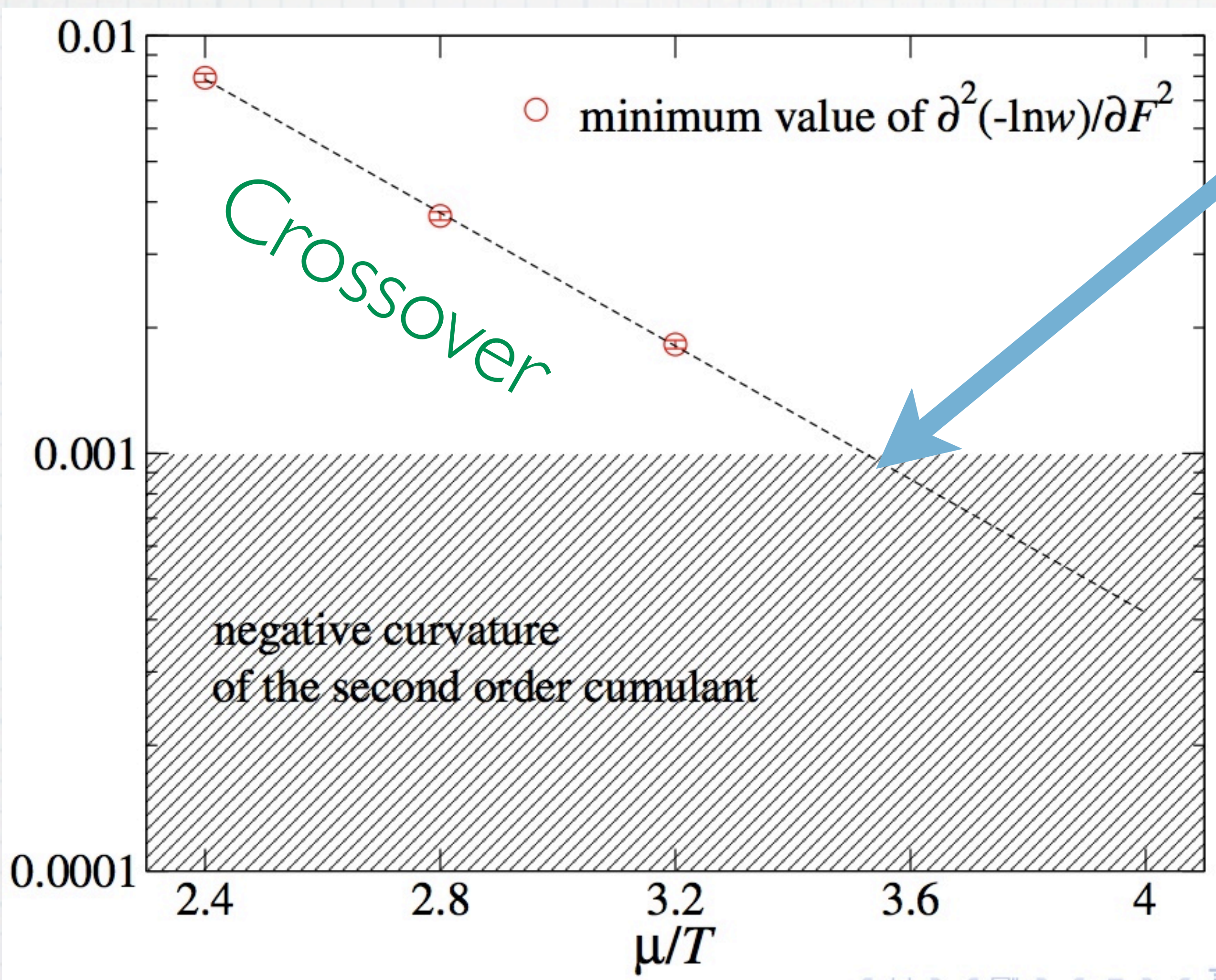


★ In the light quark region

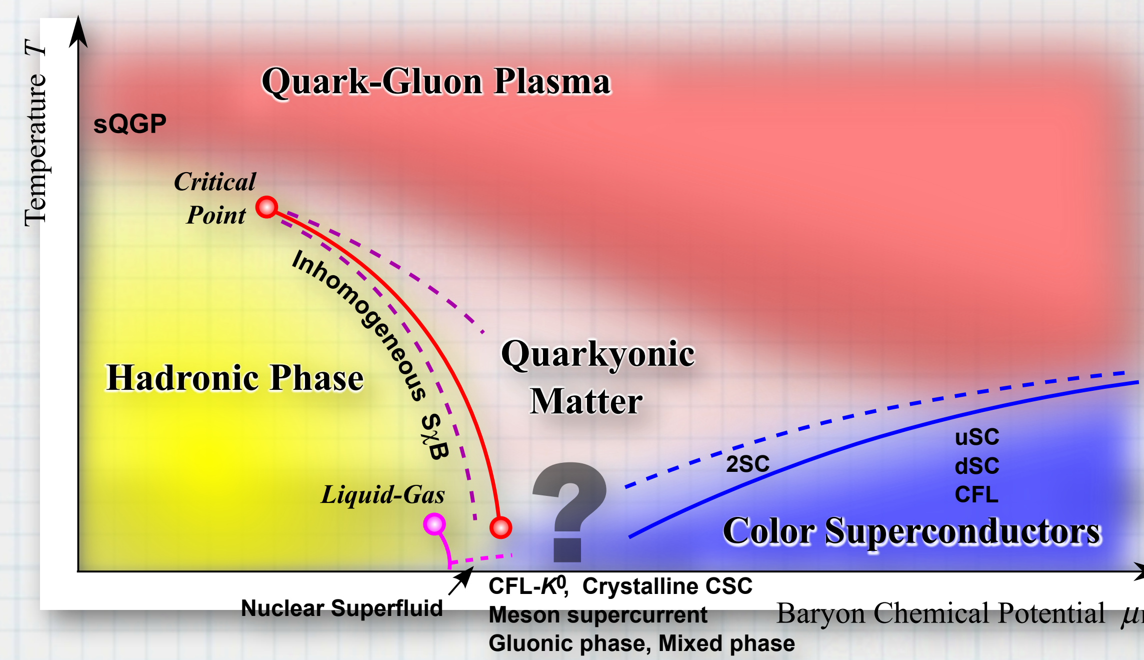
$$V(P,F;\beta,\mu) = -\ln w(P,F;\beta,\mu_0) + \frac{1}{2}\langle\theta^2\rangle_c(P,F;\beta,\mu,\mu_0)$$







Critical point around here ???

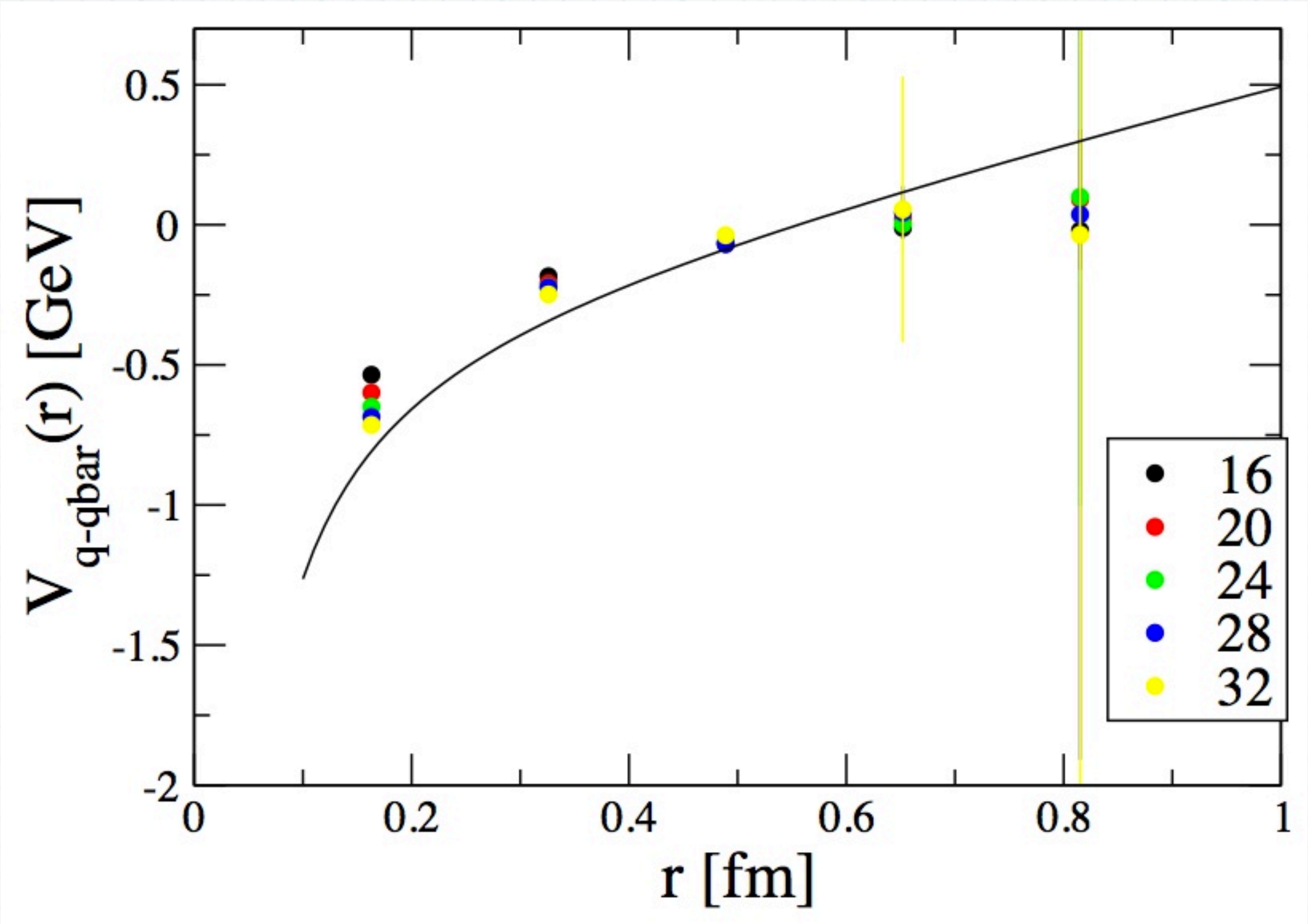


The method looks feasible. Investigation under way.

other topics

heavy quark potential

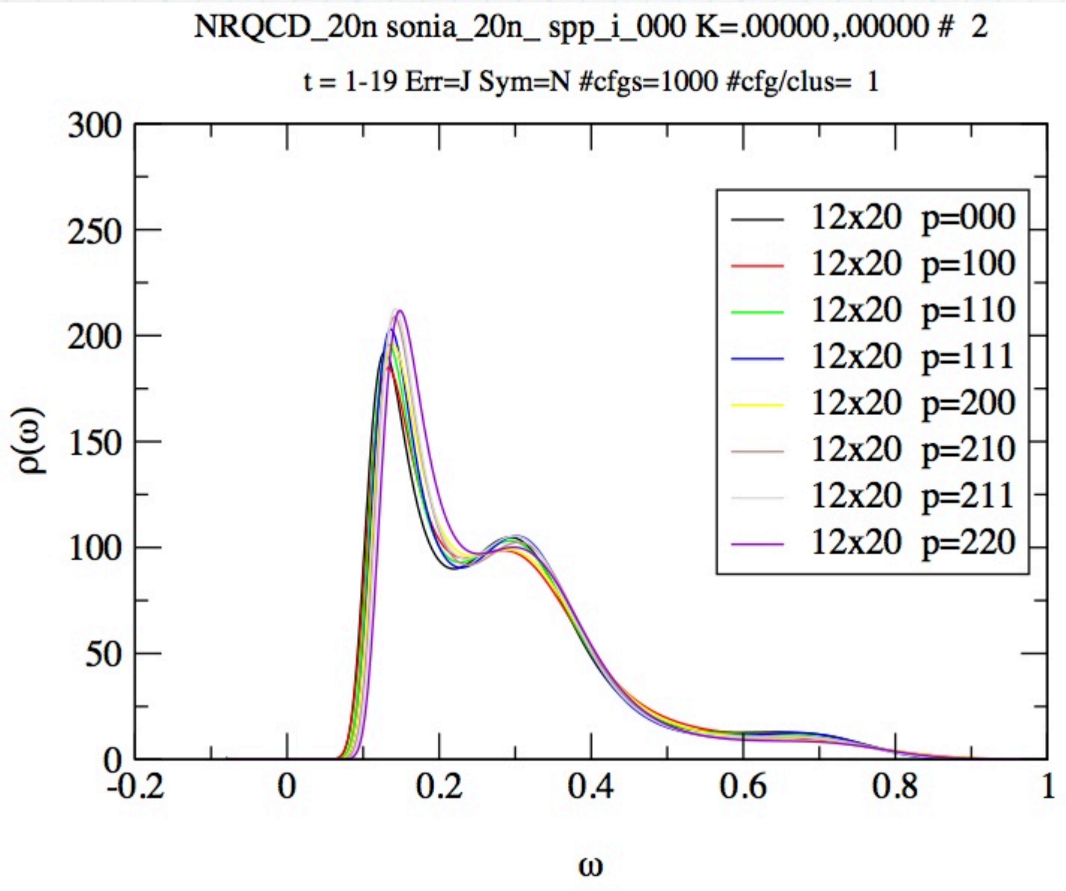
* Allton
  HAL-QCD method to compute V(r)
  T>0, N_F=2, anisotropic ξ≈6, V=12³



preliminary

moving Υ at T>0

* S.Y. Kim
  heavy S-wave state moving in a thermal bath
  MEM with NRQCD on $N_F=2$ $12^3 \times N_t$ anisotropic



$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega')$$

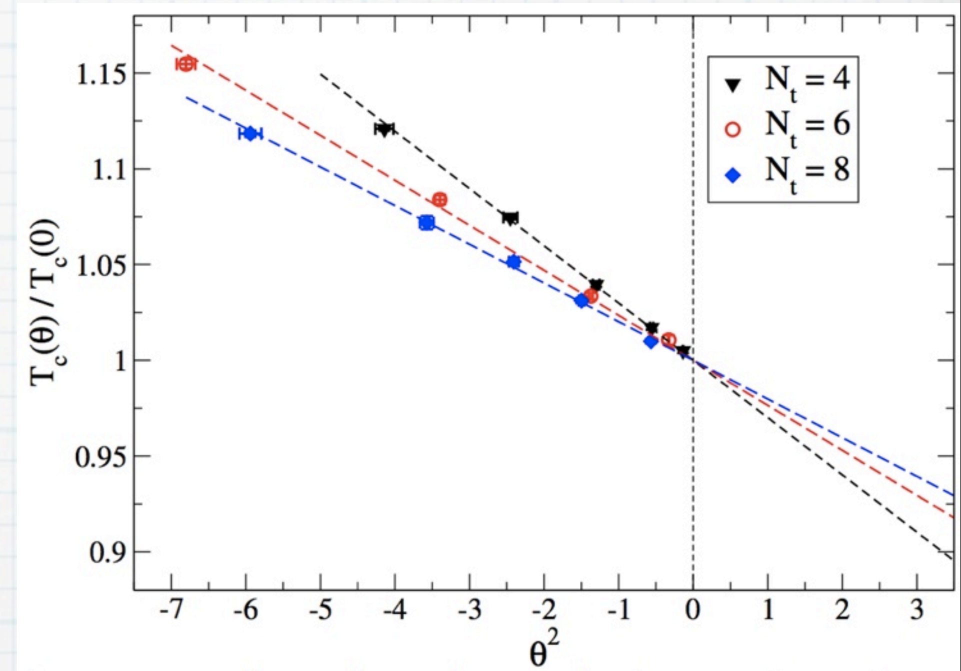
unlike QCD case, the NRQCD kernel is independent of T

$T/M << 1$,

- Temperature effect is more important than the heavy quark mass effect in S-wave bottomonium at the temperature around a few T_c

theta-dependence

* Negro (arXiv:1205.0538)
  simulation with imaginary theta-term
  => analytic continuation to real



$$Z(T,\theta) = \int D[U] e^{-S^L_{YM}[U] - \theta_L Q_L[U]}$$

β_c from Polyakov-loop susceptibility

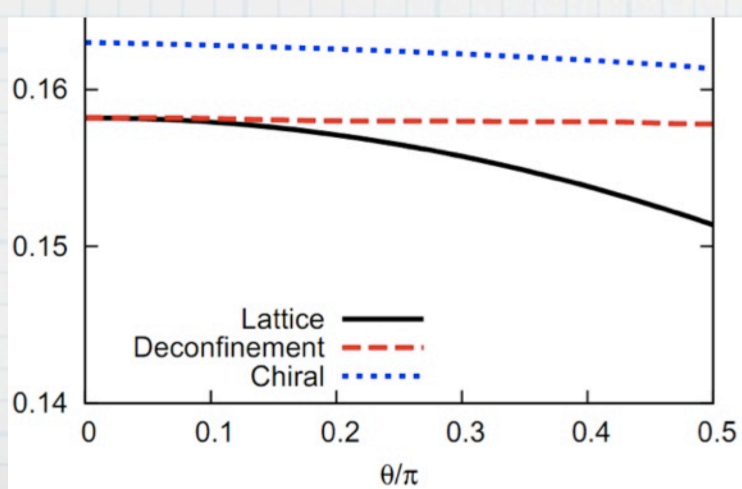
$$\frac{T_c(\theta)}{T_c(0)} = 1 - R_\theta \theta^2 + O(\theta^4)$$

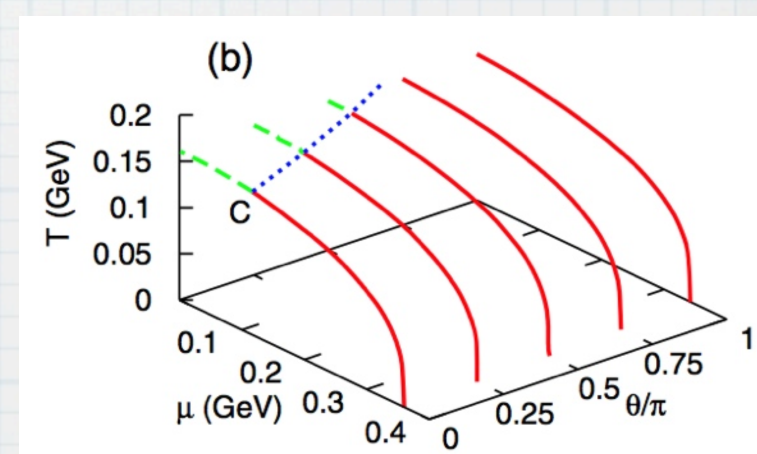
$$R_\theta^{\rm cont} = 0.0175(7)$$

$$R_\theta^{{\rm large}\, N_c}(N_c = 3) = 0.0281(62)$$

T. Sasaki (PRD85) EntanglementPNJ model

T_c





QCD transition at zero chemical potential may be 1st order when theta is large.

minimal-doubling

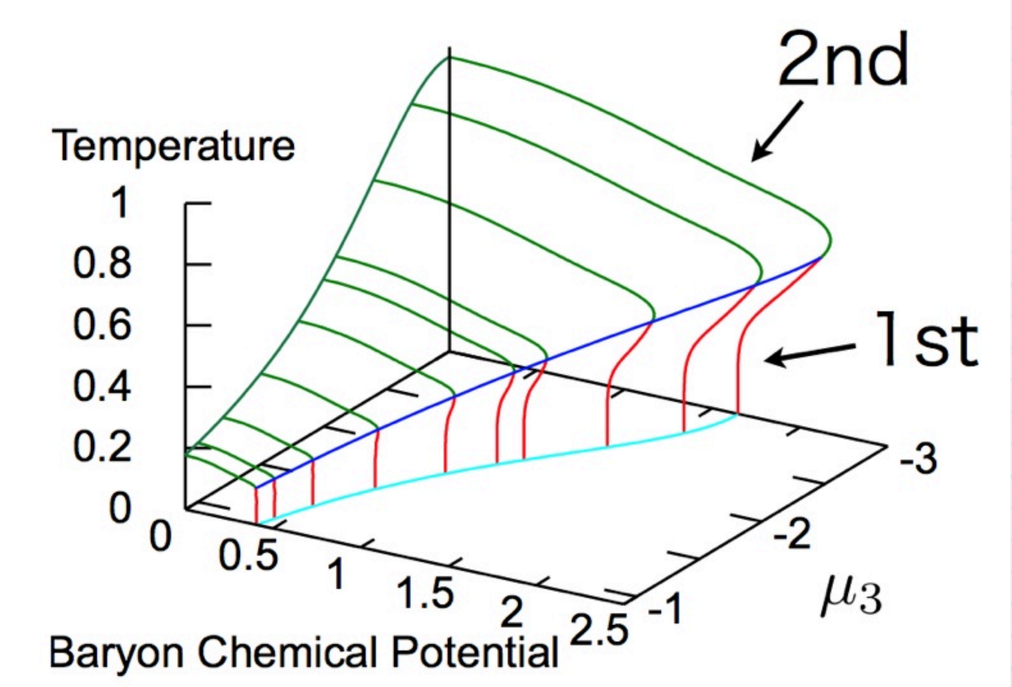
* T. Kimura (arXiv:1206.1977)
  Karsten-Wilczek fermion: 2 doublers
  => N_F=2 at $\mu \neq 0$ keeping (part of) chiral symmetry

$$S_{\mathrm{KW}} = \sum_x \left[\frac{1}{2} \sum_{\mu=1}^{4} \bar{\psi}_x \gamma_\mu (U_{x,x+\hat{\mu}} \psi_{x+\hat{\mu}} - U_{x,x-\hat{\mu}} \psi_{x-\hat{\mu}}) + \frac{r}{2} \sum_{j=1}^{3} \bar{\psi}_x i\gamma_4 (2\psi_x - U_{x,x+\hat{j}} \psi_{x+\hat{j}} - U_{x,x-\hat{j}} \psi_{x-\hat{j}}) \right]$$

+

Counter term : $\mu_3 \bar{\psi}_x i\gamma_4 \psi_x$

strong-coupling analysis =>



summary

Lattice 2012:

several updates/developments since Lattice 2011

- Improved staggered quarks: precision consistency checks near the phys. point
- More efforts are being payed to Wilson and chiral quarks: larger and lighter lattices. => next Lattice conferences.
- $U(1)_A$ recover at $T>T_c$: need to check the V-dependence
- 1st order vs. 2nd order scenario?

- $\mu \neq 0$: 1st order trans. observed for $N_F=4$. on small lattice.
- Critical point by the histogram method soon?

thank you !!