

Magnetic monopole loops generated from calorons with nontrivial holonomy

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共同研究:

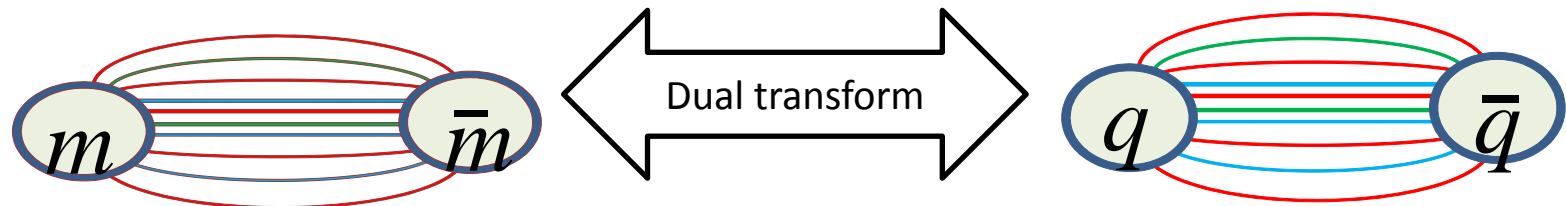
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Based on arXiv:1205.4976v1 [hep-th]

Introduction

- The dual superconductor picture is the promising mechanism for explaining quark confinement.
 - quark confinement is realized by squeezing color electric fields connecting quark and antiquark due to the dual Meissner effect which originates from condensation of magnetic monopoles.
 - there must exist a Yang-Mills field configuration from which a magnetic monopole originated draws a trajectory of a closed loop in four-dimensional space-time (as guaranteed from the magnetic current conservation), while a magnetic monopole is a point-like object in three-dimensional space-time



Introduction (cont') : static potential

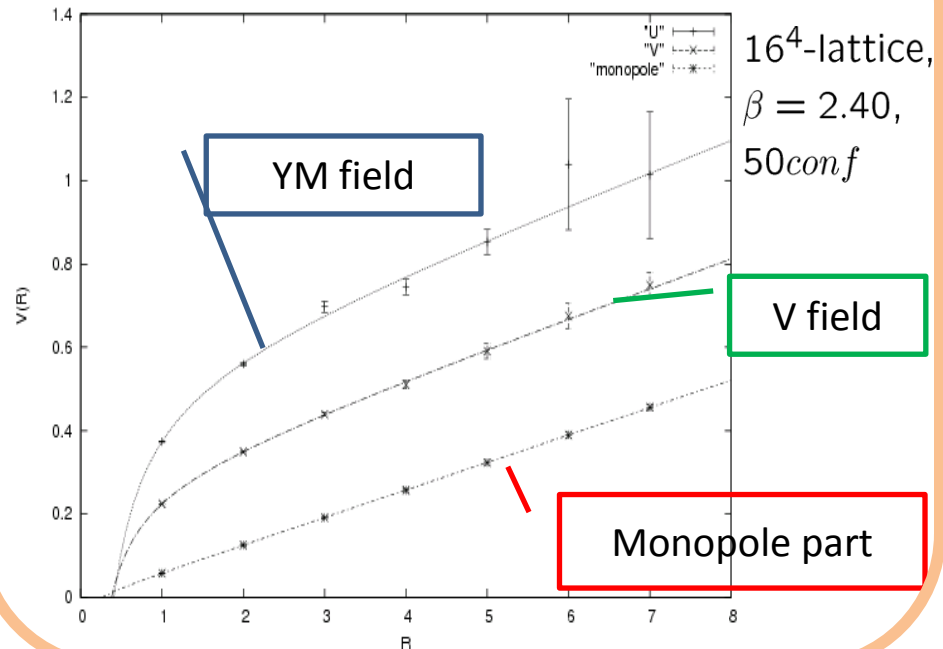
- quark-antiquark potential from Wilson loop operator
- *gauge-independent* “Abelian” Dominance
- The decomposed V field reproduced the potential of original YM field.
 $\sigma_{full} \sim \sigma_V$ ($93 \pm 16\%$)
- *gauge-independent monopole dominance*
- The string tension is reproduced by only magnetic monopole part.

$$\sigma_V \sim \sigma_{monopole} \quad (94 \pm 9\%)$$

$$\sigma_{full} \sim \sigma_{monopole} \quad (88 \pm 13\%)$$

arXiv:0911.0755 [hep-lat]

$$V(R) = c + \frac{\alpha}{R} + \sigma R$$



Introduction (cont')

What is a **Yang-Mills field configuration which contributes to the quark confinement, or causes monopole condensations.**

- A candidate for such a Yang-Mills field configuration will be **the classical solution of the Yang-Mills field equation**, which is expected to give a dominant contribution to the path-integral of the Yang-Mills theory.
- we look for the Yang-Mills field having a nontrivial topological invariant with a desire that it could be related to the magnetic charge in the dual description of the Yang-Mills theory.

Study of topological classical solutions of the Yang-Mills field equation for zero temperature case

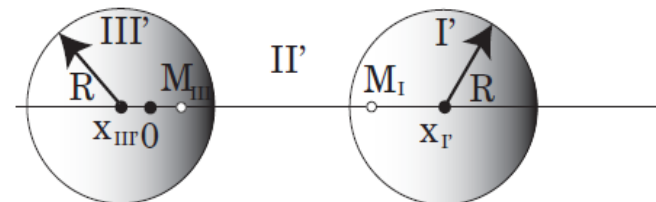
- We **cannot** find any loop of magnetic monopole generated from **one-instanton** and **one-meron** in a numerical way
- The **2-meron (dimeron)** as a non-self-dual solution generates closed loops of magnetic monopole, which go through two poles of the 2-meron in analytical and numerical way.

[Phys.Rev.D78:065033,2008](#)

[PoS LAT2009 \(2009\) 232](#)

Two-meron

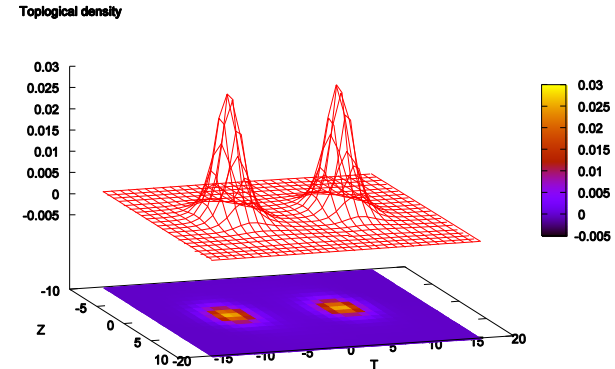
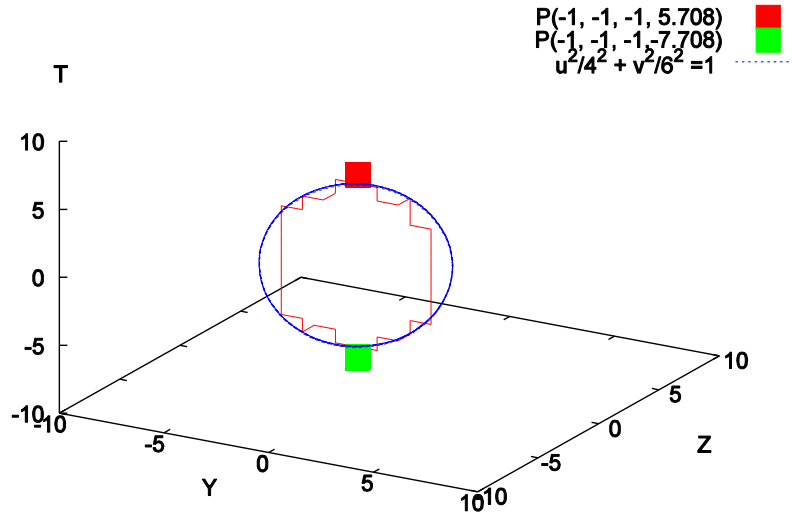
$$\mathbf{A}^{II}(x) = \frac{\sigma^A}{2} \eta_{\mu\nu}^A \left\{ \frac{(x-x'_I)_\nu}{(x-x'_I)^2} + \frac{(x-x'_{III})_\nu}{(x-x'_{III})^2} \right\}$$



$$\mathbf{A}^{I/III}(x) = \frac{\sigma^A}{2} \eta_{\mu\nu}^A \frac{(x-X')_\nu}{(x-X)^2}$$

Monopoles from two-meron

PoS LAT2009 (2009) 232



The plot of a **magnetic-monopole loop** generated by a pair of (smeared) merons in 4-dimensional Euclidean space where, **the gap of the energy between region II and I/III smoothed by using sooling method**. The 3-dimensional plot is obtained by projecting the 4-dimensional dual lattice space to the 3-dimensional one, i.e., $(x; y; z; t) \rightarrow (y; z; t)$. The positions of two meron sources are described by solid boxes, and the monopole loop by red solid line. In the lattice of the volume $[-10, 10]^3 \times [-16, 16]$ with a lattice spacing $2 = 1$, the two-merons are located at $(-1; -1; -1; -1 \pm 6.078)$, and are smeared with the instanton cap of size $R = 3.0$ ($d = 12$, $R1 = 2.833$ and $R2 = 50.833$). The monopole loop is confined in the 3-dim. space $x = -1$ and in a 2-dim. plane rotated about t-axis by 0.46rad . (For guiding the eye, the monopole loop is fitted by an ellipsoid curve (blue dotted line) with the long radius 6 and the short radius 4.)

- We have discovered in a numerical way that a loop of magnetic monopole is from the **two-instanton solution of the Jackiw-Nohl-Rebbi (JNR) type**.

Phys.Rev. D82 (2010) 045015

Two-instanton of 't Hooft type does not generate the magnetic monopole loop. We have clarified why the using a fact that the two-instanton of 't Hooft type is obtained as a special limit of JNR solution.

CHIBA-EP-194-KEK-PREPRINT-2012-10

arXiv:1205.4972 (to be published in PRD)

Two-instanton solution

$$A_{\mu}^B(x) = \eta_{\mu\nu}^B \partial_{\nu} \log \phi(x)$$

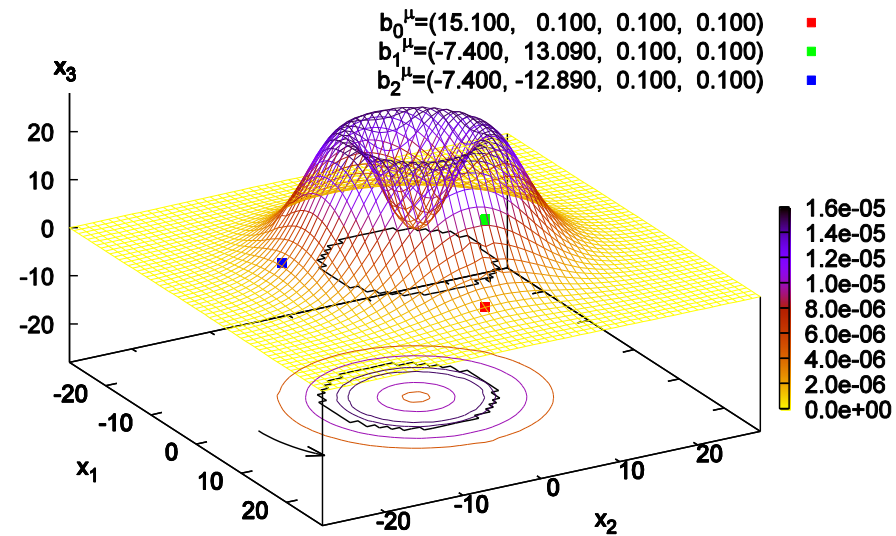
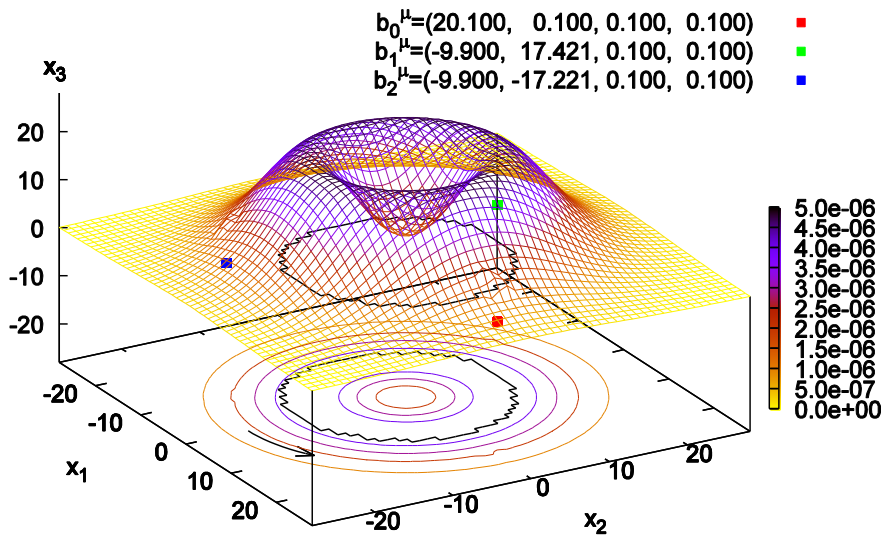
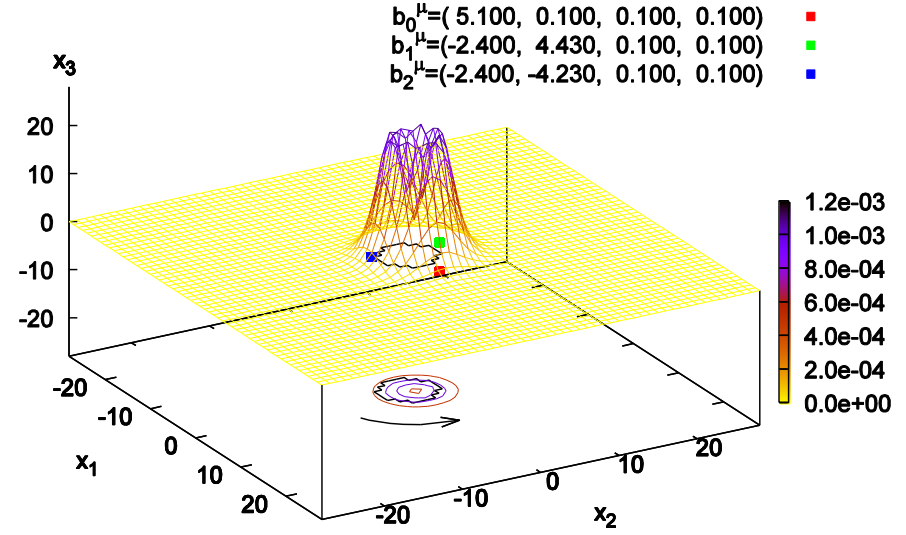
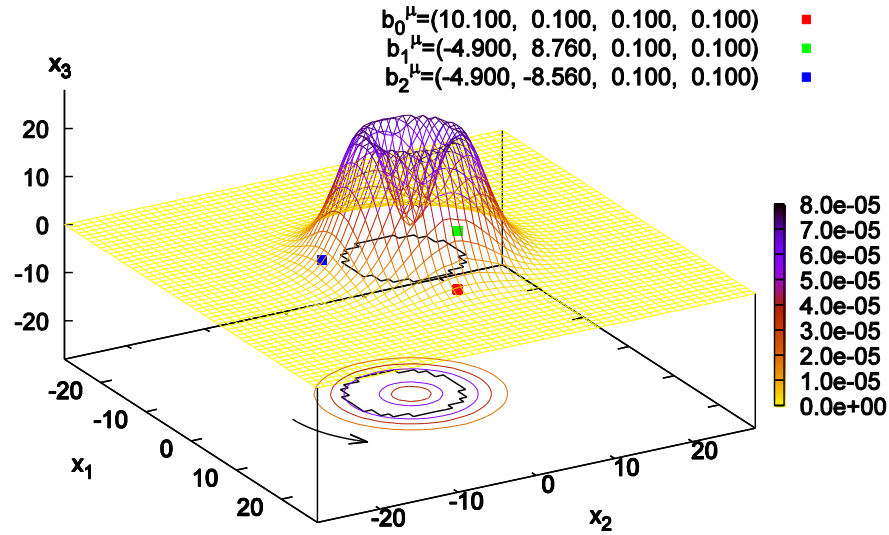
't Hooft type

$$\phi(x) = 1 + \sum_{k=1}^2 \frac{\rho_k^2}{(x-x_k)^2}$$

JNR type

$$\phi(x) = \sum_{k=0}^2 \frac{\rho_k^2}{(x-x_k)^2}$$

JNR type



Caloron as a classical solution at finite temperature

- To understand the **confinement/deconfinement phase transition** in the Yang-Mills theory at finite temperature from the viewpoint of magnetic monopoles.
- We study in a numerical way whether or not the caloron can be a source for loops of magnetic monopoles defined in the space with the period β on $\mathbb{R}^3 \times S^1$.
- Focusing on the β dependence on the behavior of the generated loops.

Caloron

The solution of the self-dual equation for the YM field \mathbf{A} on $\mathbb{R}^3 \times S^1$:

$$*\mathbf{F}_{\mu\nu}(x) = \mathbf{F}_{\mu\nu}(x),$$

where $\mathbf{F}_{\mu\nu}$ is the field strength of \mathbf{A}_μ and $*\mathbf{F}_{\mu\nu}$ is the Hodge dual of $\mathbf{F}_{\mu\nu}$

$$\mathbf{F}_{\mu\nu}(x) = \partial_\mu \mathbf{A}_\nu(x) - \partial_\nu \mathbf{A}_\mu(x) - ig[\mathbf{A}_\mu(x), \mathbf{A}_\nu(x)],$$

$$*\mathbf{F}_{\mu\nu}(x) = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \mathbf{F}_{\rho\sigma}(x).$$

Here the periodic boundary condition should be understood on the gauge field $\mathbf{A}_\mu(x)$ defined on $\mathbb{R}^3 \times S^1$: $\mathbf{A}_\mu(\vec{x}, t + \beta) = \mathbf{A}_\mu(\vec{x}, t)$; $\vec{x} \in \mathbb{R}^3$, $t \in [0, \beta]$, $x = (\vec{x}, t)$, where β is the periodicity or the circumference of S^1

A caloron is classified by

- the topological charge Q with the charge density D :

$$Q = \int d^3x \int_0^\beta dt D(\vec{x}, t) := \frac{1}{16\pi^2} \int d^3x \int_0^\beta dt \text{tr}[\mathbf{F}_{\mu\nu}(\vec{x}, t) *\mathbf{F}_{\mu\nu}(\vec{x}, t)],$$

- A holonomy:

$$H = \lim_{|\vec{x}| \rightarrow \infty} P \exp \left\{ ig \int_0^\beta dt \mathbf{A}_4(\vec{x}, t) \right\},$$

where P represents a path-ordered product along S^1 .

Numerical Method

- To extract magnetic monopoles from a caloron, we use our new formulation of Yang-Mills theory [Prog. Theor. Phys. 115, 201 \(2006\)](#), [Eur. Phys. J. C 42 475 \(2005\)](#), [Phys. Rev. D 74, 125003 \(2006\)](#).
 - This method enables one to extract magnetic monopoles from the original Yang-Mills field [without breaking the gauge symmetry](#).
 - For SU(2), this is originally formulated by Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.
 - For SU(2), this is a gauge-invariant extension of the Abelian projection invented by 't Hooft, while, for SU(3) this method does not necessarily reduce to the Abelian projection and there appear non-Abelian magnetic monopoles.
- To perform in the numerical way, we introduce the lattice version of the decomposition. [Phys. Lett. B 632, 326 \(2006\)](#), [Phys. Lett. B 645, 67 \(2007\)](#), [Phys. Lett. B 653, 101 \(2007\)](#), [Proc. Sci., LATTICE2007 \(2007\) 331](#), [Proc. Sci., LATTICE2008 \(2008\) 268](#), [PoS LAT2009:232,2009](#)

Numerical Method: gauge configuration on a lattice

- A fundamental variable on a lattice is a link variable, and a gauge configuration is presented by the link variable.
- To represent the instant solution on a lattice, we introduce a finite volume hyper cubic lattice:
 - Open boundary condition for space direction $\Leftrightarrow \mathbb{R}^3$
 - Periodic boundary condition with size β for temporal direction $\Leftrightarrow \mathbb{S}^1$

A link variable $U_{x,\mu}$ is related to a gauge field \mathbf{A}_μ in a continuum theory by

$$U_{x,\mu} = P \exp \left\{ -ig \int_x^{x+a\hat{\mu}} dy \mathbf{A}_\mu(y) \right\} = P \prod_{j=0}^{N-1} U_{x+j,\mu}$$

where a is a lattice spacing and $\hat{\mu}$ represents the unit vector in the μ direction. A link variable is calculated by path ordered product of the B-divided parallel transporter, where j -th element is defined by

$$U_{x+j,\mu} = \exp \frac{-ig\epsilon}{2N} \left\{ A_\mu(x + \frac{j}{N}\epsilon\hat{\mu}) + A_\mu(x + \frac{j+1}{N}\epsilon\hat{\mu}) \right\}$$

The decomposition of link variables: SU(2)

$$W_C[U] := \text{Tr} \left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \text{Tr}(\mathbf{1})$$

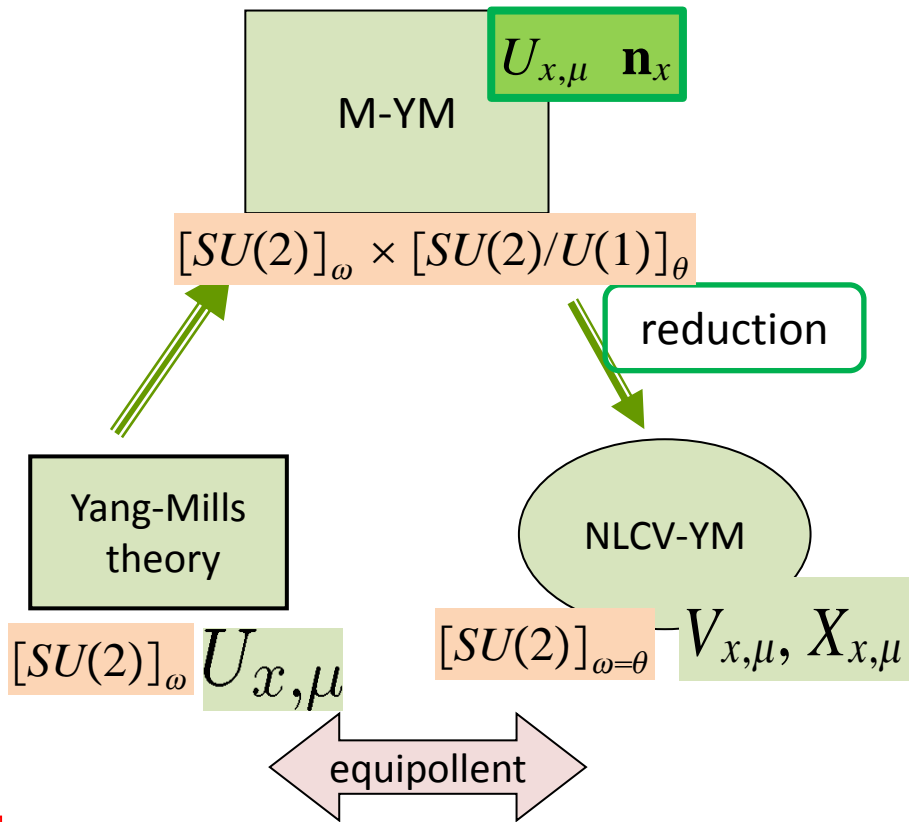
$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger$$

$$W_C[V] := \text{Tr} \left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \text{Tr}(\mathbf{1})$$



$$W_C[U] = \text{const.} W_C[V] !!$$

Reduction condition: determining color field \mathbf{n}

- By minimizing the functional

$$F[\mathbf{n}_x; U_{x,\mu}] = \sum_{x,\mu} \text{tr} \left[(D_\mu^\epsilon[U] \mathbf{n}_x)^\dagger (D_\mu^\epsilon[U] \mathbf{n}_x) \right]$$

- Because of the **finite size lattice**, we need to decide a **boundary condition** of the \mathbf{n} configuration:

We recall that one-caloron configuration approaches a pure gauge at spatial infinity $|\vec{x}| \rightarrow \infty$:

$$g\mathbf{A}_\mu(\vec{x}, t) \rightarrow ih^\dagger(\vec{x}, t)\partial_\mu h(\vec{x}, t) + O(|\vec{x}|^{-2}).$$

Then, $\mathbf{n}(x)$ as a solution of the reduction condition is supposed to behave asymptotically as

$$\mathbf{n}(\vec{x}, t) \rightarrow h^\dagger(\vec{x}, t)T_3h(\vec{x}, t) + O(|\vec{x}|^{-\alpha}),$$

for a certain value of $\alpha > 0$. Under this idea, we adopt a boundary condition:

$$\mathbf{n}_x^{\text{bound}} := h^\dagger(\vec{x}, t)T_3h(\vec{x}, t), \quad \vec{x} \in \partial V_{\mathbb{R}^3}.$$

CFNS decomposition on a lattice (cont')

- We obtain the solution (see [Phys.Lett.B691:91-98,2010](#))

After obtaining the \mathbf{n}_x configuration for given configurations $U_{x,\mu}$ in this way, we introduce a new link variable $V_{x,\mu}$ on a lattice corresponding to the restricted field (ref: V) by

$$V_{x,\mu} = \frac{L_{x,\mu}}{\sqrt{\frac{1}{2} \operatorname{tr}[L_{x,\mu} L_{x,\mu}^\dagger]}},$$
$$L_{x,\mu} := U_{x,\mu} + \mathbf{n}_x U_{x,\mu} \mathbf{n}_{x+a\hat{\mu}}.$$

Note that if the YM gauge link is fixed to the MA gauge, the color field is given by $\mathbf{n}_x = T^3$ and the decomposed variable V_x is obtained by Abelian projection

$$V_{x,\mu} = \frac{u_0 \mathbf{1} + i u_3 \sigma_3}{\sqrt{u_0^2 + u_3^2}} \quad \text{with } U_{x,\mu} = u_0 \mathbf{1} + i \sum_{k=1}^3 u_k \sigma_k$$

Wilson loop operator & magnetic monopole on a lattice

Non-Abelian Stokes' theorem e.g. K.-I. Kondo PRD77 085929(2008)

$$W_C[\mathbf{A}] = \text{tr} \left[P \exp ig \oint_C dx^\mu A_\mu(x) \right] / \text{tr}(\mathbf{1}) = \int [d\mu(\xi)]_\Sigma \exp \left\{ \int_{S:C=\partial S} dS^{\mu\nu} \mathcal{F}_{\mu\nu}[V] \right\}$$

$$= \int [d\mu(\xi)]_\Sigma \exp \left\{ ig \sqrt{\frac{N-1}{2N}} (k, \Xi_\Sigma) + ig \sqrt{\frac{N-1}{2N}} (j, N_\Sigma) \right\}$$

$$\Xi_\Sigma := *d\Theta_\Sigma \Delta^{-1} = \delta * \Theta_\Sigma \Delta^{-1}, N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$$

$$D\text{-dimensional Laplacian } \Delta = d\delta + \delta d$$

Θ_Σ : the vorticity tensor with support on the surface Σ_C spanned by Willson loop C

$$\Theta_\Sigma^{\mu\nu}(x) = \int_\Sigma dS^{\mu\nu}(X(\sigma)) \delta^D(x - X(\sigma))$$

lattice
version

$$\langle W_C[V] \rangle = \langle W_C[\text{Mono}] \rangle = \left\langle \exp \left\{ i \sum_{x,\mu} k_{x,\mu} \Xi_{x,\mu} \right\} \right\rangle$$

$$n_{x,\mu} = \frac{1}{2\pi} k_\mu = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_\nu \mathcal{F}_{x,\rho\sigma}$$

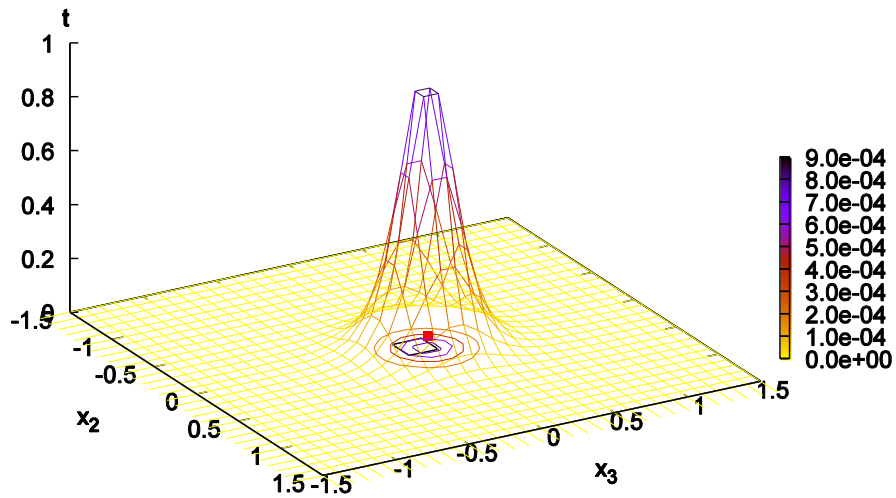
$$\mathcal{F}_{x,\mu\nu} \equiv \text{argTr}[(\mathbf{1} + \mathbf{n}_x) V_{x,\mu} V_{x+\hat{\mu},\nu} V_{x+\hat{\nu},\mu}^\dagger V_{x,\nu}^\dagger]$$

HS caloron

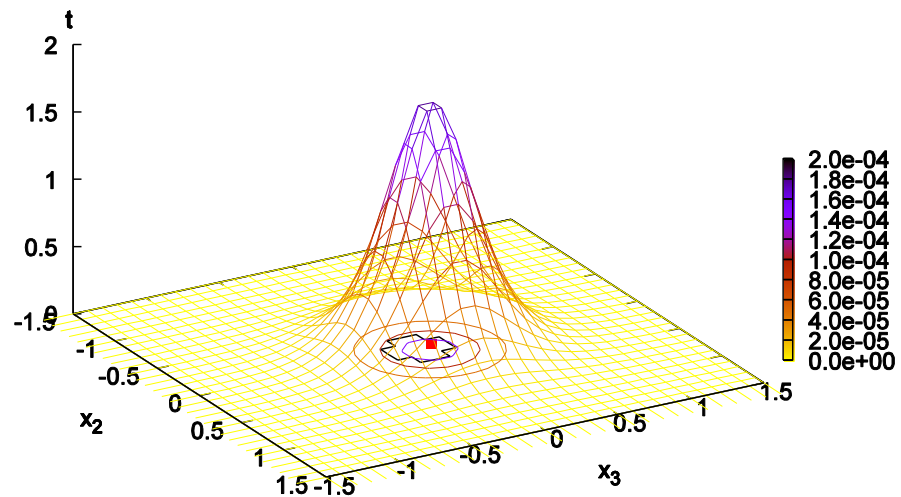
	L_t	β	$ k_{x,\mu} > 1$	$k_{x,\mu} = -1$	$k_{x,\mu} = 0$	$k_{x,\mu} = 1$	Q_V
(a)	10	1.0	0	4	13729992	4	0.952
(b)	20	2.0	0	8	13729984	8	0.975
(c)	30	3.0	0	8	13729984	8	0.980
(d)	40	4.0	0	8	13729984	8	0.983

TABLE I: The distribution of generated configurations of $k_{x,\mu}$ and the charge Q_V for the HS caloron on the lattice with a volume $V = (2aL_x)^3 aL_t$ with fixed $L_x = 35$ and various $L_t = 10, 20, 30, 40$ (with $a = 0.1$).

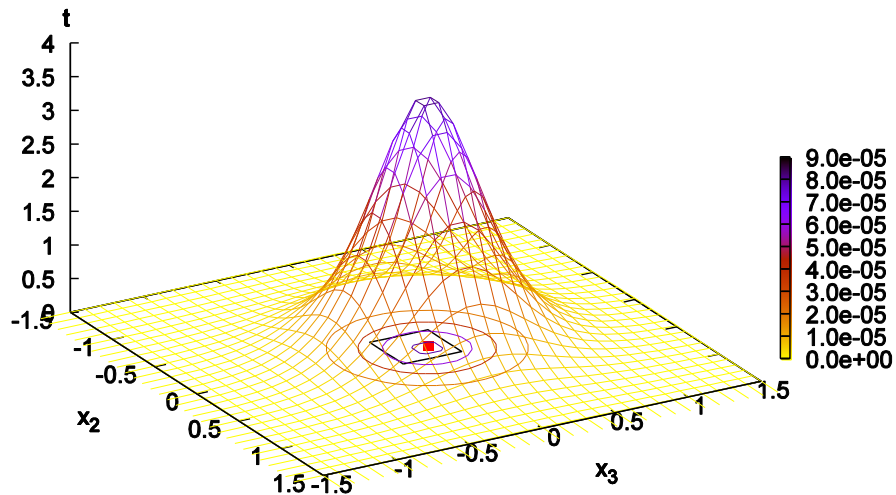
$$x_0^\mu = (0.050, 0.050, 0.050, 0.050)$$



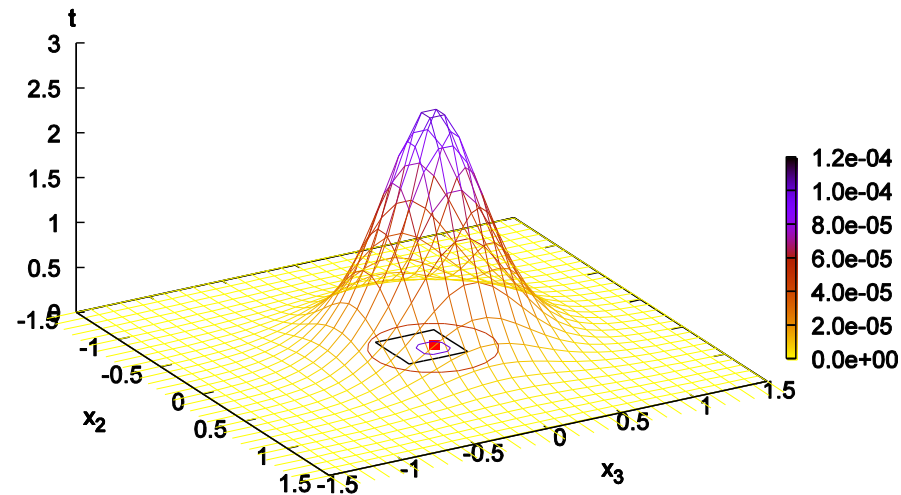
$$x_0^\mu = (0.050, 0.050, 0.050, 0.050)$$



$$x_0^\mu = (0.050, 0.050, 0.050, 0.050)$$



$$x_0^\mu = (0.050, 0.050, 0.050, 0.050)$$



HS caloron

B. J. Harrington and H. K. Shepard ; [PRD17, 2122--2125\(1978\)](#).

- one-caloron having a unit topological charge $Q=1$
- trivial holonomy $H=1$ for the gauge group $SU(2)$

$$g\mathbf{A}_\mu(\vec{x}, t) = -\eta_{\mu\nu}^{A(\mp)} T^A \partial_\nu \log \phi(\vec{x}, t),$$

$$\phi(\vec{x}, t) = 1 + \frac{\lambda \rho^2}{2|\vec{x} - \vec{x}_0|} \frac{\sinh(\lambda|\vec{x} - \vec{x}_0|)}{\cosh(\lambda|\vec{x} - \vec{x}_0|) - \cos(\lambda(t - t_0))},$$

where $T_A = \sigma_A/2$ (σ_A : Pauli matrices) and $\eta_{\mu\nu}^{A(\pm)}$ is the symbol defined by

$$\eta_{\mu\nu}^{A(\pm)} = \epsilon_{A\mu\nu 4} \pm \delta_{A\mu} \delta_{\nu 4} \mp \delta_{A\nu} \delta_{\mu 4}.$$

Here $x_0^\mu = (\vec{x}_0, t_0)$ is the center parameter and ρ is the size parameter, and λ is the parameter associated with β by $\lambda = 2\pi/\beta$.

KvBLL caloron

C. Kraan, van Baal [NPB 533 627\(1998\)](#) & K.Lee, C.Lu [PRD58 025011\(1998\)](#)

- one-caloron having a unit topological charge $Q=1$
- trivial holonomy $H \neq 1$ for the gauge group $SU(2)$

$$g\mathbf{A}_\mu = -\left(\eta_{\mu\nu}^{3(+)}\partial_\nu \log \Phi + v\delta_{\mu 4}\right)T_3 - \Phi \text{Re}\left[\left(\eta_{\mu\nu}^{1(+)} - i\eta_{\mu\nu}^{2(+)}\right)(T_1 + iT_2)(\partial_\nu + iv\delta_{\nu 4})\zeta\right],$$

$$\text{Re}M = (M + M^\dagger)/2 \text{ for the matrix, } \Phi = \frac{\psi}{\hat{\psi}},$$

$$\hat{\psi} = -\cos(\mu(t - t_0)) + \cosh(w|\vec{r}|) \cosh(v|\vec{s}|) + \frac{\vec{r} \cdot \vec{s}}{|\vec{r}||\vec{s}|} \sinh(w|\vec{r}|) \sinh(v|\vec{s}|),$$

$$\psi = \hat{\psi} + \frac{\mu^2 \rho^4}{4|\vec{r}||\vec{s}|} \sinh(w|\vec{r}|) \sinh(v|\vec{s}|) + \frac{\mu \rho^2}{2s} \cosh(w|\vec{r}|) \sinh(v|\vec{s}|) + \frac{\mu \rho^2}{2|\vec{r}|} \sinh(w|\vec{r}|) \cosh(v|\vec{s}|),$$

$$\zeta = \frac{\mu \rho^2}{2\psi} \left(e^{-i\mu(t-t_0)} \frac{\sinh(v|\vec{s}|)}{|\vec{s}|} + \frac{\sinh(w|\vec{r}|)}{|\vec{r}|} \right),$$

$$\mu = \frac{2\pi}{\beta}, \quad v = \frac{\theta}{\beta}, \quad w = \frac{2\pi - \theta}{\beta} = \mu - v,$$

$$\vec{r} = \vec{x} - \vec{x}_0 + \frac{\rho^2}{2\beta} \vec{\theta}, \quad \vec{s} = \vec{x} - \vec{x}_0 - \frac{\rho^2}{2\beta} \frac{w}{v} \vec{\theta}, \quad \vec{\theta} = (0, 0, \theta),$$

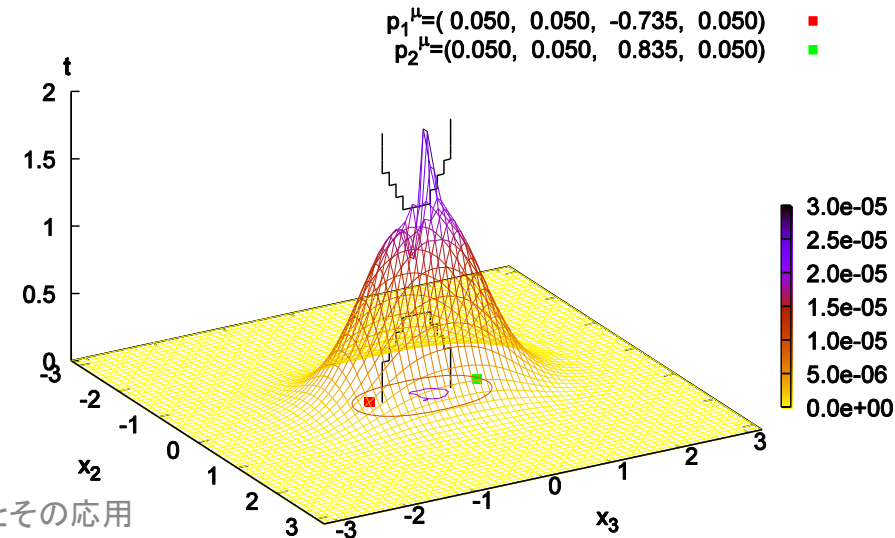
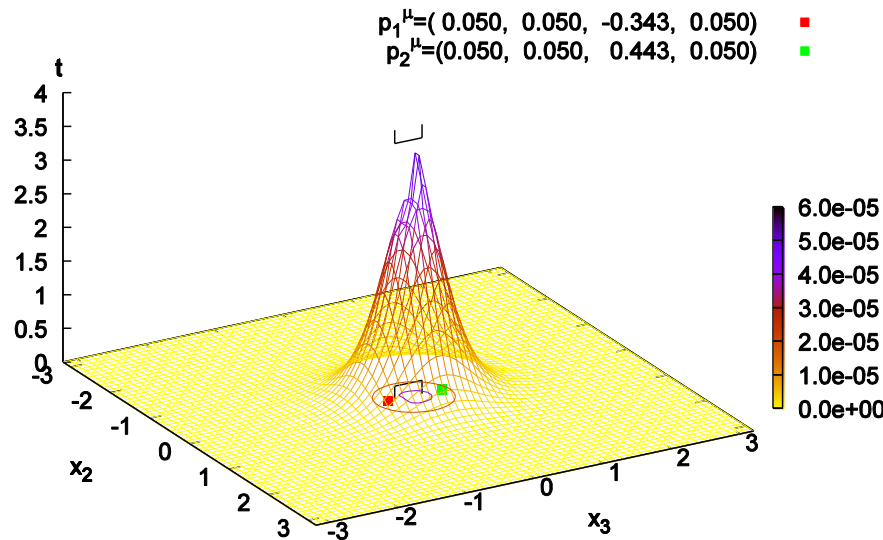
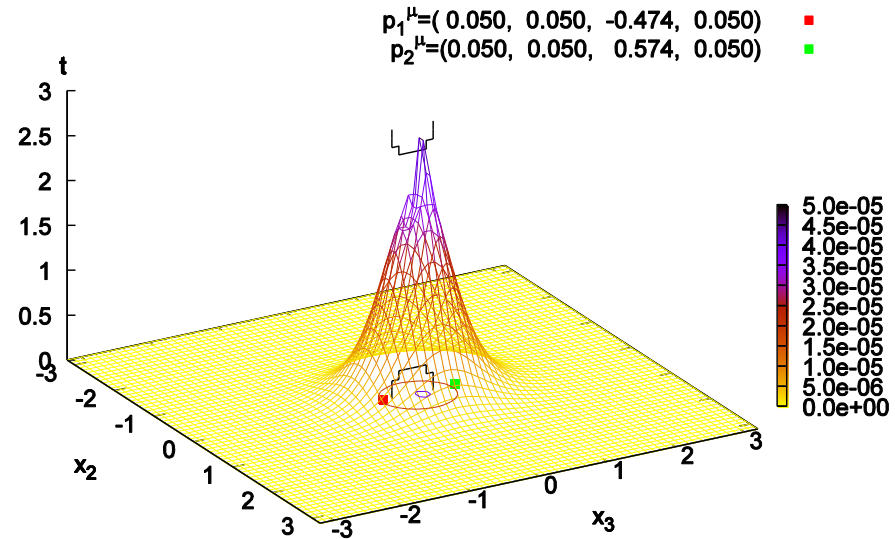
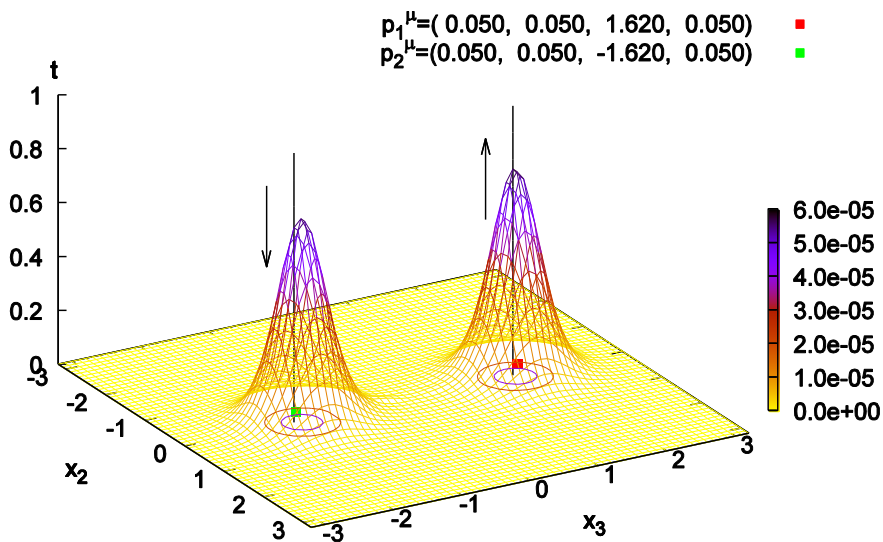
where $x_0^\mu = (\vec{x}_0, t_0)$ and ρ are respectively the center and the size parameters, and θ is the parameter related to the holonomy. The KvBLL caloron is characterized by the parameters

KvBLL caloron

nontrivial holonomy H is fixed by taking $\theta=\pi$.

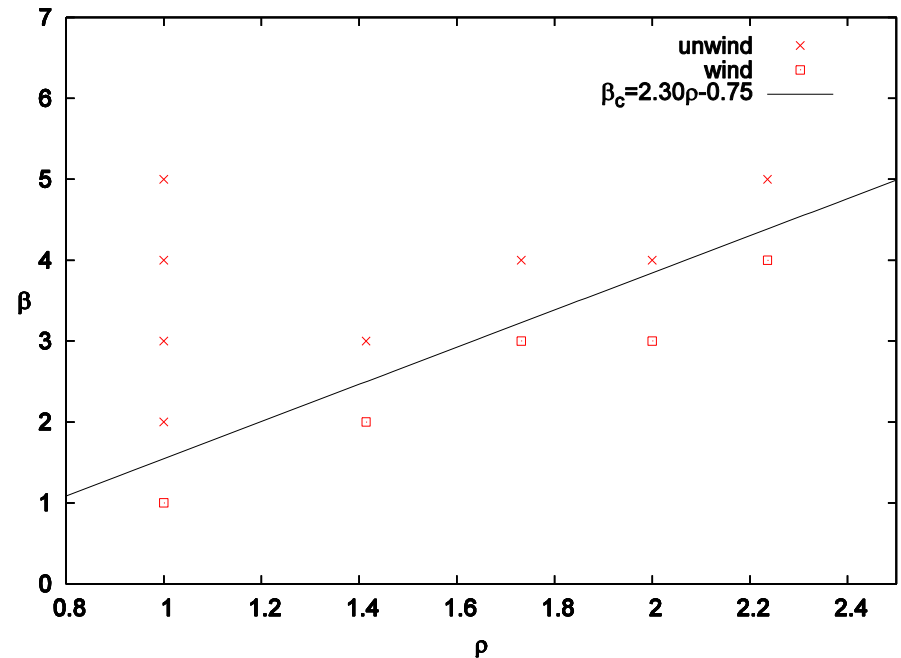
	L_t	β	$ k_{x,\mu} > 1$	$k_{x,\mu} = -1$	$k_{x,\mu} = 0$	$k_{x,\mu} = 1$	Q_V
(a)	10	1.0	0	10	13719980	10	0.973
(b)	20	2.0	0	22	13719956	22	0.986
(c)	30	3.0	0	12	13719976	12	0.987
(d)	40	4.0	0	8	13719984	8	0.987

TABLE II: The distribution of generated configurations of $k_{x,\mu}$ and the charge Q_V for the KvBLL caloron on the lattice with a volume $V = (2aL_x)^3 aL_t$ with fixed $L_x = 35$ and various $L_t = 10, 20, 30, 40$ (with $a = 0.1$).



β dependence of monopole loops

- ρ in the range $\rho^2=1,2,3,4,5$. FIG. shows whether the resulting magnetic loop winds or not along S^1 for various choices of Lt and ρ .
- FIG. indicates that the critical circumference β_c at which the winding number of the loop varies exists and that β_c depends on ρ : β_c and ρ have a positive correlation, which is schematically shown in FIG.



Summary and Discussion

- We have investigated the possible magnetic monopole content in the one-caloron solution, i.e., a periodic self-dual solution of the Yang-Mills field equation with the period β defined on $\mathbb{R}^3 \times S^1$.
- We have shown in the numerical way
 - the one-caloron solution with nontrivial holonomy, i.e., KvBLL caloron, can be a source of the closed loop of magnetic monopoles
 - while the one-caloron with trivial holonomy, i.e., HS caloron, does not generate the magnetic monopole loop.
- The magnetic loop generated from the KvBLL caloron changes its topological behavior depending on the magnitude of the periodicity β , which is the length of the circumference of S^1 in $\mathbb{R}^3 \times S^1$.

- Since the β is identified with the inverse temperature T^{-1} in the Yang-Mills theory at finite temperature,
 - this result could be a clue to understand the phase transition from confinement phase to the deconfinement phase at finite temperature from the viewpoint of magnetic monopole according to the dual superconductor picture for the QCD vacuum