基研研究会「熱場の量子論とその応用」

冷却フェルミ原子の実験を通じて中性子物質を探る

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科研費 KAKENHI

新学術領域研究
“実験と観測で解き明かす中性子星の核物質”
「冷却原子を用いた中性子過剰な低密度核物質の状態方程式」

2012/8/24@京都大学 基礎物理学研究所 湯川記念館 Panasonic 国際交流ホール

Contents

- **Neutron star and our new project**
- Universal many-body Fermi system using cold atoms
- Experimental method to measure the EOS
- Summary

Neutron stars

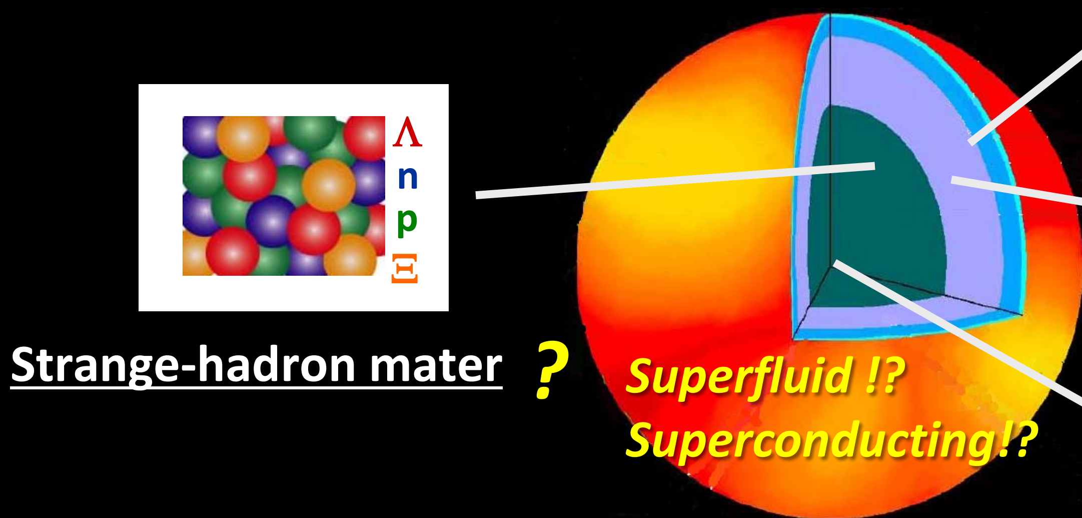
- The highest dense matter of the observable universe

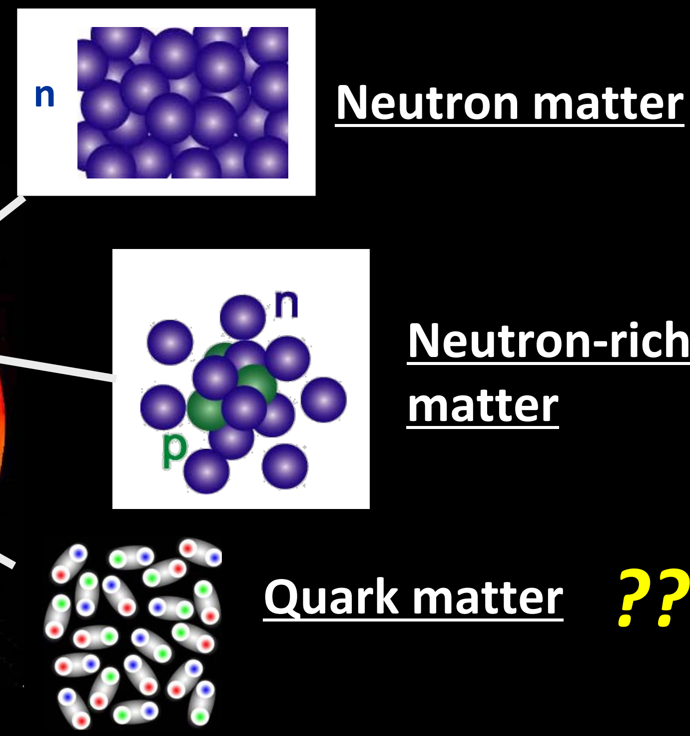
  Mass: 1~2 $M_\odot$ (solar mass) 、 Radius : ~10 km?

  ⇒ Density in the core : 3~10ρ_0 (ρ_0=0.16fm^{-3})

  = Giant nuclei floating in space

- Treasure box of many-body physics

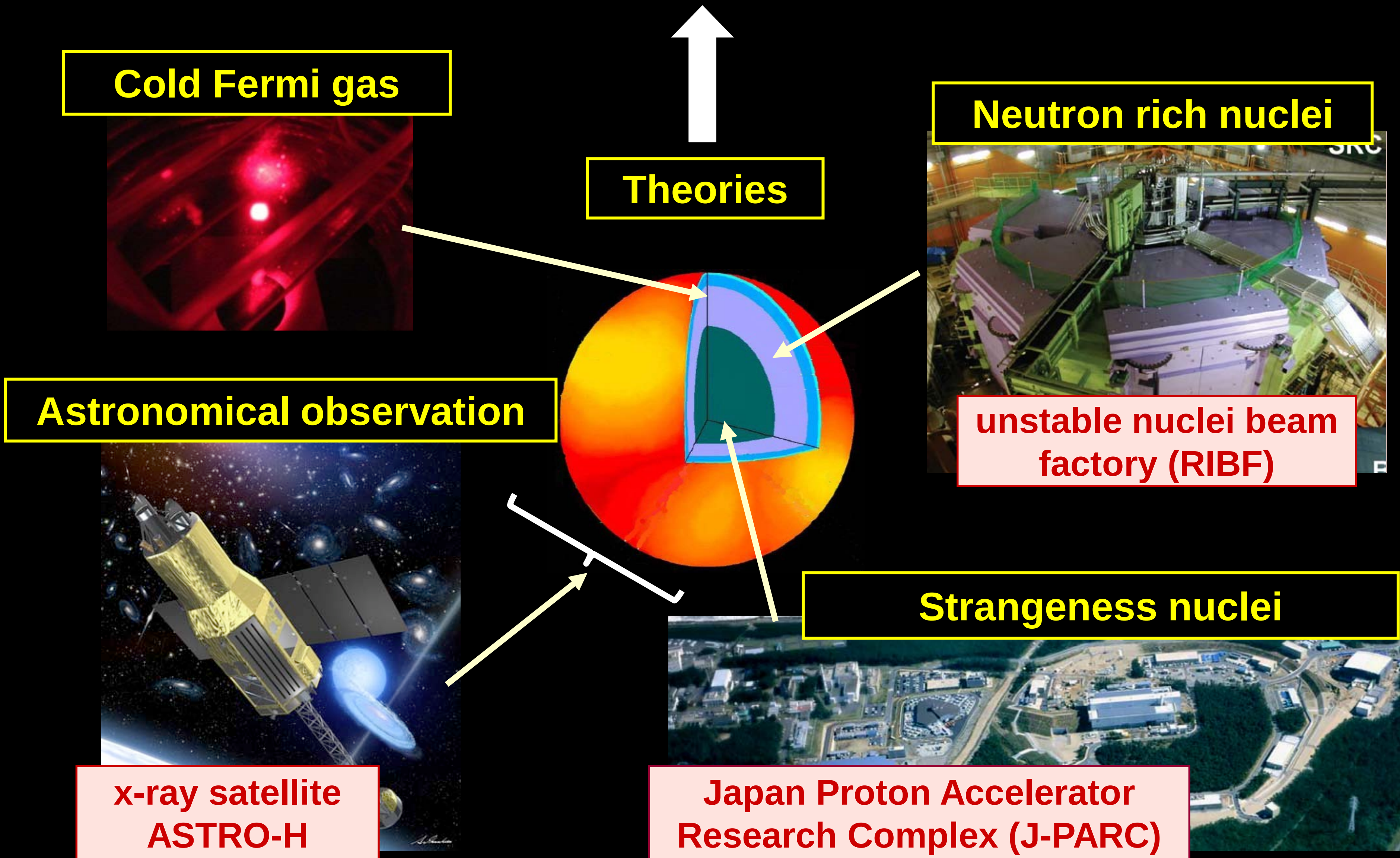




Internal structures and the EOS to sustain the star ?

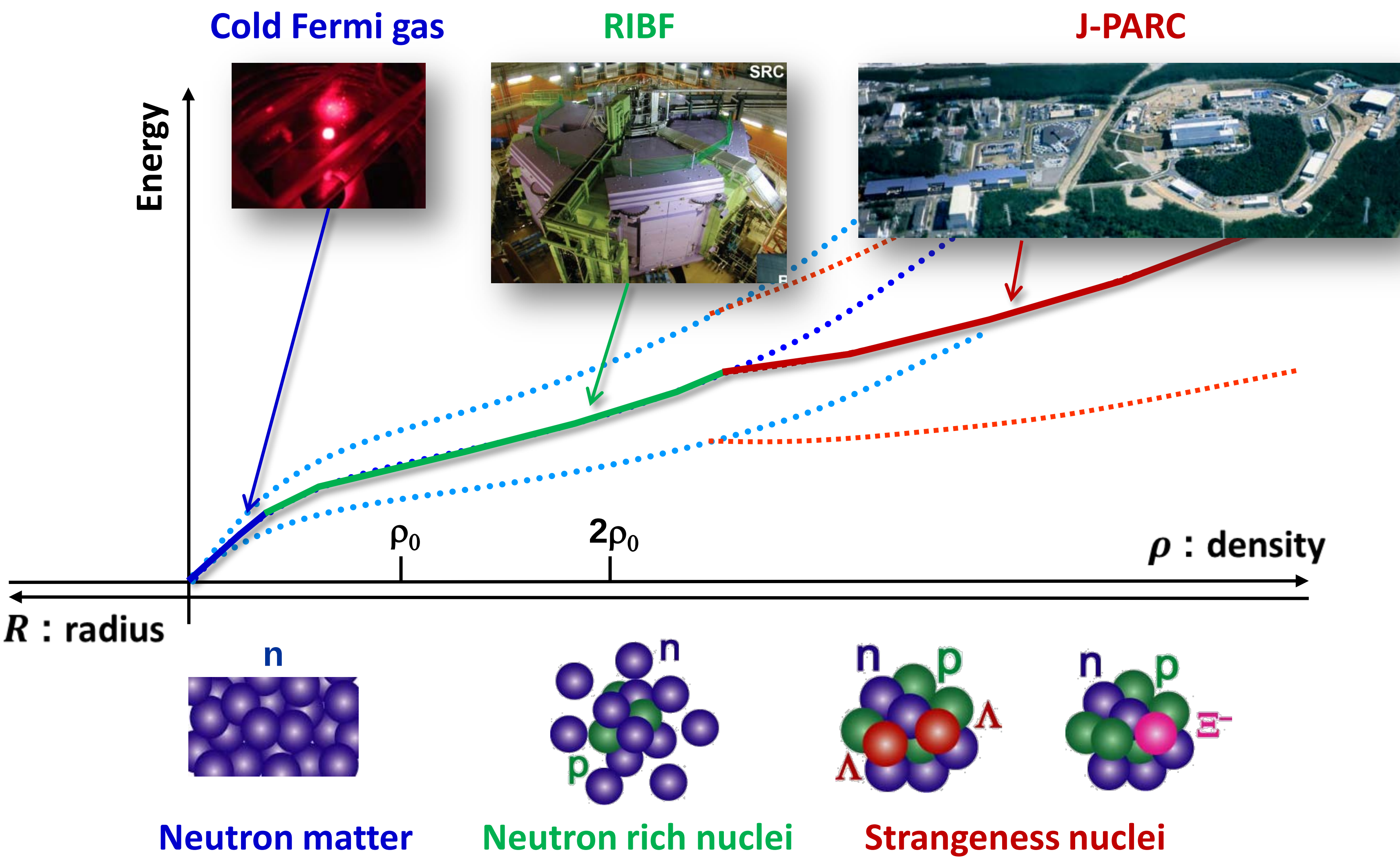
Combination of Experiment - Observation - Theory

Determination and verification of the EOS



Ground experiment : Measurement of the EOS

Equation of state of nuclear matter : $E = f(\rho, (n_n, n_p, n_\Lambda, \cdots))$

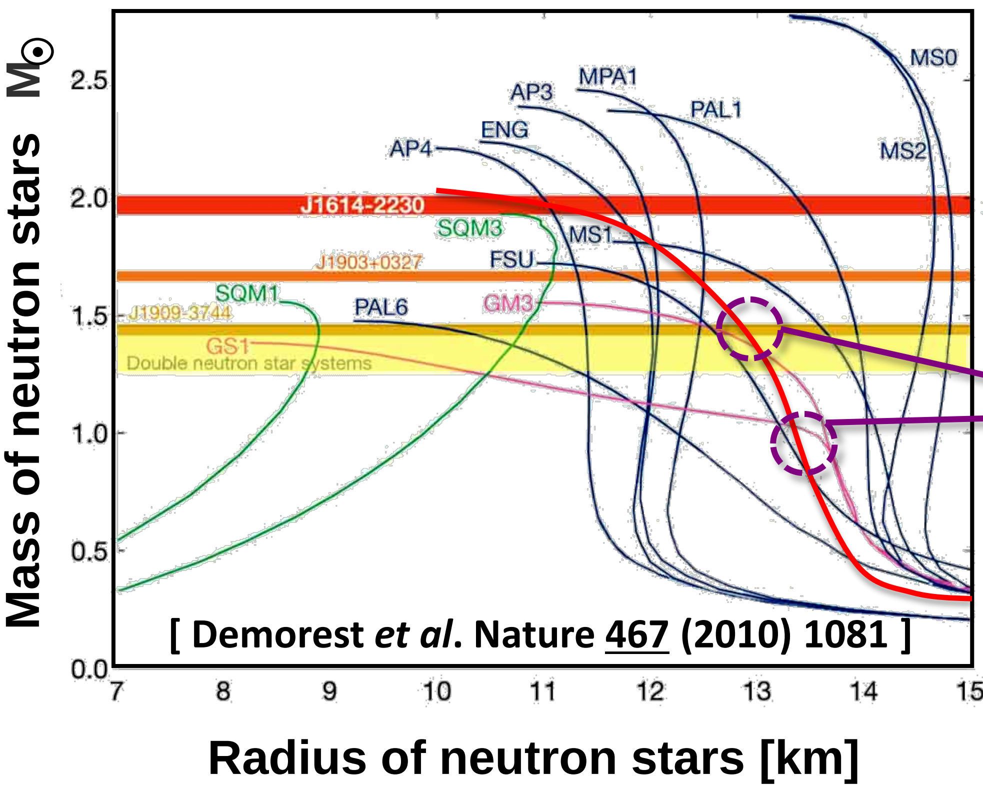


Astronomical observation : Verification of the EOS

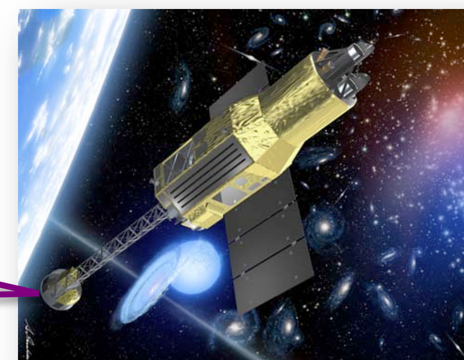
Equation of state of nuclear matter : $E = f(\rho, (n_{\rm n}, n_{\rm p}, n_{\Lambda}, \cdots))$

Balance between gravity and pressure

Mass-Radius curve



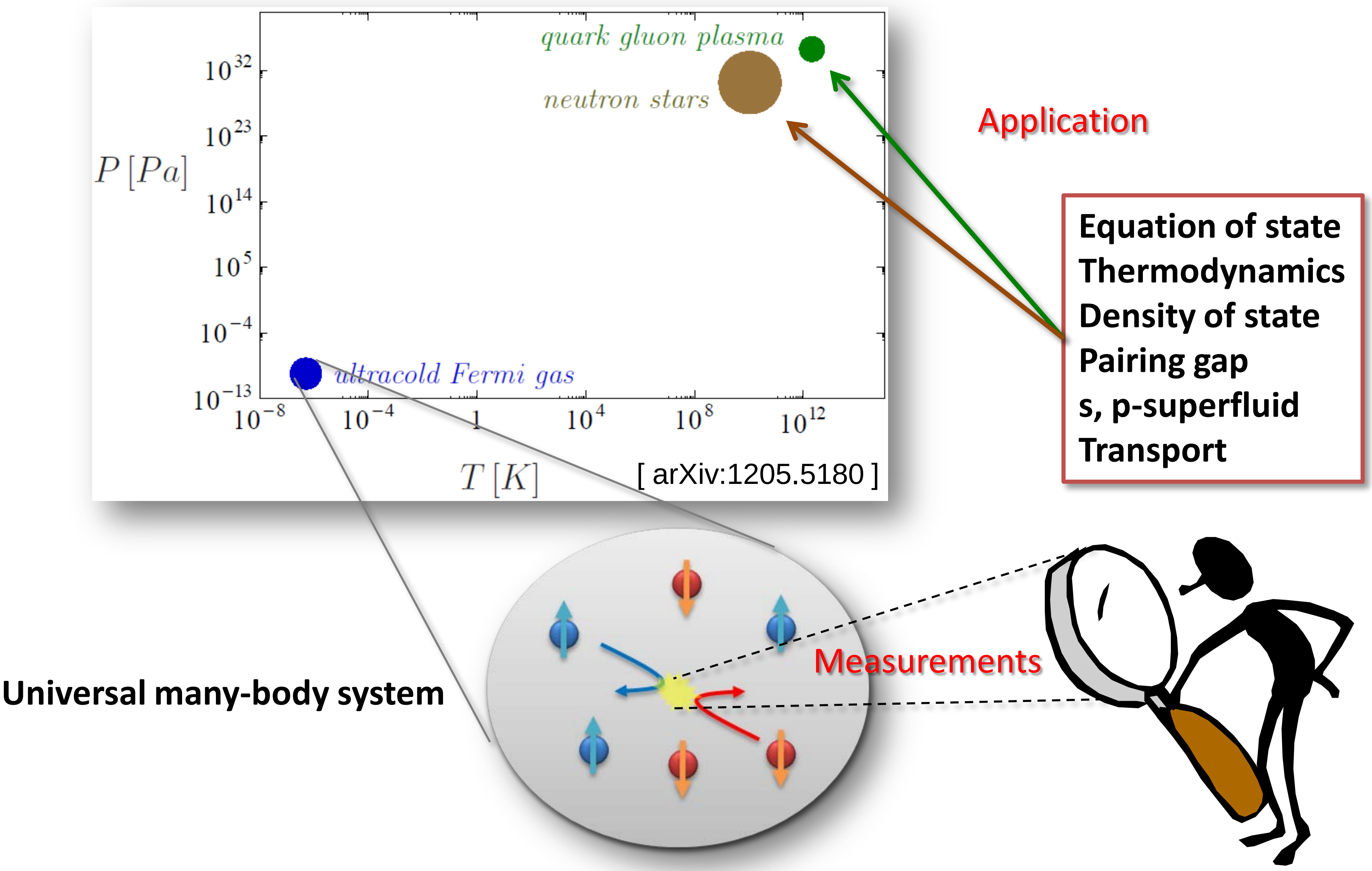
x-ray satellite ASTRO-H (launched in 2014)



Verification of the EOS

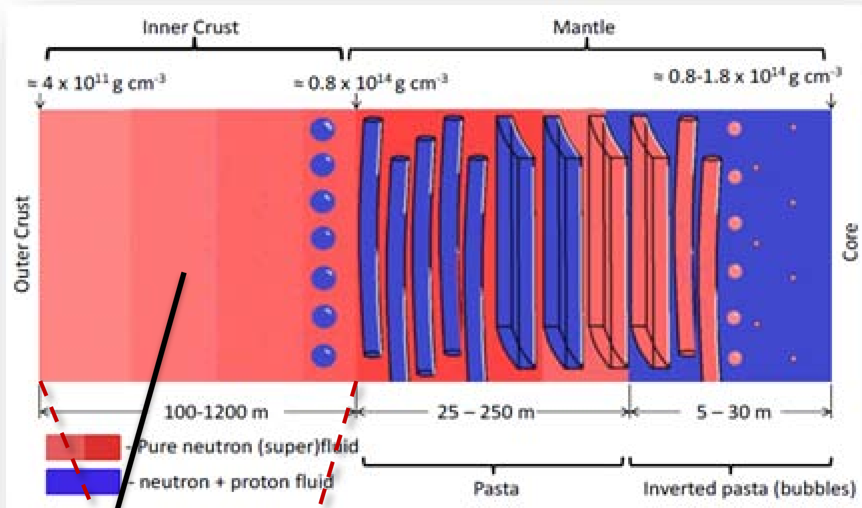
Mission of our cold atom team

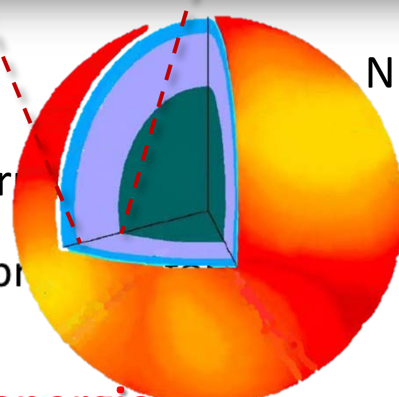
Measurement of universal many-body physics and applications



How cold atoms helps to understand neutron stars?

Internal structure of neutron stars [arXiv:1112.2018]



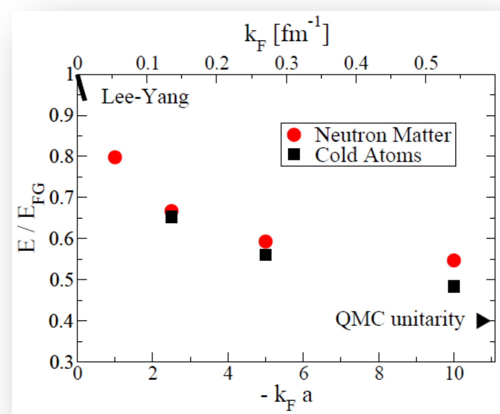


Numerical calculations of energies @ *T*=0 [arXiv:1109.4946]

Homogeneous 2-spin Fer

Challenging many-body pr 1

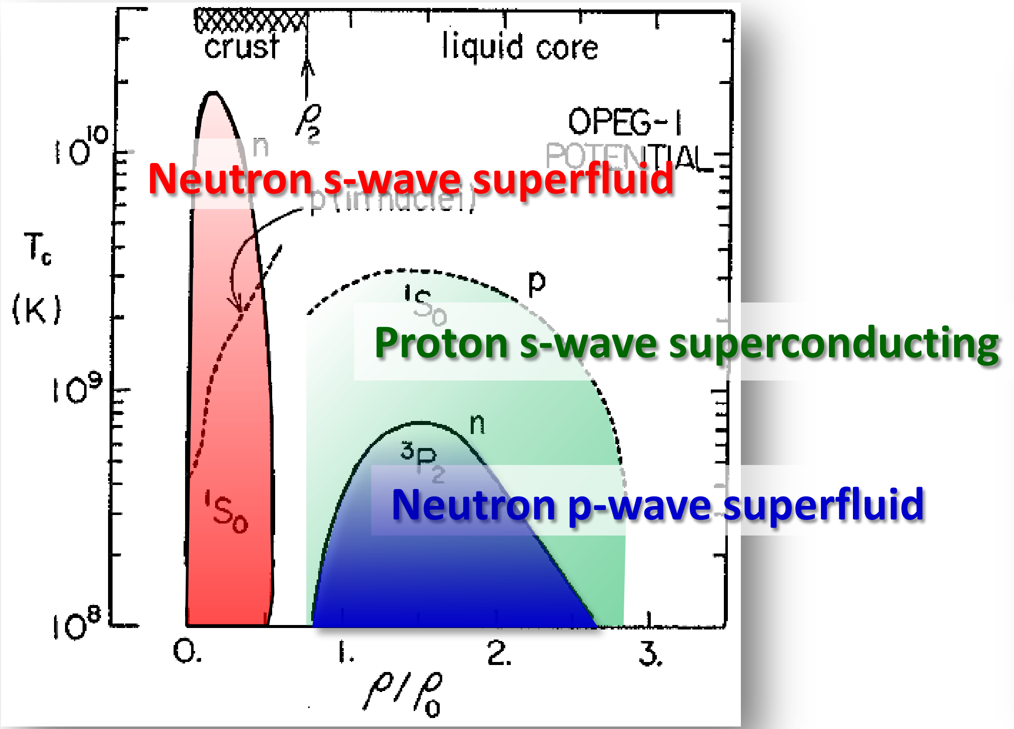
- **Lower limit of energies**
- **Benchmark of many-body theories**



How cold atoms helps to understand neutron stars?

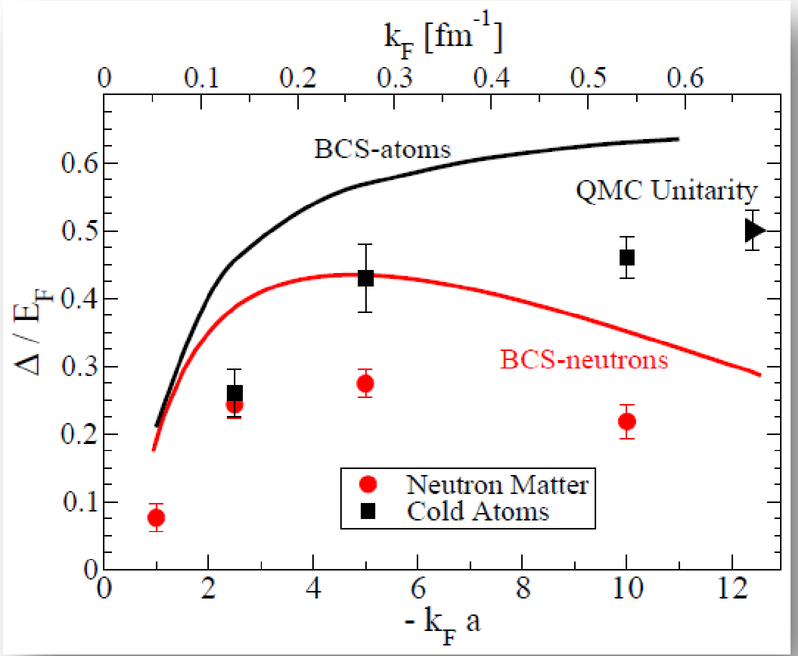
Superfluidity and pairing gaps

Superfluid phase diagram in neutron stars



[T. Takatsuka and R. Tamagaki, Progress of Theoretical Physics Supplement 112, 27 (1993)]

Numerical calculations of gaps [arXiv:1109.4946]



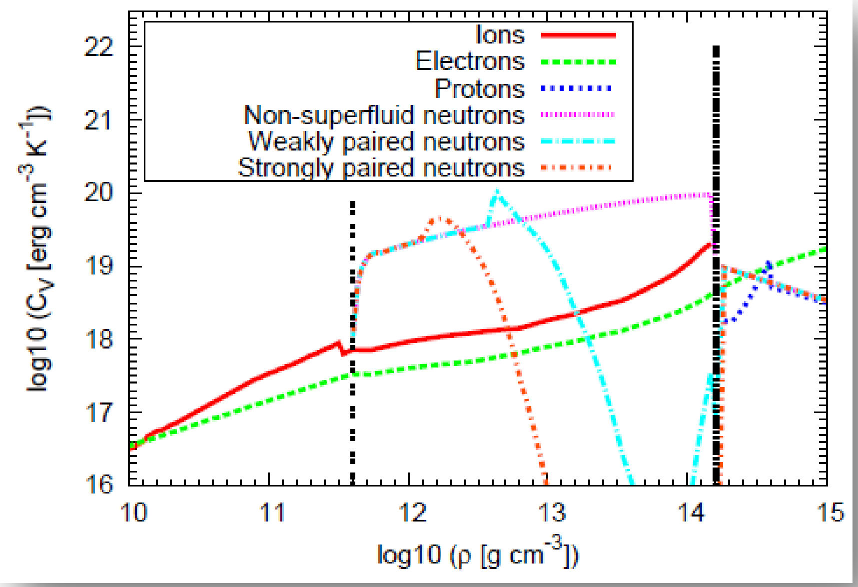
Effect of the effective range is not negligible

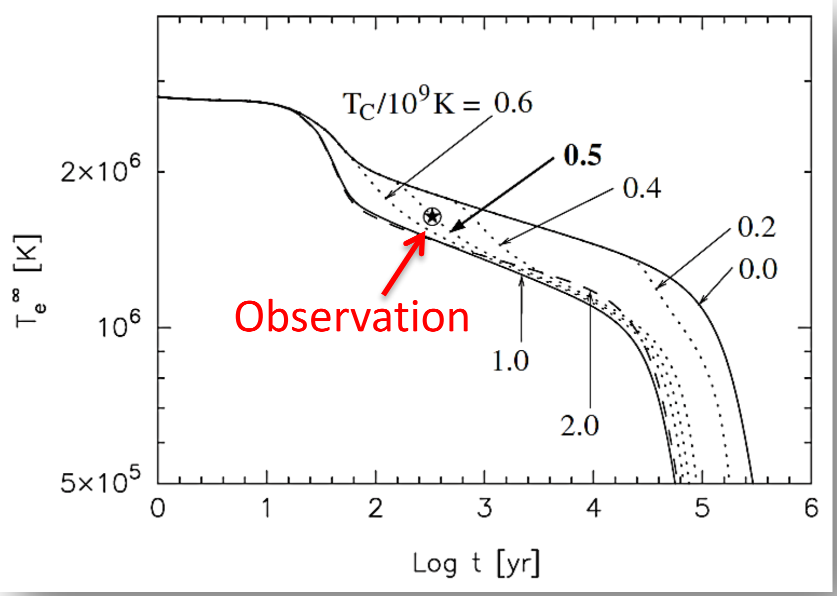
How cold atoms helps to understand neutron stars?

Specific heat and cooling curve

Specific heat in the crust of neutron stars @ T=10^9K
[arXiv:1201.2774]

Cooling curve of a neutron star
[PRL 106, 081101 (2011)]





Finite temperature experiment using cold atoms →

Thermodynamic properties
Critical temperature
Superfluid density
Gaps

Curve of the neutron star ←

How cold atoms helps to understand neutron stars?

Origin of high-T_C Fermi superfluid (superconducting)

	T_C	T_C/T_F
BCS superconductors	5K	5×10^{-5}
^{3}He	2.7mK	5×10^{-4}
High-T_C superconductors	100K	10^{-2}
Neutron matter	**10^9K**	**0.1**
Atomic Fermi gases	200nK	**0.2**

- **Density of state, spectrum function**
- **Superfluid transition temperature**
- **Pairing gaps, size of cooper pairing**

“実験と観測で解き明かす中性子星の核物質”

「冷却原子を用いた中性子過剰な低密度核物質の状態方程式」

東大グループ：s波で相互作用しているフェルミ粒子系

電通大グループ：p波で相互作用しているフェルミ粒子系
（向山研）

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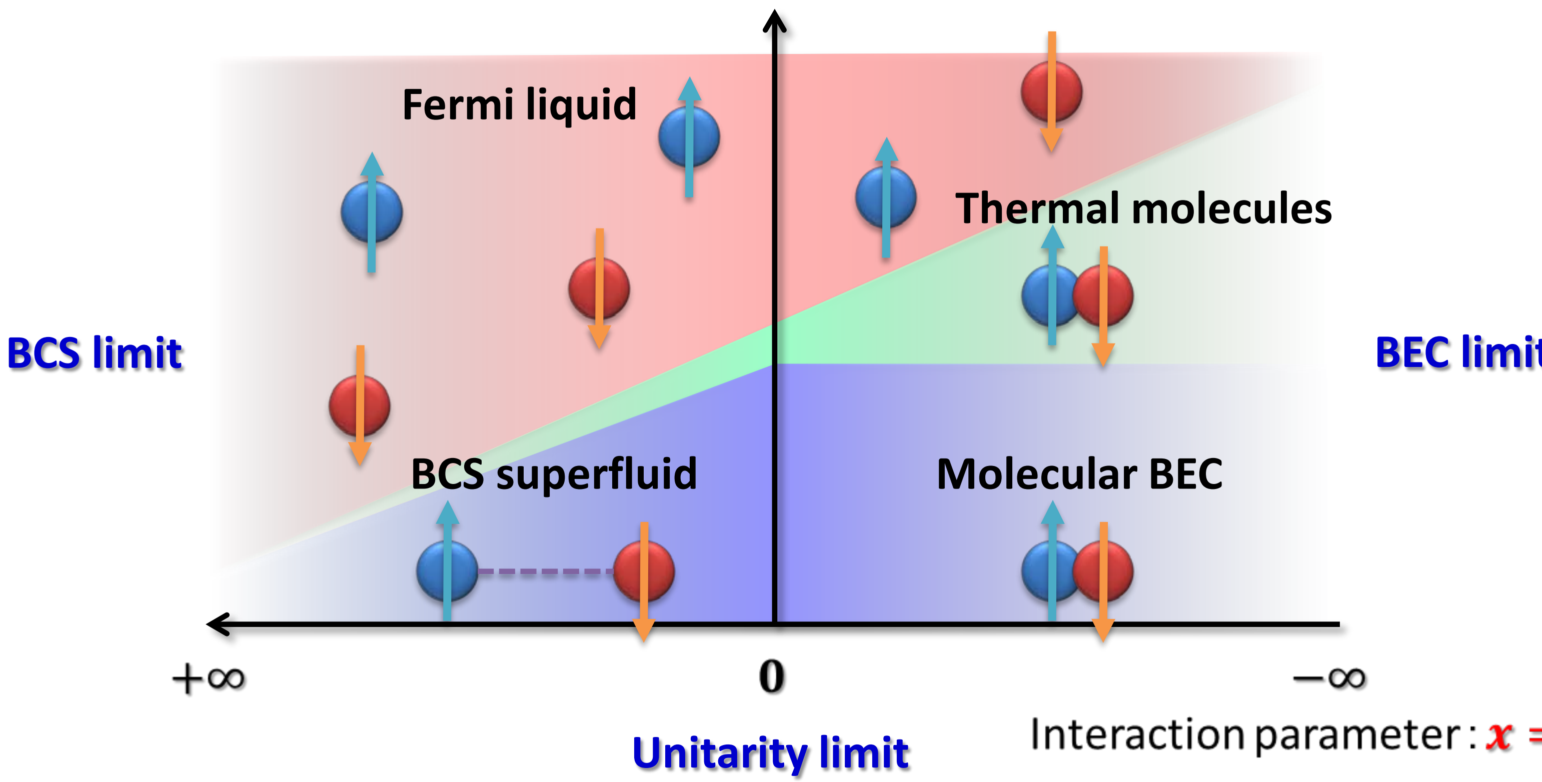
Universal many-body physics using cold atoms

BCS-BEC crossover

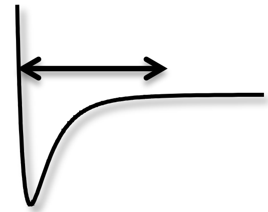
Temperature parameter : $\theta = \frac{T}{T_F}$



Interaction parameter : $x = -\frac{1}{k_F a_s}$

Lenghth scales in cold atom systems

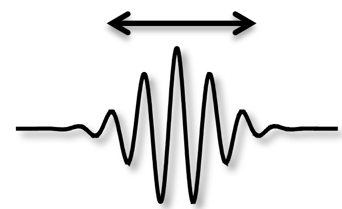
- Van der Waals length : $R_{\mathrm{vdw}} = \frac{1}{2}\left(\frac{2\mu C_6}{\hbar^2}\right)^{1/4} = 1.7\mathrm{nm}$



- Effective length : $r_{\mathrm{e}} = 4.7\mathrm{nm}$ @834Gauss (Feshbach resonance)

[Scattering amplitude : $f(k) = \dfrac{1}{-\frac{1}{a_s} + \frac{1}{2} r_e k^2 - ik}$]

particle-specific parameters



- Thermal length : $\Lambda_{\mathrm{T}} = \dfrac{\hbar}{\sqrt{2\pi m k_{\mathrm{B}} T}} \sim 100\mathrm{nm}$@1μK

- Inter-particle spacing : $n^{-1/3} \sim k_{\mathrm{F}}^{-1} \sim 100\mathrm{nm}$

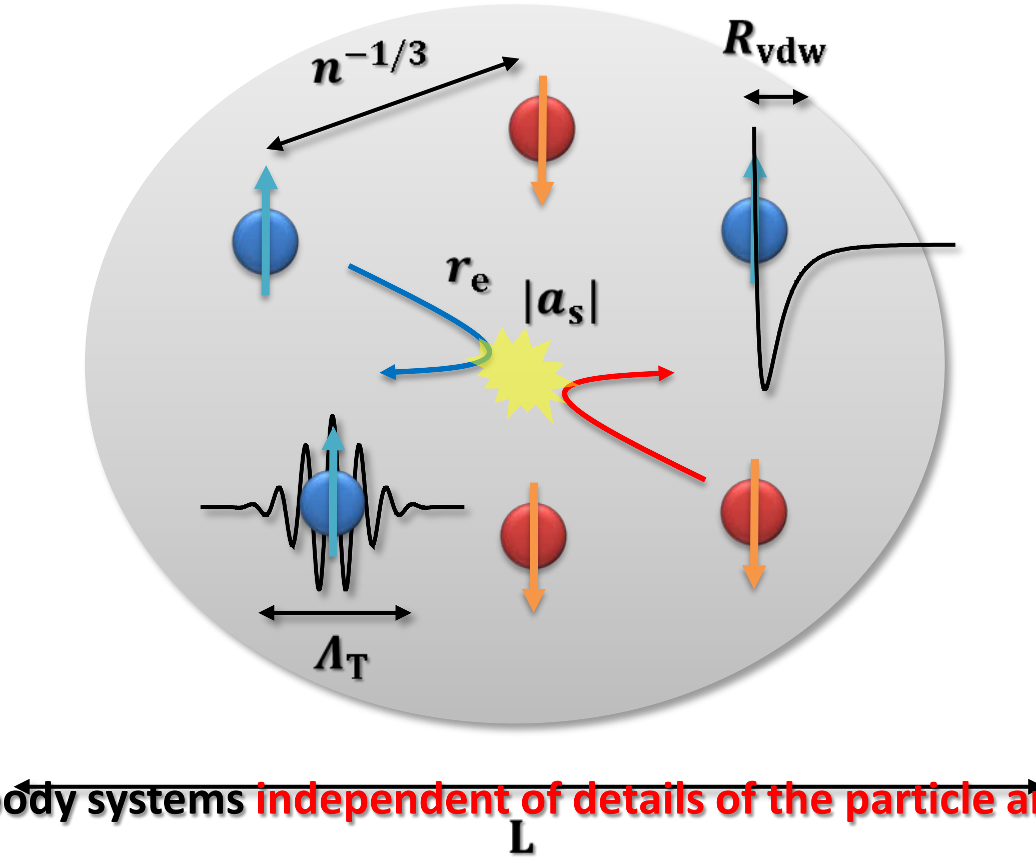
- S-wave scattering length : $|a_{\mathrm{s}}| = 0 \sim \infty$ (by Feshbach resonances)

- Size of potential : $\mathrm{L} \sim 10\mu\mathrm{m}$ } System-specific parameter

Universal many-body system

Length scales : $R_{\mathrm{vdw}} < r_{\mathrm{e}} \ll \Lambda_{\mathrm{T}},\ n^{-\frac{1}{3}},\ |a_{\mathrm{s}}| \ll \mathrm{L}$

Energy scales : $k_B T,\ \ \varepsilon_F,\ \frac{\hbar^2}{m a_s^2}$



Universal many-body systems independent of details of the particle and the system

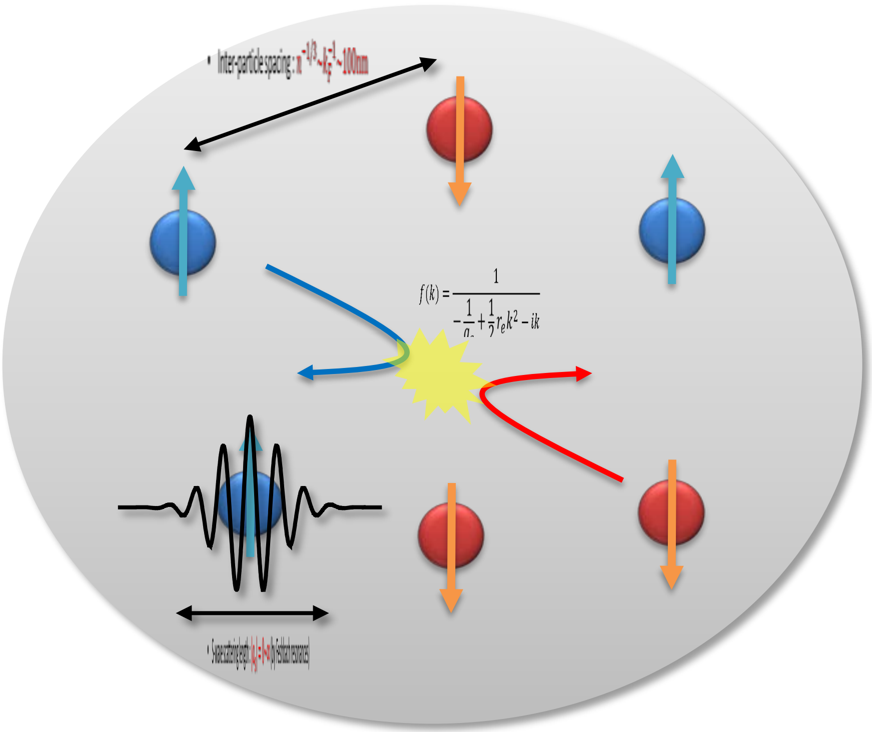
T=0, $|a_s|$=∞

One length scale : ε_F

Internal energy : $E_0^{Unitary} = N\varepsilon_F \times Const = \xi \times E_0^{Ideal}$

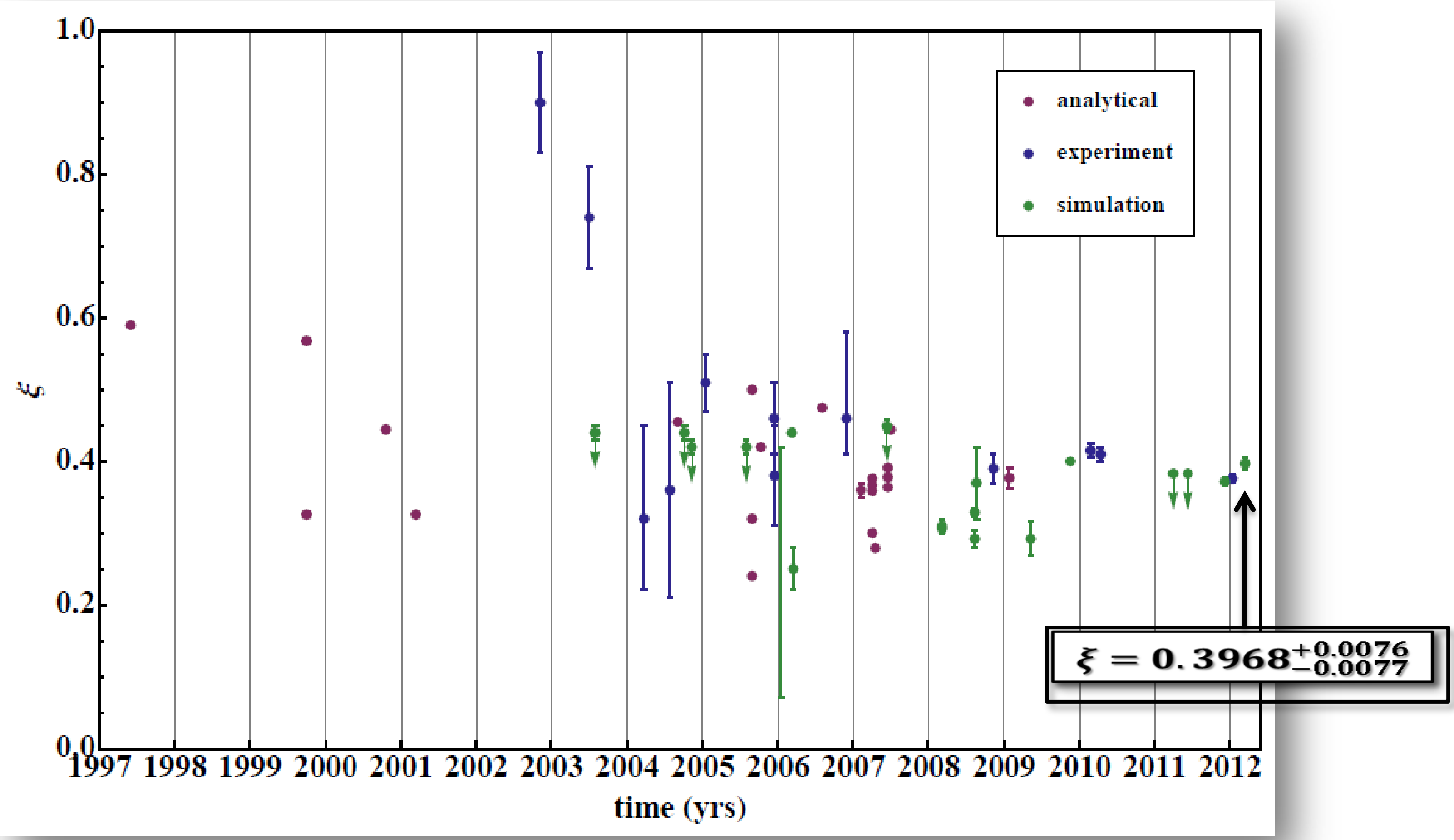
Bertsch parameter
effect of interactions

Length scales : $R_{vdw} < r_e \ll \Lambda_T, n^{-\frac{1}{3}}, |a_s| \ll L$



Bertsch parameter

$$\xi = \frac{E_0^{Unitary}}{E_0^{Ideal}} < 1 \longrightarrow \text{Attractive interactions}$$

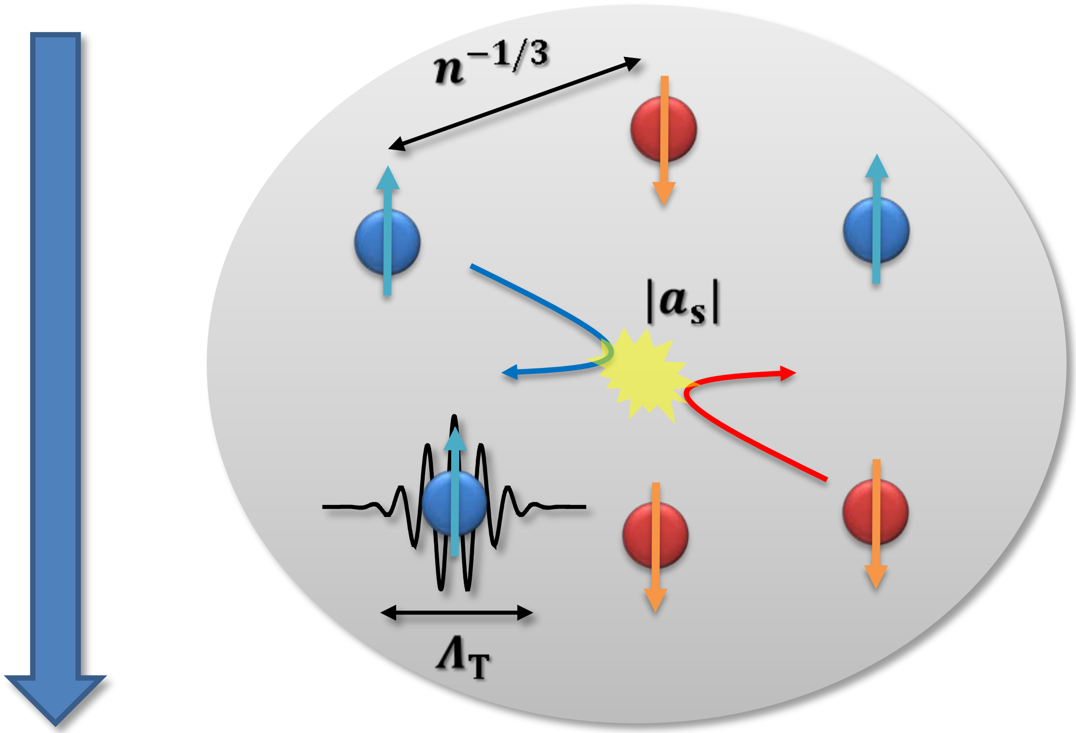


[arXiv:1203.3169]

How about finite *T* and *a* ?

Equation of state

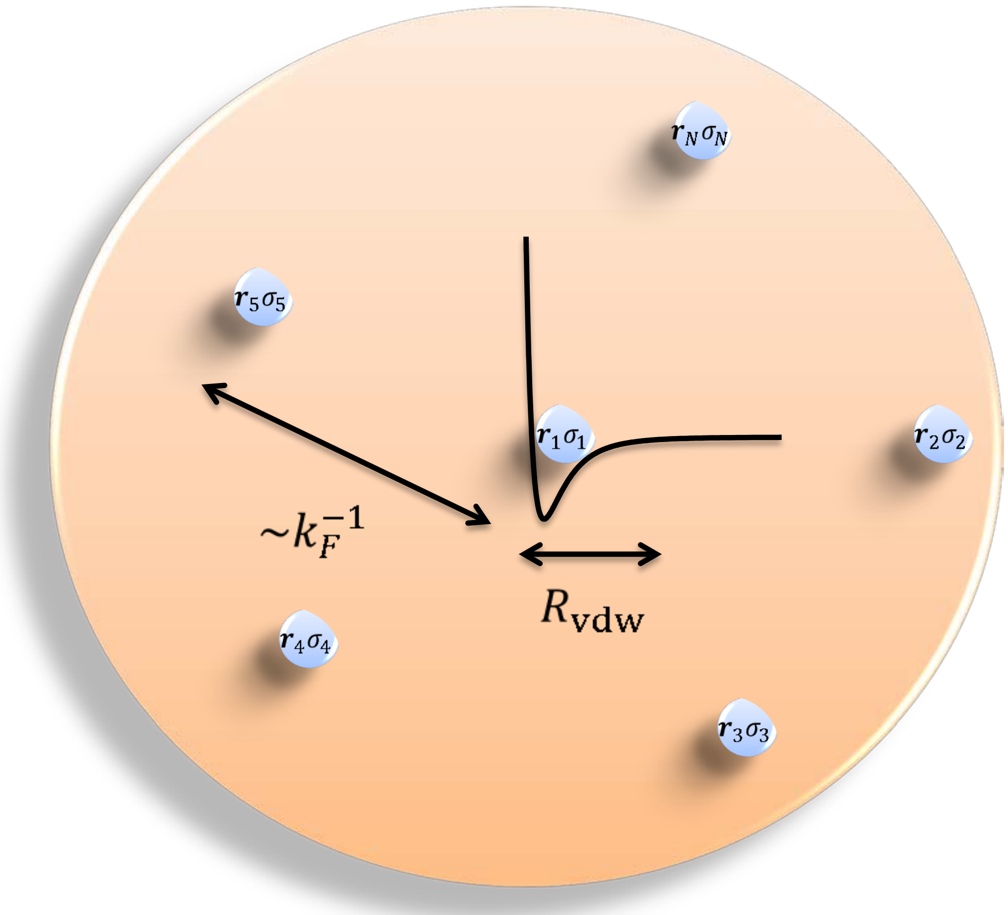
- Non-interacting gas : $PV = -\Omega(V, T, \mu)$



- Interacting Fermi gas : $PV = -\Omega(V, T, \mu, ?)$

The many-body system

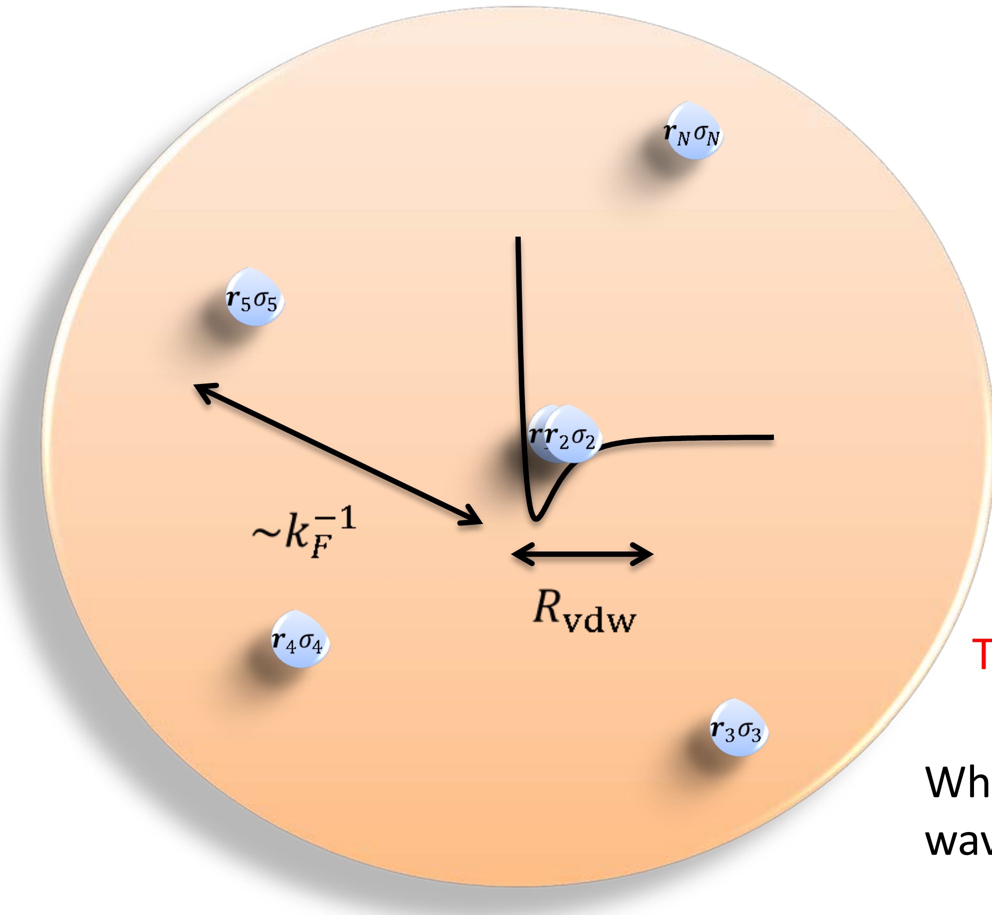
$$\Psi_N^{(n)}(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2, \cdots, \boldsymbol{r}_N\sigma_N)$$



- Two components : σ =↑ or ↓
- Balanced :$N_\uparrow = N_\downarrow = N/2$
- Only s-wave scattering between ↑ and ↓
- No 3-body collision (diluteness, Pauli exclusion)
- BCS-BEC crossover region :$k_F a_s \gg 1$

Contact interaction

$$\lim_{|r_1 - r_2| \lesssim R_{vdw}} \Psi_N^{(n)}(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2, \cdots, \boldsymbol{r}_N\sigma_N)$$



Other N-2 particles do not interact with the two particles at the moment

1. Diluteness
2. Pauli exclusion

$$\propto \phi(\boldsymbol{r}_1 - \boldsymbol{r}_2)\Phi_{N-2}^{(n)}(\boldsymbol{r}_3\sigma_3, \boldsymbol{r}_4\sigma_4, \cdots, \boldsymbol{r}_N\sigma_N)$$

Two-body wave function

What is the expectation value of the two-body wave function in such a system?

Universal many-body function and Tan's contact

- Two-body density matrix at short range :

$$|\phi_{\text{pair}}(r)|^2 = \langle \psi_\uparrow^\dagger(\boldsymbol{r}_1)\psi_\downarrow^\dagger(\boldsymbol{r}_2)\psi_\downarrow(\boldsymbol{r}_2)\psi_\uparrow(\boldsymbol{r}_1)\rangle$$

$$\xrightarrow{a,k_{\text{F}}^{-1} \gg r \equiv |r_1 - r_2| \gtrsim R_{\text{vdw}}} 4\pi k_{\text{F}} N \cdot h(x,\theta) \cdot \left|\frac{\phi(r)}{4\pi}\right|^2 \quad , \left(\begin{array}{c} x \equiv -\frac{1}{k_{\text{F}} a}, \theta \equiv \frac{T}{T_{\text{F}}} \\ \phi(r) = \frac{1}{r} - \frac{1}{a} \end{array} \right)$$

Universal many-body function

[Zhang and Leggett, PRA 79, 023601 (2009)]

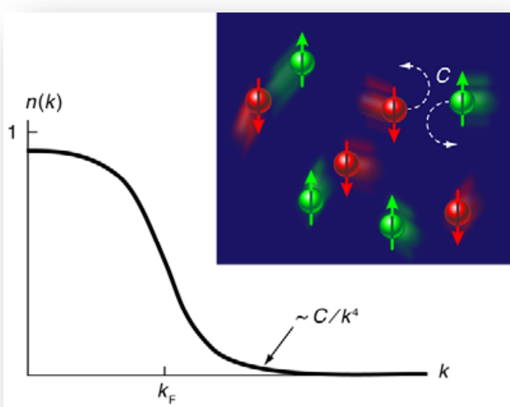
- Tail of the momentum distributions of particles :

$$n_{\uparrow \text{or} \downarrow}(k_{\text{F}}, |a|^{-1}, \Lambda_{\text{T}}^{-1} < k < R_{\text{vdw}}^{-1}) = \left| \int d^3 r e^{ikr} \phi_{\text{pair}}(r) \right|^2$$

$$= 4\pi k_{\text{F}} N h(x,\theta)/k^4 \equiv \frac{C}{k^4}$$

Tan's contact

[S. Tan, Ann. Phys. 323, 2952 (2008)]



Physics **3**, 48 (2010)

Universal equation of state (EOS) for a dilute Fermi system

- Adiabatic relation : $\left(\frac{dE}{da^{-1}}\right)_{S,V,N} = -\frac{\hbar^2}{4\pi m}C \equiv -I$

- Total differential of the internal energy : $dE = -pdV + TdS + \mu dN - Ida^{-1}$

  New thermodynamic variable

- Grand canonical potential : $\Omega = E - TS - \mu N$

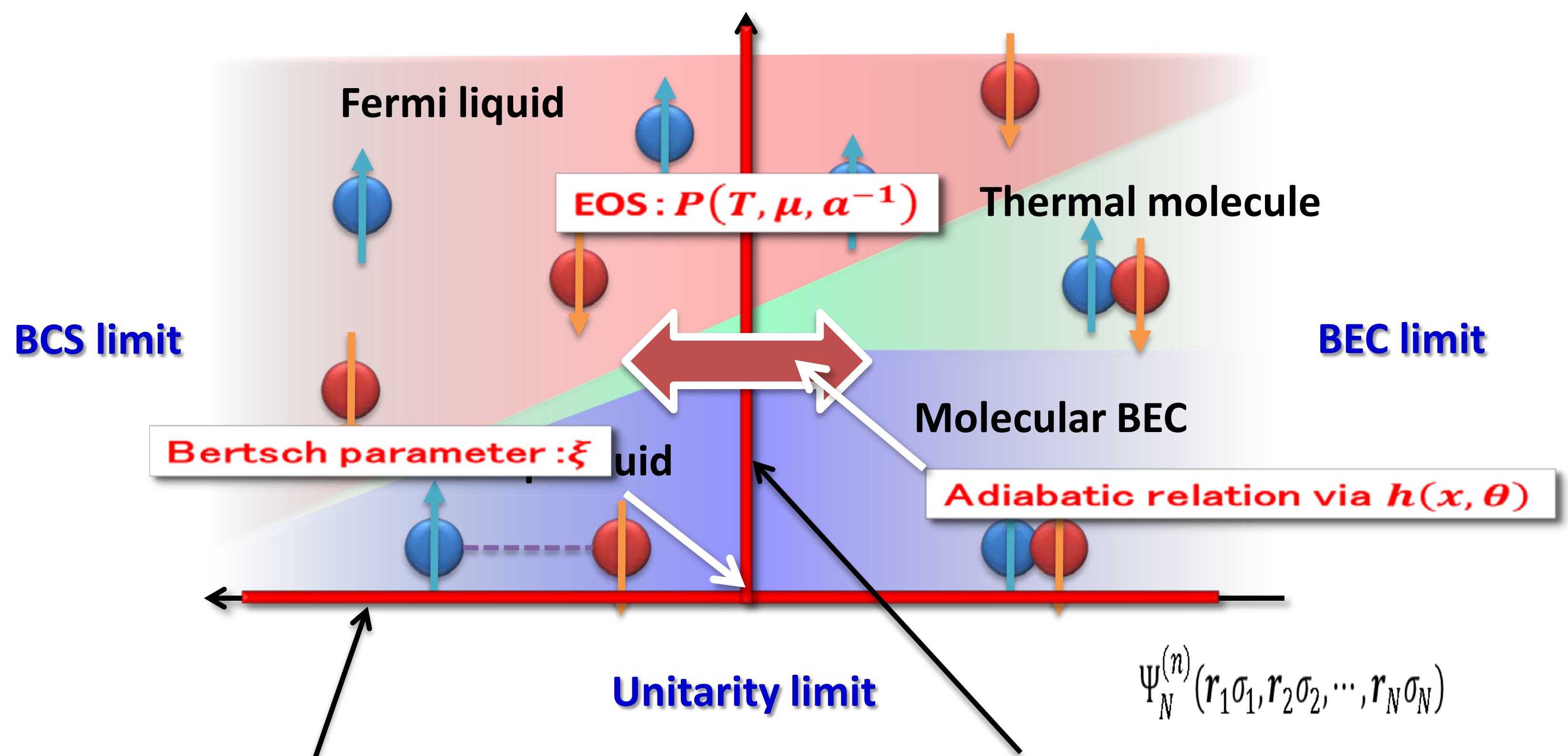
- Total differential of the grand canonical potential :

$$d\Omega = dE - d(TS) - d(\mu N) = -pdV - SdT - Nd\mu - Ida^{-1}$$

EOS of the universal many-body system : $\boldsymbol{\Omega(V, T, \mu, a^{-1}) = -p(T, \mu, a_s^{-1})V}$

BCS-BEC crossover in cold atom systems

Temperature parameter : $\theta = \frac{T}{T_F}$



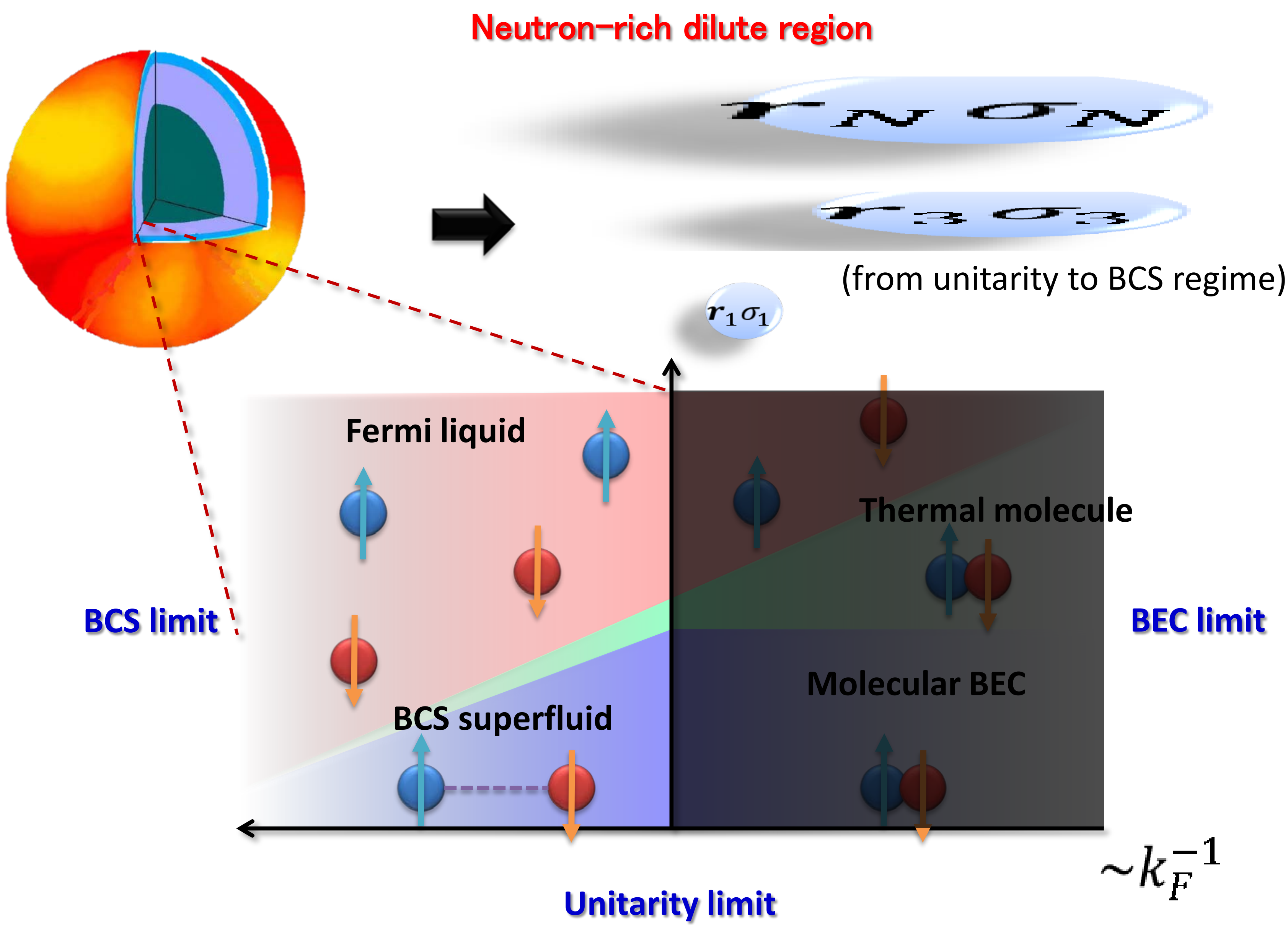
[N. Navon, *et al.*, Science **328**, 729 (2010)]

[M. Horikoshi, *et al.*, Science **327**, 442 (2010)]

[S. Nascimbène, *et al.*, Nature **463**, 1057 (2010)]

[M. Ku, *et al.*, Science **335**, *563* (2012)]

Cold Fermi gases and the inner crust of neutron stars



Physics undetermined by experiments

- EOS over the crossover: $P(T, \mu, a^{-1})$
- Thermodynamic functions : E, F, S
- Universal many-body function: $h(x, \theta)$

Today's topics

- Density of state, spectrum function
- Superfluid transition temperature
- Superfluid density
- Pairing gaps, size of cooper pairing
- Transport（thermal conductivity, viscosity）

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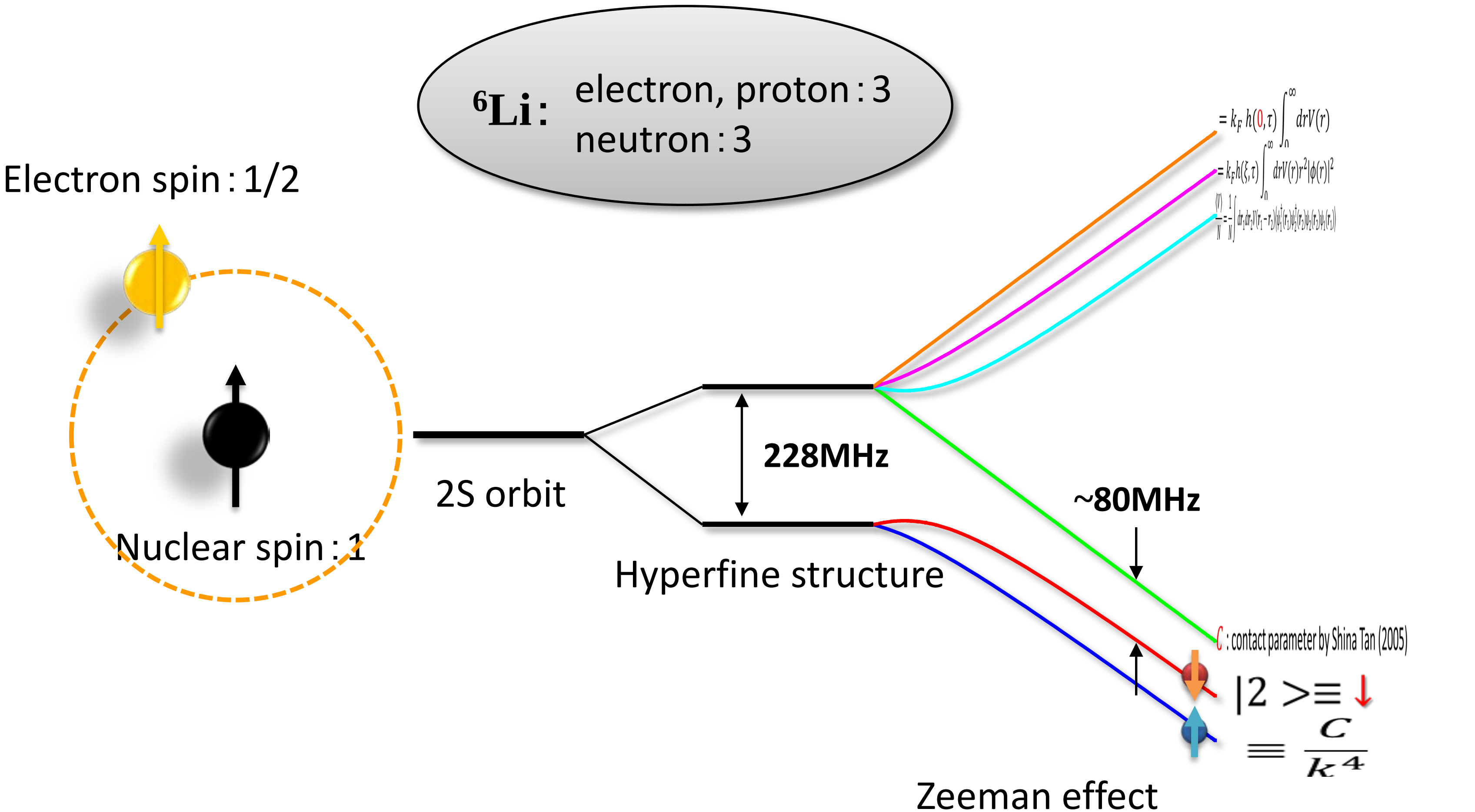
Fermi atom : ^{6}Li

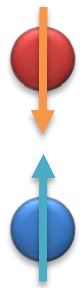
Two components system is realized using two different **Internal states**



Another candidate is ^{40}K

<u>Feshbach resonance</u>

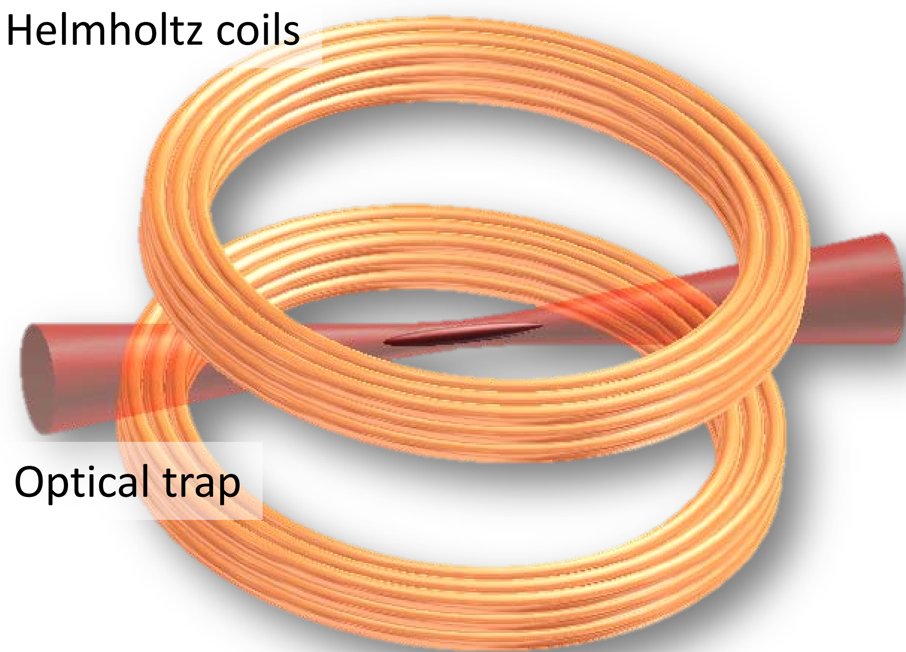
Magnetically tunable scattering length between the spin states

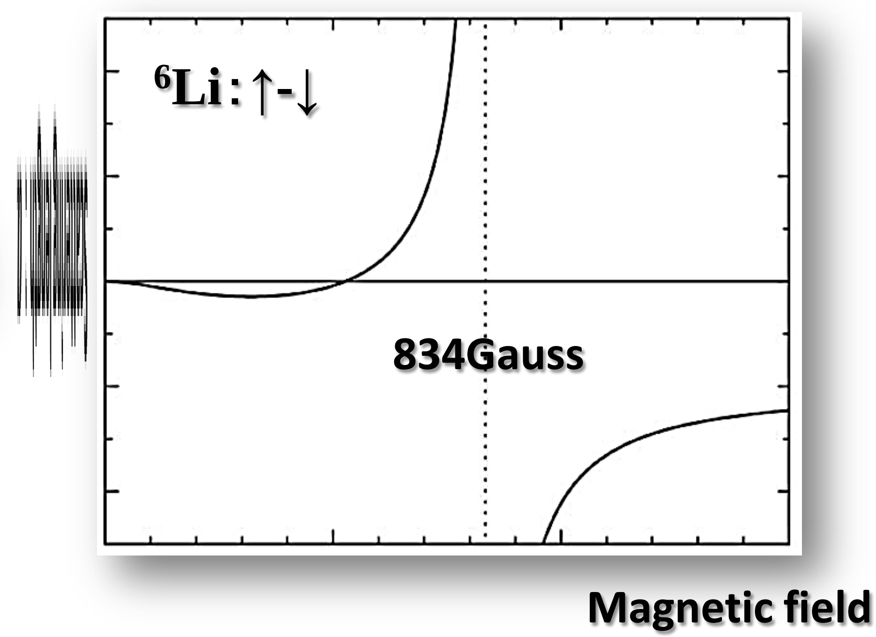


$$\equiv |F = 1/2, m_F = +1/2\rangle$$

$$\equiv |F = 1/2, m_F = -1/2\rangle$$

$$a(B)$$

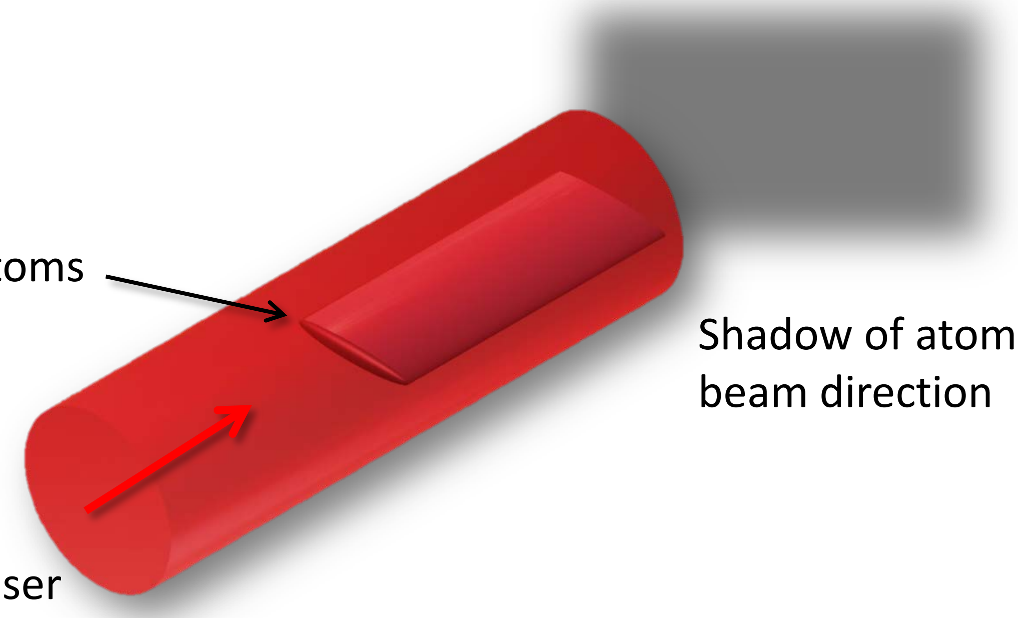




Method of observation

Absorption imaging

Cold atoms



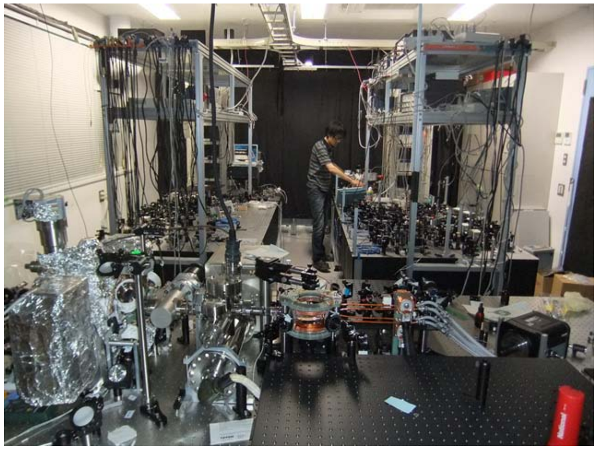
Shadow of atoms integrated along the probe beam direction

Resonant laser

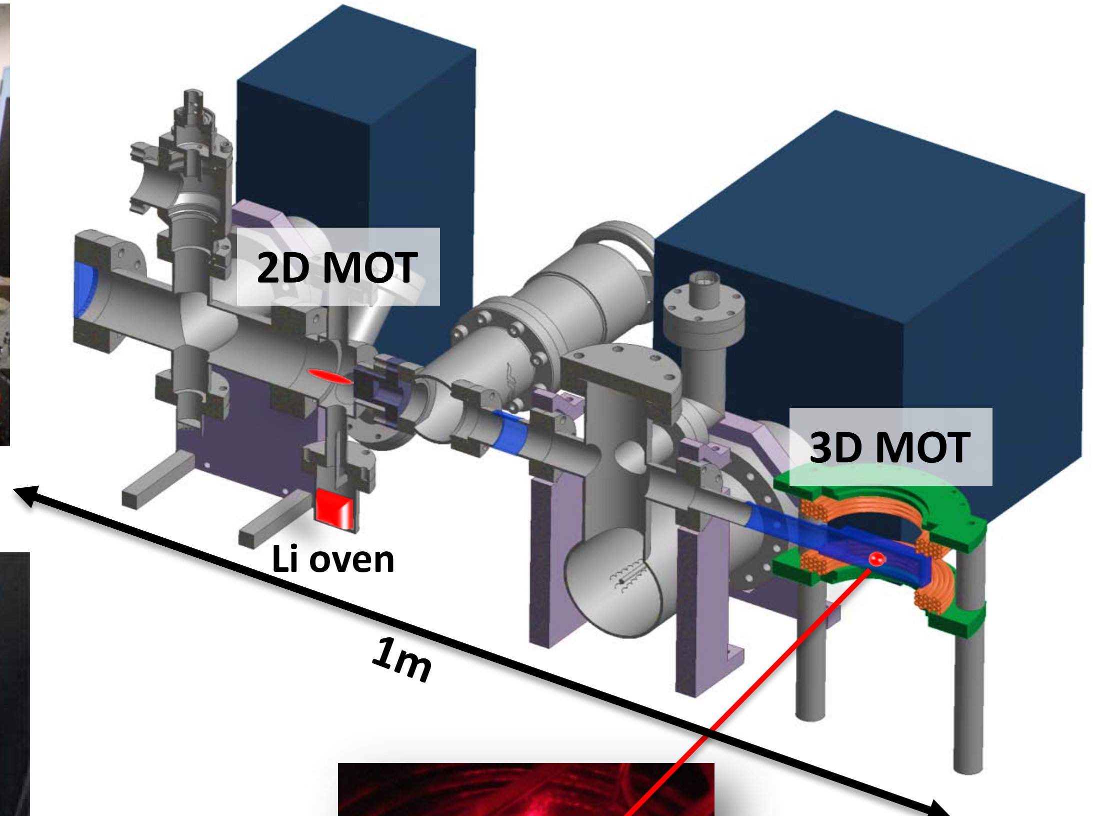
In situ imaging → density distribution

After expansion → momentum distribution

Our laboratory @ Photon Science Center of University of Tokyo



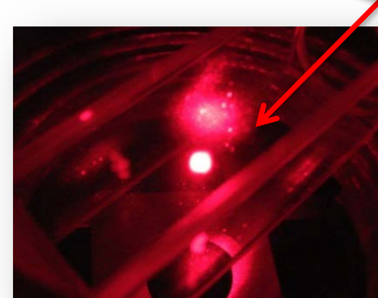
Since April, 2011



Prof. Gonokami

Graduate studentt : Togashi

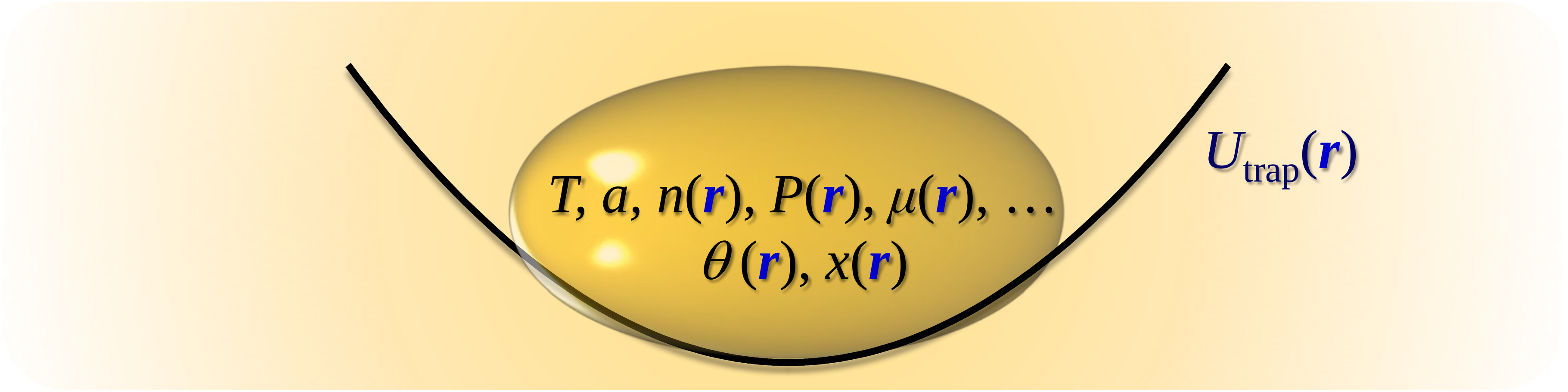
Undergraduate student : Ito



Simultaneous MOT of ^{6}Li and ^{7}Li

How to measure $P(T, \mu, a^{-1})$ and $h(x, \theta)$ using cold atoms ?

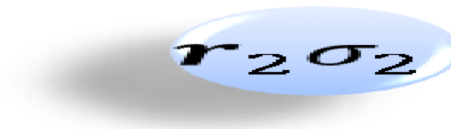
The most serious problem is inhomogeneity of the gas trapped in a harmonic trap



- Thermodynamic quantities are position dependent
- Measured momentum distributions are averaged values over the trap
- Thermometer, Pressure meter, **Chemical potential meter？**

Our previous route to determine the EOS at the unitarity limit

[M. Horikoshi, et al., Science 327, 442 (2010)]

- **Force balance :** $\nabla P(r) + n(r)\nabla U_{trap}(r) = 0$
- **Pressure – energy relation :** $PV = \frac{2}{3}E$
- **Internal energy :** 

↓ Thermodynamic relationship

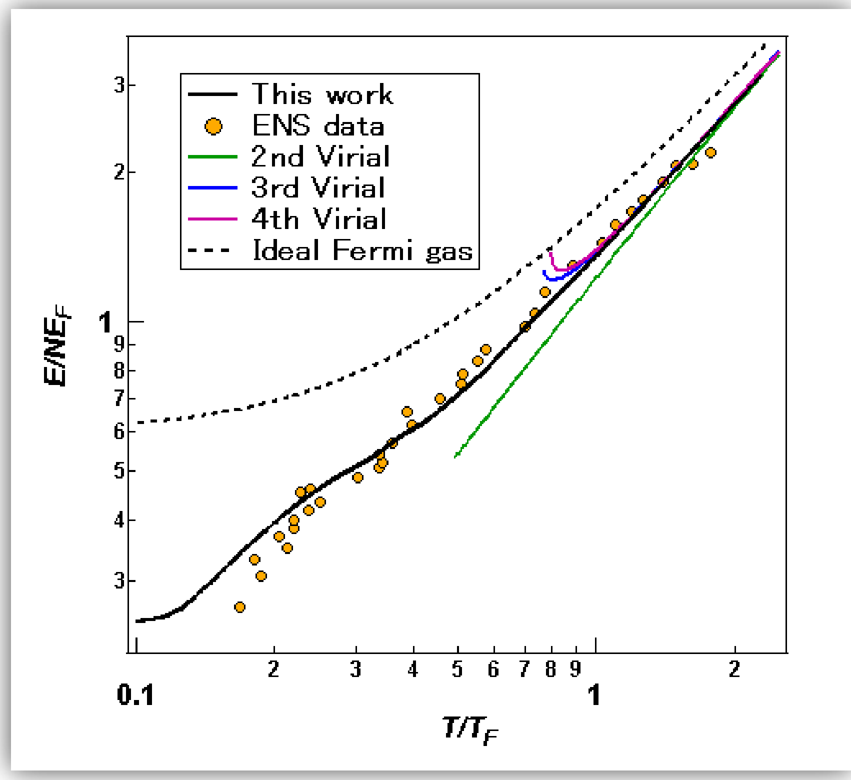
EOS at the unitarity : $P(T, \mu, a^{-1} = 0)$

Improved our data

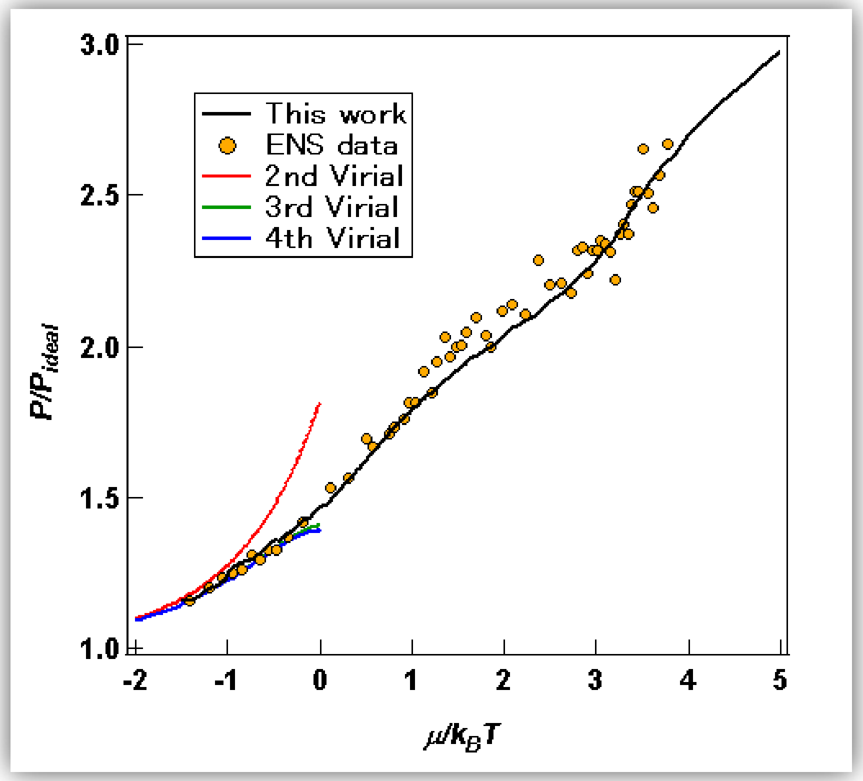
Recently, thermometry provided by J. Thomas's group has been improved, especially, at high T/T_F region.

[arXiv:1105.2496]

Internal energy



EOS at unitarity



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[M. Horikoshi, et al., Science 327, 442 (2010)]

- **Force balance :** $\nabla P(r) + n(r)\nabla U_{trap}(r) = 0$
- **Pressure - energy relation :** $PV = \frac{2}{3}E$
- **Internal energy density :** $E = N\varepsilon_F(n) f_E\left(\frac{T}{T_F(n)}\right)$

↓ Thermodynamic relationship

EOS at the unitarity : $P(T, \mu, a^{-1} = 0)$

Problems for $a^{-1} \neq 0 : PV = \frac{2}{3}(E - N\varepsilon_F x h(x,\theta))$

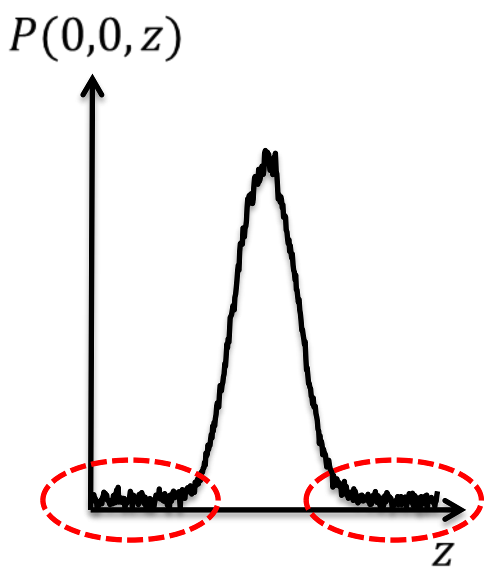
unknown

ENS's route to determine the EOS at the unitarity limit

[S. Nascimbène, *et al.*, Nature **463**, 1057 (2010)]

EOS at the unitarity : $\boldsymbol{P(T,\mu,a^{-1}=0)}$

- **Pressure :** $\nabla P(r)+n(r)\nabla U_{trap}(r)=0 \longrightarrow P(0,0,z)=\dfrac{m{\omega_r}^2}{2\pi}\bar{n}(z)$
- **Temperature :** direct measurement by mixing ^{7}Li into ^{6}Li
- **Chemical potential :**



- $P(\mu,T,0)\xrightarrow{\xi=\exp\left(\frac{\mu}{k_BT}\right)<1}\dfrac{2k_BT}{{\lambda_T}^3(T)}\left[\xi+\left(-2^{-5/2}+\sqrt{2}b_2\right)\xi^2\right]$
- $b_2=1/2$ at the unitarity limit
- Local density approximation : $\mu(z)=\mu_0-U_{\mathrm{trap}}(z)$

Fitting

Construct higher terms

- $P(0,0,z)=\dfrac{2k_BT}{{\lambda_T}^3(T)}\left[\xi(z,\mu_0)+\dfrac{3}{4\sqrt{2}}\xi(z,\mu_0)^2\right]$

ENS's route to determine the EOS at the unitarity limit

[S. Nascimbène, *et al.*, Nature **463**, 1057 (2010)]

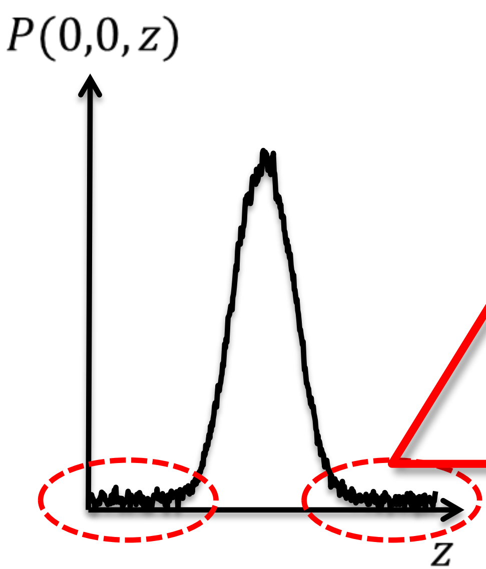
EOS at the unitarity : $P(T, \mu, a^{-1} \neq 0)$

- **Pressure :** $\nabla P(r) + n(r)\nabla U_{trap}(r) = 0 \longrightarrow P(0,0,z) = \frac{m{\omega_r}^2}{2\pi}\bar{n}(z)$
- **Temperature :** direct measurement by mixing ^{7}Li into ^{6}Li
- **Chemical potential :**

$$b_2(T,a) = \sum_b e^{|E_b|/k_BT} - \frac{\mathrm{sgn}(a)}{2}(1-\mathrm{erf}(x))e^{x^2}$$

$$x = \frac{\Lambda_T}{\sqrt{2\pi}a}$$

[Tin-Lun Ho and Erich J. Mueller, Phys. Rev. Lett. 92, 160404 (2004)]



- $P(0,0,z) = \frac{2k_BT}{{\lambda_T}^3(T)}\left[\xi(z,\mu_0) + \left(-2^{-5/2} + \sqrt{2}b_2(T,a)\right)\xi(z,\mu_0)^2\right]$

We have to keep the same temperature of gases

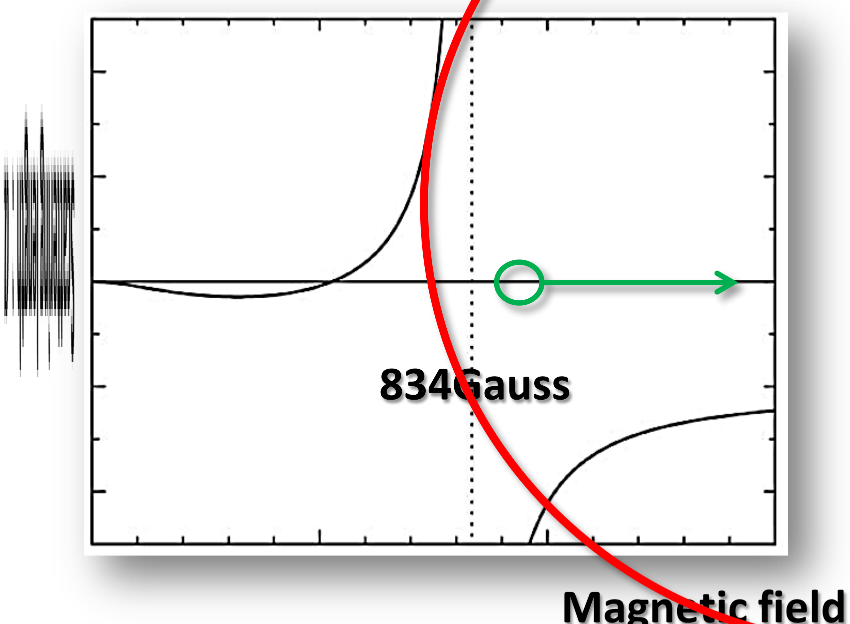
Our new route to determine the EOS at finite scattering length

[Togashi and Horikoshi, 67th JPS meeting (2012)]

◆ **Local chemical potential from observables integrated over the trap**

Under thermal equilibrium, each local position satisfy : $\mathcal{E} = Ts + \mu n - P$

Integrate using a harmonic potential



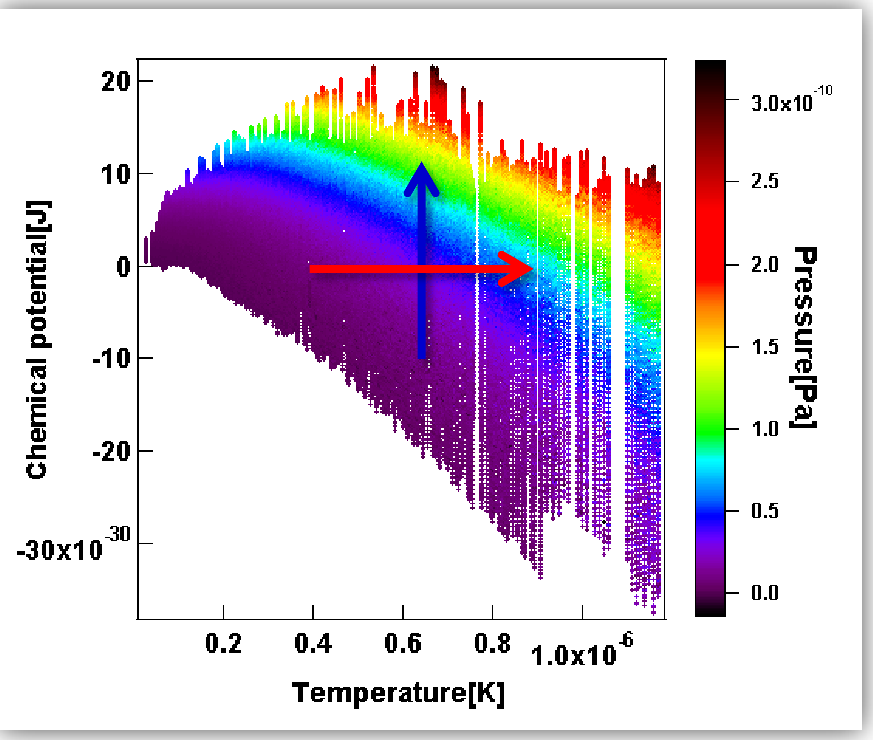
$$E_{\mathrm{rel}} = TS + \mu_0 N - \frac{5}{3} E_{\mathrm{pot}}$$

LDA : $\mu(r) = \mu_0 - U_{\mathrm{trap}}(r)$

We can measure all of the quantities except for $\mu(0)$

Derivation of the internal energy

EOS via ENS's route : $P(\mu, T, a=\infty)$



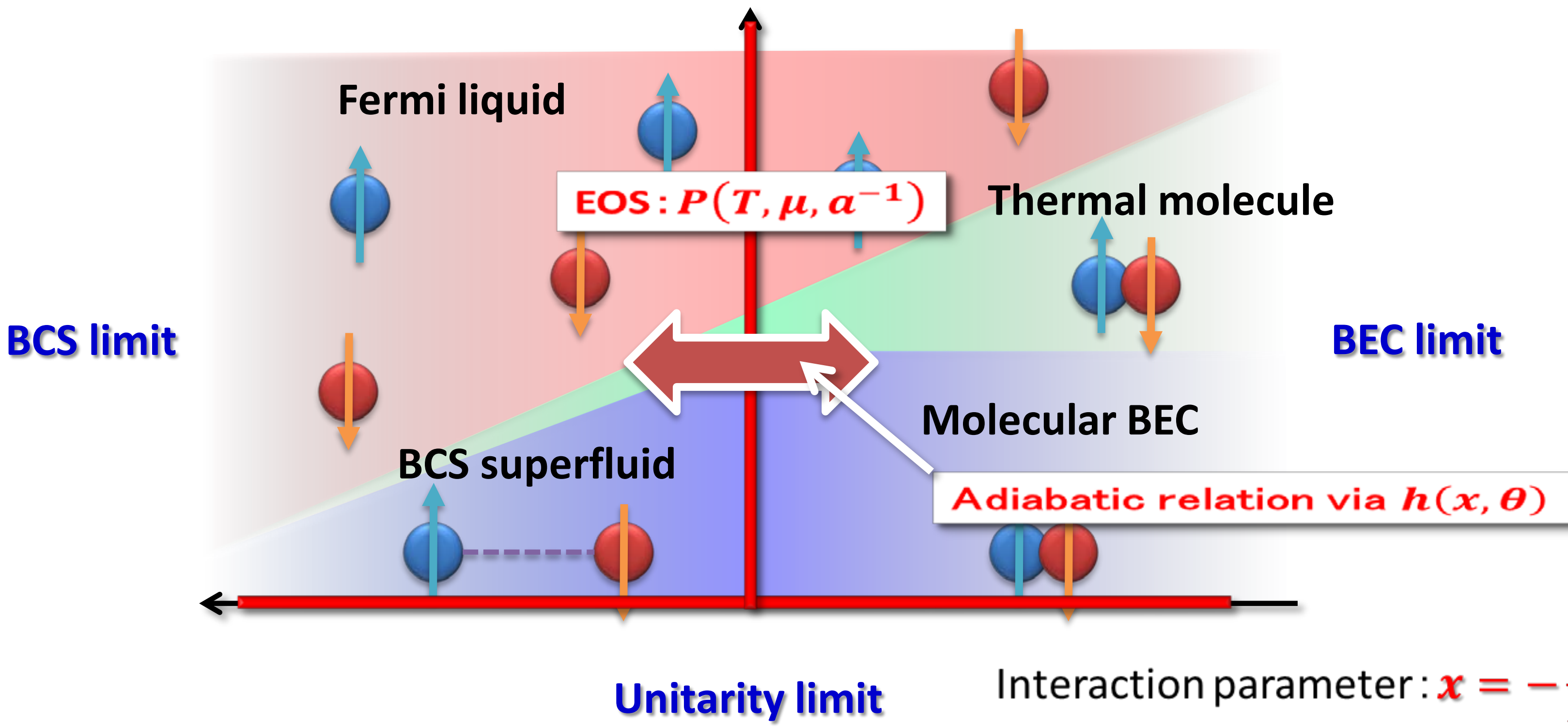
Particle density : $n=(dP/d\mu)_{T,a}$

Entropy density : $s = (dP/dT)_{\mu,a}$

Internal energy density : $\varepsilon = Ts + \mu n - P$

Verification of the measured EOS

Temperature parameter : $\theta = \frac{T}{T_F}$



Interaction parameter : $x = -\frac{1}{k_F a}$

- Pressure – energy relation : $PV = \frac{2}{3}(E - N\varepsilon_F x h(x, \theta))$
- Adiabatic relation : $\left.\frac{\partial E}{\partial x}\right|_\theta = 2\varepsilon_F N h(x, \theta) > 0$

Correction of the effective range of neutrons

Necessary condition for universality : $r_e \ll \Lambda_T,\ k_F^{-1},\ |a_s| \ll \mathrm{L}$

^{6}Li atom : $r_e \sim 5\mathrm{nm} \ll k_F^{-1} \sim 100\mathrm{nm}$ $\longrightarrow$ $k_\mathrm{F} r_\mathrm{e} \sim 0.05$

Correction

Neutron : $r_e \sim 1\mathrm{fm} < k_F^{-1} \sim 2\mathrm{fm}$ $\longrightarrow$ $k_\mathrm{F} r_\mathrm{e} \lesssim 0.5$

non-negligible value

1. **Direct control of the effective range via electric field** [PRL 100, 153201 (2008)]

2. **Adiabatic relation for an effective range** : $$\frac{\partial E}{\partial r_\mathrm{e}} = -\frac{4\pi\hbar^2}{m}(A, B)$$

[arXiv:1204.3204]

$$n_\sigma(k) \xrightarrow[k\to\infty]{} \frac{C}{k^4} + \frac{D}{k^6} + \cdots$$

Effect of the lattice of neutron-rich nuclei is another issue

Theoretical support is necessary

Contents

- Neutron star and our new project
- Universal many-body Fermi system using cold atoms
- Experimental method to measure the EOS
- **Summary**

Summary

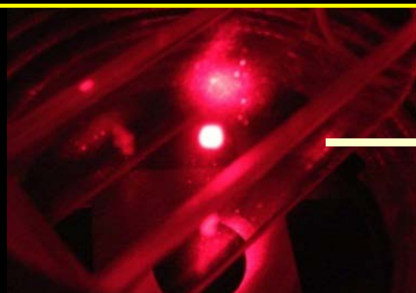
- EOS over the BCS-BEC crossover : $P(T, \mu, a^{-1})$
- Universal many-body function : $h(x, \theta)$
- Thermodynamic functions
- T_C curve

- Pressure-energy relation : $PV = \frac{2}{3}(E - N\varepsilon_F x h(x, \theta))$
- Adiabatic relation : $\left.\frac{\partial E}{\partial x}\right|_\theta = 2\varepsilon_F N h(x, \theta) > 0$

Summary

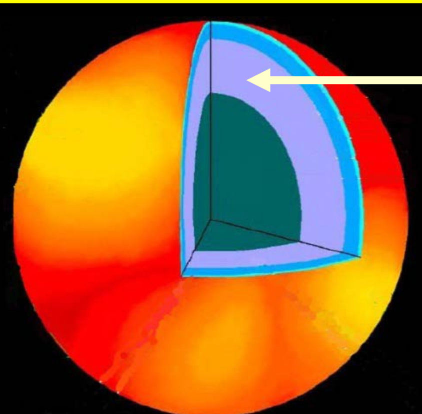
Simulation of neutron-rich dilute nuclear matter using ultracold Fermi gases

Cold Fermi gas



Universal EOS : $P(T, \mu, a^{-1})$
Universal many-body function : $h(x, \theta)$
Critical temperature, Pairing gap, …

Inner crust of neutron stars



Correction of the effective range
Lattice of neutron-rich nuclei
Protons

Theories

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