多軌道を持つ引力フェルミ原子気体における相分離

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Introduction: ultra cold fermi gas

Controllability to unusual degree of freedom

configuration of trap potential (harmonic trap, optical lattice)

multi-component

population imbalance (spin imbalance)

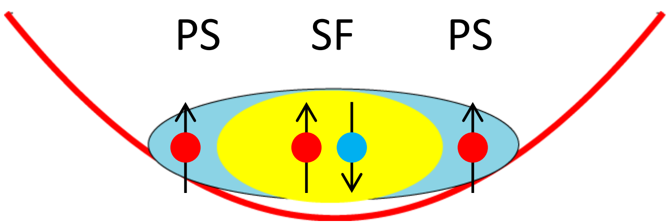
interaction strength (repulsive to attractive)

Superfluidity of two component fermi gas

BCS-BEC crossover

FFLO state

Phase separation of polarized component in trap potential



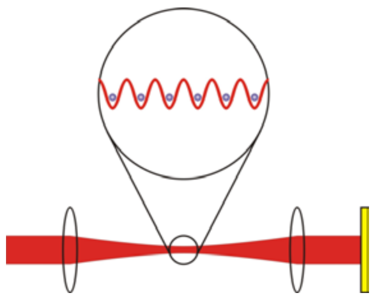
Two component fermi gas in harmonic trap potential

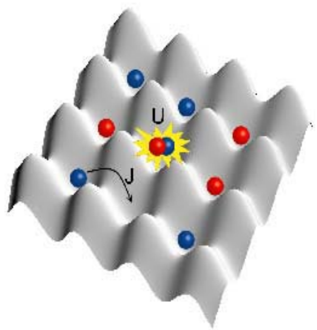
A. E. Feiguin and F. Jeidrich-Meisner Phys. Rev. B. **76,** 220508(R) (2007).

Masaki Tezuka, and Masahiko Ueda, Phys. Rev. Lett. 100, 110403 (2008).

Introduction: optical lattice

Optical lattice: periodic potential created by laser beams





multi-band(p-band)

Experimentally atom gas with p-band is achieved

Phys. Rev. Lett. 99, 200405 (2007)
Nature. Phys, Vol7, 147, (2011)

Various phenomena are expected
(Orbital order, magnetization with high spin etc.…)

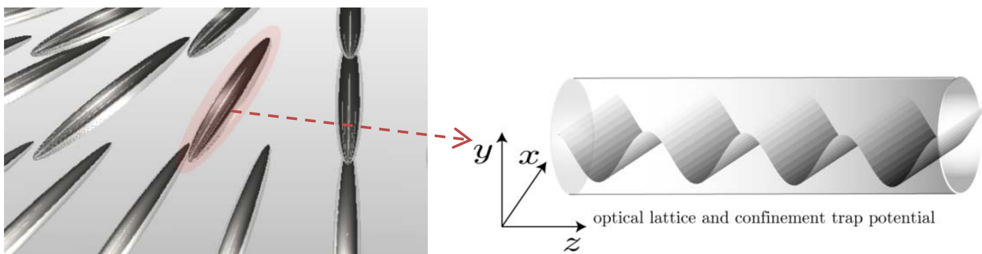
Phys. Rev. B 77, 174431 (2008)
Phys. Rev. Lett, 100, 200406 (2008)

Multi band Superfluidity

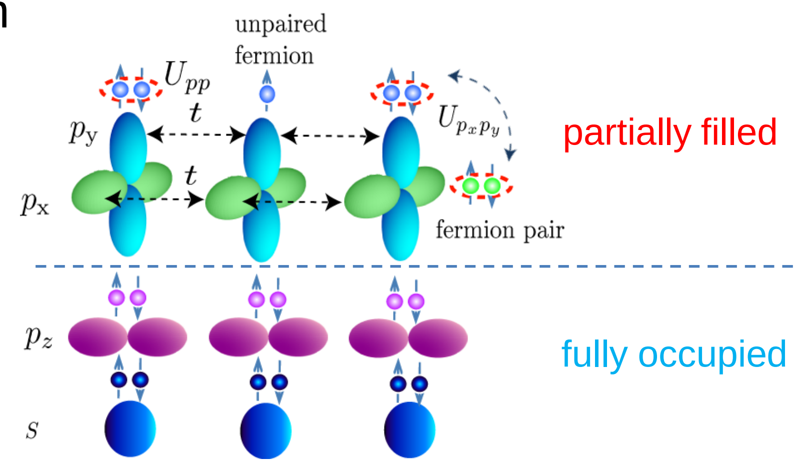
Multi-band effects of attractive fermi gas in 1D optical lattice

contents

1D p-orbitals (two degenerate multi-band) system



1-D optical lattices



Approach

Density matrix renormalization study

Effective Hamiltonian in strong coupling limit

Results

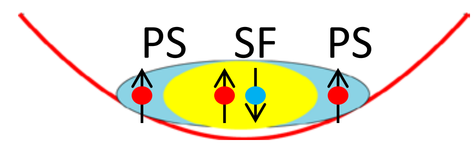
Haldane insulator phase

Phys. Rev. Lett. 97, 260401 (2006), Phys. Rev. B. 81, 020408(R) (2010).

Luther-Emery phase

Phys. Rev. Lett. 33, 589 (1974).

Strong phase separation of polarized component without trap potential

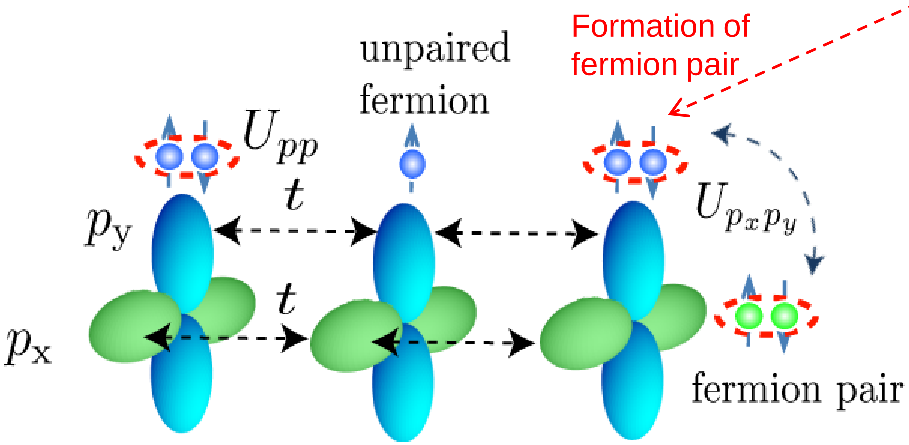


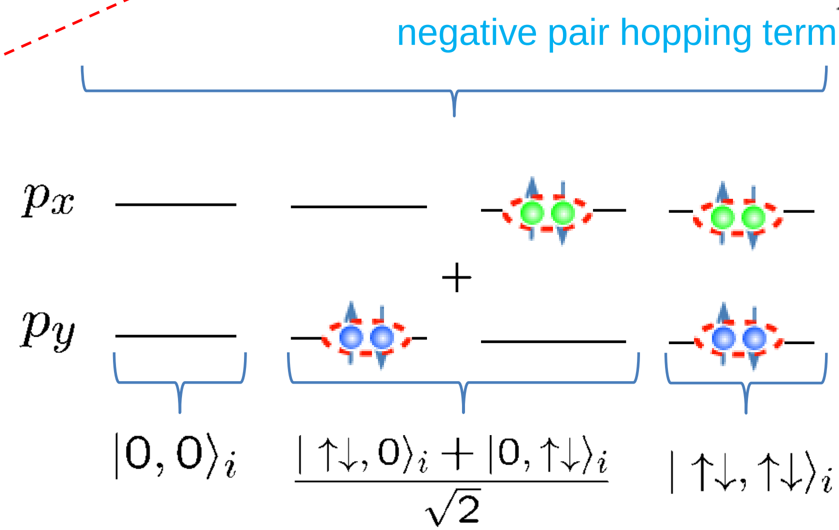
Phase separation in harmonic trap potential (single band)

Model and Formulation

p-orbitals tight binding Hamiltonian with $U_{pp}, U_{p_x p_y} < 0$

$$H - \mu N = -\sum_{p,\sigma} \sum_{<i,j>} t c^{\dagger}_{p,\sigma,i} c_{p,\sigma,j} - \sum_{p,\sigma,i} \mu n_{p,\sigma,i} + \sum_i \Big[\sum_p U_{pp} n_{p,\uparrow,i} n_{p,\downarrow,i}$$
$$+ \sum_{p \neq p'} U_{pp'} \left\{ c^{\dagger}_{p,\uparrow,i} c^{\dagger}_{p',\downarrow,i} c_{p,\downarrow,i} c_{p',\uparrow,i} + n_{p,\uparrow,i} n_{p',\downarrow,i} + c^{\dagger}_{p,\uparrow,i} c^{\dagger}_{p,\downarrow,i} c_{p',\downarrow,i} c_{p',\uparrow,i} \right\} \Big]$$





$$t = -\int dz w_{p,i+1} \left[\frac{-\hbar^2 \nabla^2}{2M} + V_{\text{opt}} \right] w_{p,i}$$

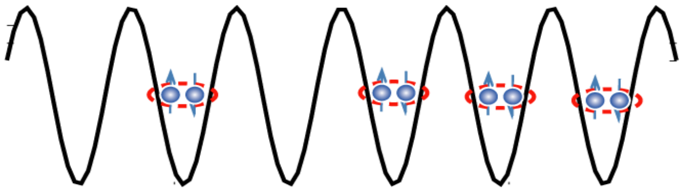
$$U_{pp'} = g \int d^3x w^2_{p,i} w^2_{p',i}$$

Wannier function (p-orbital)

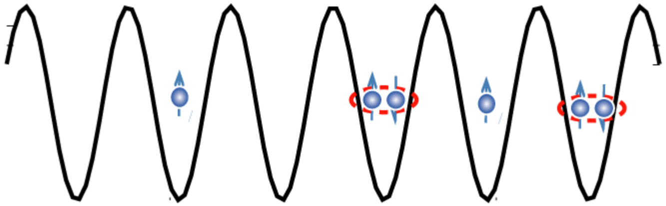
pseudo spin 1 states

DMRG study

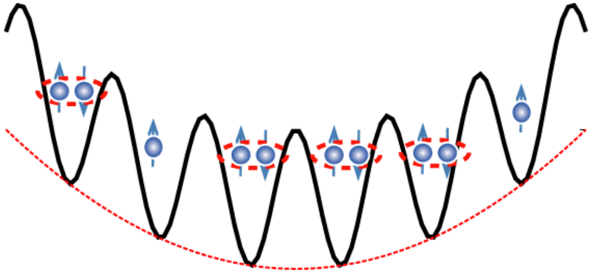
- Without spin imbalance



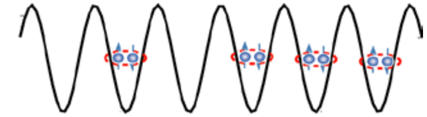
- With spin imbalance



- Effect of harmonic trap potential



Result (without spin imbalance)



Open boundary condition

Total lattice site number: $L = 100$

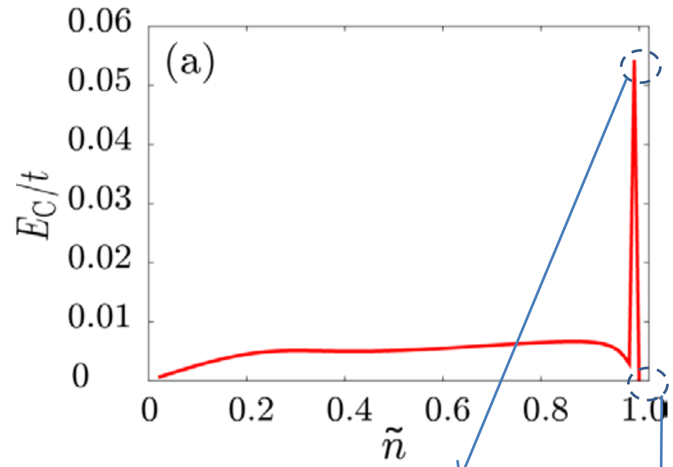
Filling rate: $\tilde{n} = \sum_{p,i,\sigma} n_{p,i,\sigma}/2L \ (0 \leq \tilde{n} \leq 2)$

Inter-band interaction: $U_{pp}/t = -10$

Intra-band interaction: $U_{p_x p_y}/t = 4U_{pp}/9$

Spin imbalance: $P = \sum_{p,i}(n_{p,\uparrow,i} - n_{p,\downarrow,i}) = 0$

charge gap: $E_{\mathrm{C}} \equiv E(N + \uparrow\downarrow) + E(N - \uparrow\downarrow) - 2E(N)$



charge gap

gapless excitation

Effective Hamiltonian (large U limit)

1-D pseudo spin 1 XXZ model

$$H_{\mathrm{eff}} = J_{\mathrm{ex}} \sum_{<i,j>} \left(\rho_i^{(z)} \rho_j^{(z)} - \rho_i^{(+)} \rho_j^{(-)}/2 \right) - \sum_i 2\bar{\mu}\rho_i^z$$

pseudo spin 1 operator

$$\rho_i^{(+)} = \sum_p c^\dagger_{p,\uparrow,i} c^\dagger_{p,\downarrow,i}$$

$$\rho_i^{(-)} = [\rho_i^{(+)}]^\dagger$$

$$\rho_p^{(z)} = \sum_p (\sum_\sigma n_{p,\sigma,i} - 1)/2$$

$$\bar{\mu} = \mu + \frac{|U_{pp}| + |U_{p_x p_y}|}{2}$$

$$J_{\mathrm{ex}} = \frac{2t^2}{|U_{pp}| + |U_{p_x p_y}|}$$

Haldane phase

spin gap

edge 1/2 spin

Phys. Lett. A 93, 464 (1983)

Phys.Rev. Lett. 59, 799 (1987)

Haldane insulator phase

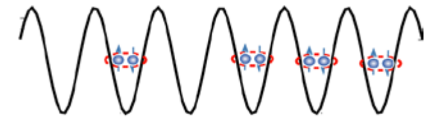
pseudo spin gap

→charge gap

edge pseudo 1/2 spin

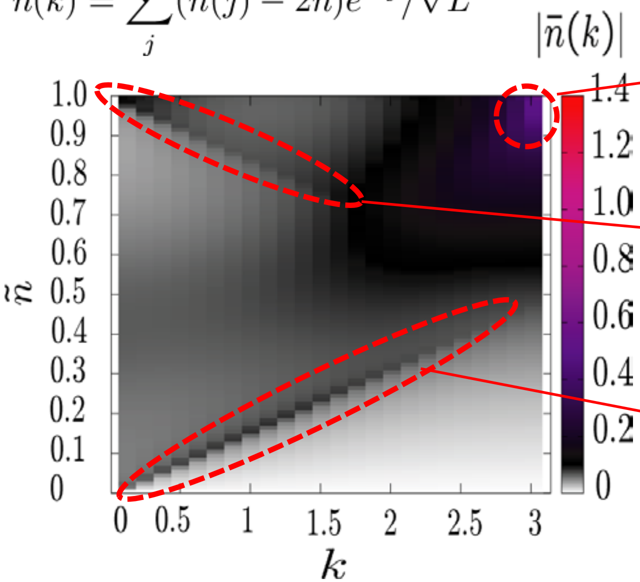
→gapless excitation at the edges

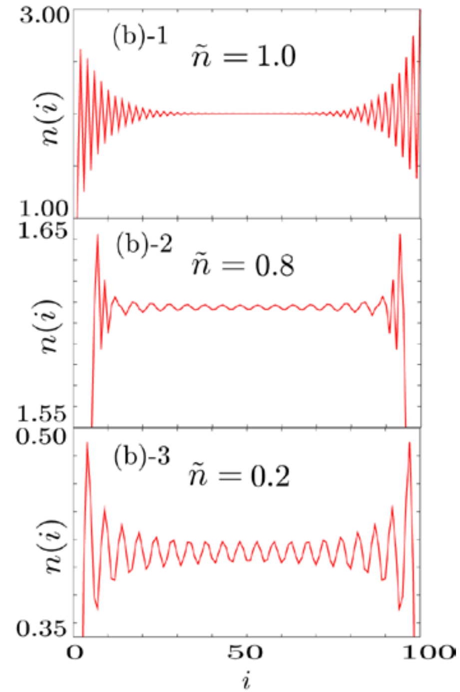
Result (without spin imbalance)



Absolute value of Fourier transformation of density fluctuation

$$\tilde{n}(k) = \sum_j (n(j) - 2\tilde{n}) e^{ikj} / \sqrt{L}$$





strong peak

edge state: staggered CDW with exponential decay

weak peak

CDW

Periodicity of CDW

$$H_{\text{eff}} = J_{\text{ex}} \sum_{<i,j>} \left(\rho_i^{(z)} \rho_j^{(z)} - \rho_i^{(+)} \rho_j^{(-)}/2 \right) - \sum_i 2\bar{\mu} \rho_i^z$$

$$\rho_i^{(+)} = \sum_p c_{p,\uparrow,i}^{\dagger} c_{p,\downarrow,i}^{\dagger} \quad \rho_i^{(-)} = [\rho_i^{(+)}]^{\dagger} \quad \rho_p^{(z)} = \sum_p (\sum_\sigma n_{p,\sigma,i} - 1)/2$$

$$2k_{\text{F}} = 2\pi(1 - |\sum_i \rho_i^{(z)}|) = 2\pi\tilde{n}$$

Fermi vector in equivalent spinless fermion

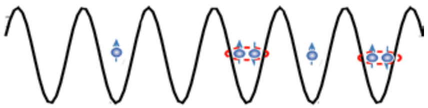
Spin 1 Heisenberg model with magnetic field
[Gabor Fath, Phys. Rev. B **68**, 134445, (2003).]

Luther-Emery phase

Superconducting pair correlation $\langle \rho_i^{(+)} \rho_0^{(-)} \rangle$
Density-density correlation $\langle \rho_i^{(z)} \rho_0^{(z)} \rangle$

developed with power-low decay

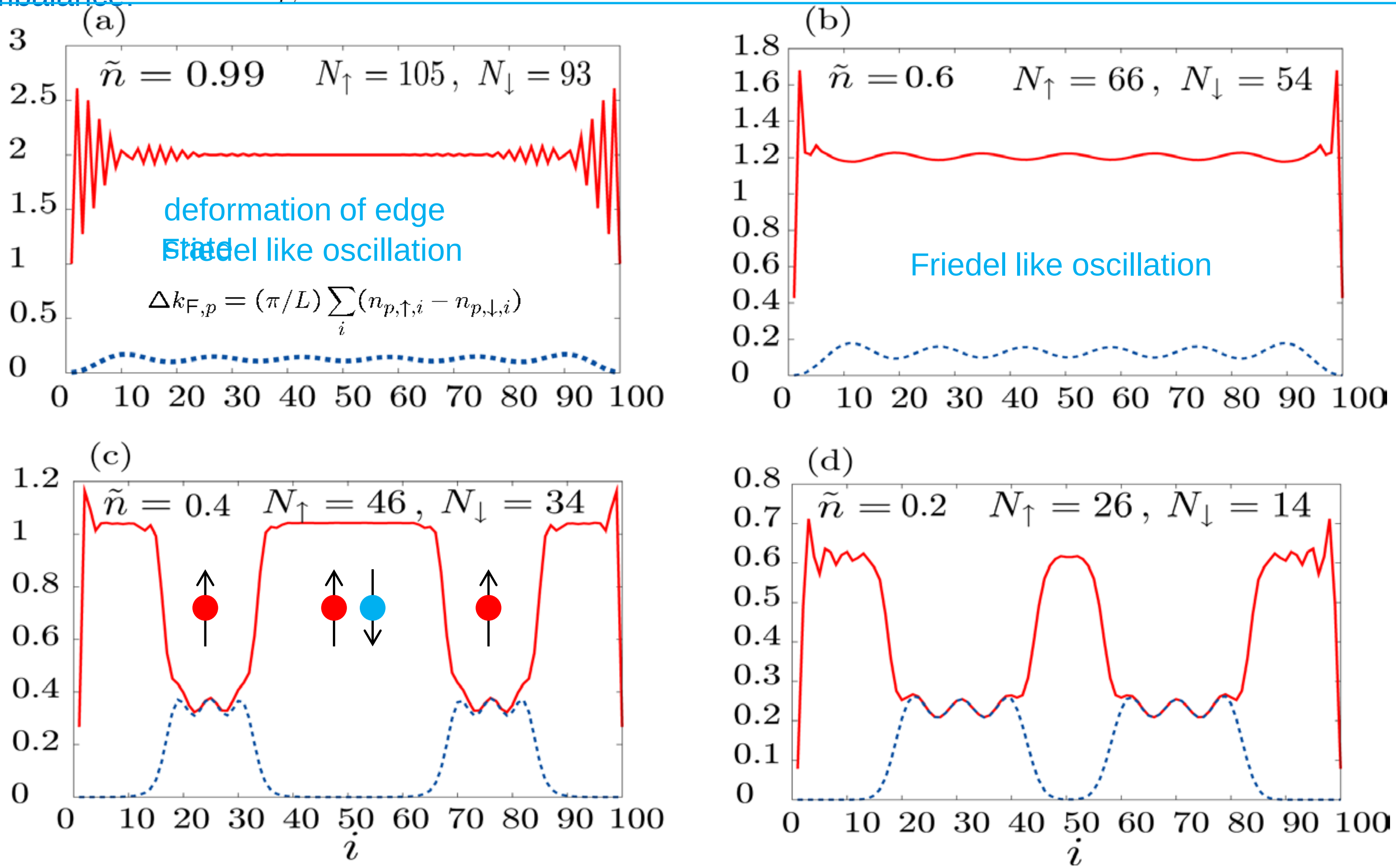
Result (with spin imbalance)



Spin imbalance: $P = \sum_{p,i} (n_{p,\uparrow,i} - n_{p,\downarrow,i}) = 12$

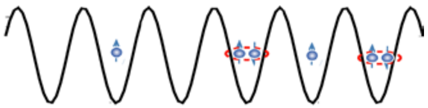
density profile $n_{\uparrow}(i) + n_{\downarrow}(i)$ (solid red line)

spin density profile $n_{\uparrow}(i) - n_{\downarrow}(i)$ (dotted blue line)



Strong phase separation of polarized component without trap potential

Result (with spin imbalance)



Effective Hamiltonian with spin imbalance (large U limit)

$$H_{\rm eff} = P\left[-\sum_{p,\sigma}\sum_{<i,j>} \underline{t c^\dagger_{p,\sigma,i} c_{p,\sigma,j}} - 2\bar\mu\sum_i \rho_i^{(z)} - \frac{J_{\rm ex}}{2}\sum_{<i,j>}\rho_i^{(+)}\rho_j^{(-)} + J_{\rm ex}\sum_{<i,j>}\rho_i^{(z)}\rho_j^{(z)}\right]P$$

$$-\sum_{pp'} P\left[J_{\rm ex}\left(n^{(z)}_{p,\uparrow,i}+\frac{1}{2}\right)\left(n^{(z)}_{p',\uparrow,j}+\frac{1}{2}\right)\right]P$$

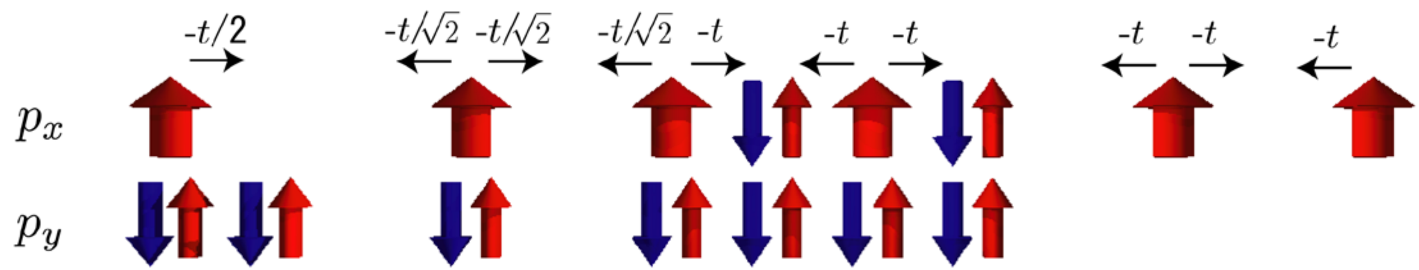
$$\bar\mu = \mu + \frac{|U_{pp}|+|U_{p_xp_y}|}{2} \qquad J_{\rm ex} = \frac{2t^2}{|U_{pp}|+|U_{p_xp_y}|}$$

Projection operator P

pseudo spin 1 state $\mathcal{H}_{\rho=1} = \{|0,0\rangle, (|\uparrow\downarrow,0\rangle + |0,\uparrow\downarrow\rangle)/\sqrt{2}, |\uparrow\downarrow,\uparrow\downarrow\rangle\}$

unpaired fermion $\mathcal{H}_{\uparrow} = \{|\uparrow,0\rangle, |0,\uparrow\rangle, |\uparrow,\uparrow\downarrow\rangle, |\uparrow\downarrow,\uparrow\rangle, |\uparrow,\uparrow\rangle\}$

hopping process of unpaired fermion



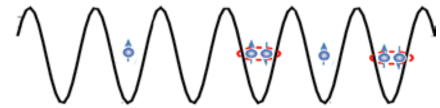
$\rho_i^{(z)} =$	1/2	0	-1	1/2	-1	1/2	1	1/2	1	-1	-1/2	-1	-1/2	
$n(i) =$	3	2	0	3	0	3	4	3	4	0	1	0	1	

CDW like configuration — high density — low density

Transfer energy of unpaired fermion is reduced

unpaired fermion tend to form low (or high) density region

Result (with spin imbalance)



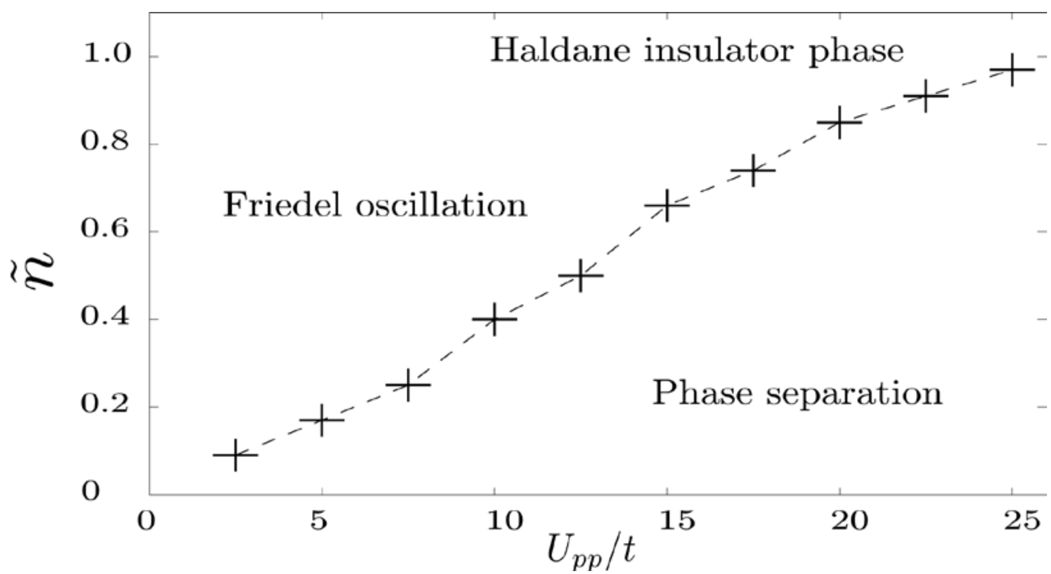
$$H_{\text{eff}} = P\left[-\sum_{p,\sigma}\sum_{<i,j>} t c^{\dagger}_{p,\sigma,i} c_{p,\sigma,j} - 2\bar{\mu}\sum_{i}\rho_i^{(z)} - \frac{J_{\text{ex}}}{2}\sum_{<i,j>}\rho_i^{(+)}\rho_j^{(-)} + J_{\text{ex}}\sum_{<i,j>}\rho_i^{(z)}\rho_j^{(z)}\right]P$$

$$-\sum_{pp'} P\left[J_{\text{ex}}\left(n^{(z)}_{p,\uparrow,i}+\frac{1}{2}\right)\left(n^{(z)}_{p',\uparrow,j}+\frac{1}{2}\right)\right]P$$

phase diagram

$2\bar{\mu}\rho_i^{(z)}$

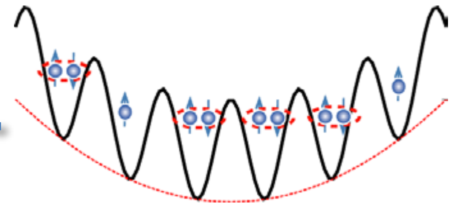
pseudo spin-spin interaction
become irrelevant



$$J_{\text{ex}} = \frac{2t^2}{|U_{pp}| + |U_{p_x p_y}|}$$

pseudo spin-spin interaction
become irrelevant

Result (effect of harmonic trap potential)



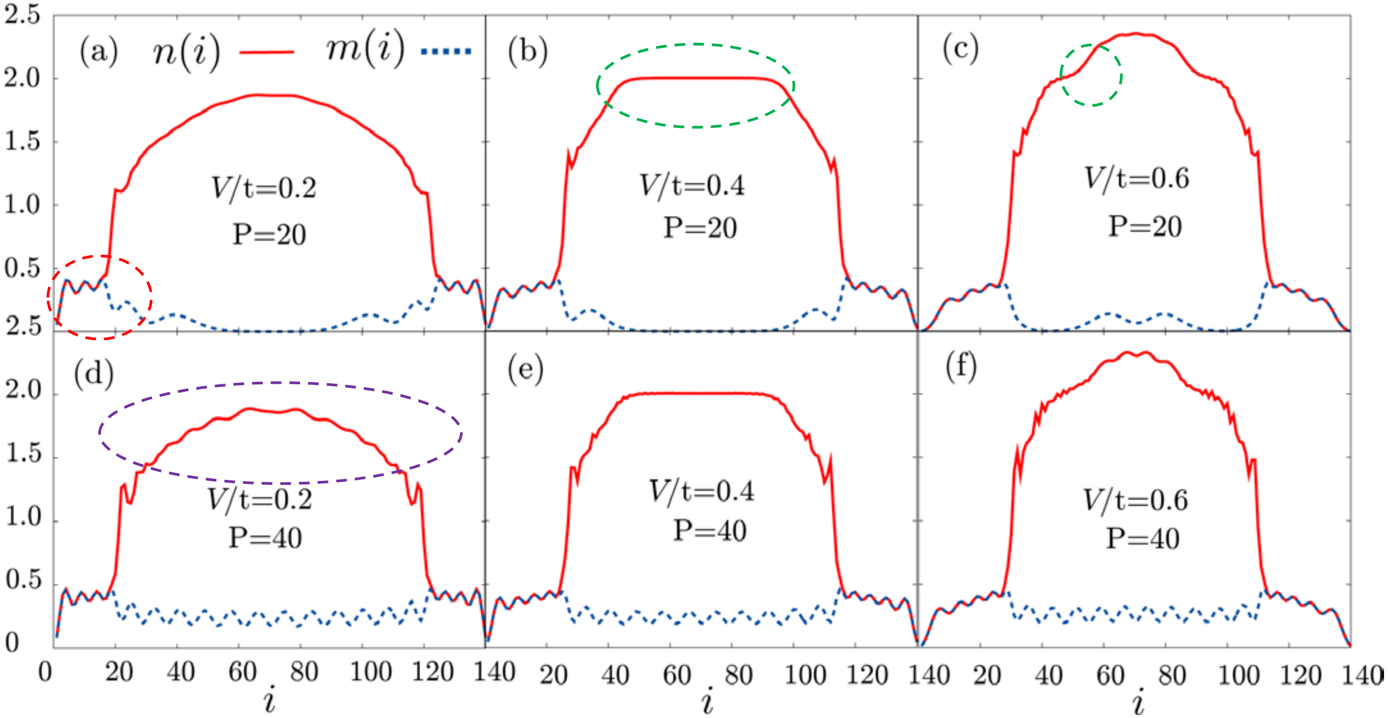
Multi-band Hamiltonian + harmonic trap potential

$$\sum_{p,\sigma,i} V_{\text{ho}}(i) n_{p,\sigma,i}$$

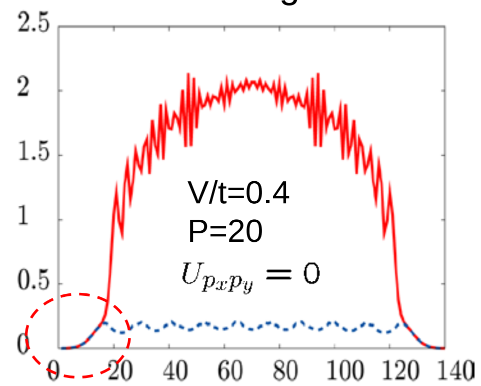
$$V_{\text{ho}}(i) = V(2/(L-1))^2 (i-(L+1)/2)^2$$

$L = 140$
$U_{pp}/t = -10 \quad U_{p_x p_y}/t = 4U_{pp}/9$
$P = \sum_{p,i} (n_{p,\uparrow,i} - n_{p,\downarrow,i})$

Result of multi-band



Result of single-band



Strong phase separation

Mott core like structure: Haldane insulator phase(polarization is excluded)

Friedel like oscillation (polarization and density)

Summary

1D p-orbitals fermi gas with attractive interaction

Various quantum phase

Haldane insulator phase
Luther-Emery phase
Friedal oscillation of density and spin density profiles
Strong phase separation of polarized component with out trap potential

Effect of trap potential

Various phase can coexist by controlling filling rate, strength of trap potential and spin imbalance rate.