Equilibration of Scaler Fields in an Expanding System

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Abstract

We present numerical analyses of nonequilibrium field theoretical approach of $O(N)$ scaler model with longitudinal expansion in 2+1 dimensions. We include Next-to-Leading Order of $1/N$ expansion as self energy in the presence of background classical field. We compare quantum dynamics and classical statistical approximation. Then we see the difference of two approaches in final distribution functions in strongly coupled regimes, where Boltzmann tail of distribution function is given only in quantum dynamics.

1 Introduction

Recently experiments have been done to create and study Quark-Gluon Plasma (QGP) by colliding two nuclei at center-of-energy 200GeV at RHIC and 2.76TeV at LHC. For produced QGP nearly ideal hydrodynamics succeeds in describing dynamics after thermalization of Glasma, for which initial condition is given by classical longitudinal color electric and magnetic fields with vacuum quantum fluctuations. Its success is based on early thermalization of Glasma $t_{\rm eq} = 0.6\text{-}1.0\text{fm}/c$ [1]. This time scale is comparable with the formation time of partons [2]. Then normal parton picture might fail to describe thermalization of Glasma, where parton picture estimates 2-3 fm$/c$[3]. Hence it is necessary to adopt dynamics beyond parton picture. Furthermore other instability or classical statistical approaches do not describe late-time Bose-Einstein distribution. Thus we should adopt approaches which describe late time true thermalization. As a candidate of approaches that are beyond the parton picture and describe late time Bose-Einstein distribution, we adopt nonequilibrium quantum field theoretical approach, which is represented by Kadanoff-Baym(KB) equation [4] with equation of motion of classical fields.

One of the merits of solving these equation is that field-particle conversion occurs. If particles are produced from classical fields, they collide each other, so that late time Bose-Einstein distribution is realized. The other merit is presence of the spectral function with finite decay width, which induces rapid change of distribution function due to 2-to-2 collisions compared with semi-classical Boltzmann equation. They might play a significant role in describing early thermalization of gluons [5].

Section 2 is devoted to introduction of time evolution equation for classical fields and quantum fluctuations. In Sec. 3 we give numerical results. We summarize our work in Sec. 4.

2 Time evolution equation

In this section we write down KB equation and time evolution equation of classical fields. First we start with action of scalar $O(N)$ model,

$$S = \int d^{d+1}x \left[\frac{1}{2}\partial\phi_a\partial\phi_a - \frac{1}{2}m^2\phi_a\phi_a - \frac{\lambda}{24N}(\phi_a\phi_a)^2\right], \tag{1}$$

where particle components a runs over $1, \cdots, N$ and d represents the spatial dimension. The merit of adopting this model is to cover all time evolution of instability by use of $1/N$ expansion. Then the equations of motion of classical fields $\bar{\phi}_a \equiv \langle \phi_a \rangle = \bar{\phi}\delta_{a1}$ and quantum fluctuations $F_{ab}(x,y) \equiv \frac{1}{2}\langle\{\tilde{\phi}_a(x), \tilde{\phi}_b(y)\}\rangle$ $\rho_{ab}(x,y) \equiv \langle[\tilde{\phi}_a(x), \tilde{\phi}_b(y)]\rangle$ (that are Fourier transformed), where $\tilde{\phi}_a = \phi_a - \bar{\phi}_a$, are given by

$$\left[\partial_\tau^2 + \frac{1}{\tau}\partial_\tau + m^2 + \frac{\lambda}{6N}\left(\bar{\phi}(\tau)^2 + F_{11}(\tau,\tau) + \sum_{b\neq 1} F_{bb}(\tau,\tau)\right)\right]\bar{\phi}(\tau) = -\int_{\tau_0}^{\tau} \tau' d\tau' \Sigma_{11}^{\rho}(\tau,\tau')\bar{\phi}(\tau'), \quad (2)$$

$$G_0^{-1}F(\tau,\tau',p) = -\int_{\tau_0}^{\tau} \tau'' d\tau'' \Sigma^{\rho}(\tau,\tau'',p)F(\tau,\tau',p) + \int_{\tau_0}^{\tau'} \tau'' d\tau'' \Sigma^{F}(\tau,\tau'',p)\rho(\tau'',\tau',p), \quad (3)$$

$$G_0^{-1}\rho(\tau,\tau',p) = -\int_{\tau'}^{\tau} \tau'' d\tau'' \Sigma^{\rho}(\tau,\tau'')\rho(\tau,\tau',p), \quad (4)$$

$$G_0^{-1} \equiv \left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} + \frac{p_\eta^2}{\tau^2} + p_T^2\right]\delta_{ab} + M_{ab}^2(\bar{\phi})$$

Here we adopt proper time $\tau = \sqrt{t^2 - z^2}$, rapidity $\eta = \tanh^{-1}\frac{z}{t}$ and its Fourier transformed p_η to treat longitudinal expansion (we adopt spatial homogeneity) and set initial time τ_0, local mass shift $M_{ab}(\bar{\phi})$ and Next-to-Leading Order self-energy $\Sigma^{F,\rho}$ of $1/N$ expansion which contains 2-to-2 collisions non-perturbatively.

As an initial condition, we set $\bar{\phi}_a(\tau_0) = \sqrt{\frac{6N}{\lambda}}\sigma\delta_{a1}$ with vacuum quantum fluctuations for F and ρ. Numerical simulations are done in 2+1 dimensions for strongly coupled $\lambda = 10$ regimes in an expanding system. Then we compare quantum dynamics and classical statistical approximation which omit $\rho\rho$ terms in self-energy [5, 6, 7].

3 Numerical analyses in 2+1 dimension

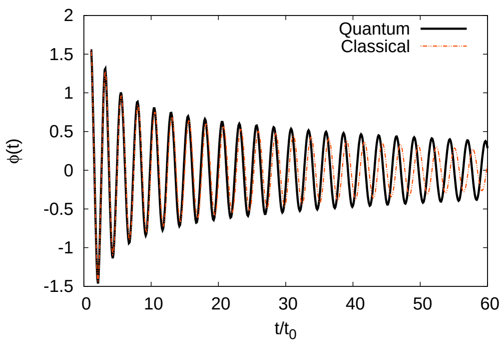

Figure 1: Time evolution of the classical field $\bar{\phi}$ for quantum (black solid line) and classical statistical approximation (red dashed line).

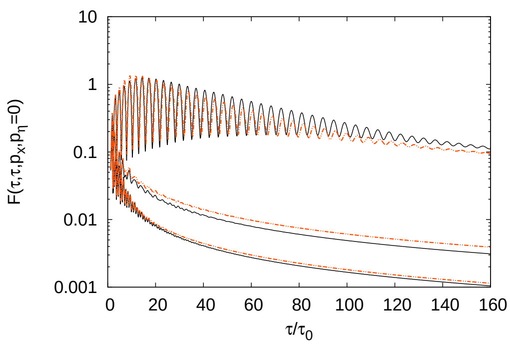

Figure 2: Time evolution of $F_{11}(\tau,\tau,p_\eta = 0, p_T = 0, \pi\sigma/8, \pi\sigma/4)$ for for quantum (black solid line) and classical statistical approximation (red dashed line).

In this section we show time evolution of classical field $\bar{\phi}(\tau)$ (Fig. 1) and statistical functions $F_{11}(\tau,\tau,p)$ (Fig. 2). In Fig.1 classical fields damps (due to $\frac{\partial_\tau}{\tau}\bar{\phi}$ term in Eq. (2)) with oscillation.

Then field particle conversion occurs, which means that particle production occurs due to decay of classical fields. Time evolution of F in Fig.2 shows this particle production, where F changes from $O(1)$ to $O(1/\lambda)$. Finally we present the distribution function n_p (Fig. 3) by use of functional fit $F = \frac{C}{\tau\sqrt{m_{\rm eff}^2+p_T^2}}\left(n_p + \frac{1}{2}\right)$ where C and $m_{\rm eff}$ are fitting parameter. In quantum dynamics the distribution function is Bose-Einstein type and shows Boltzmann tail ($n_p = \frac{1}{e^{p_T/T}-1}$ with temperature T) , while it gives power law $n_p = \frac{T}{p_T} - \frac{1}{2}$ in classical statistical approximation. In classical approach true thermalization is not realized.

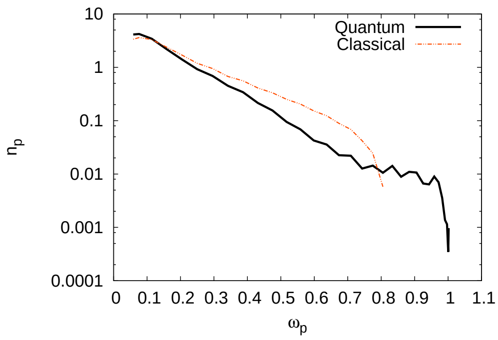

Figure 3: Number distribution function n_p at late time $\tau/\tau_0 = 150$.

4 Summary

In this work we have solved the Kadanoff-Baym equation and time evolution equation of classical fields for $O(N)$ scalar model in a spatially homogeneous expanding system in 2+1 dimensions. We have included NLO nonlocal self-energy representing 2-to-2 by use of $1/N$ expansion which covers all time evolution of instability of F from $O(1)$ to $O(1/\lambda)$. In strongly coupled regimes $\lambda = 10$, while quantum dynamics shows Bose-Einstein distribution, classical statistical approximation gives power law behavior in distribution function, which is not true thermalization. Thus we need quantum dynamics in order to realize the Bose-Einstein distribution.

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