Creation of D9-brane–$\overline{\text{D9}}$-brane Pairs from Hagedorn Transition of Closed Strings ∼ Cylinder Amplitude and Sphere Amplitude

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The Hagedorn transition of closed strings has been proposed as a phase transition via condensation of this winding tachyon. On the other hand, we have previously shown that a phase transition occurs near the Hagedorn temperature and D9-brane–$\overline{\text{D9}}$-brane pairs become stable. We present a conjecture that *D9-brane–anti-D9-brane pairs are created by the Hagedorn transition of closed strings.* We show some circumstantial evidences for this conjecture.

1 Hagedorn Transition of Closed Strings

It is well known that perturbative string gas has a characteristic temperature called the Hagedorn temperature. We can compute the one-loop free energy of strings by using path integral in Matsubara method. The one-loop free energy of strings diverges above this temperature. With respect to closed strings, it has been said that the Hagedorn temperature is associated with a phase transition, in analogy to the deconfining transition in QCD. This is because we can reach the Hagedorn temperature by supplying finite energy in the closed string case.

One explanation for this divergence is that a 'winding mode' in the Euclidean time direction becomes tachyonic above the Hagedorn temperature. Atick and Witten have proposed the Hagedorn transition of closed strings via condensation of this winding tachyon [1]. They advocate that the Hagedorn temperature is not really a limiting temperature but rather is associated with a phase transition. Atick and Witten argued further from the worldsheet point of view. At low temperature, sphere worldsheet does not contribute to the free energy, since it cannot wrap the compactified Euclidean time. But if we consider the condensation of winding tachyon above the Hagedorn temperature, the sphere worldsheet is no longer simply connected and it contributes to the free energy above the Hagedorn temperature. This is because the insertion of the winding tachyon vertex operator means the creation of a tiny hole in the worldsheet which wraps around the compactified Euclidean time, and the condensation of winding tachyon induces an infinite number of tiny holes in the worldsheet. It should be noted that this mode appears only in Matsubara formalism. We cannot identify which modes condensate to what extent in Lorentzian time when this winding tachyon condensate in the Euclidean time.

Significant effort has been devoted to find out the stable minimum of the potential of this winding tachyon. But we have not succeeded in finding it out so far. It is difficult to compute the potential of closed string tachyon because this potential has to be calculated by closed string field theory and this theory has not been well-established.

2 Brane–anti-brane Pairs at Finite Temperature

We have previously discussed the behavior of brane-antibrane pairs at finite temperature in the constant tachyon background [2]. At zero temperature, the spectrum of open strings on these unstable branes contains a tachyon field T. In the brane–antibrane configuration, we have $T = 0$, and the potential of this tachyon field has a local maximum at $T = 0$. The tachyon potential has a non-trivial minimum, which is called closed string vacuum, and the tachyon falls into it at zero temperature. Sen conjectured that the potential height of the tachyon potential exactly cancels the tension of the original brane-antibrane pairs [3]. This implies that these unstable brane systems disappear at the end of the tachyon condensation.

Although brane-antibrane pairs are unstable at zero temperature, there are the cases that they become stable at finite temperature. We have calculated the finite temperature effective potential of open strings on these branes based on boundary string field theory. For the D9-brane–$\overline{\text{D9}}$-brane pairs, a phase transition occurs at slightly below the Hagedorn temperature and the D9-brane–$\overline{\text{D9}}$-brane pairs become stable above this temperature. On the other hand, for the Dp-brane–$\overline{\text{D}p}$-brane pairs with $p \leq 8$, such a phase transition does not occur. We thus concluded that not a lower dimensional brane-antibrane pairs but D9-brane–$\overline{\text{D9}}$-brane pairs are created near the Hagedorn temperature. Let us call this phase transition brane-antibrane pair creation transition.

3 Creation of D9-brane–$\overline{\text{D9}}$-brane Pairs from Hagedorn Transition of Closed Strings

Let us consider the relationship between above two phase transitions. Atick and Witten argued about the meaning of the condensation of the winding tachyon [1]. The insertion of the winding tachyon vertex operator corresponds to the creation of a tiny hole in the worldsheet which wraps around the compactified Euclidean time. But what is the hole of closed string worldsheet? Let us try to think about it from a different point of view. Since the boundaries of holes wind around the Euclidean time direction, taking a time slice of a sphere worldsheet with some winding tachyon insertion, we obtain open strings. Therefore, this worldsheet represents open strings propagating in the Euclidean time direction. Then we can identify the boundary of a hole created by winding tachyon vertex operator with a boundary of an open string on a D9-brane–$\overline{\text{D9}}$-brane pair, and the insertion of winding tachyon vertex operator means the insertion of the boundary of open strings in the tiny hole limit, which wraps the compactified Euclidean time once. If we enlarge the size of this hole, we can describe open strings with arbitrary boundary. Therefore, we present a following conjecture :

> *D9-brane–$\overline{\text{D9}}$-brane pairs are created by the Hagedorn transition of closed strings.*

That is, above two phase transitions are two aspects of one phase transition. In the sense that $T = 0$ is the perturbative vacuum of open strings, this is a phase transition from closed string vacuum to open string vacuum. In other words, the stable minimum of the Hagedorn transition is the open string vacuum.

Here we describe some circumstantial evidences for this conjecture. First, if we consider the thermodynamic balance on D9-brane–$\overline{\text{D9}}$-brane pairs, we can show that energy flows from closed strings to open strings and open strings dominate the total energy. This is because we can reach the Hagedorn temperature for closed strings by supplying finite energy, while we need infinite energy to reach the Hagedorn temperature for open strings on these branes.

Secondly, we show that, in the Matsubara formalism, some types of amplitude of open strings approaches to closed string ones if we take an appropriate limit. The cylinder amplitude of open strings close to the closed string vacuum has the form of the propagator of winding tachyon. The sphere amplitude for two winding tachyons vanishes, and the cylinder amplitude also vanishes if we take the closed string vacuum limit together with the Hagedorn temperature limit under appropriate condition. In these limits, the cylinder amplitude with a single massless boson insertion approaches to the sphere amplitude with two winding tachyons and a single massless boson insertion. We also show that the cylinder amplitude with two winding tachyons insertion approaches to the sphere amplitude with four winding tachyons insertion. We are now investigating whether the cylinder amplitude with $2n$ winding tachyons insertion approaches to the sphere amplitude with $(2n + 2)$ winding tachyons insertion. These are examples that we can identify the open string amplitude in the closed string vacuum limit with the closed string sphere amplitude with some winding tachyons insertion. It seems reasonable to conclude that we can identify the winding tachyon as the closed string vacuum limit of the boundary of an open string, which winds once around the compactified Euclidean time.

Thirdly, we show that the potential energy at the open string vacuum decreases limitlessly as the temperature approaches to the Hagedorn temperature. From this we may say that the open string vacuum becomes the global minimum in entire space of the open string tachyon field near the Hagedorn temperature. This is the property that the stable minimum of the Hagedorn transition is expected to have.

References

[1] J. J. Atick and E. Witten, *Nucl. Phys.* **B310** 291 (1988).

[2] K. Hotta, JHEP **0212** (2002) 072, hep-th/0212063; JHEP **0309** (2003) 002, hep-th/0303236; Prog. Theor. Phys. **112** (2004) 653, hep-th/0403078.

[3] A. Sen, "Tachyon Condensation on the Brane Antibrane System," JHEP **9808** (1998) 012, hep-th/9805170.