Direct observations of thermal fluctuations below shot noise levels

Kenichiro Aoki and Takahisa Mitsui
Research and Education Center for Natural Sciences and Hiyoshi Dept. of Physics, Keio University, Yokohama 223–8521, Japan

We explain a general method for obtaining measurements below shot noise levels. We apply this to surface thermal fluctuations to obtain their spectra to previous unseen precision. The physics behind the spectra and the broad applicability of the noise reduction method is discussed.

Thermal fluctuation phenomena are truly ubiquitous — for instance, all surfaces we observe, solid, liquid or otherwise fluctuate thermally. In this note, we describe some direct quantitative measurements of these fluctuations occurring in common place materials at room temperatures, to precision unachieved up to now. In doing so, we can test our theoretical understanding of the basic physics principles and properties of materials observed through thermal fluctuations to hitherto untested precision. More importantly, by exploring new territory, we can search for new physics phenomena. These thermal fluctuations are at atomic scale and one obstacle that needs to be overcome to achieve such precision is the prevalence of noise from various sources. We describe a new method for achieving measurements at previously unseen low noise levels below. For such measurements, light scattering methods are most commonly used, in which shot noise arising in photoconversion is unavoidable. Shot noise is a fluctuation in the photocurrent spectrum of the form $\langle(\Delta I)^2\rangle = 2eI\,\Delta f$, where e, I, f are the electron charge, current and the frequency. Shot noise can be a limiting factor for measurements that require high precision, such as the attempts to measure gravitational waves. Previous efforts to reduce shot noise using squeezed states of light have achieved reductions by around a factor of two[1, 2].

Let us briefly outline the principle underlying the noise reduction we employ[3, 4]: In any measurement, $D_1 = S + N_1$, signal S is accompanied by some noise N_1. The spectrum obtained from this measurement is $\langle|\tilde{D}_1|^2\rangle = \langle|\tilde{S}|^2\rangle + \langle|\tilde{N}_1|^2\rangle$, where tildes denote the Fourier transform. If S has no known periodicity, there is no way to separate out the signal from the noise, even in principle. However, if we make another measurement D_2 of the same signal, uncorrelated noise can be statistically reduced in the correlation $\langle\overline{\tilde{D}_1}\tilde{D}_2\rangle \to \langle|\tilde{S}|^2\rangle$. The relative error here is $\sim 1/\mathcal{N}$, $\mathcal{N}$ being the number of averagings. This simple principle is *not* limited to optical measurements, surface fluctuations nor to shot noise. Any uncorrelated noise including shot noise, amplitude modulation noise, amplification noise and so on, are reduced in this approach. In practice, we obtain a factor of 10^{-3} reduction from the shot noise level. The limitations are that we need to arrange the measurements so that the unwanted noise is uncorrelated in them and that the signal needs to be stable enough for the averagings to be effective.

We first discuss liquid surface thermal fluctuations, sometimes called 'ripplons'. Liquid surface fluctuations have been predicted [5] and have been experimentally observed some time ago[6, 7]. The fluctuation spectrum of simple liquid surfaces have been derived from hydrodynamical considerations[8, 9]. The spectrum has qualitatively different behavior for liquids with weak and strong dissipation. Any simple liquid has strong dissipation and is highly viscous at high frequencies, which is intuitively natural. For liquids with low dissipation, there is a peak in the dispersion relation, leading to a well defined wave. For liquids with high dissipation, no such peak nor simple wave behavior exists, precluding the application of traditional observation methods, which use the waves effectively as gratings. We study these phenomena mainly for the following reasons: First, these basic physics phenomena are of interest on their own. Second, since the fluctuation spectrum has been derived previously, we can confirm that our measurement method is effective at sub-shot noise levels and furthermore, we can explore the limitations of the traditional hydrodynamical approach.

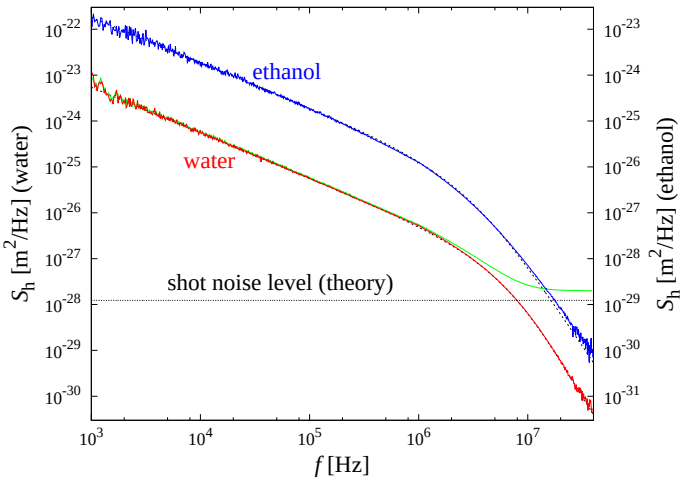

Figure 1: Experimentally observed surface height fluctuation spectra for water (red) and ethanol (blue). Respective theoretical spectra are also shown (black dashed), which agree well with the experimental results so as to be almost invisible. For comparison, observed data for a single differential detection, $\langle|\tilde{D}_1|^2\rangle$ is shown for water (green). This is clearly dominated at higher frequencies by the shot noise, whose theoretical value is indicated (black, dotted).

In Fig. 1, direct measurements of the fluctuation spectra of water and ethanol are shown. We obtained these measurements by applying the noise reduction method explained above to the classic Michelson interferometry[10]. In the measurements, we use the coherent state of light and *not* squeezed states. A single light source is used and the noise in the multiple measurements are uncorrelated due to the quantum property of light. The observed shot noise level agrees with theory and our measurement can be seen to go down three orders of magnitude below it. The theoretical spectra are seen to agree

with the measured spectra to a high degree, for these liquids.

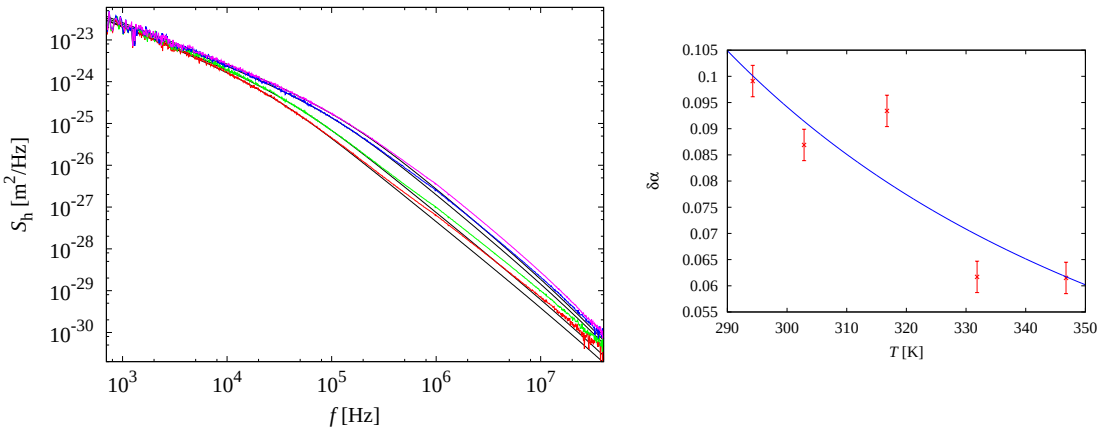

Figure 2: (Left) Observed surface fluctuation spectra for oil at various temperatures, T= 294 (red), 303 (green), 332 (blue), 347 (magenta) [K] with their theoretical predictions (black dashed). Fluctuations are larger at higher temperatures. (Right)Temperature dependence of the deviation from theory of the high frequency fall off $f^{-\alpha}$ ($f \gtrsim 1\,\mathrm{MHz}$).

In Fig. 2, the surface fluctuation spectra of oil are shown for various temperatures. Oil is highly viscous and its spectrum differs from that of water and ethanol qualitatively. In particular, the fall off at high frequencies seen here are $\sim f^{-2}$ for oil, in contrast to $\sim f^{-4}$ for water and ethanol. Unlike the case of water, experimental spectra of oil surface fluctuations visibly differ from the theoretical spectra obtained solely from hydrodynamical considerations. These deviations are larger at lower temperatures and are likely caused by the more 'complex' nature of the fluid.

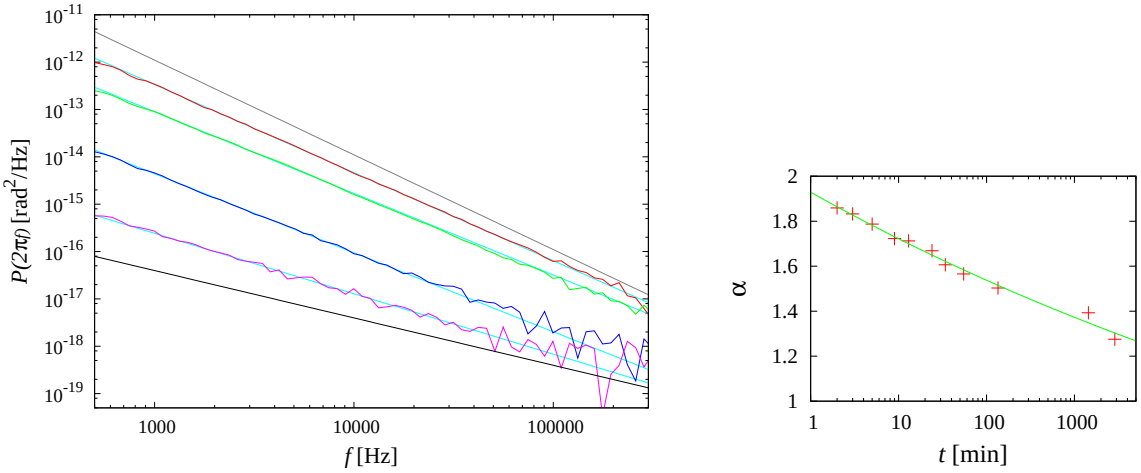

Figure 3: (Left) Epoxy adhesive surface fluctuations 2(red), 9(green), 24(blue) minutes and 2 days(magenta) after application, together with fits to them to functions of the form const.$\times f^{-\alpha}$ (thin lines, cyan), which work quite well. As a guide, f^{-2} (grey) and (black) are also shown. (Right) Time dependence of α, f dependence of the spectrum $\sim f^{-\alpha}$. Gradual decrease from $\alpha = 2$ to $\alpha = 1$ is seen.

The measurement requires only a short time so that it can be applied to obtaining the time dependence of fluctuation spectra. In Fig. 3, we plot the surface fluctuation spectra of epoxy adhesive at various times after its application[3]. As the adhesive 'hardens', the fluctuations become smaller and its frequency dependence is seen to change. Since a viscous simple liquid surface behaves as f^{-2} and an elastic solid surface behaves as f^{-1} in this frequency region, the spectrum changes roughly from liquid to solid behavior with time. In this measurement, the noise reduction is crucial for obtaining spectra below the shot noise level which is $\sim 4\times10^{-15}$ [rad^2/Hz] here. This result is obtained with an optical lever with noise reduction, which reflects the versatility of the noise reduction method.

In this note, we explained the principle of noise reduction that allows us to measure at levels orders of magnitude below the shot noise level. This allows us to directly measure fluctuation spectra of surfaces to previously unseen precision and some of the results were presented here. The results required only a small power ($\sim 0.5\,$mW), small sample size ($\sim \mu m$) and short measurement times (few s). The method has a wide area of applicability including measurements that require non-invasiveness. Our method can be applied to various materials and situations to observe hitherto unseen phenomena, which we intend to report on, in the near future.

References

[1] C.M. Caves, Phys. Rev. D23, 1693 (1981).

[2] K. McKenzie, B.C. Buchler, D.A. Shaddock, P.K. Lam, D.E. McClelland, Class. Quantum Grav. 21, S1037 (2004).

[3] T. Mitsui, K. Aoki, Phys. Rev. E 80, 020602(R)-1 (2009).

[4] K. Aoki, T. Mitsui, Phys. Rev. E 86, 011602 (2012).

[5] M. von Schmoluchowski, Ann Physik 25, 225 (1908); L. Mandelstam, Ann. Physik 41, 609 (1913).

[6] R.H. Katyl, U. Ingard, Phys. Rev. Lett. 19, 64 (1967).

[7] D. Langevin, *"Light scattering by liquid surfaces and complementary techniques"*, Marcel Dekker, New York (1992).

[8] V.G. Levich, *"Physicochemical Hydrodynamics"*, Prentice-Hall, Englewood Cliffs (1962).

[9] M.-A. Bouchiat, and J. Meunier, J. de Phys. 32, 561 (1971).

[10] T. Mitsui, K. Aoki, `arXiv:1206.3828`