

Interweaving Chiral Spirals

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Abstract

The interweaving chiral spirals (ICS), defined as superposition of differently oriented chiral spirals, break the chiral, translational, and rotational symmetry in quark matter at finite density. We further postulate that the quark mass gap produced by the ICS enlarges the region of the confined phase, by tempering the growth of quark fluctuations near the Fermi surface. The EOS becomes stiffer by increasing of the pressure by $\sim N_c \Lambda_{\text{QCD}} \times (\Lambda_{\text{QCD}} \times 4\pi\mu_q^2)$.

1 Impacts of the inhomogeneous chiral condensates

Recently, it has been argued that there is a new state of QCD matter at high baryon density and low to intermediate temperatures [1].

This novel state is called Quarkyonic matter, with the Fermi sea mainly composed of quarks, but with the confined excitations near the Fermi surface. The domain of the applicability of this picture is given by $M_N/N_c \leq \mu_q \leq N_c^{1/2}\Lambda_{\text{QCD}}$; the former characterizes the scale of the quark matter formation where the hadronic-nuclear-quark matter formation occurs rapidly within a small change in μ_q , while the latter gives the scale of deconfinement where the strength of quantum quark fluctuations becomes comparable to those of gluons due to the enlarged phase space near the Fermi surface.

In this talk, we ask what happens to the chiral symmetry at density mentioned above. So far most of theoretical calculations have suggested that the chiral restoration occurs shortly after the formation of the quark Fermi sea, provided that the chiral condensate is homogeneous [2]. For this reason, the chiral restoration line is usually placed very close to the chemical freezeout line, which should be closely related to the quark matter formation.

However, recently it was recognized that the chemical freezeout line may be separated from the chiral restoration line, according to the lattice QCD estimates for the μ_q -dependence of chiral and quark number susceptibilities [3]. One of the possibilities to fill this gap might be the inhomogeneous chiral condensates [4, 5, 6]. In fact several calculations based on the chiral models as well as confining models suggest that inhomogeneous chiral condensates can break the chiral symmetry even after the quark matter formation, if the gluon exchange force remains strong enough. Furthermore, such condensates can create the mass gap for quarks near the Fermi surface, suppressing the screening effects for the gluons. Thus the inhomogeneous chiral condensates have a big impact on the

deconfinement phenomena. All excitations except the Nambu-Goldstone bosons acquire the mass gaps.

2 A single Chiral Spiral

At zero density, the chiral condensate is formed by condensations of particles and anti-particles. At finite density, however, the Pauli-blocking requires large energy for the creation of an anti-particle, thus the particle-antiparticle condensation is not favored. Instead, more proper ingredients of condensates are particle-holes near the Fermi surface. The excitations near the Fermi surface do not cost energy much, despite they have large momenta, $|\vec{p}| \sim \mu_q$. Any condensates made of pairings must contain particles (holes) with hard momenta, so hard momenta appear either relative or total momenta of the pairs.

Below we focus on the possibility of the chiral density wave (CDW) pairing which evolves in a particular direction. It contains a co-moving particle-hole pair with a large total momentum of $\sim 2\mu_q$, while the relative momentum between them is small and $\sim \Lambda_{\text{QCD}}$. Actually the chiral density wave solution can be always interpreted as the chiral spirals (CSs). A key observation is that once we have a condensation of a pair moving to, say, $+z$ -direction, there is also a pair moving to $-z$ direction. Mathmatically, one can project out fermion components moving to $\pm z$ directions by operating the projection matrices [5],

$$\psi_{\pm} \equiv \frac{1 \pm \gamma_0 \gamma_z}{2} \psi . \quad (1)$$

Then we have two types of the chiral condensates, $\langle \bar{\psi}_- \psi_+ \rangle \sim \Delta e^{2i\mu_q z}$, $\langle \bar{\psi}_+ \psi_- \rangle \sim \Delta e^{-2i\mu_q z}$, whose sum and difference give

$$\langle \bar{\psi} \psi \rangle \sim \Delta \cos(2\mu_q z), \quad \langle \bar{\psi} i \gamma_0 \gamma_z \psi \rangle \sim \Delta \sin(2\mu_q z), \quad (2)$$

with a fixed radius of Δ of the order Λ_{QCD}^3 , which is given by solving the gap equation. These condensates obviously break the chiral symmetry, translational invariance, rotational invariance, and the second condensate further breaks parity *locally*.

3 Interweaving Chiral Spirals

We have seen that the CS must have a particular orientation. Let us ask: Can chiral pairs be formed in such a way to cover the entire Fermi surface, and can differently oriented CSs be interweaved in a consistent way?

The answer to these questions depends on the models, mainly due to the interactions among differently oriented CSs. The attempts to construct the ICS for the NJL-model can be found in [7], but the authors have concluded that the single CDW is energetically favored compared to the ICS. In fact, the interactions among CDWs appear to be repulsive, so the inclusion of many CDWs destroy condensates one another.

However, the conclusion may be altered if we take the asymptotic free nature of microscopic interactions into account. The gluon exchange should be strong for small momentum transfer, while should be weak for hard momentum transfer. If one uses models with this feature, such a momentum dependence is converted into the form factor effects for the interactions among quarks and condensates (or mean fields), which drastically reduces the energetic cost due to interactions among differently oriented CSs [8].

As a consequence, we can find an optimized number of CSs, which depends upon quark density. It means that as quark density increases, there generate more CSs, leading to the sequential phase transitions. Such transitions are likely to be first order, and might be related to the star quake.

Since the entire Fermi surface is covered by the condensates, the reduction of the free energy is very large, $\sim -N_c \Lambda_{\text{QCD}} \times (\Lambda_{\text{QCD}} \times 4\pi\mu_q^2)$, where the former factor comes from the size of the quark mass gap, while the latter comes from the number of pairs. In other words, the quark matter with ICSs give much higher pressure than the quark matter with free quarks, leading to stiffer EOS. This feature is welcomed for the explanation of recently found neutron star with the mass about twice of the solar mass.

References

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