Effects of Vertex correction in the Schwinger-Dyson equation in QED3

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1 Introduction

Chiral order parametr $\langle\overline{\psi}\psi\rangle$ is gauge invariant in QED,QCD.However if we evaluate this quantity by the Schwinger-Dyson equation for example,we must take into account the vertex correction to satisfy Ward-Takahashi-identy which is the consequence of gauge invariance.Ward-Takahashi identity is written as

$$(p-q)_{\mu}\Gamma_{\mu}(p,q)=S^{-1}(q)-S^{-1}(p), \tag{1}$$

where Γ is a three-point vertex,and S is a fermion propagator

$$S(p)=\frac{i}{A(p)p\cdot\gamma-B(p)}. \tag{2}$$

Reliable Ansatz for three point vertex in QED shoud satisfy Ward-Takahashi identity.Following Ball-Chiu Ansatz[3] we have

$$\Gamma_{\mu}(p,q)=\Gamma_{\mu}^{T}(p,q)+\frac{A(p)+A(q)}{2}\gamma_{\mu}+\frac{A(p)-A(q)}{2(p^{2}-q^{2})}(p+q)\cdot\gamma(p+q)_{\mu}-\frac{B(p)-B(q)}{p^{2}-q^{2}}(p+q)_{\mu}, \tag{3}$$

where $\Gamma_{\mu}^{T}(p,q)$ is an arbitrary transverse part that can be added to the vertex,without dsiturbing the Ward-Takahashi identity.

2 Schwinger-Dyson equation

In QED3 and Chern-Simon QED[2],Schwinger-Dyson equation for the fermon propagator with vertex correction is given

$$S^{-1}(p)=S_{0}^{-1}(p)-ie^{2}\int\frac{d^{3}k}{(2\pi)^{3}}\gamma_{\mu}S(k)\Gamma_{\nu}(p,k)D_{\mu\nu}(p-k), \tag{4}$$

where D is the photon propagator

$$D_{\mu\nu}(p)=\frac{-i(g_{\mu\nu}-p_{\mu}p_{\nu}/p^{2}-i\mu\epsilon_{\mu\nu\rho}p_{\rho}/p^{2})}{p^{2}-\mu^{2}}-id\frac{p_{\mu}p_{\nu}}{p^{4}},$$

, μ is a gauge invariant mass,and d is a gauge fixing parameter.Following

$$iS(p)^{-1} = A(p)\gamma \cdot p - B(p) = \gamma \cdot p - \Sigma(p)$$

for 2-spinor

$$2B(p) = tr(\Sigma(p)), 2p^2(A(p) - 1) = tr(\gamma \cdot p\Sigma(p)) \quad (5)$$

we obtain coupled integral equation for $A(p), B(p)$.

3 Numerical analysis

If we solve the Dyson-Schwinger equation with longitudinal vertex in QED3,we find that the gauge dependence of chiral order parameter $\langle\overline{\psi}\psi\rangle$ is very small.At least for weak coupling it is estimated

$$\langle\overline{\psi}\psi\rangle = -(3.2 - 3.4) \times 10^{-3} e^4 \quad (6)$$

for $d = 0..2$,here d is a gauge fixing parameter.The above value agrees quite well with the results in [1].In three dimension QED is superrenormalizable and ultraviolet finite but infrared singular.We find that correction enhance the infrared singularity of the wave function $A(p)$ and mass $B(p)$.The following term in the vertex enhances the mass and vacuum expectation value

$$-\frac{B(p) - B(q)}{p^2 - q^2}(p + q)_\mu. \quad (7)$$

Gauge dependent term has infrared singular too.In Chern-Simon QED gauge field has mass.So that in the Landau gauge infrared divergence does not appear.However in other gauge we find the infrared divergence as in QED3.For 2-spinor case vacuum expectation value $\langle\overline{\psi}\psi\rangle$ is a spin density. In QED dynamical mass $B(p) \propto 1/p^2$. This is modified by Chern-Simon term of the photon propagator as $B(p) \propto 1/p$ at high-energy.This yields logarithmic divergence of the spin density $\langle\overline{\psi}\psi\rangle \propto \ln \Lambda$,where vacuum expectation value is written as

$$\langle\overline{\psi}\psi\rangle = -\int \frac{d^3p}{(2\pi)^3} \frac{2B(p)}{A^2(p)p^2 + B(p)^2}. \quad (8)$$

For $\Lambda = 10^3, \mu = 1.0$,we get $\langle\overline{\psi}\psi\rangle = .08e^4$,$d = 0..3$.In the Figures we see the Landau gage solution of wave function renormalization $Z = 1/A(p)$ and effective mass $M(p) = B(p)/A(p)$ with $\mu = 1$ and the scale $p = \exp(\pi/2 \sinh(n/155)$.In the near future detailed analysis will appear[6].

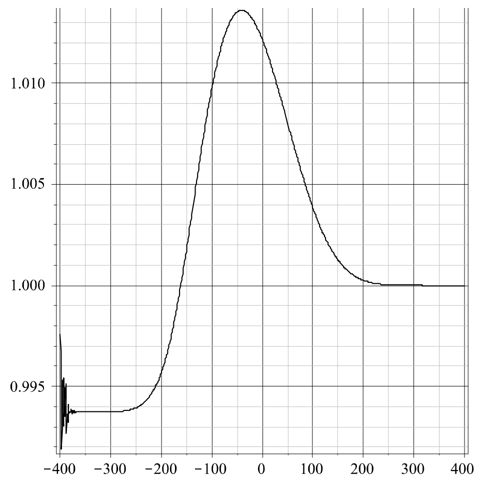

Fig.1 $Z = 1/A(p)$ in $d = 0$ gauge.

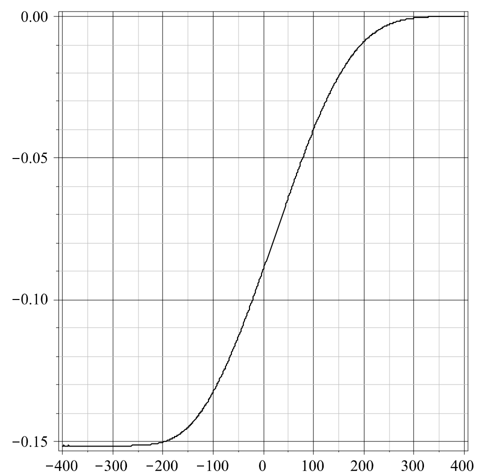

Fig.2 $M(p) = B(p)/A(p)$ in $d = 0$ gauge.

4 References

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