一次元量子気体の二重極振動の減衰における量子位相滑り

段下 一平

京大基研

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YUKAWA INSTITUTE FOR
THEORETICAL PHYSICS

I. Danshita, Phys. Rev. Lett. 111, 025303 (2013)

熱場の量子論とその応用
京都大学基礎物理学研究所
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Outline:

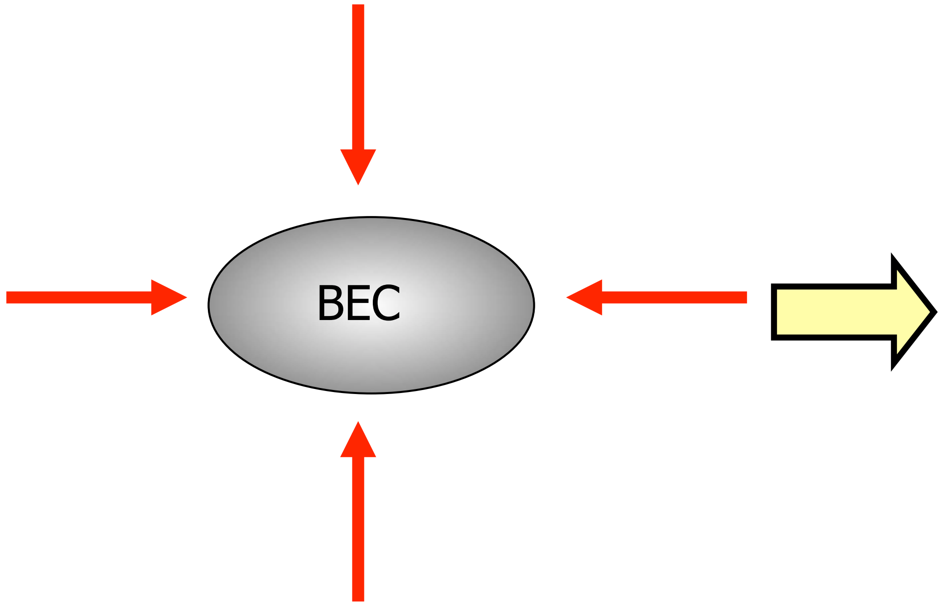
Damping rate ⇔ Nucleation rate

1. Introduction
2. Hand-waving picture
3. Numerical corroboration in the hardcore boson limit
4. Mechanism of the damping in softcore bosons
5. Finite temperature effects
6. Conclusions

Special thanks to Nikolay Prokof'ev (UMass Amherst)

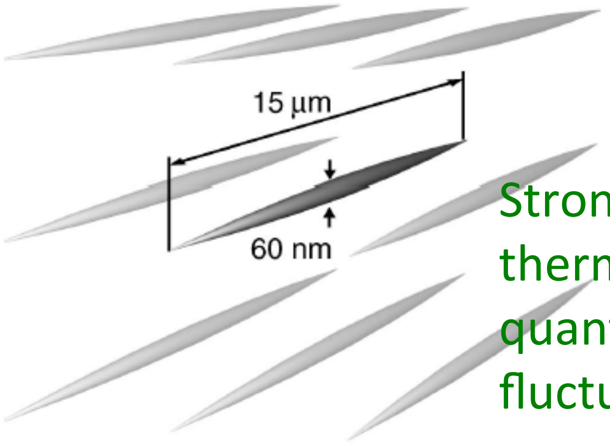
1. Introduction

1.1. Experiments on transport of 1D lattice bosons



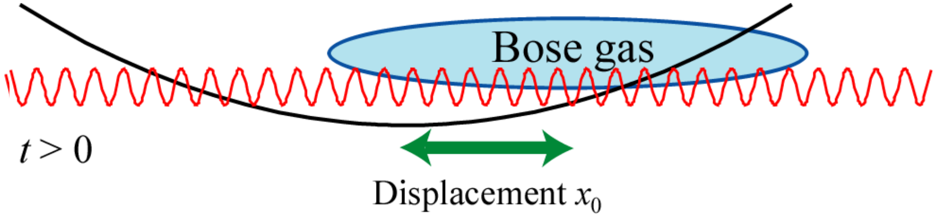
A 2D array of 1D Bose gases

H. Moritz et al., PRL (2003)



Strong thermal & quantum fluctuations

Dipole oscillations:



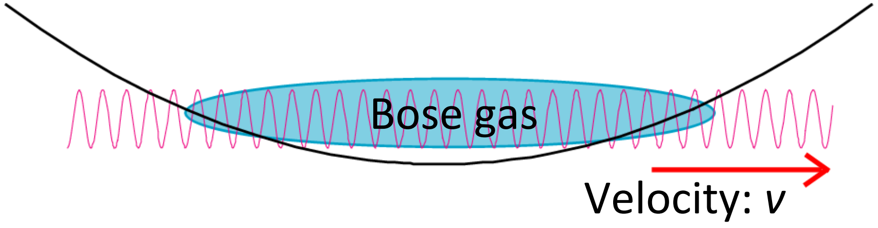
ETH: T. Stöferle et al., PRL (2004)

NIST: C. D. Fertig et al., PRL (2005)

Innsbruck: E. Haller et al., Nature (2010)

Stony Brook: B. Gadway et al., PRL (2011)

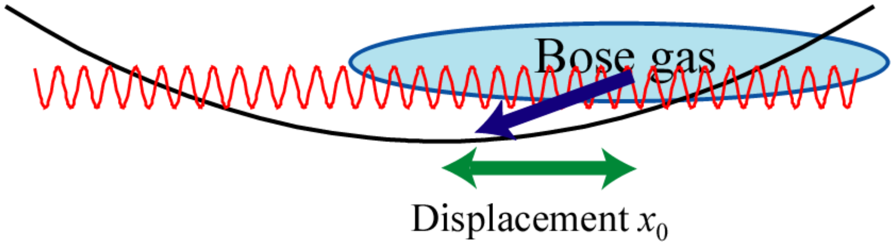
Moving optical lattices:

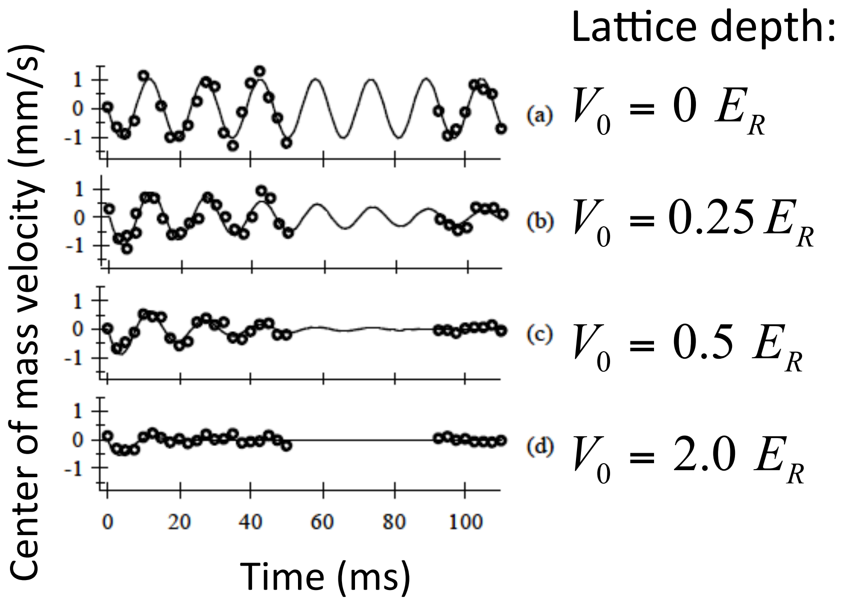


MIT: J. Mun et al., PRL (2007)

1.2. Damped dipole-oscillation

NIST: C. D. Fertig et al., PRL (2005)





- They observed a **dissipative flow** (significant damping) even though the flow velocity is much smaller than the critical value predicted by the Gutzwiller mean-field theory.

Maximum velocity MF critical velocity for $V_0 = 2.0\ E_R$

$$\sim 0.1\ E_{\rm R}d/\hbar \ll 0.3\ E_{\rm R}d/\hbar \sim$$

- The damping rate rapidly increases when deepening the lattice.

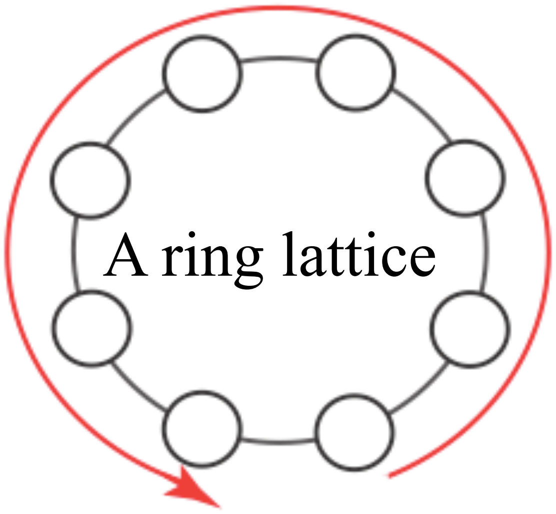
Interpretation by Polkovnikov et al. PRA (2005):

This breakdown of superfluidity is due to phase slips via thermal activation or quantum tunneling.

1.3. Superflow decay via phase slips

N. Giordano, PRL (1988)

A supercurrent is flowing

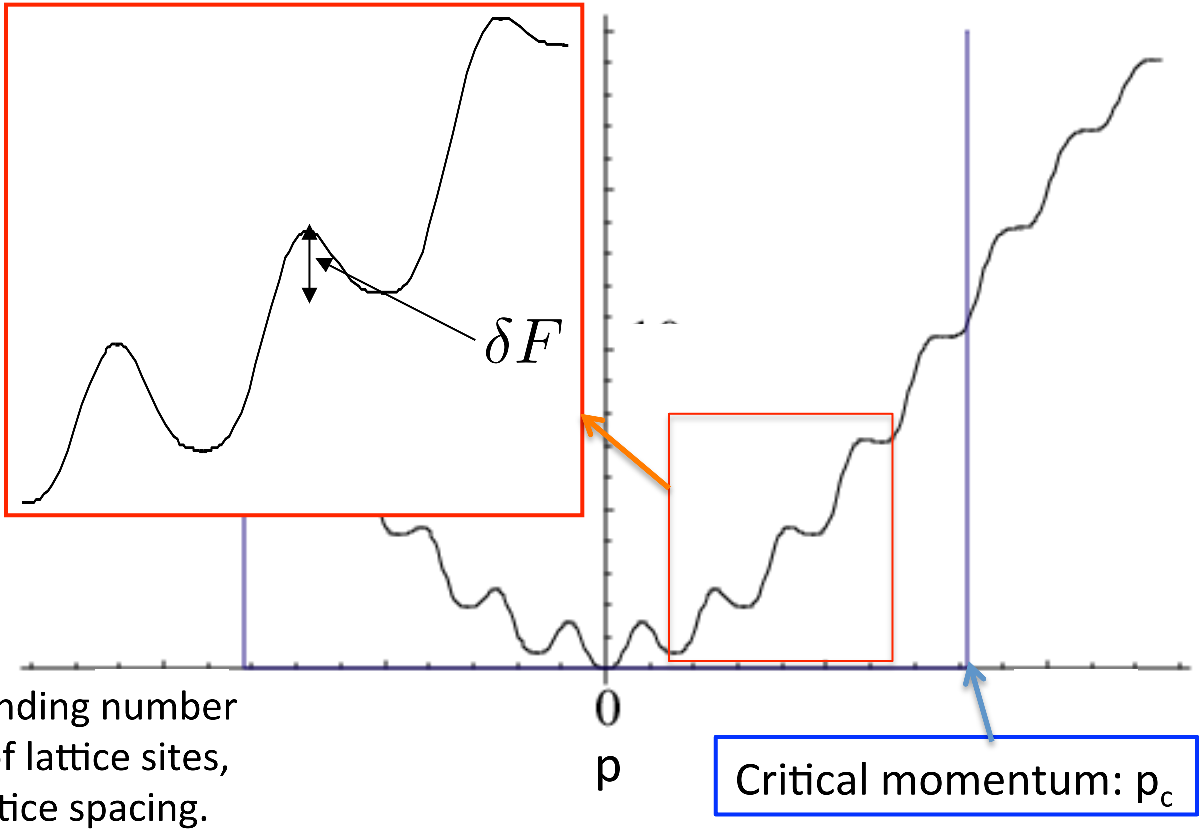


The flow momentum is quantized as

$$p_n = \frac{2\pi\hbar}{Ld} n$$

n: winding number
L: # of lattice sites,
d: lattice spacing.

Free energy: F(p)



A state with finite flow velocity can be metastable. → Persistent current

The momentum at which δF reaches zero. → Superfluid critical momentum

Strong thermal or quantum fluctuations → Superflow decay via phase slips

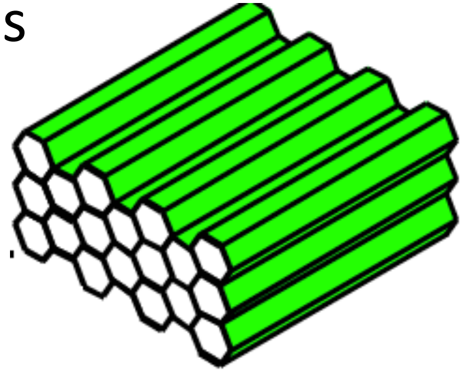
Phase-slip nucleation rate: $\Gamma = \underset{\text{Thermal}}{\underline{\Gamma_{\mathrm{AT}}}} + \underset{\text{Quantum}}{\underline{\Gamma_{\mathrm{QT}}}}$

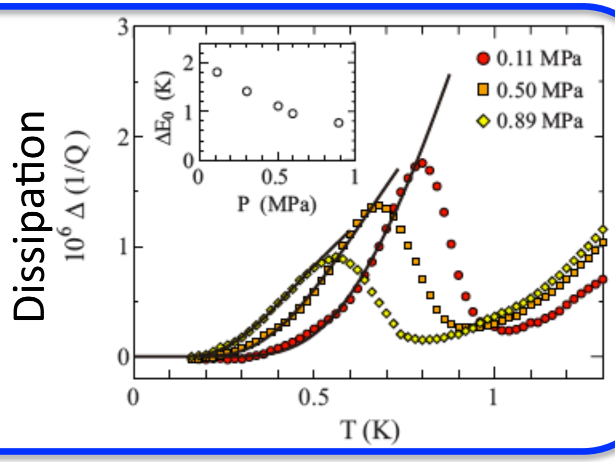
1.4. Other 1D superfluids (superconductors)

Liquid ^{4}He in nanopores

R. Toda et al., PRL (2007)
J. Taniguchi et al., PRB (2010)
T. Eggel et al., PRL (2011)

etc.

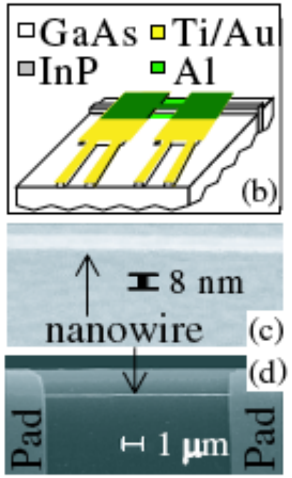


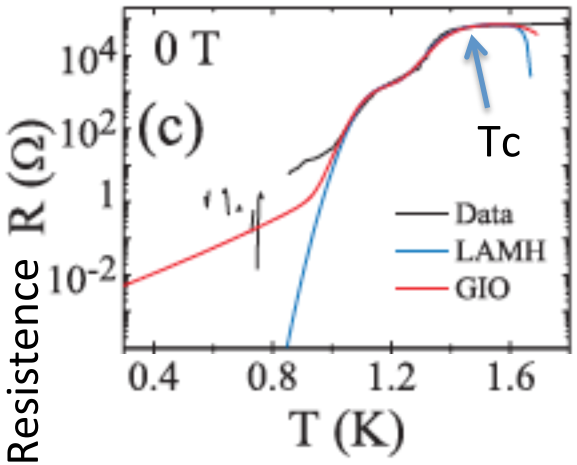


Superconducting nanowires and nanotubes

A. Bezryadin et al., Nature (2000)
F. Altomare et al., PRL (2006)
K. Yu. Arutyunov et al., Phys. Rep. (2008)
M. Kociak et al., PRL (2001)
Z. Wang et al., Nanoscience (2012)

etc.



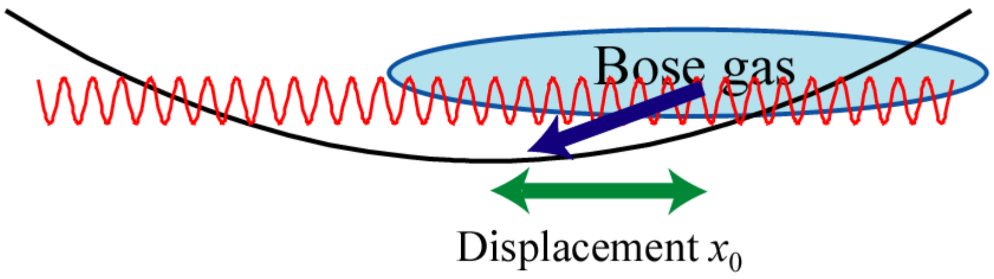


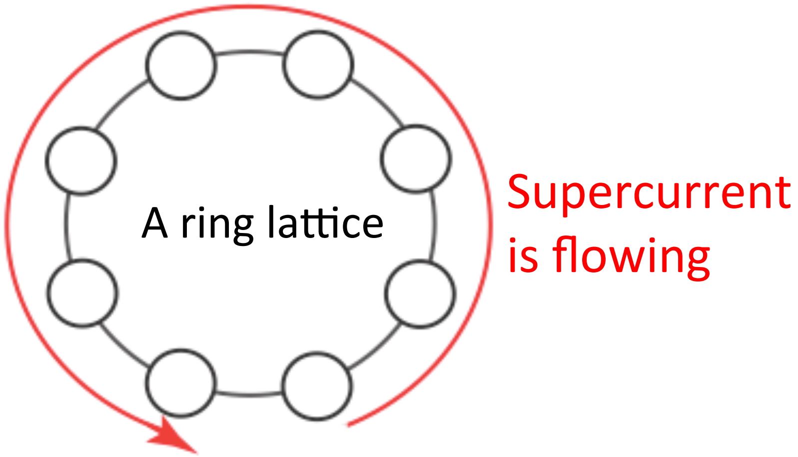
The concept of phase slip is central also to the understanding of 1D superfluids (superconductors) in these condensed matter systems.

QPS in ultracold atoms

- Unified view of 1D superfluids
- Study of QPS in a highly controllable manner

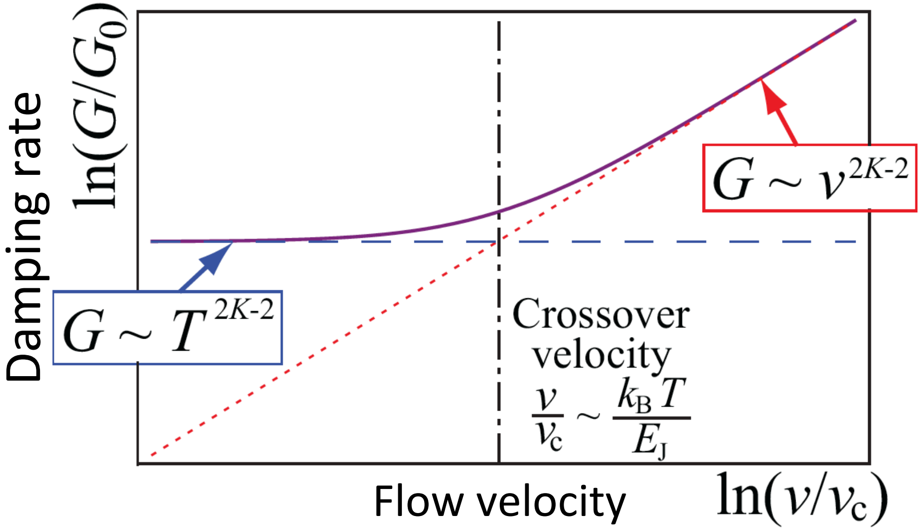
1.5. What we do in this work





Damping rate : G ⟷ ? ? ? Phase-slip nucleation rate: Γ

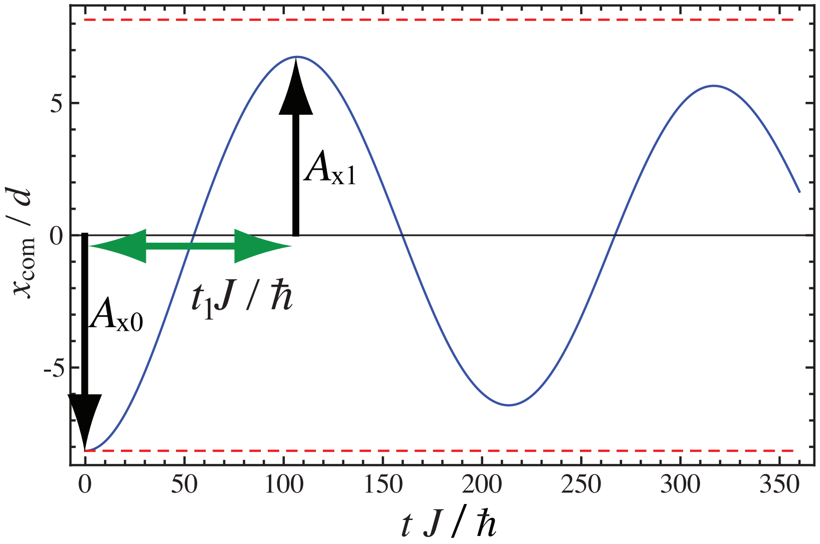
- We find the relation between *G* and Γ: $G(v) \propto \Gamma(v)/v$
- Using the relation, we elucidate the mechanism of the damping at T=0.
- Universal damping behavior:



where v_c is the MF critical velocity

2. Hand-waving picture

- **Relation between the nucleation rate Γ and the damping rate G**



The definition of the damping rate:

$$G = \frac{\log(A_{\rm x1}/A_{\rm x0})}{t_1}$$

The maximum velocity:

$$A_{\rm v1} \simeq \omega A_{\rm x0}$$

(a) The lost potential energy

$$E_{\rm loss} = \frac{1}{2} M\omega^2 (A_{\rm x0}^2 - A_{\rm x1}^2)$$

$$\boxed{A_{\rm x0} \gg A_{\rm x0} - A_{\rm x1}}$$

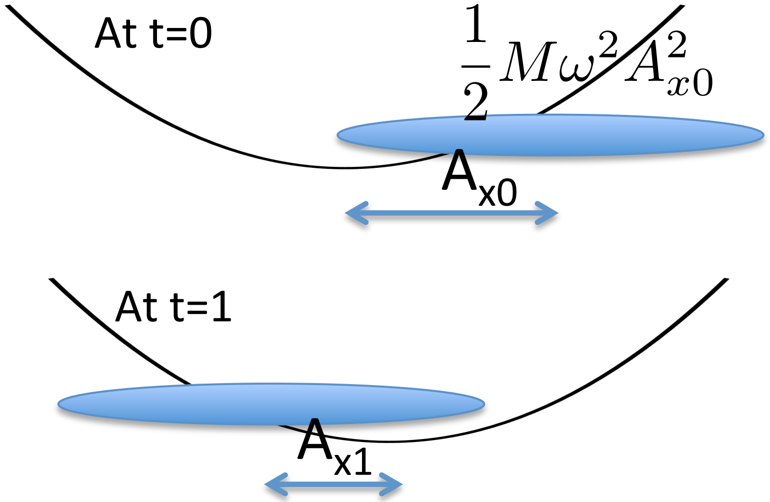
$$\simeq M\omega^2 A_{\rm x0}^2 (1 - A_{\rm x1}/A_{\rm x0})$$

$$\boxed{Gt_1 \ll 1}$$

$$\boxed{A_{\rm x1}/A_{\rm x0} = e^{-Gt_1} \simeq 1 - Gt_1}$$

$$\simeq M A_{\rm v1}^2 G t_1$$

Potential energy:

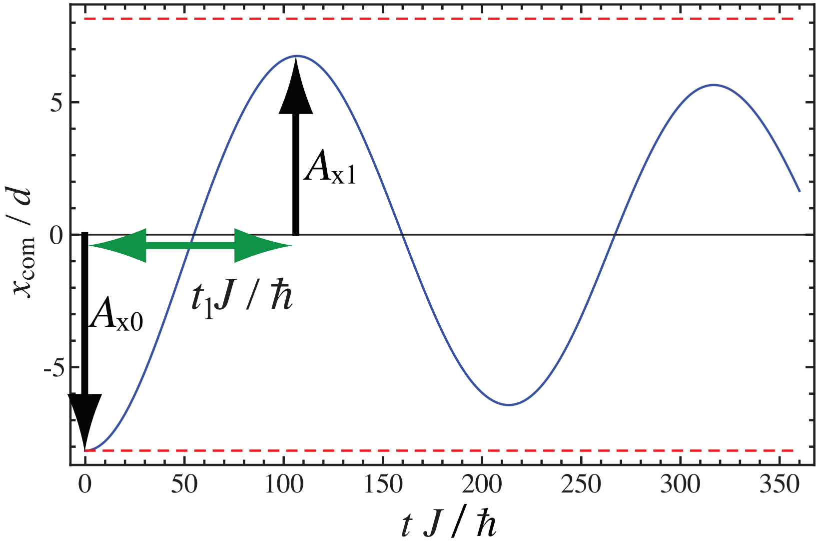


$$\frac{1}{2} M\omega^2 A_{x0}^2$$

Potential energy: $\frac{1}{2} M\omega^2 A_{x1}^2$

2. Hand-waving picture

- **Relation between the nucleation rate Γ and the damping rate G**



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$$Gt_1 \ll 1$$

$$A_{\rm x1}/A_{\rm x0} = e^{-Gt_1} \simeq 1 - Gt_1$$

$$\simeq MA_{\rm v1}^2 Gt_1$$

(b) The Joule heat

$$E_{\rm loss} = P \times t_1$$

$$= RI^2 \times t_1$$

$$I \sim n_{\rm 1D} A_{\rm v1}$$

$$R = 2\pi\hbar\frac{\Gamma}{I}$$

$$\sim 2\pi\hbar n_{\rm 1D} A_{\rm v1}\Gamma \times t_1$$

(a) & (b)
lead to

Damping rate: $G \sim 2\pi\hbar\frac{n_{\rm 1D}}{M}\frac{\Gamma}{A_{\rm v1}} \propto \frac{\text{Nucleation rate}}{\text{Flow velocity}}$

3. Numerical corroboration of the relation

3.1. 1D hardcore Bose-Hubbard model with a single barrier potential

$$\hat{H} = -J\sum_j(\hat{c}_j^\dagger \hat{c}_{j+1} + \mathrm{h.c.}) + V\sum_j \hat{m}_j\hat{m}_{j+1} + \sum_j\left[\Omega(j - X_c(t)/d)^2 + \lambda\delta_{j,0}\right]\hat{m}_j,$$

J : Hopping energy, V : Nearest neighbor interaction, λ : Strength of the barrier potential

Ω : Curvature of the trapping potential, X_c: Displacement of the trap center,

Advantages:

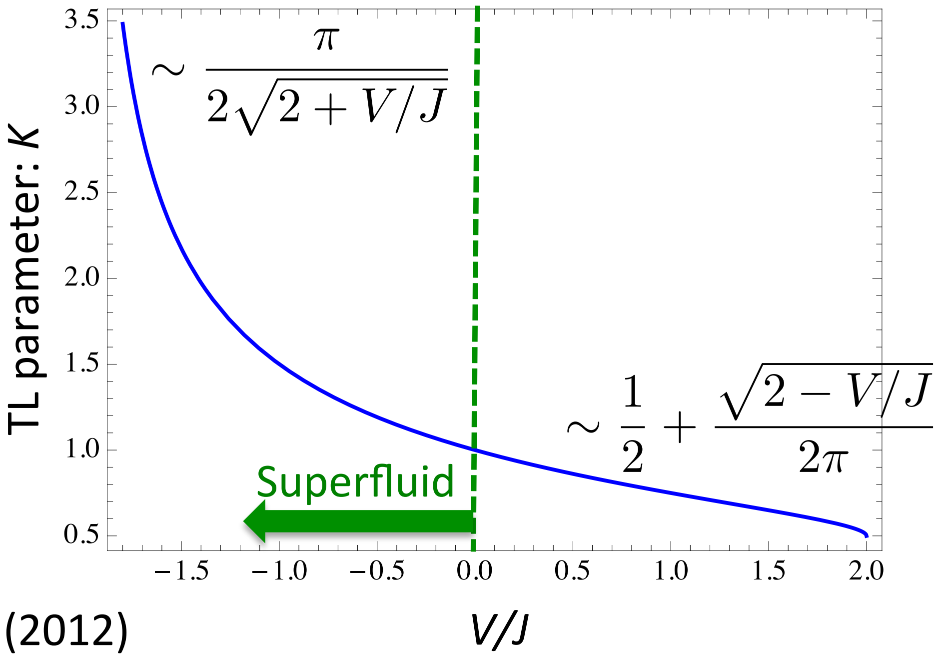
- Nucleation rate of a quantum phase slip at T=0: $\Gamma \propto v^{2K-1}$ when $v \ll v_c$ & $K > 1$

(λ<<J) Yu. Kagan et al., PRA (2000)
(λ>>J) H. P. Büchler et al., PRL (2001)

- The Tomonaga-Luttinger (TL) parameter *K* at ν=0.5 is related to *V/J* as

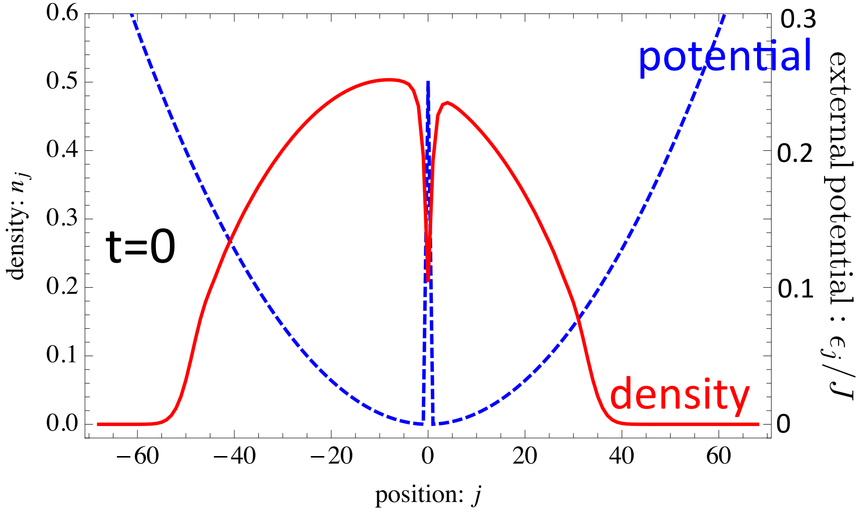
$$\boxed{K = \left[2 - \frac{2}{\pi}\arccos\left(\frac{V}{2J}\right)\right]^{-1}}$$

e.g. M. Cazalilla et al., RMP (2011)

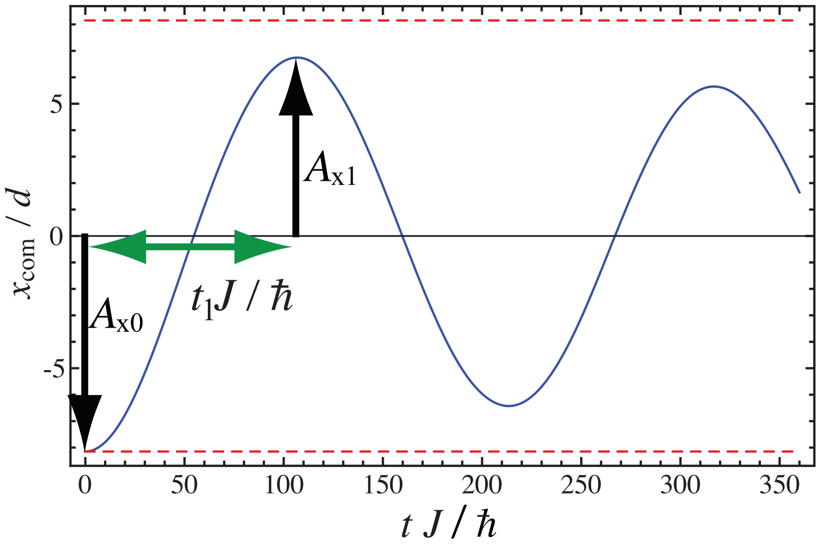


- This model is numerically solvable with TEBD [G. Vidal, PRL (2004)], which can precisely capture quantum phase slips.
  I. Danshita & A. Polkovnikov, PRB (2010); PRA (2012)

3.2. Dipole oscillation and damping rate



Time evolution of the COM position:



$$\Omega = 3.2 \times 10^{-4} J\,,\, X_c = 8d$$

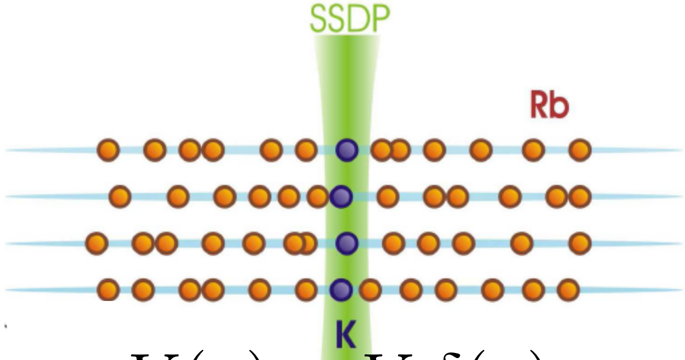
$$V = -1.4J,\ \lambda = 1.0J,\ N = 31$$

※ The trap curvature is set such that $n_{\rm max} \simeq 0.5$

Assuming the under-damped oscillation,
we define the damping rate as

$$G = \frac{\log(A_{\rm x1}/A_{\rm x0})}{t_1}$$

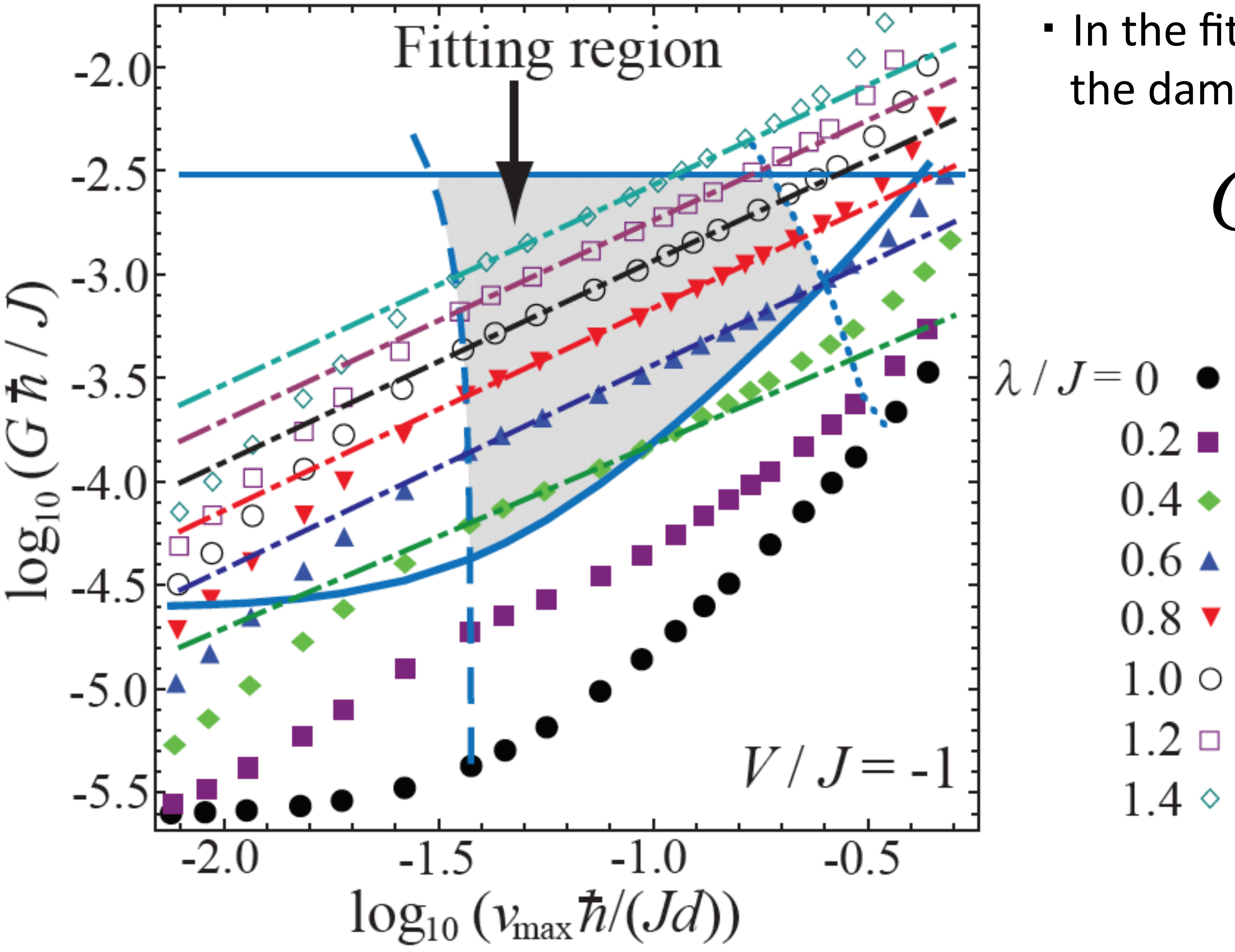
We expect $\boxed{G \propto \Gamma / v \propto v^{2K-2}}$ $\because\ \ \Gamma \propto v^{2K-1}$



$$V(x) \simeq V_0 \delta(x)$$

Single impurity potential
Florence: J. Catani et al., PRA (2012)

3.3. The damping rate vs the velocity (hardcore BHM)



- In the fitting region, the damping rate obeys

$$G \propto v^{\eta}$$

- The power η hardly changes for different λ .

$$\boxed{\eta \simeq 2K - 2}$$????

Four conditions for the fitting region:

i) $Gt_1 < 1/4$ (thin solid), ii) $G > 10G_0$ (thick solid)

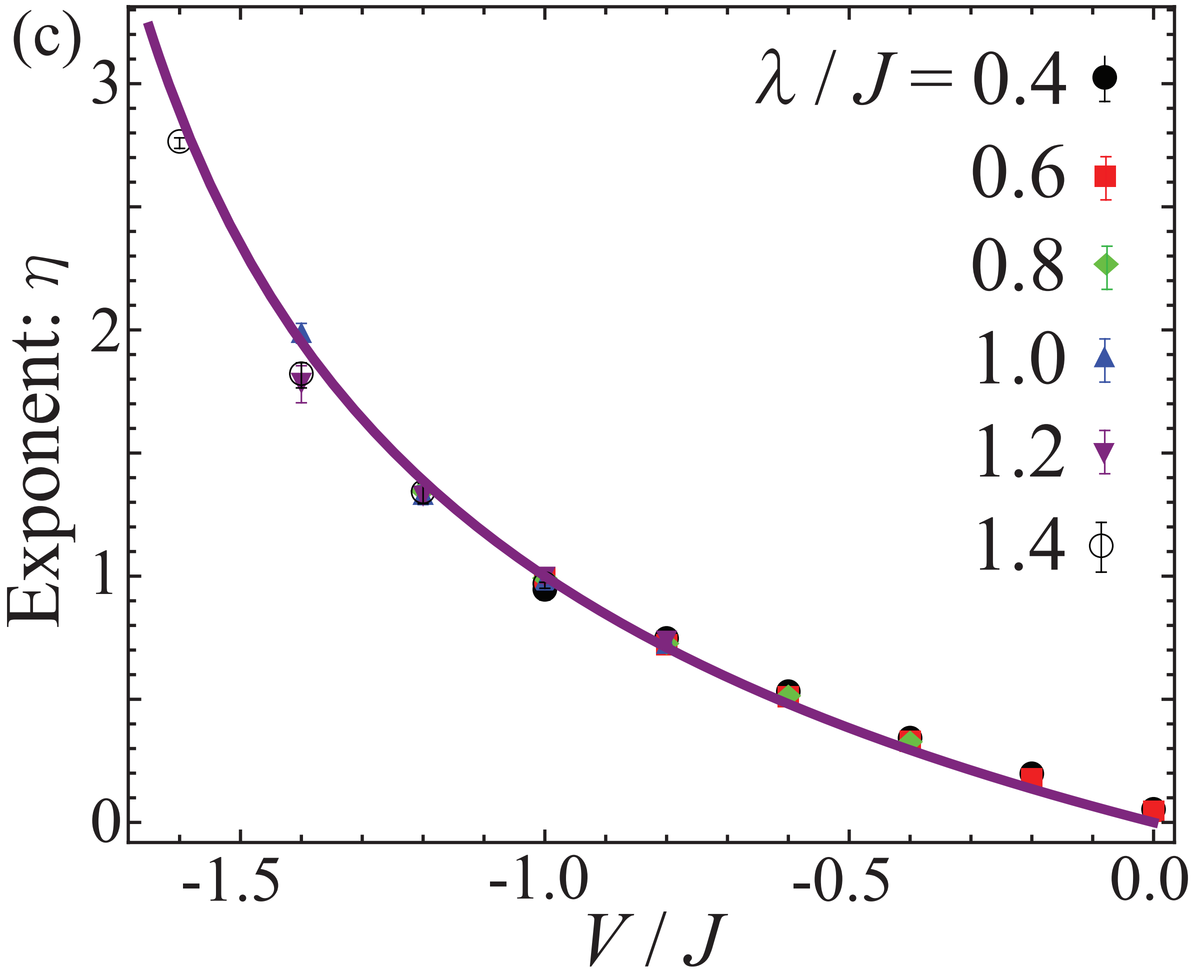
iii) $x_0 \geq d$ (dashed), iv) $v_{\max} \leq v_c/5$ (dotted)

3.4. The exponent vs *V*/*J* (hardcore BHM)

Using the fitting function,

$$f(x) = a\,x^{\eta}$$

we extract the exponent η.



The damping rate obeys the following scaling formula: $G \propto v^{2K-2} \propto \Gamma/v$

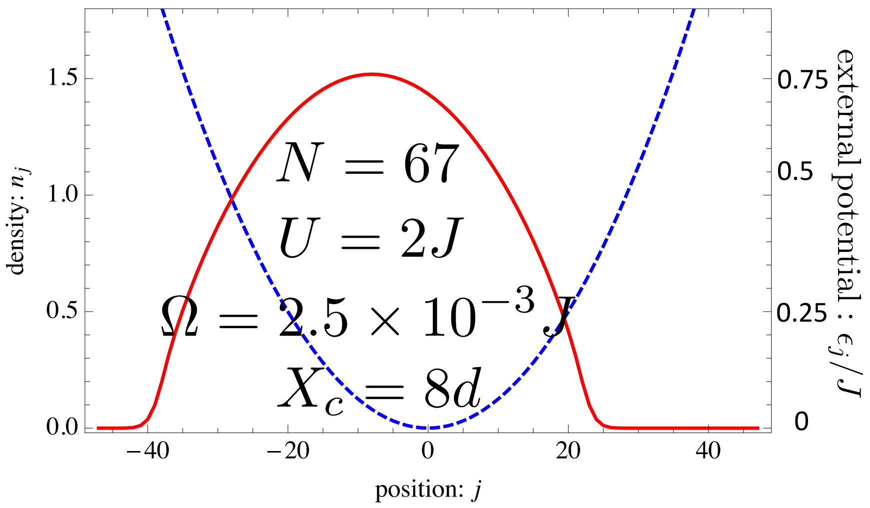
∴ The relation has been corroborated.

4. Mechanism of the damping in a 1D Bose gas in an optical lattice

4.1. The softcore Bose-Hubbard model with no barrier potential

which corresponds to actual experiments.

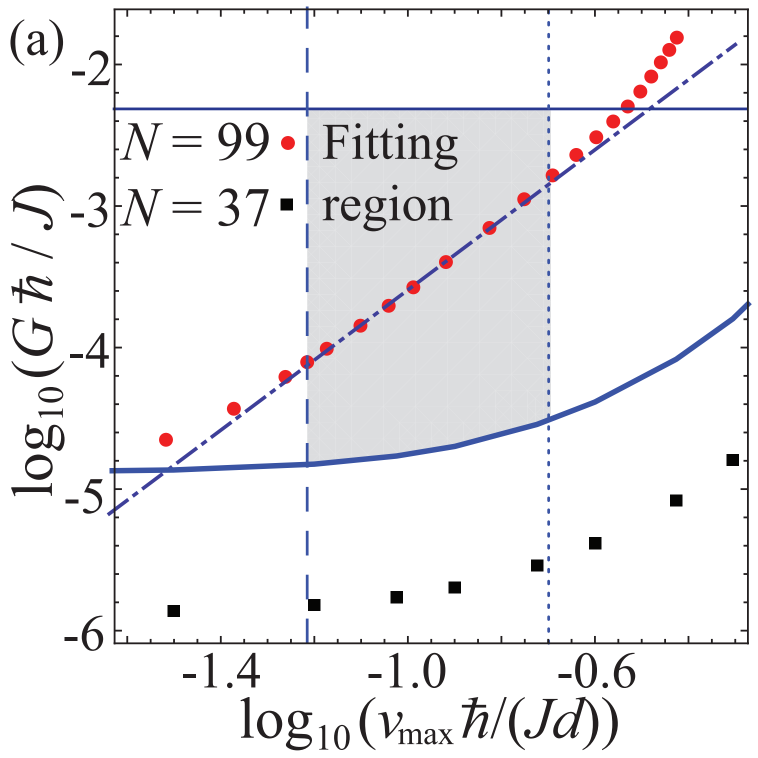
$$\hat{H} = -J\sum_j (\hat{b}_j^\dagger \hat{b}_{j+1} + \mathrm{h.c.}) + \frac{U}{2}\sum_j \hat{n}_j(\hat{n}_j - 1) + \sum_j \Omega(j - X_c(t)/d)^2 \hat{n}_j$$



The density profile is smooth.

The trap curvature Ω is set such that

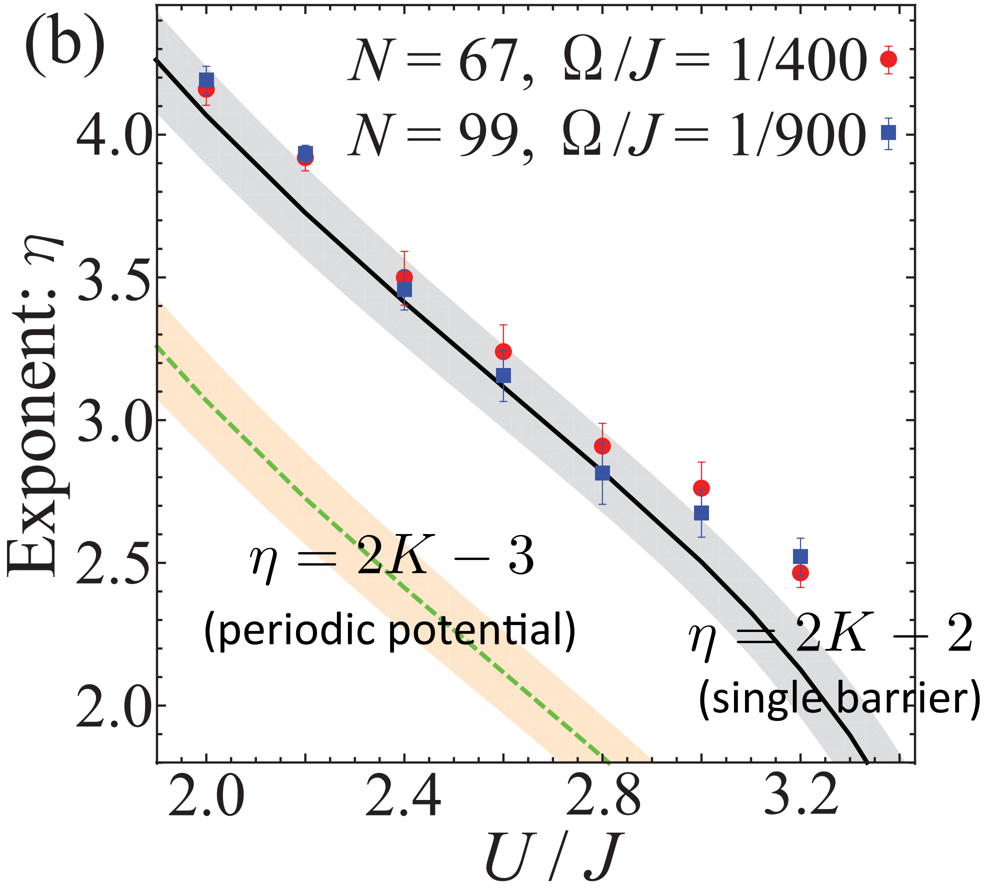
$1 < n_{\max} < 2$



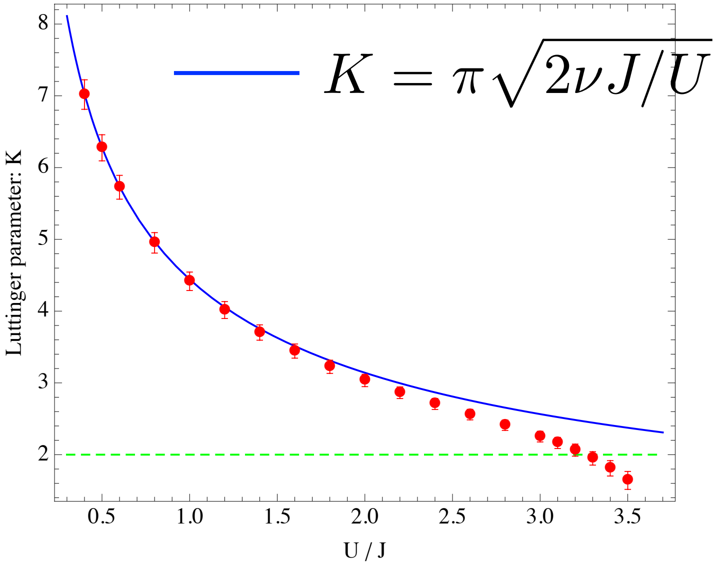
- The damping rate G obeys the power-law with respect to the momentum p.

※ Previous works have studied dipole oscillations of the same model with different parameters:
I. Danshita and C. W. Clark, PRL (2009); S. Montangero et al., PRA (2009)

4.2. The exponent vs *U*/*J* (softcore BHM)



The TL parameter is extracted from the one-body density matrix numerically calculated with TEBD.



Yu. Kagan et al., PRA (2000)
H. P. Büchler et al., PRL (2001)

The damping rate obeys the scaling formula for the quantum-phase slips in the presence of a **single impurity** :

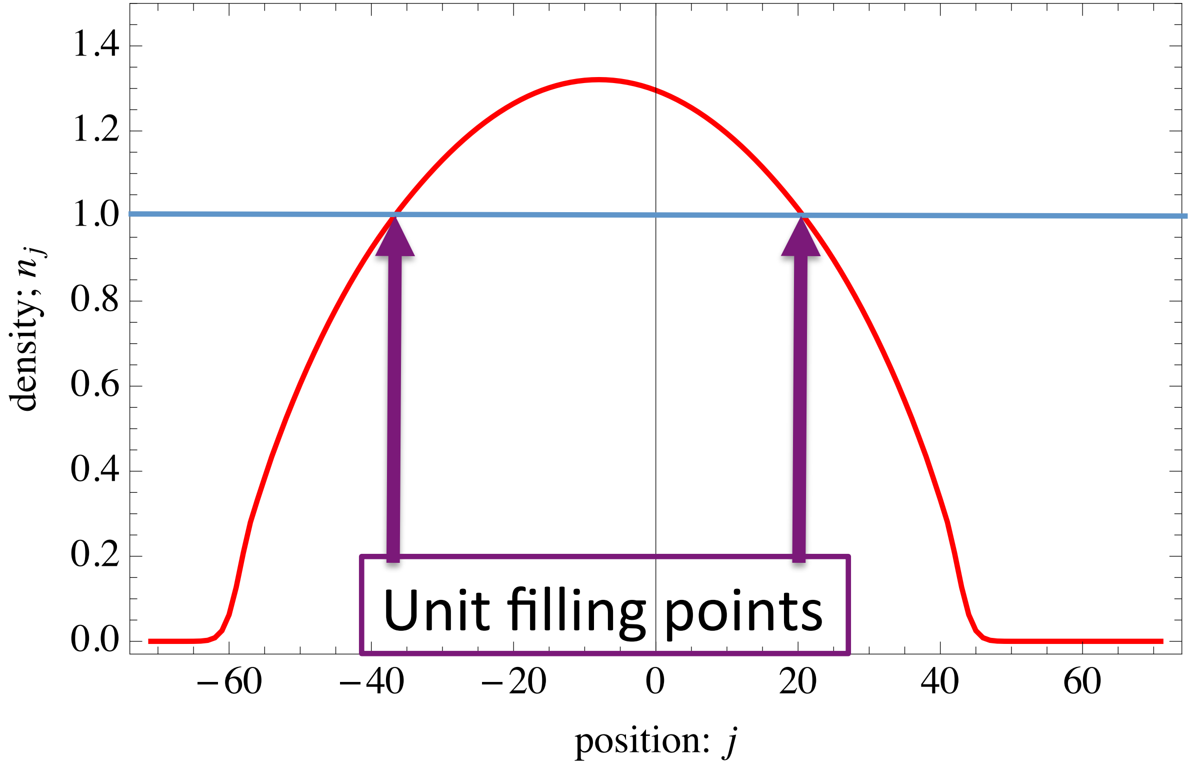
$$G \propto v^{2K-2}$$

rather than that for a periodic potential at a commensurate filling:

$$G \propto v^{2K-3}$$ ✗

I. Danshita & A. Polkovnikov, PRA (2012)

4.3. Effective impurities



Unit filling points

Transport in the regions near the unit filling points is much more suppressed than in the other regions.

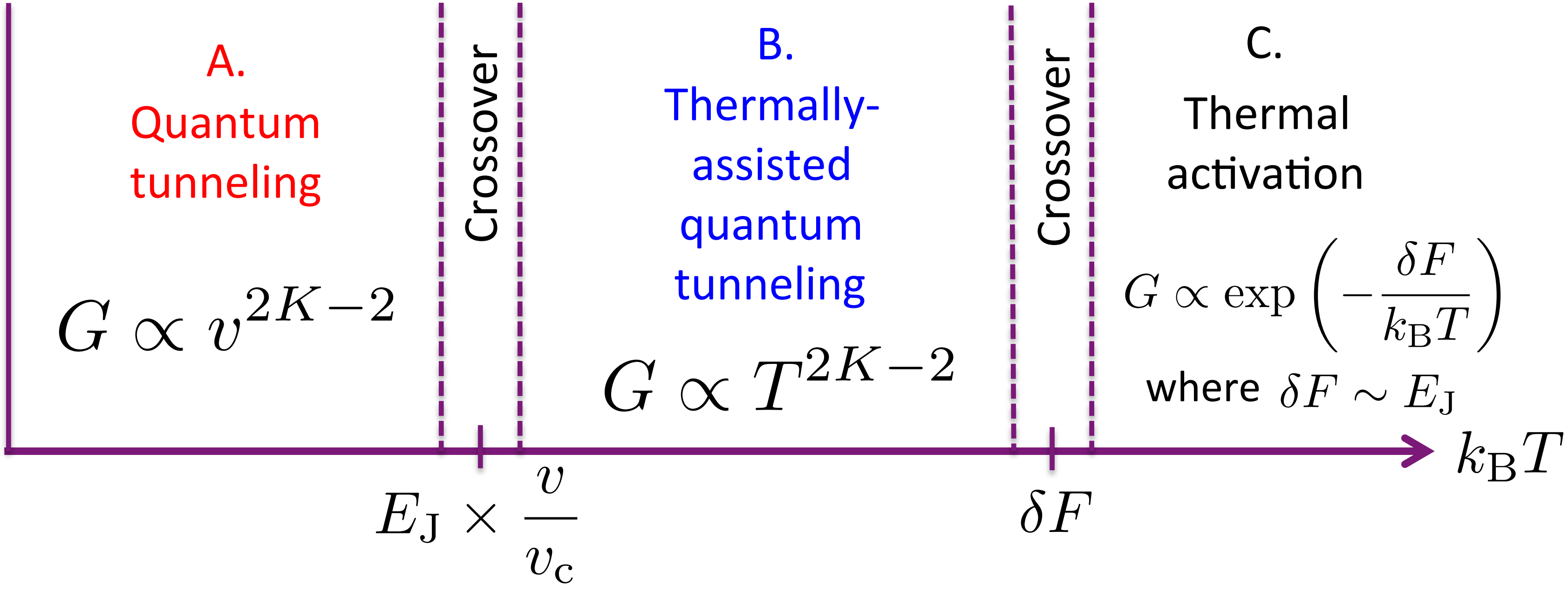
- The unit-filling regions act as barrier potentials for the other parts of the gas.

- The damping rate obeys the scaling formula for a **single impurity**.

5. Finite temperature effects

5.1. QPS at finite temperatures

QPS rate Γ is taken from Kagan et al. PRA (2000)



A. Quantum tunneling

$$G \propto v^{2K-2}$$

Crossover

B. Thermally-assisted quantum tunneling

$$G \propto T^{2K-2}$$

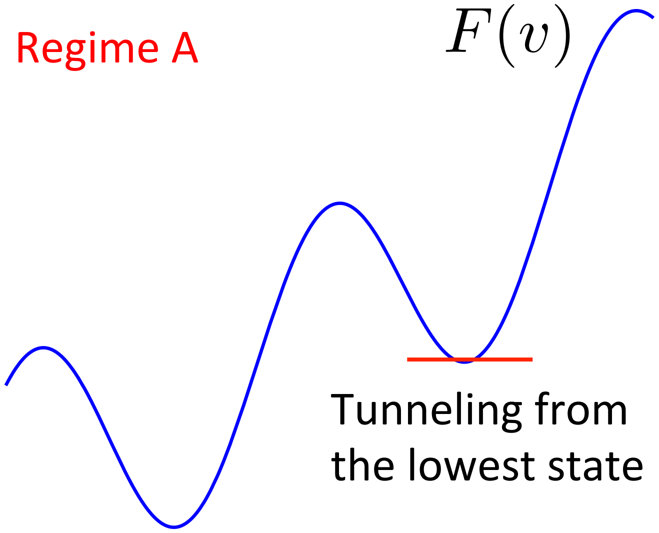
Crossover

C. Thermal activation

$$G \propto \exp\left(-\frac{\delta F}{k_\mathrm{B}T}\right)$$

where $\delta F \sim E_\mathrm{J}$

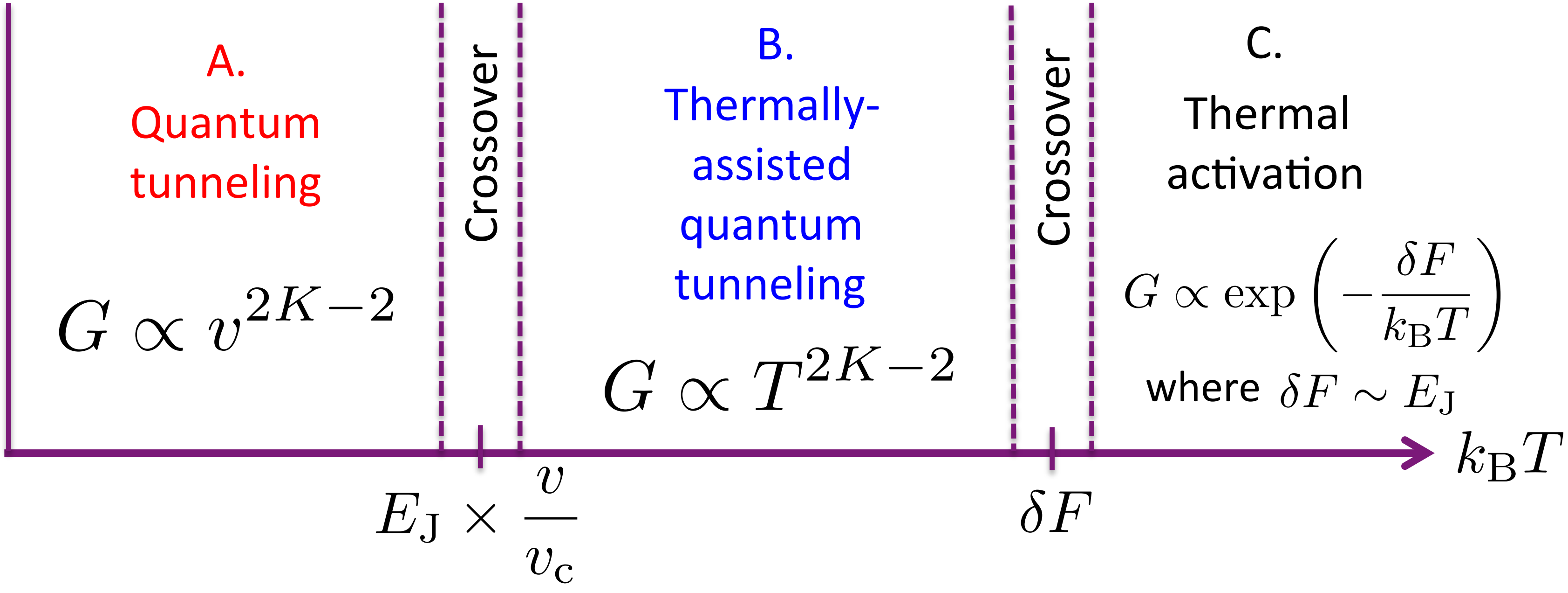
Regime A



Tunneling from the lowest state

5.1. QPS at finite temperatures

QPS rate Γ is taken from Kagan et al. PRA (2000)



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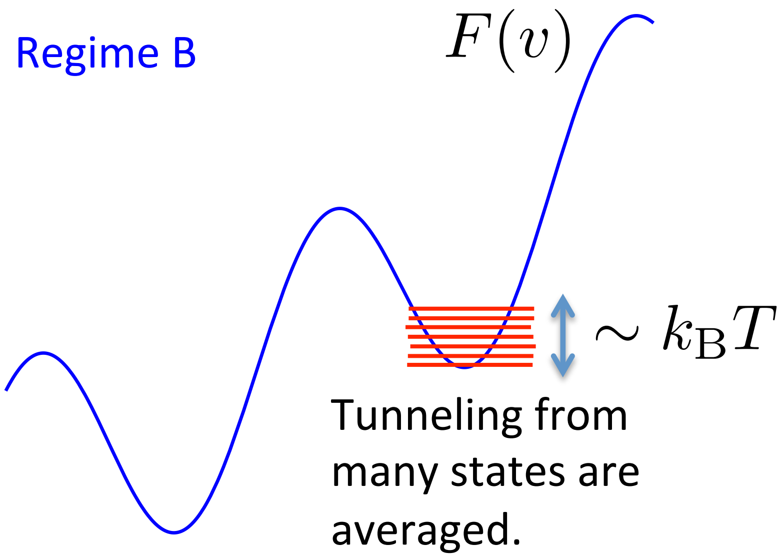
Crossover

C. Thermal activation

$G \propto \exp\left(-\frac{\delta F}{k_\mathrm{B}T}\right)$

where $\delta F \sim E_\mathrm{J}$

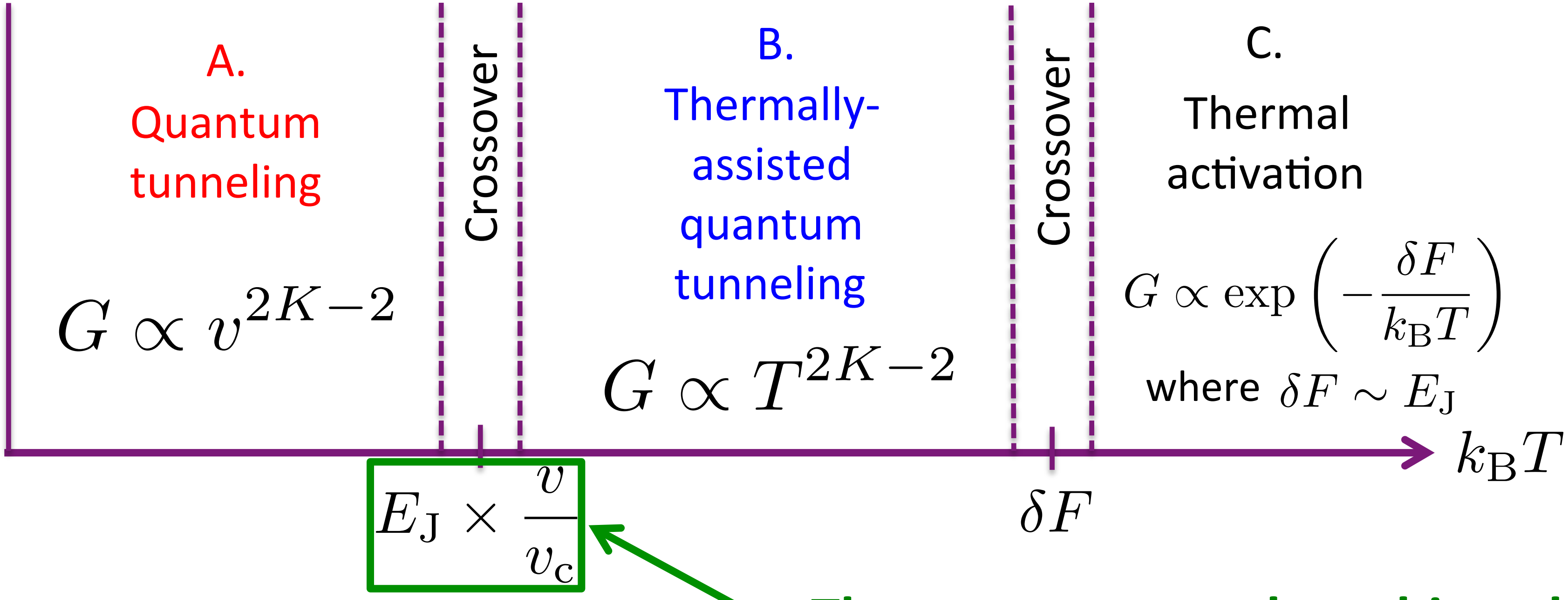
Regime B



Tunneling from many states are averaged.

5.1. QPS at finite temperatures

QPS rate Γ is taken from Kagan et al. PRA (2000)



A.
Quantum
tunneling

$$G \propto v^{2K-2}$$

Crossover

B.
Thermally-assisted
quantum
tunneling

$$G \propto T^{2K-2}$$

Crossover

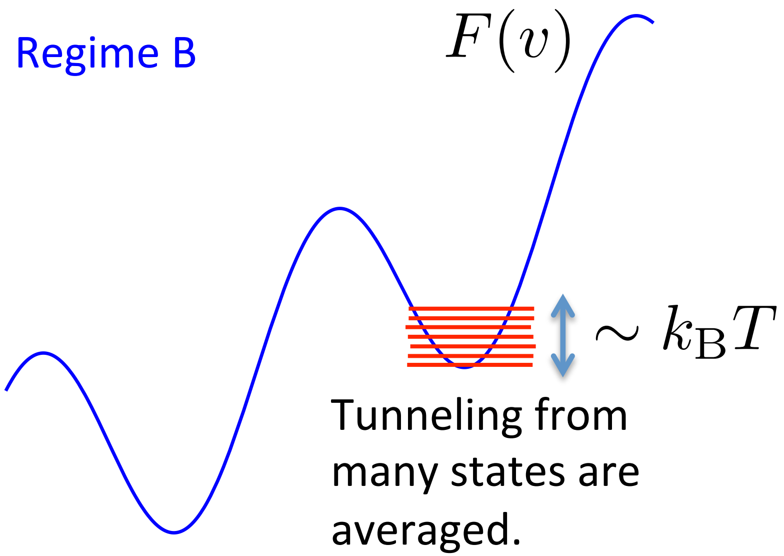
C.
Thermal
activation

$$G \propto \exp\left(-\frac{\delta F}{k_{\mathrm{B}}T}\right)$$

where $\delta F \sim E_{\mathrm{J}}$

The crossover can be achieved by changing the velocity.

Regime B



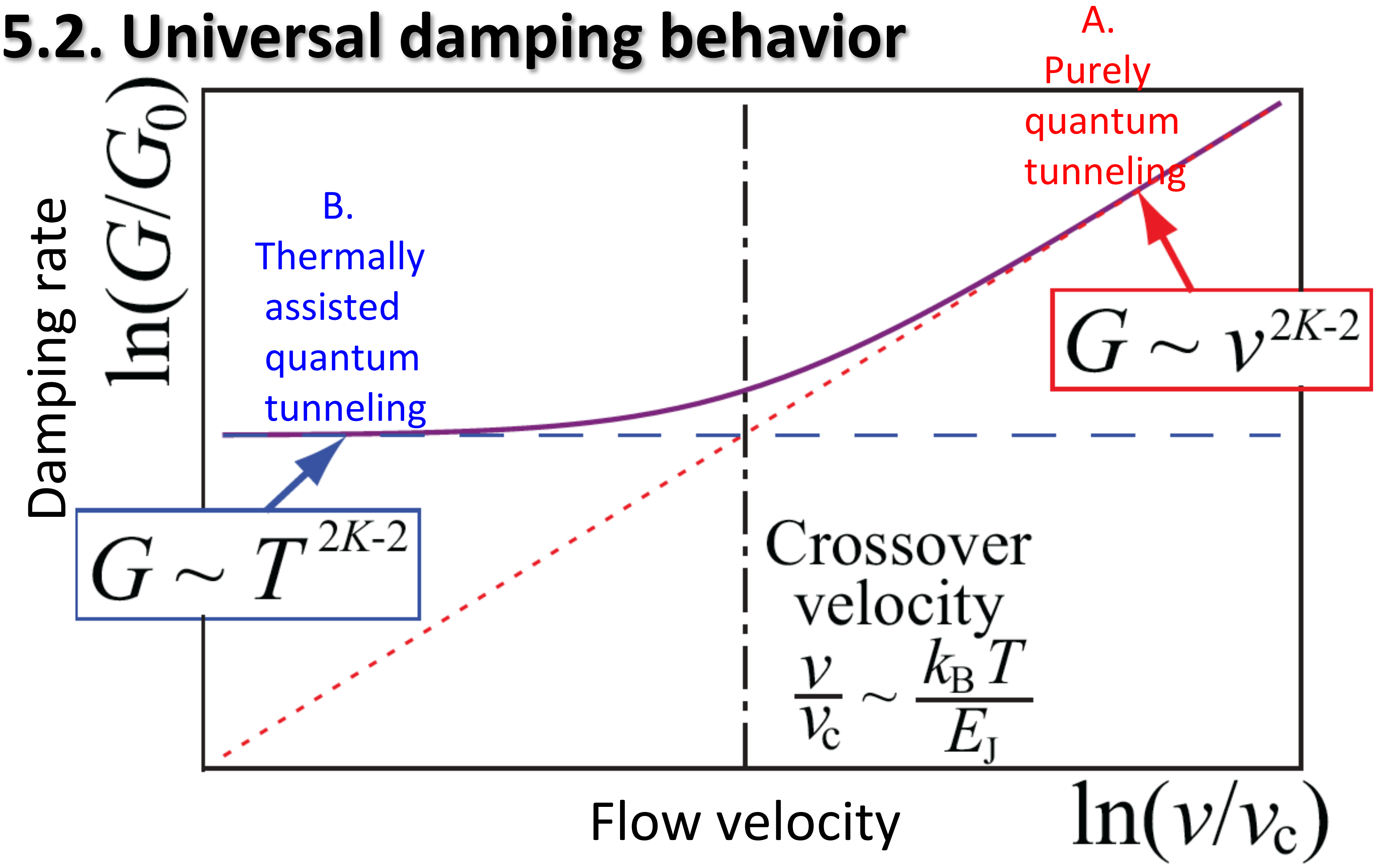
Tunneling from
many states are
averaged.

In the experiment of Fertig et al., PRL (2005),

$$E_{\mathrm{J}}/k_{\mathrm{B}} \simeq 30\mathrm{nK} \text{ and } \frac{v}{v_{\mathrm{c}}} \sim \frac{1}{3}$$

while $T \sim 10\mathrm{nK}$ in typical experiments.

5.2. Universal damping behavior

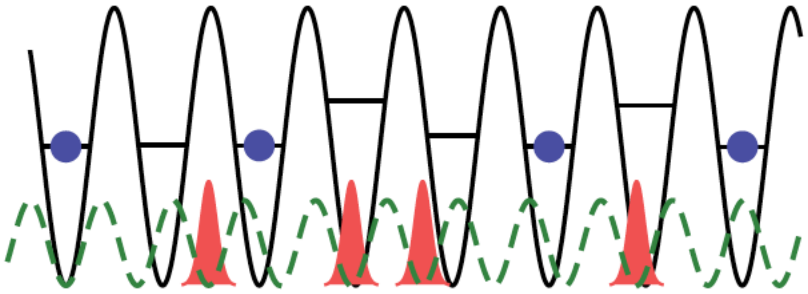


This allows for the determination of K. → **Mott transition point (K=2)**

The same behavior may be seen universally in systems whose low-energy physics can be described as the single-component TL liquid.

Note: v_c is the mean-field critical velocity and E_J is the Josephson plasma energy.

5.3. Implication to a disorder potential



Disorder potential $\lim_{x\to\infty}\langle V(x)V(0)\rangle = 0$

Stony Brook: B. Gadway et al., PRL (2011)

In the case of a **weak** disorder,

$$\Gamma \propto \begin{cases} v^{2K-1} & \text{Regime A} \\ vT^{2K-2} & \text{Regime B} \end{cases}$$

S. Khlebnikov & L. P. Pryadko, PRL (2005)

The same universal damping behavior

→ Localization transition of the Giamarchi-Schultz type (*K*=3/2)

In the case of a **strong** disorder,

$$\Gamma \sim ???$$

The TEBD-based analyses may answer this question.

→ may also address the localization transition in a strong disorder.

6. Conclusions

We have studied the transport of 1D Bose gases in strong connection with quantum nucleation of phase slips.

- We have found the relation between the damping rate G and the phase-slip nucleation rate Γ: $\boxed{G(v) \propto \Gamma(v)/v}$

- This relation allows to analyze QPS in cold atom experiments (and in the exact TEBD or tDMRG simulations).

- We corroborate that the damping of the dipole oscillation of 1D lattice bosons is due to the nucleation of QPS.

- We suggest that the damping rate vs the flow velocity exhibits the universal behavior, which can be tested in future experiments.

- Such experiments could be interpreted as a quantum simulation of 1D superfluids (or superconductors).

I. Danshita, Phys. Rev. Lett. 111, 025303 (2013)

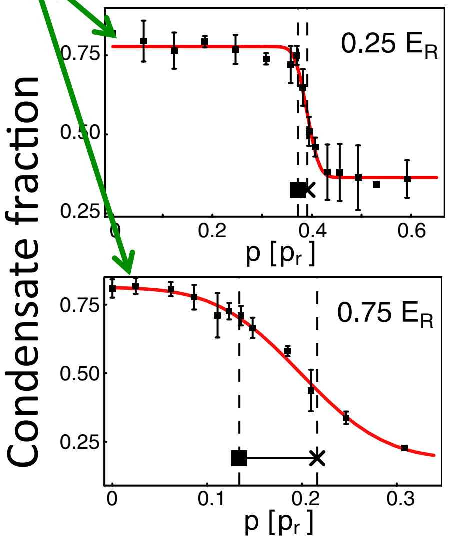
3.2. Isn’t it due to trivial finite temperature effects ??

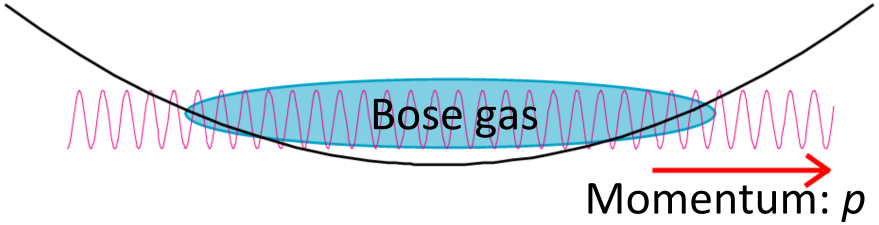
Well-known fact:

In 1D, the superfluid fraction $\rho_s \rightarrow 0$ in the thermodynamic limit at any T > 0.

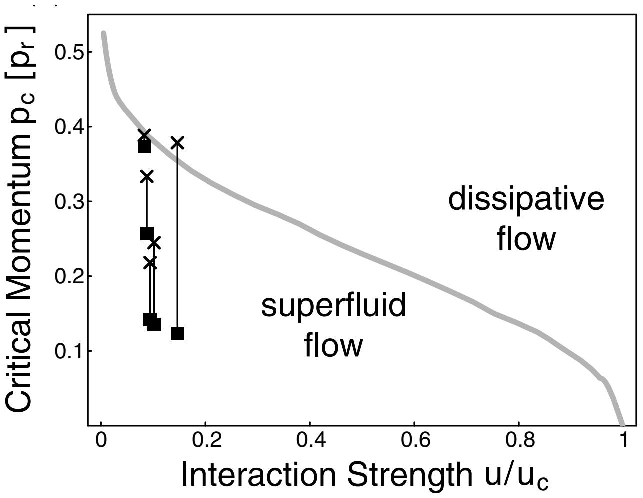
Naive guess: The system is not in the SF phase. , which is wrong.

Large fraction is condensed !!





MIT: J. Mun et al.,
PRL (2007)



Dissipationless (superfluid) flow if the velocity is sufficiently small.

The system is in the SF state.

1D Bose gases at finite temperatures exhibit superfluidity as long as the “lifetime” of superflow is longer than the time scale in experiment.

Yu. Kagan et al.
PRA (2000)

Also in the context of liquid ^{4}He: T. Eggel et al., PRL (2011)