

New analogies between xQCD and cold atoms

Yusuke Nishida (TITech)

**YITP workshop on “thermal quantum
field theories and their applications”**

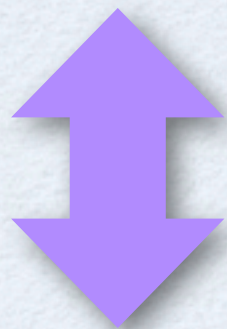
August 28 (2013)

Ultracold atoms ($\sim 10^{-9}$ K)

= nonrelativistic

point-like particles

with short-range interactions



Extreme QCD ($\sim 10^{12}$ K)

= relativistic quarks

with gauge interactions

1. “Hard probes” in cold atoms

- Use of energetic atoms to locally probe strongly-interacting atomic gases
- Y.N., Phys. Rev. A (2012) [arXiv:1110.5926]

2. “Quark-hadron continuity” in cold atoms

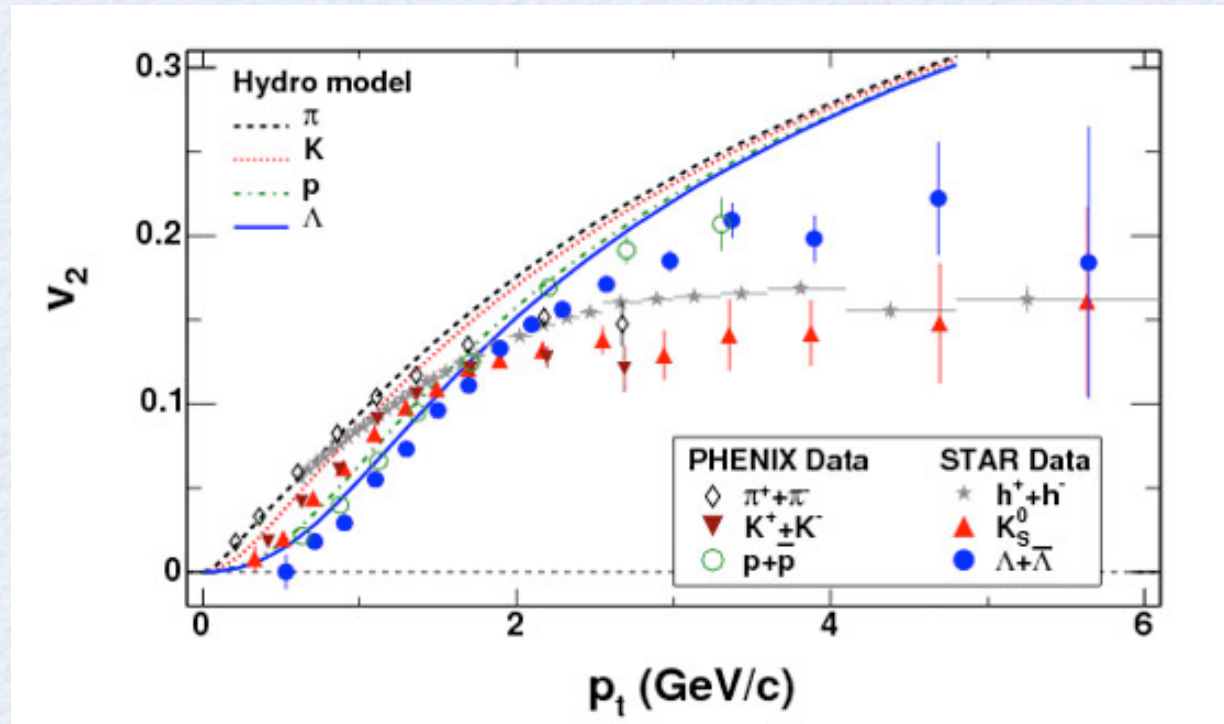
- Smooth crossover from atoms to trimers in 3-component Fermi gases
- Y.N., Phys. Rev. Lett. (2012) [arXiv:1207.6971]

**“Hard probes”
in cold atoms**

xQCD vs. cold atoms

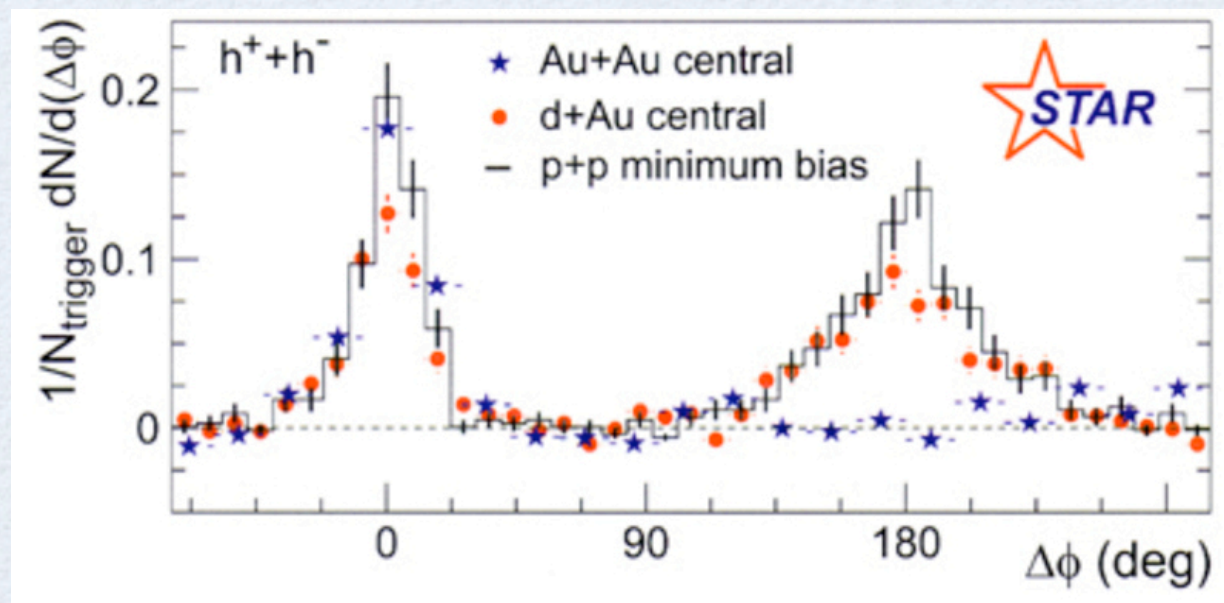
- Elliptic flow

- Small shear viscosity



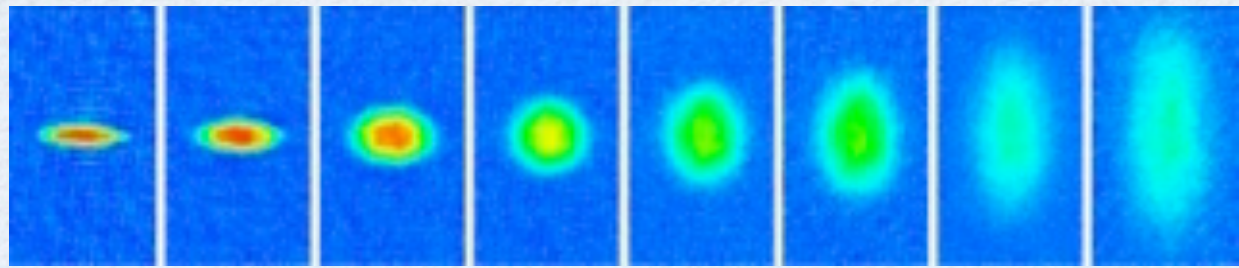
$$\frac{\eta}{s} \approx \frac{1}{4\pi}$$

- Jet quenching



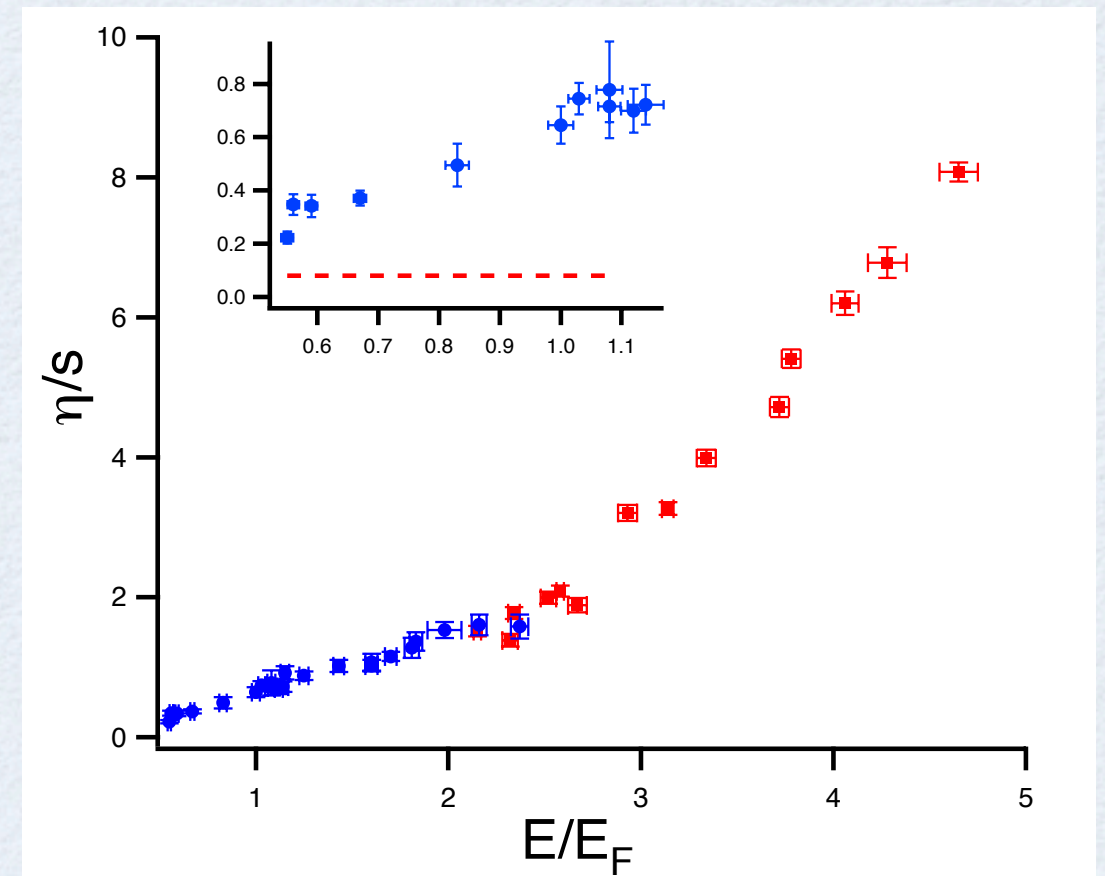
xQCD vs. cold atoms

- Elliptic flow



K. M. O'Hara et al., Science (2002)

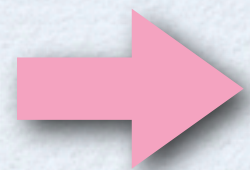
- Small shear viscosity



C. Cao et al., Science (2011)

- Jet quenching

???

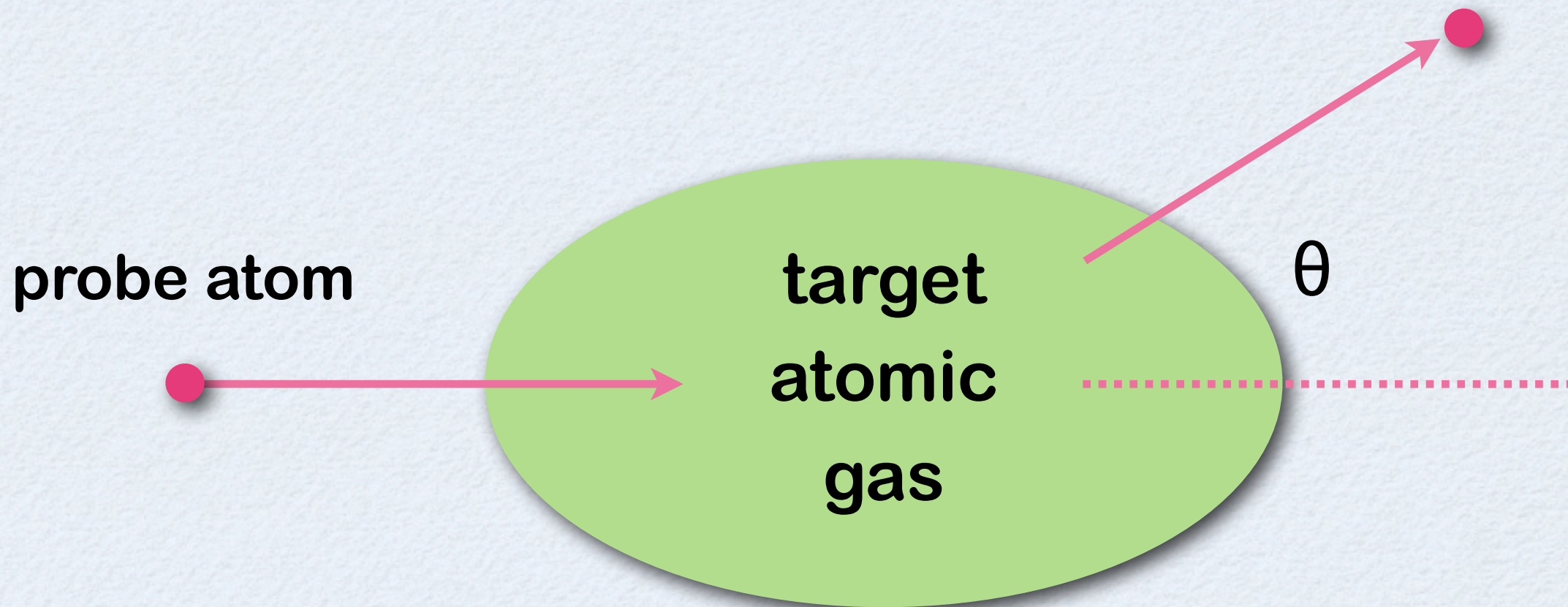


What is its analogue in cold atoms ?

Probe atomic gas with atoms

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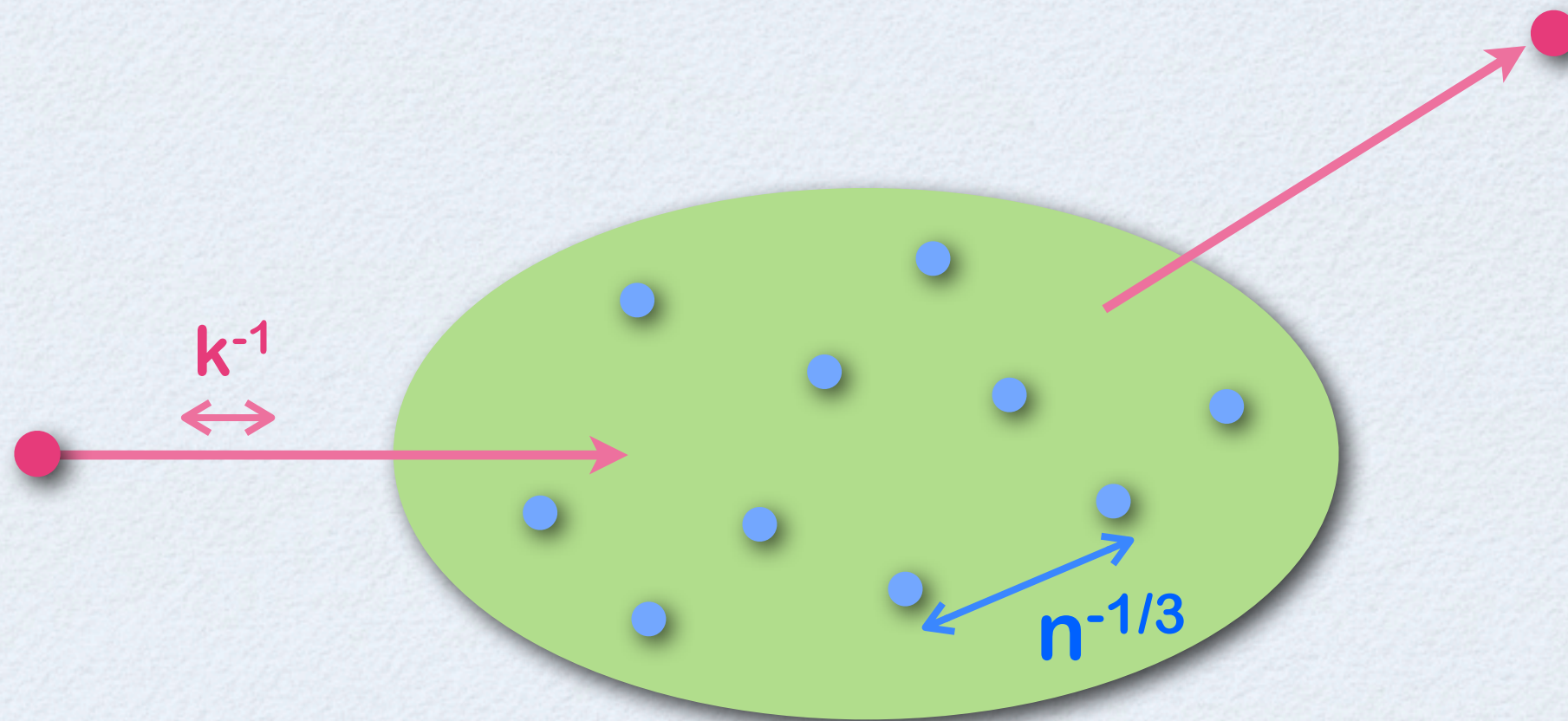
Shoot a **probe** atom into the **target** atomic gas and measure its differential scattering rate



What can we learn from the scattering data on the (strongly-interacting) target atomic gas?

Probe atomic gas with atoms

Shoot a **probe** atom into the **target** atomic gas and measure its differential scattering rate

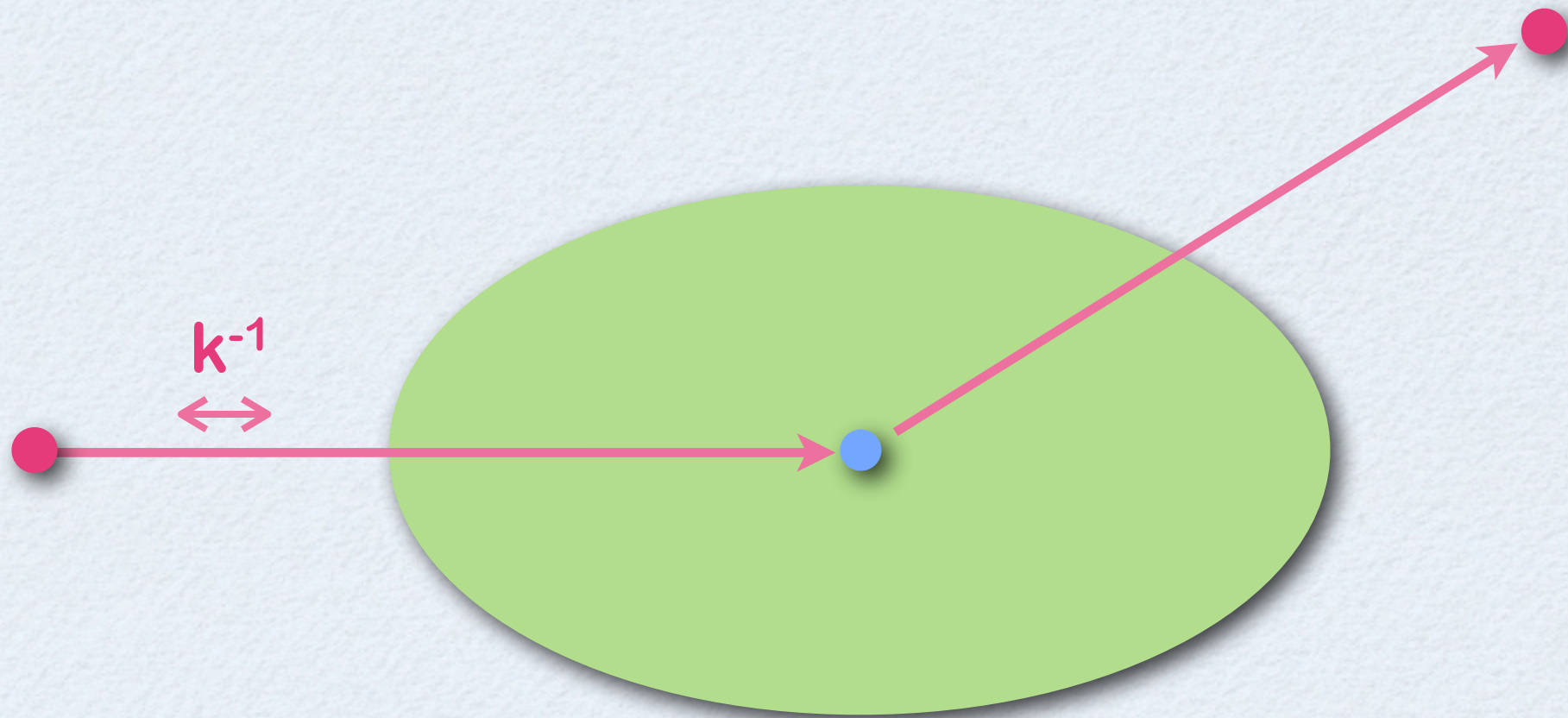


Large $k \gg n^{1/3} \Rightarrow$ Few-body scattering problems

$$\frac{d\Gamma(k)}{d\Omega} = \dots$$

Leading contribution

Shoot a **probe** atom into the **target** atomic gas and measure its differential scattering rate

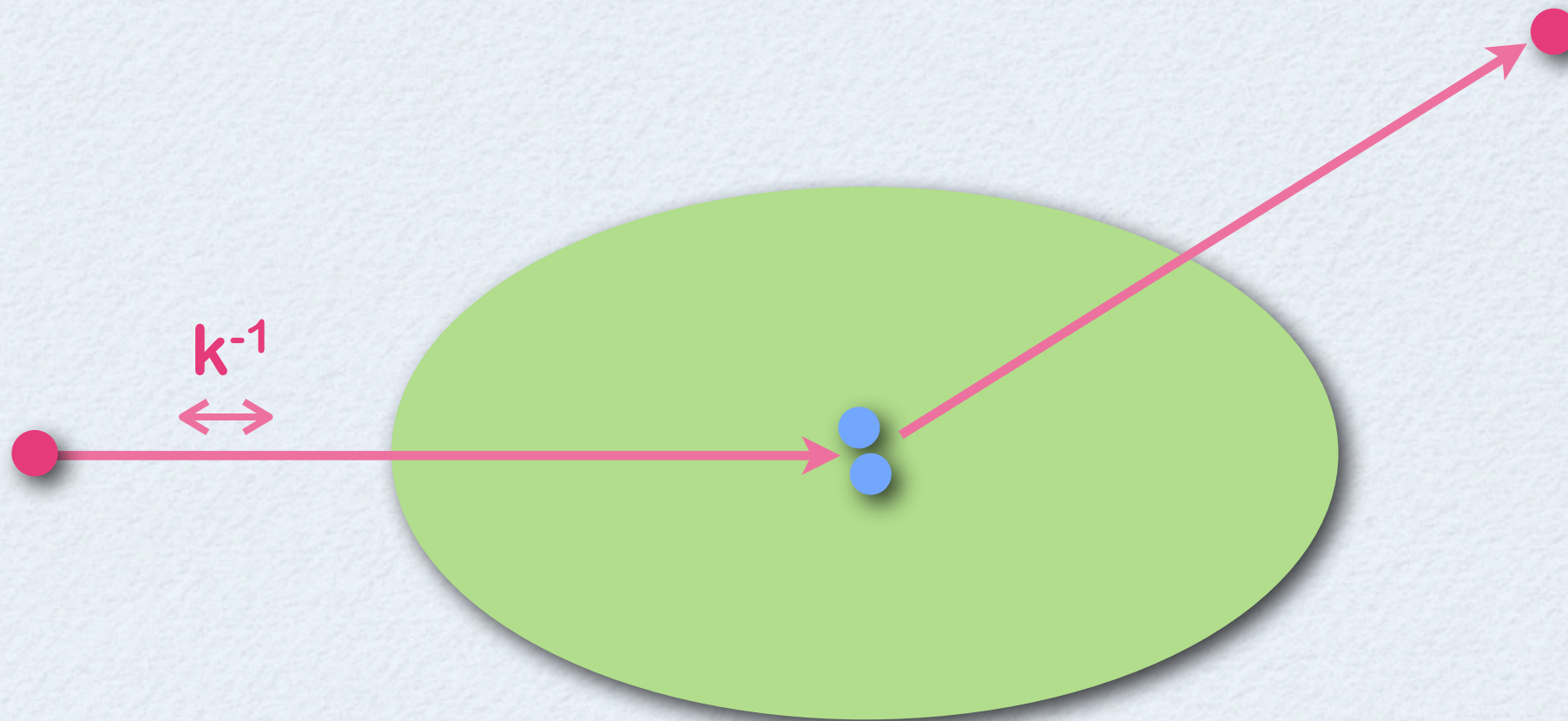


Large $k \gg n^{1/3} \Rightarrow$ Few-body scattering problems

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + \dots$$

Sub-leading contribution

Shoot a **probe** atom into the **target** atomic gas and measure its differential scattering rate



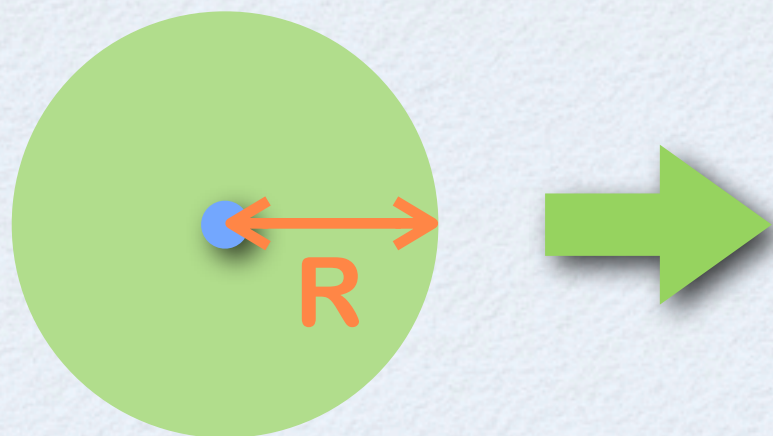
Large $k \gg n^{1/3} \Rightarrow$ Few-body scattering problems

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta) \frac{C}{k^2} + \dots$$

What is “C” ?

Probability of finding 2 particles at small separation

- noninteracting gas : $\langle \hat{n}(r) \hat{n}(0) \rangle = n^2$
- interacting gas : $\langle \hat{n}(r) \hat{n}(0) \rangle \rightarrow \frac{C}{(4\pi|r|)^2}$



The diagram shows a blue dot representing a particle at the center of a green circle representing a volume of radius R . A double-headed orange arrow labeled R indicates the radius. A large green arrow points from the diagram to the right, leading to the following equation:

$$\int_{|r| < R} \langle \hat{n}(r) \hat{n}(0) \rangle \sim \begin{cases} n^2 R^3 \\ C R \end{cases}$$

Anomalously enhanced probability is quantified by the “**contact density**” C

Important characteristic of strongly-int atomic gases

V

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Viewpoint: How the tail wags the dog in ultracold atomic gases

Eric Braaten, Department of Physics, Ohio State University, Columbus, OH 43210 USA and and Bethe Center for Theoretical Physics, University of Bonn, Bonn, Germany

Published February 2, 2009 | *Physics* 2, 9 (2009) | DOI: 10.1103/Physics.2.9






Recent calculations of the properties of ultracold atoms have revealed how two-body interactions at very short distances determine essential properties of many-body systems.

The development of the field of ultracold atoms has opened up new frontiers in both few-body and many-body physics. Of particular interest

Universal properties of the ultracold Fermi gas
Shizhong Zhang and Anthony J. Leggett
Phys. Rev. A 79, 023601 (2009)

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Subject Areas

- Atomic and Molecular Physics
- Superfluidity

Viewpoint: Fermi gases as a test bed for strongly interacting systems

Daniel E. Sheehy, Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA






Published June 7, 2010 | *Physics* 3, 48 (2010) | DOI: 10.1103/Physics.3.48

A new perspective on strongly interacting fermions emerges from the experimental confirmation of a universal formula.

Some of the most vexing present-day problems in physics center on understanding the many-body properties and phases of strongly interacting fermions. Part of the difficulty arises from the fact that while

Verification of Universal Relations in a Strongly Interacting Fermi Gas
J. T. Stewart, J. P. Gaebler, T. E. Drake, and

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Subject Areas

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- scattering rate : $\Gamma(k) = -2 \text{Im} \Sigma(k)$
- optical theorem : $\Gamma(k) = \int d\Omega \frac{d\Gamma(k)}{d\Omega}$


$$\begin{aligned} iG(k) &= \int dx e^{ikx} \langle T \psi(x) \psi^\dagger(0) \rangle \\ &= \sum_i A_i(k) \langle O_i \rangle \end{aligned}$$

$$n = \langle \psi^\dagger \psi \rangle, \quad C = \langle (\psi^\dagger \psi)^2 \rangle, \quad \dots$$

Lowest few O_i are needed at large k

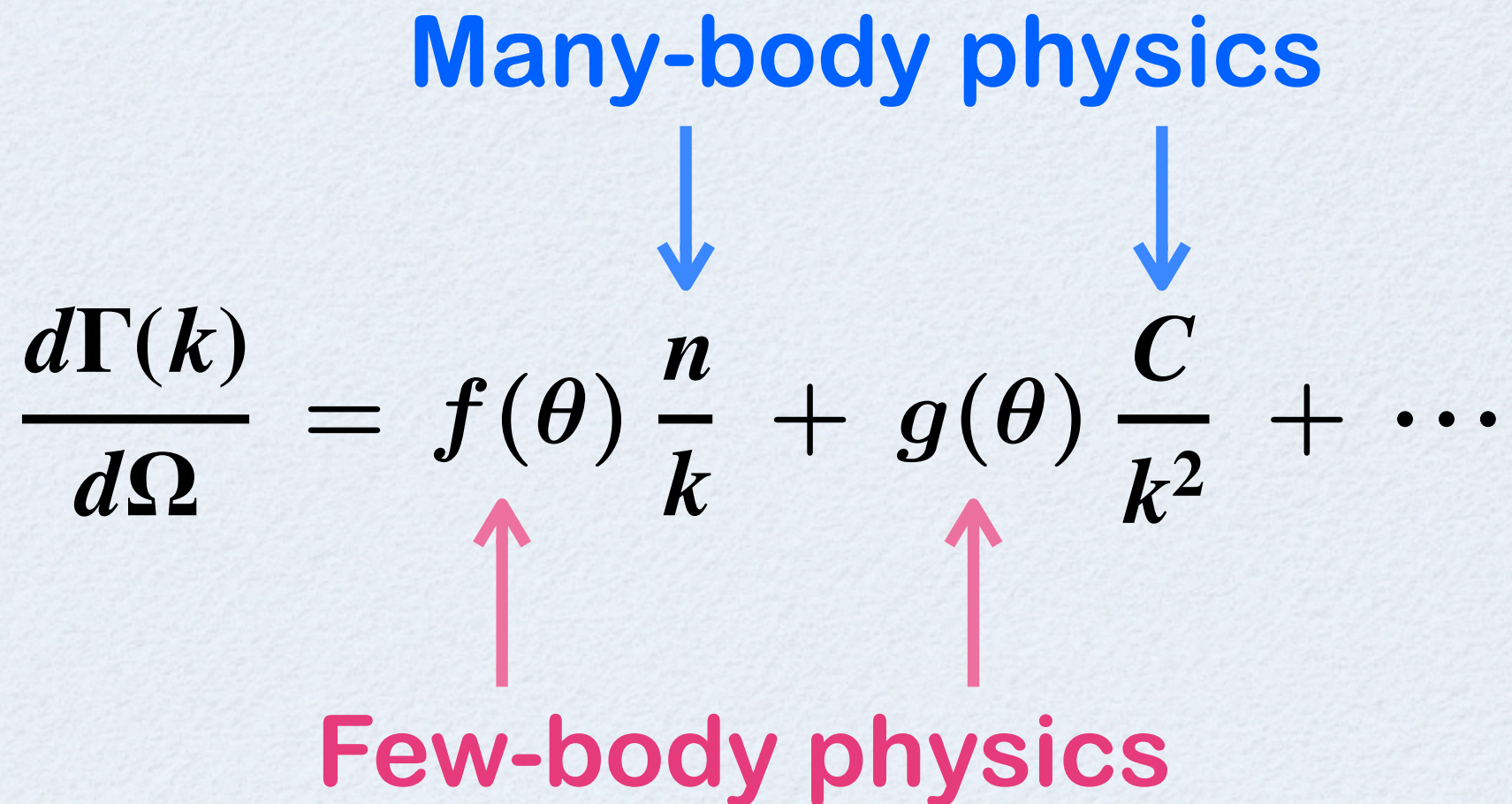


Systematic large- k expansion !

Many-body physics

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta) \frac{C}{k^2} + \dots$$

Few-body physics



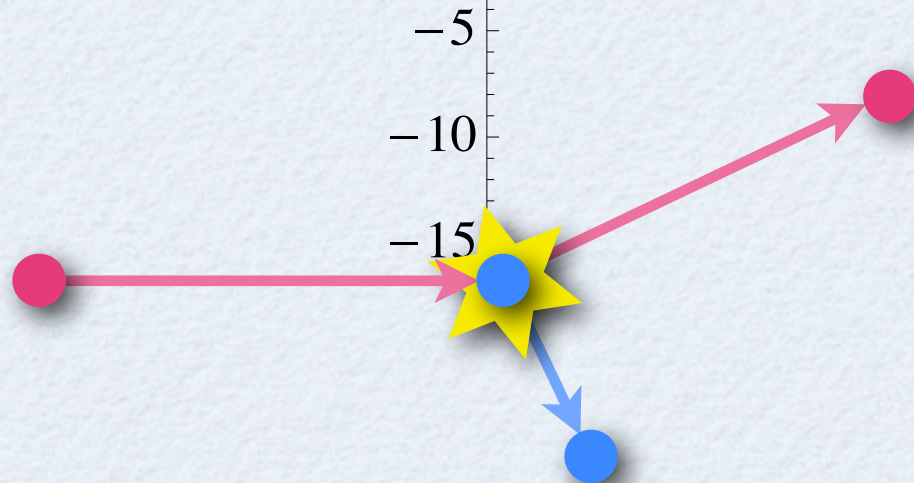
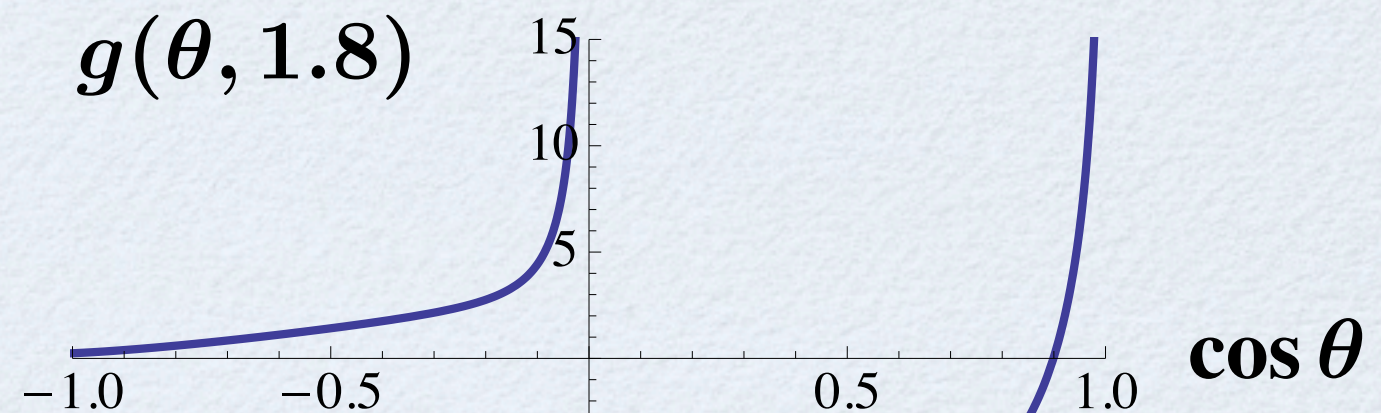
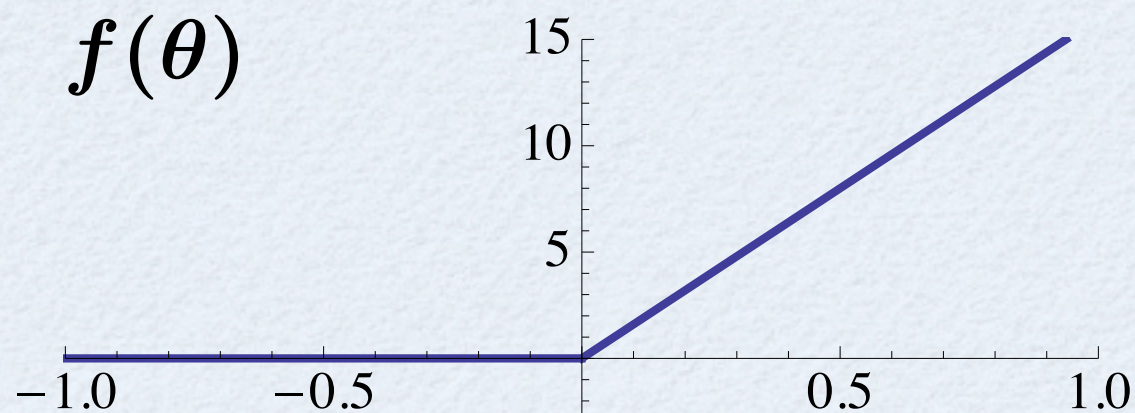
Few-body physics plays an important role to probe many-body physics !

Differential scattering rate

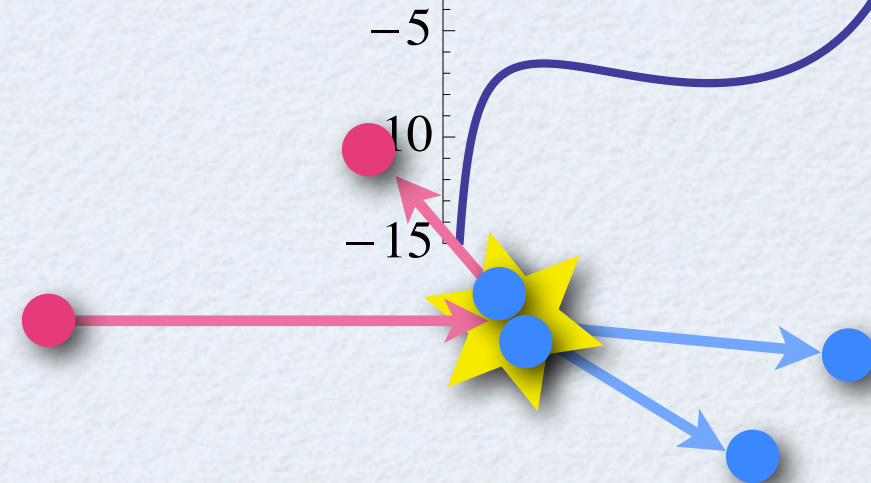
$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + \dots$$

For zero-range interactions

Efimov effect



forward scattering
($\theta < 90^\circ$) only



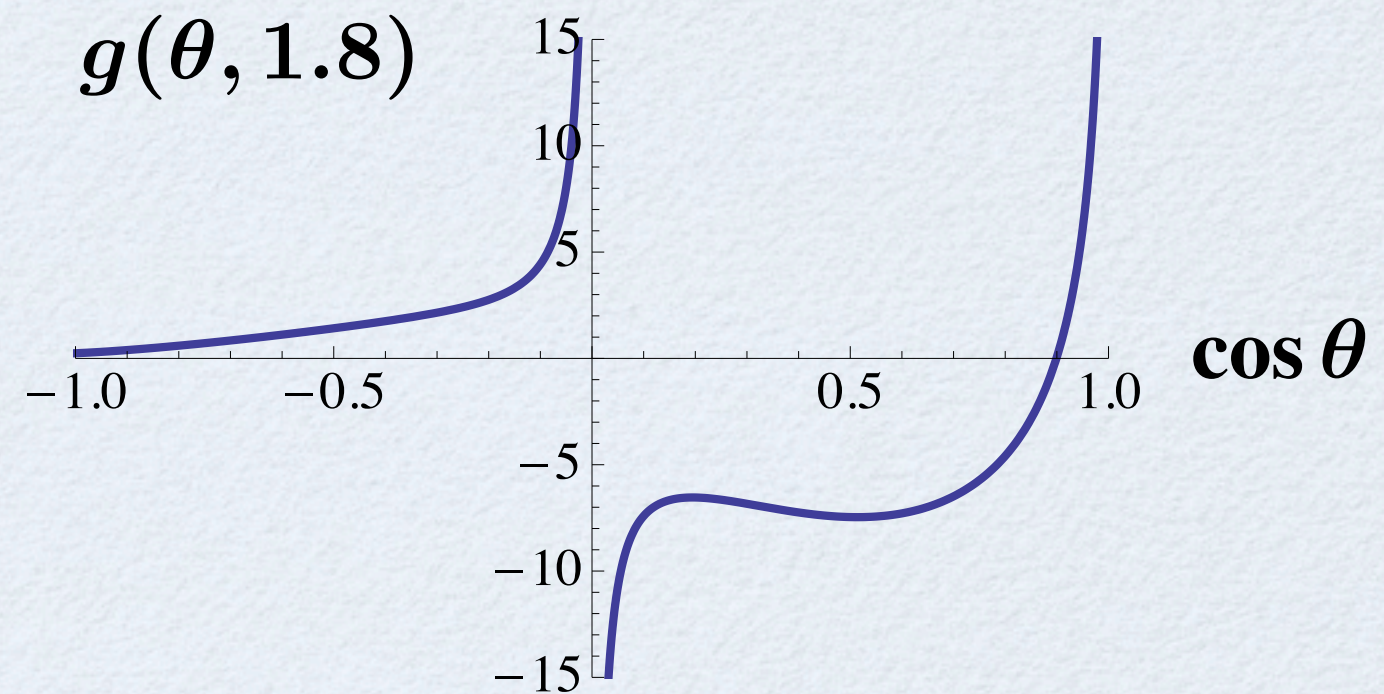
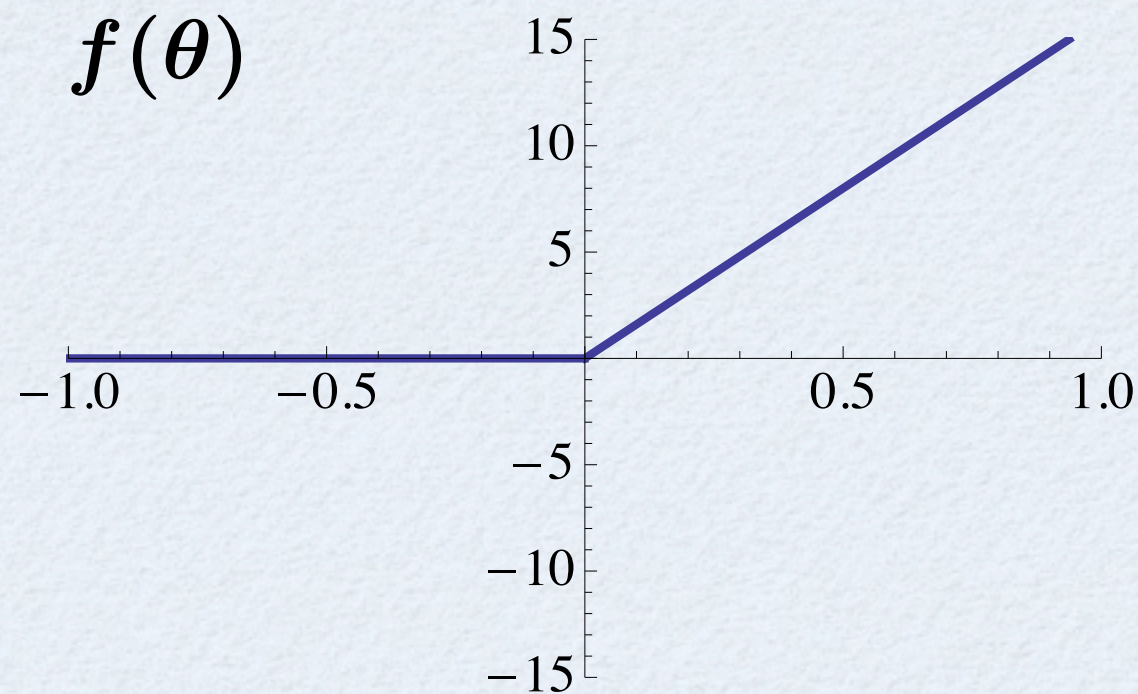
backward scattering
($\theta > 90^\circ$) possible

Differential scattering rate

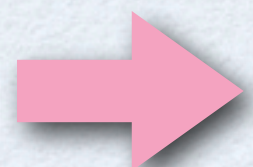
$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + \dots$$

For zero-range interactions

Efimov effect



Backward scattering rate measures contact density

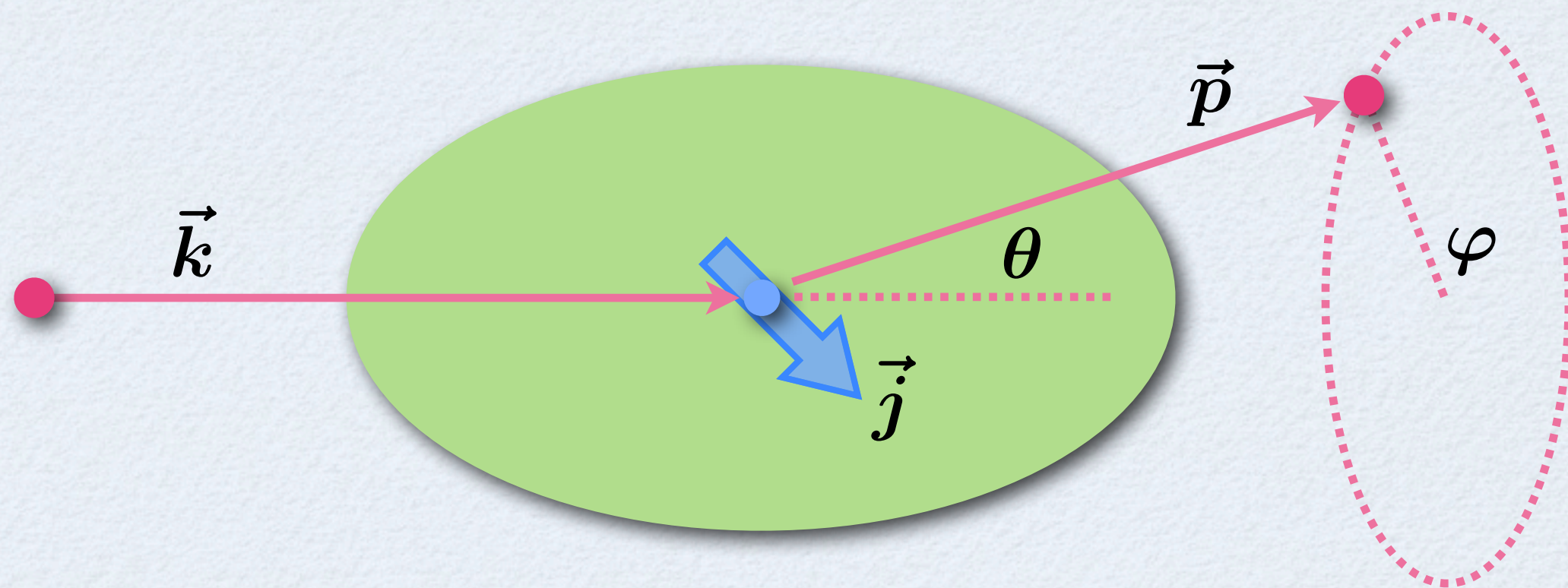


New local probe of strongly-int atomic gases

Differential scattering rate

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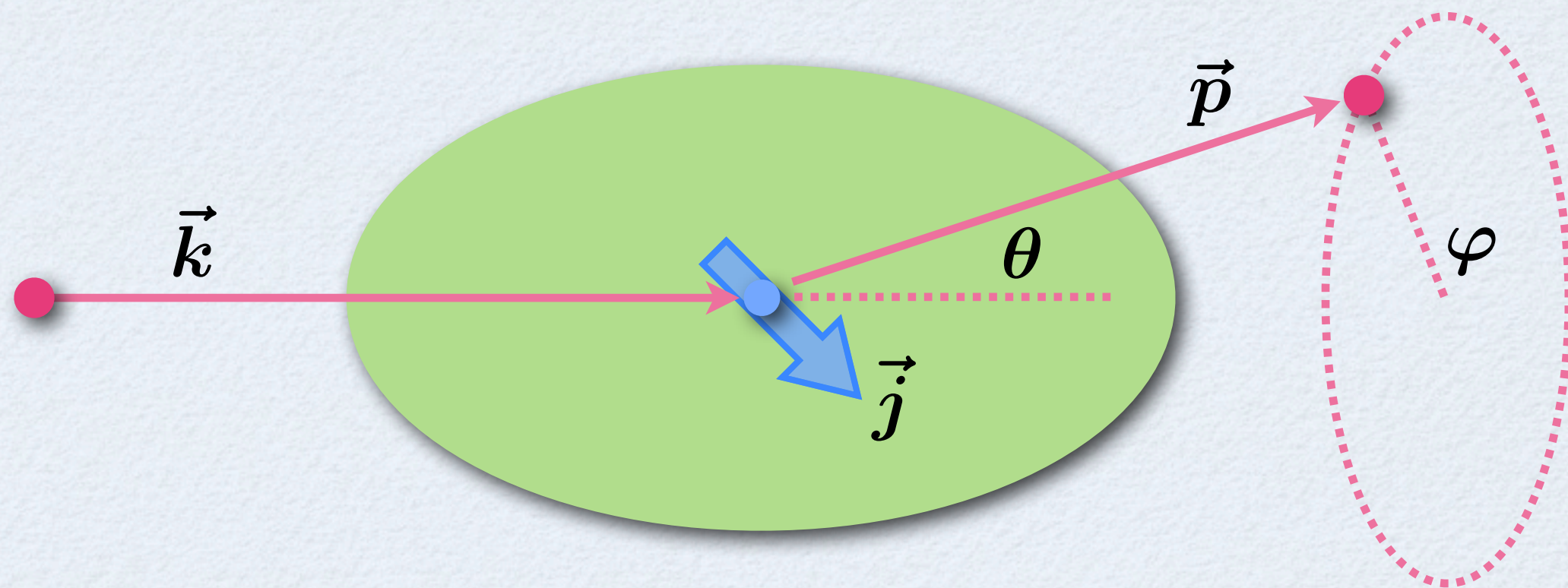
$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + 16 \Theta(\cos \theta) (2 \cos \theta \hat{k} + \hat{p}) \cdot \frac{\vec{j}}{k^2} + \dots$$



Azimuthal (φ) anisotropy reveals currents in many-body states

Differential scattering rate

$$\frac{d\Gamma(k)}{d\Omega} = f(\theta) \frac{n}{k} + g(\theta, k/\kappa_*) \frac{C}{k^2} + 16 \Theta(\cos \theta) (2 \cos \theta \hat{k} + \hat{p}) \cdot \frac{\vec{j}}{k^2} + \dots$$

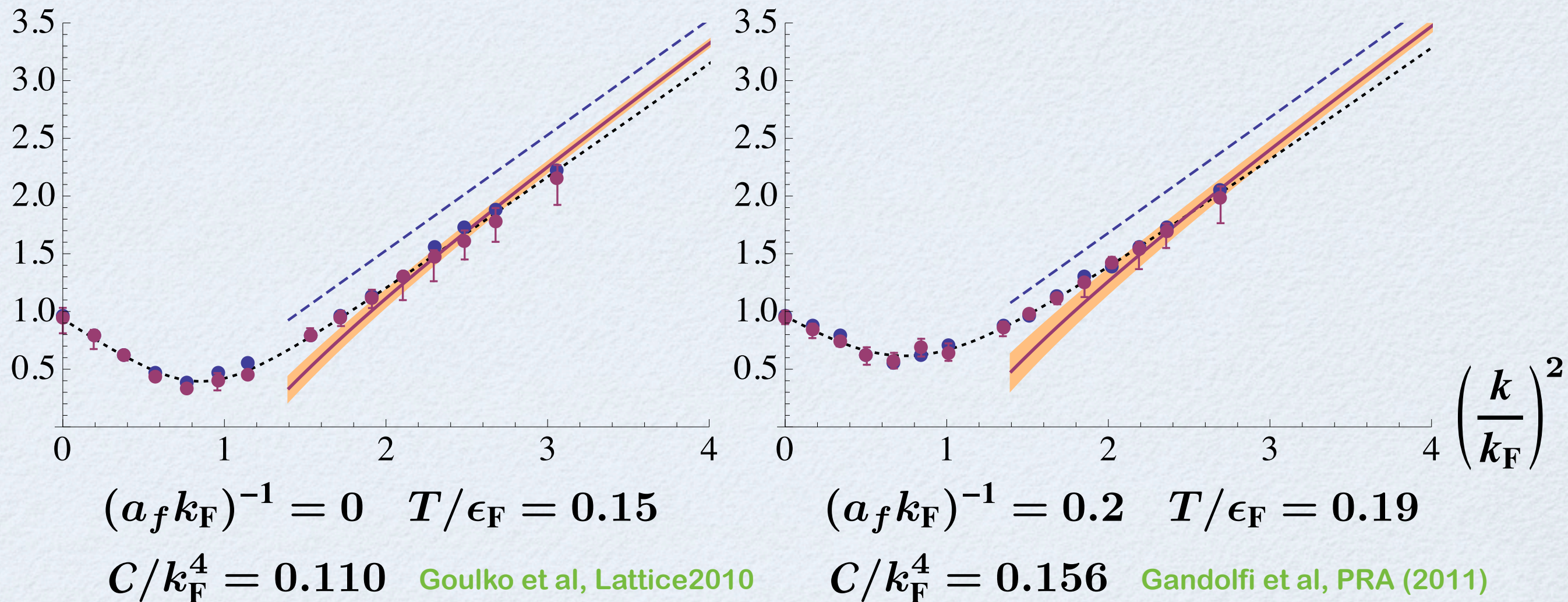


$$N_{\text{measured}}(\theta, \varphi) = \frac{1}{k} \int dl \frac{d\Gamma(k)}{d\Omega} \times N_{\text{shot}}$$

How large is large ?

$$E_{\uparrow}(k) = \left[1 + 32\pi \frac{n_{\downarrow}}{ak^4} - 7.54 \frac{C}{k^4} + O(k^{-6}) \right] \frac{k^2}{2m}$$

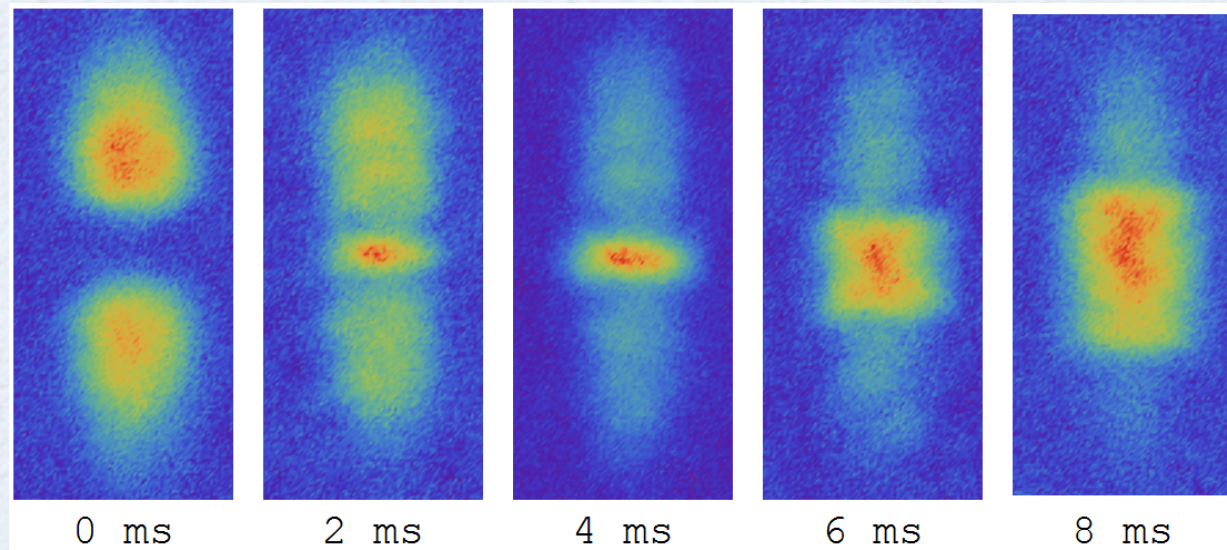
Comparison of $E(k)/\epsilon_F$ with QMC P. Magierski et al., PRL (2011)



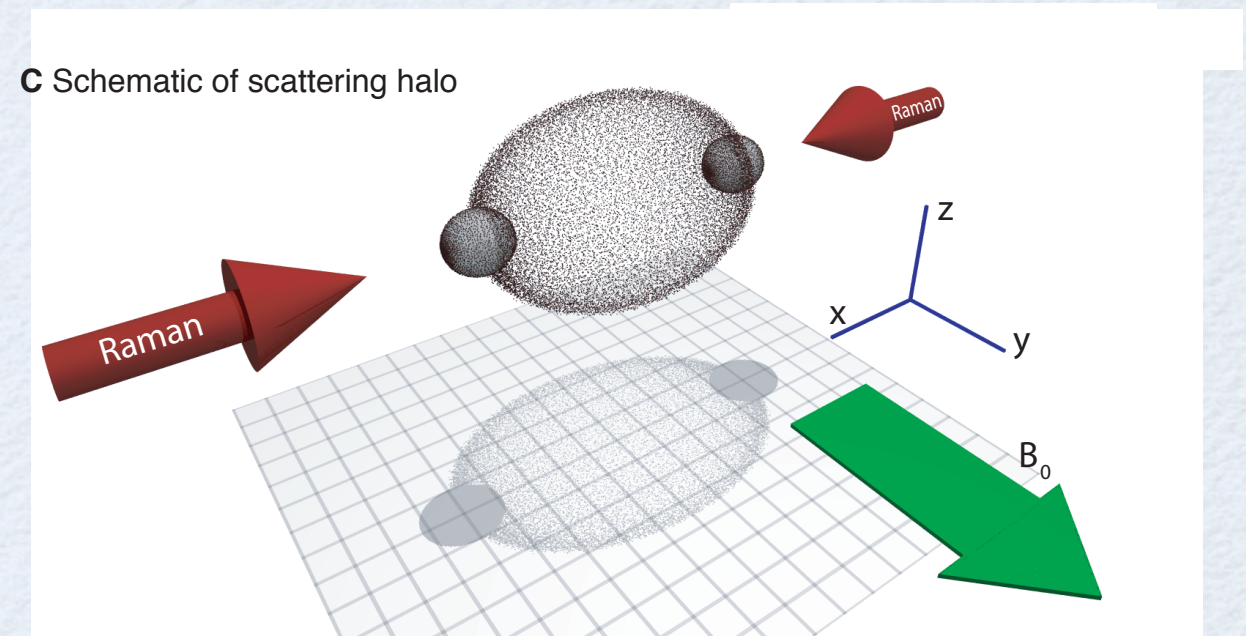
Reasonable agreement even at $k/k_F > 1.5$!

Ultracold atom "colliders"

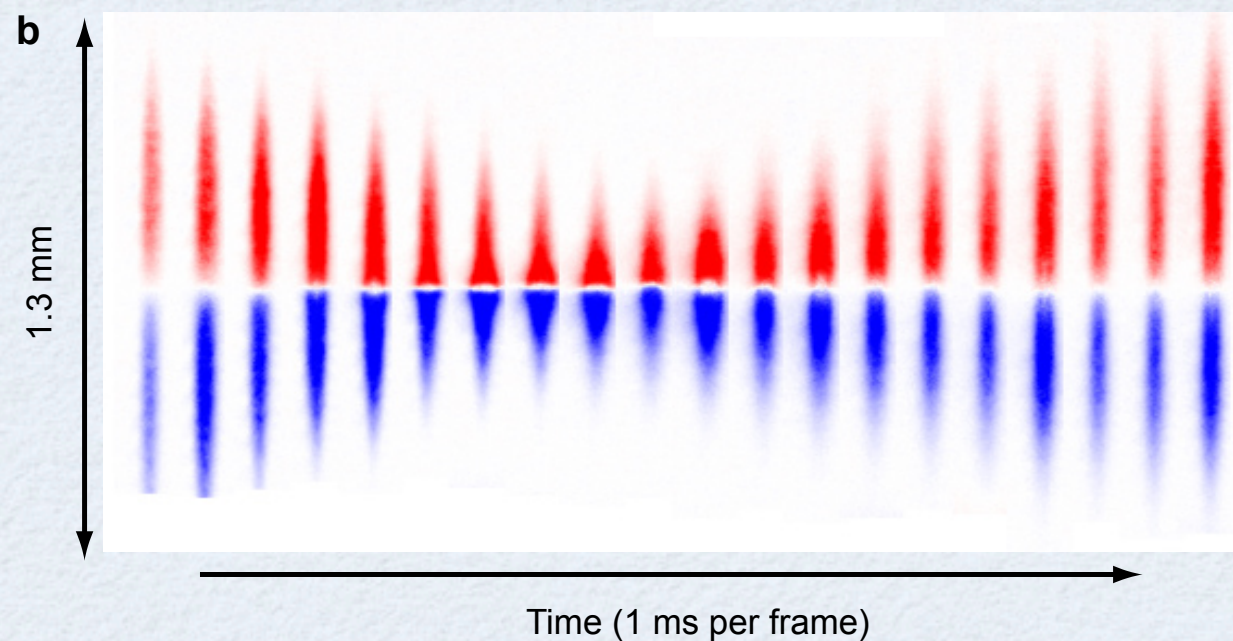
Duke (2011)



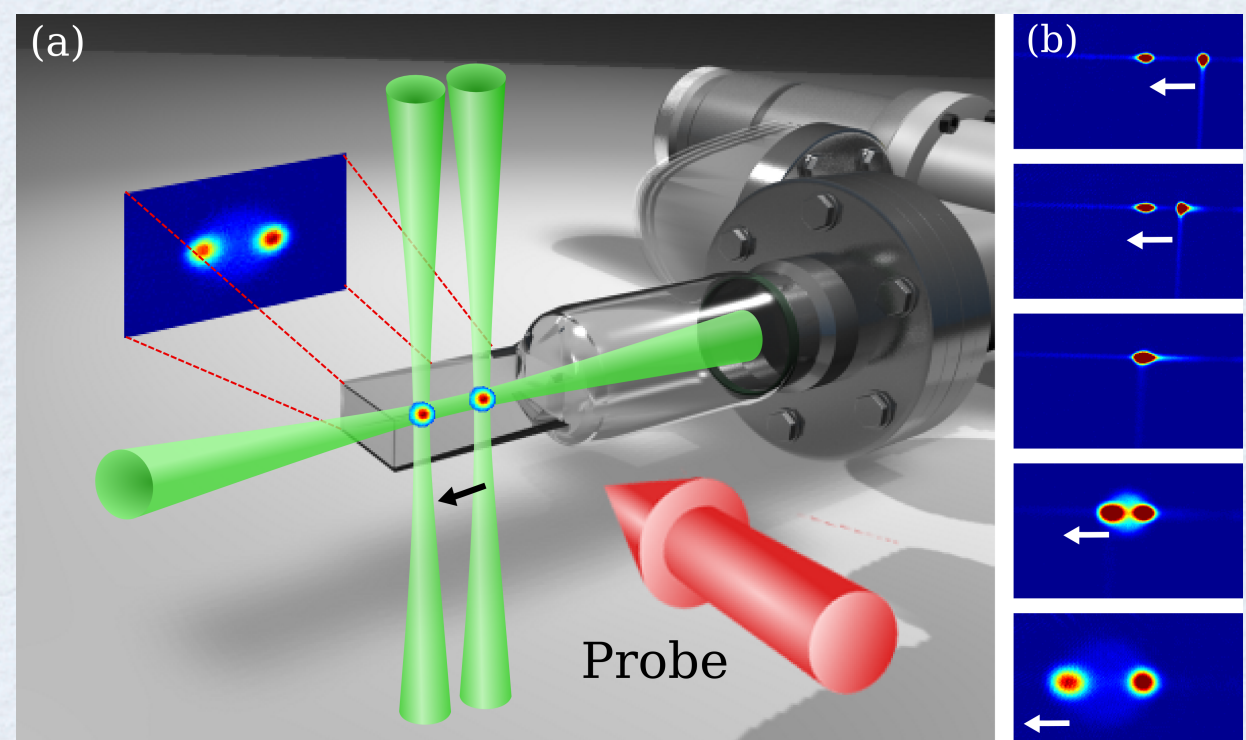
NIST (2012)



MIT (2011)

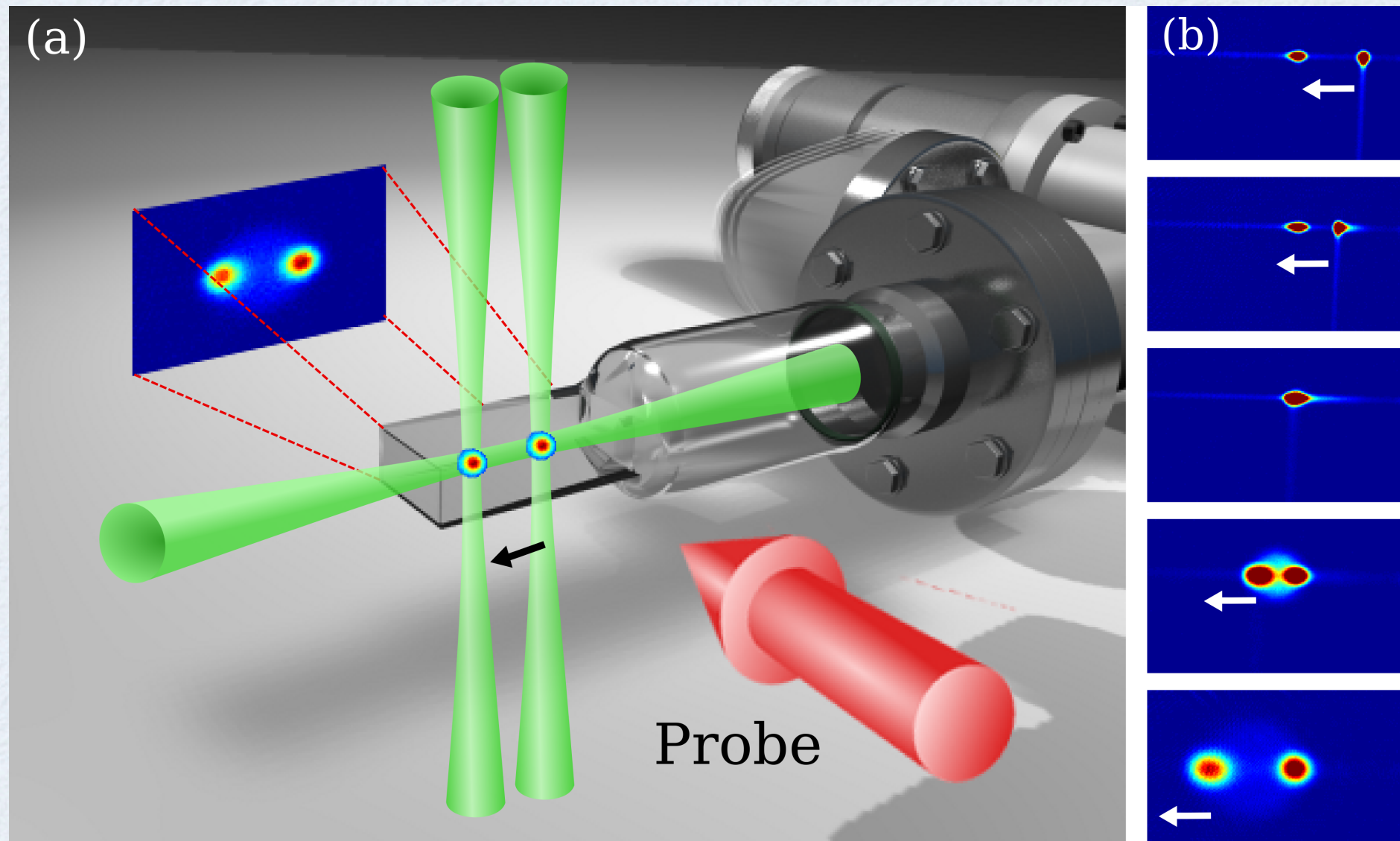


Otago (2012)

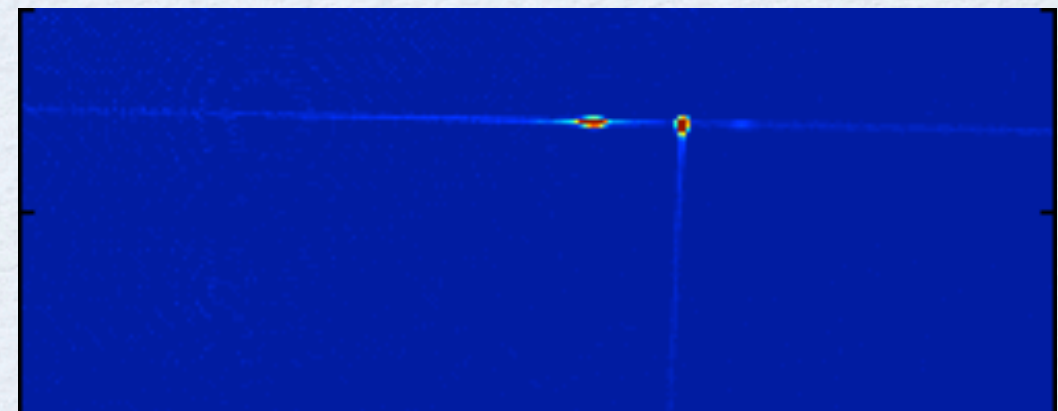


Ultracold atom “colliders”

“A laser based accelerator for ultracold atoms”

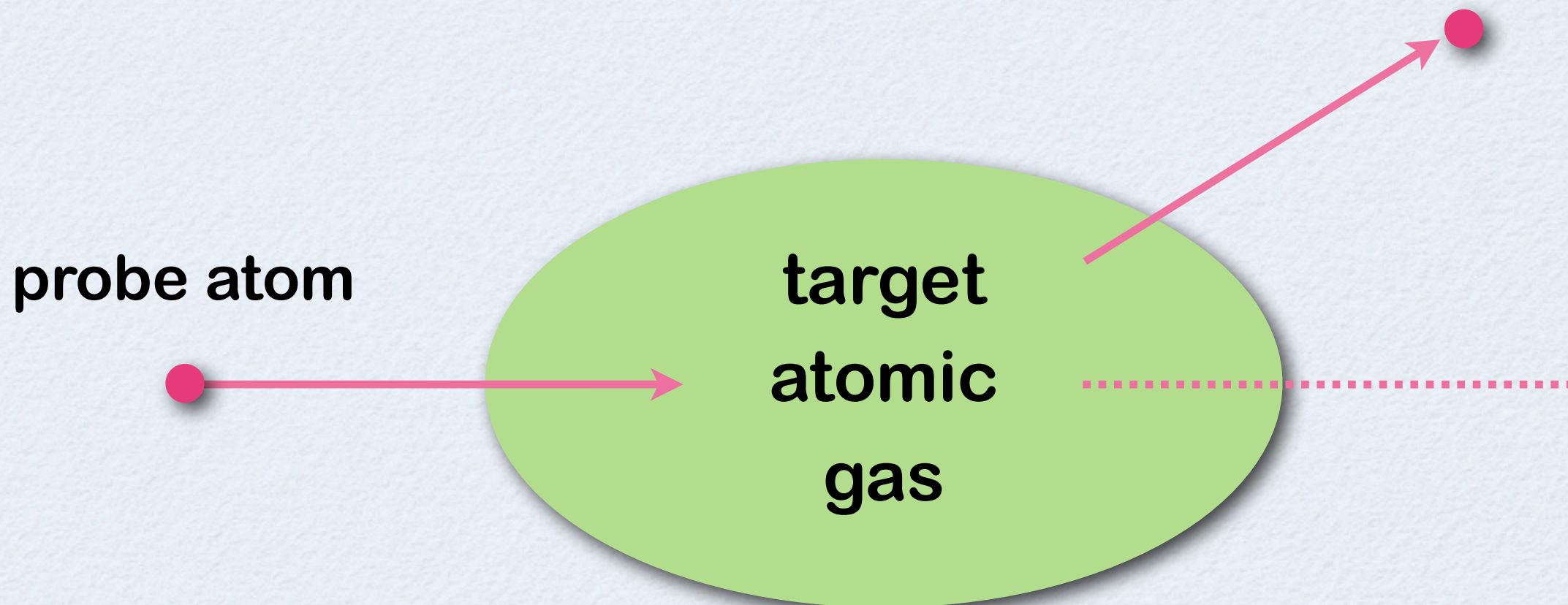


University of Otago
(New Zealand)
Optics Letters (2012)



Short summary

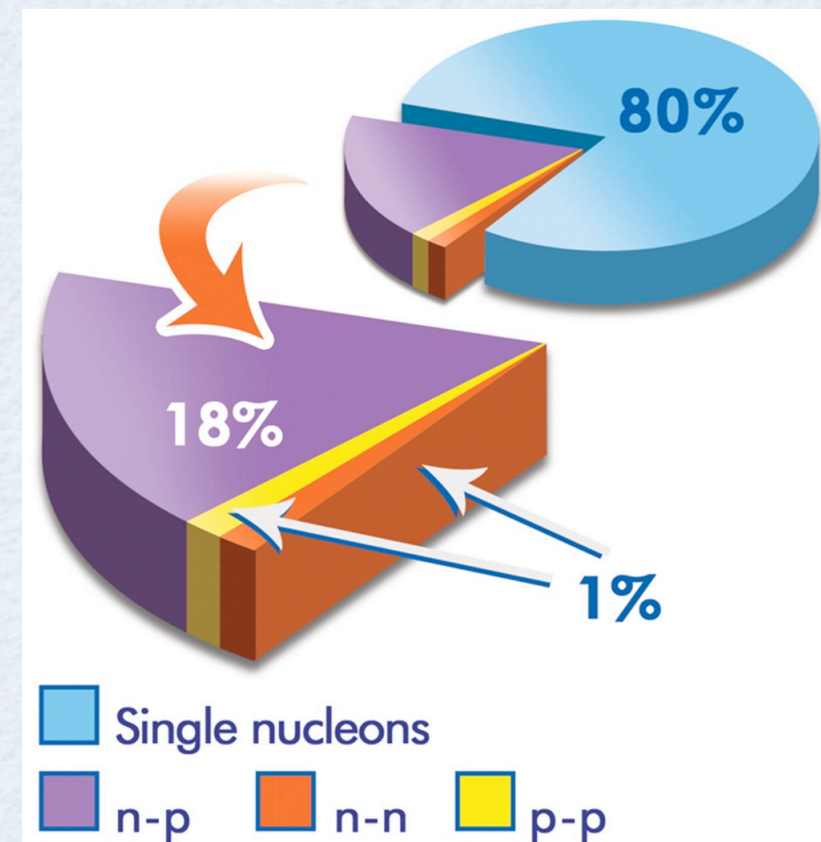
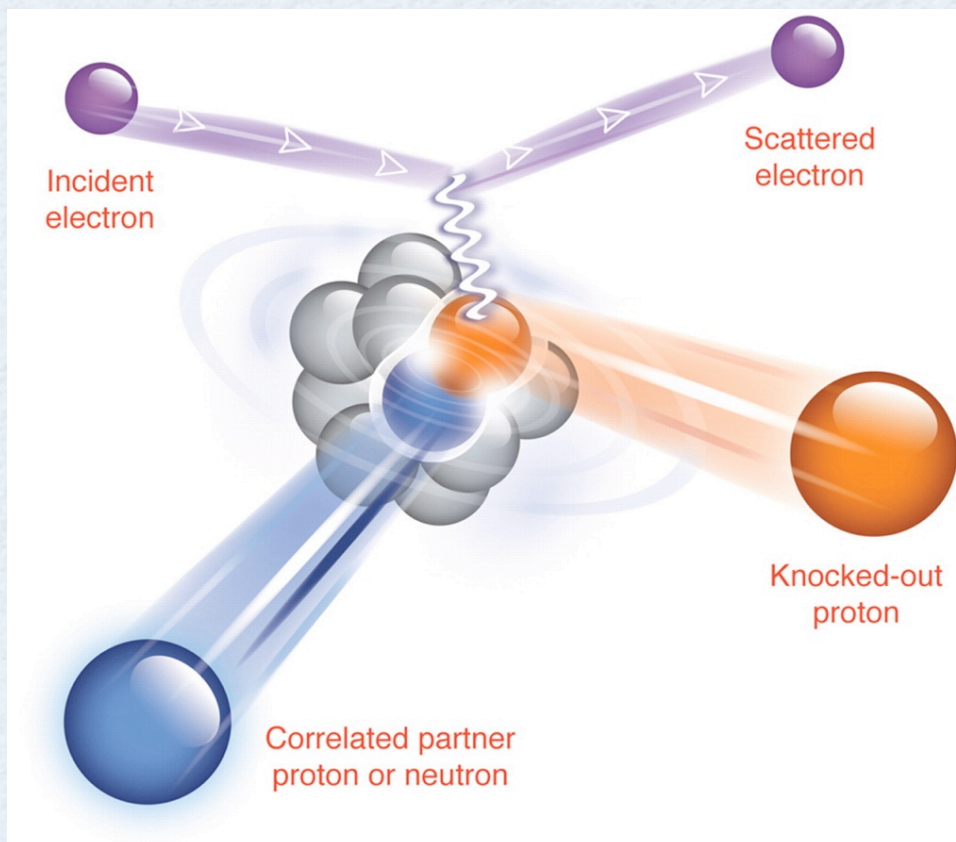
- Energetic atoms \Rightarrow New tool to locally probe strongly-interacting atomic gases
- Systematic large- k expansions are possible
 - ✓ backward scattering \Rightarrow contact density
 - ✓ azimuthal anisotropy \Rightarrow current density



Short summary

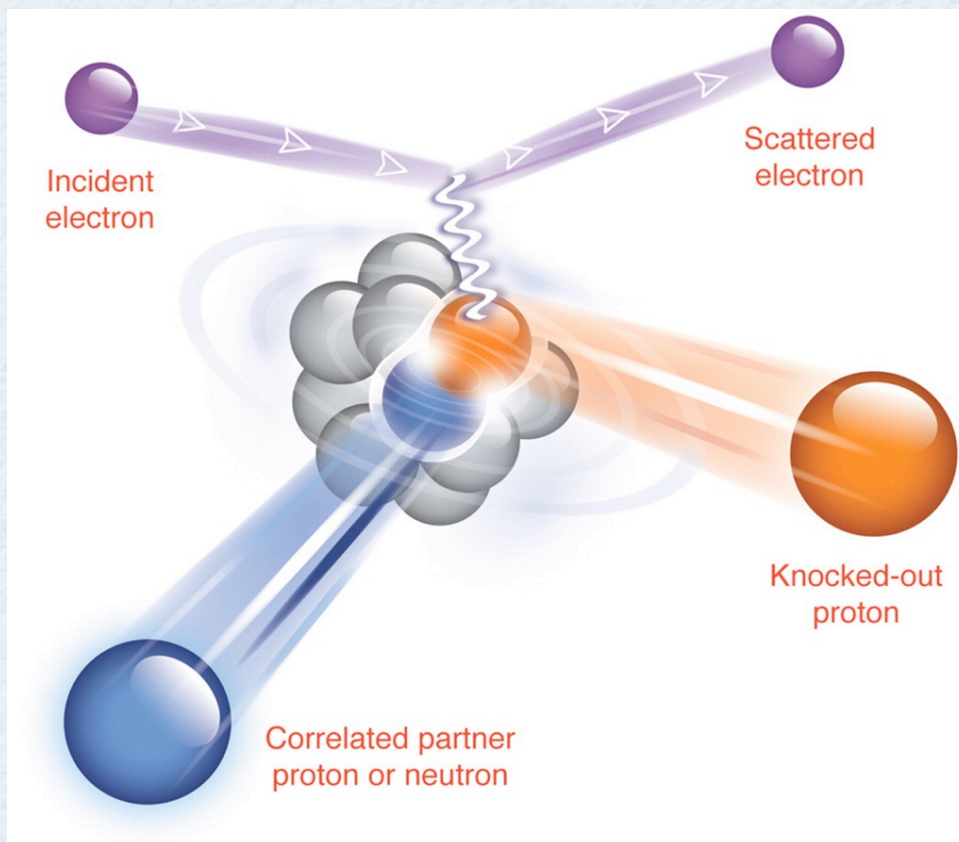
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- Close connection to nuclear/particle physics

JLab, Science 320, 1476 (2008)



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JLab, Science 320, 1476 (2008)



“Hard probes” are useful to reveal short-range pair correlations both in atomic gases and nuclei (QGP?)

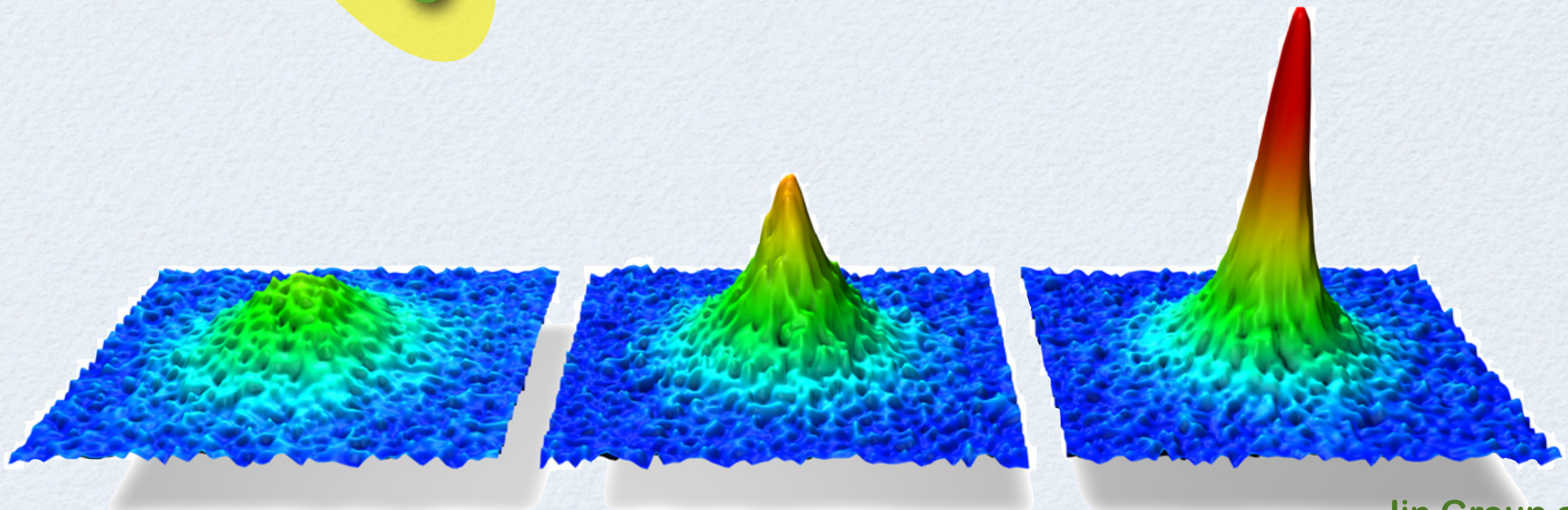
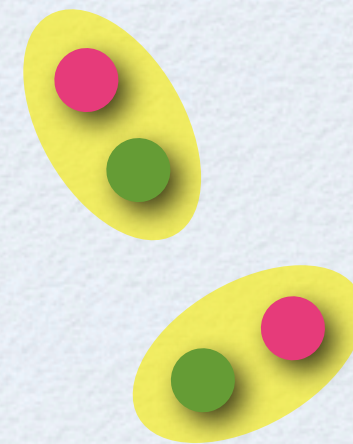
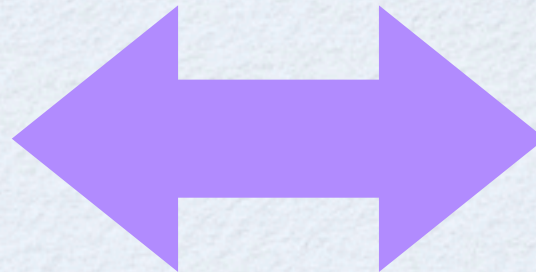
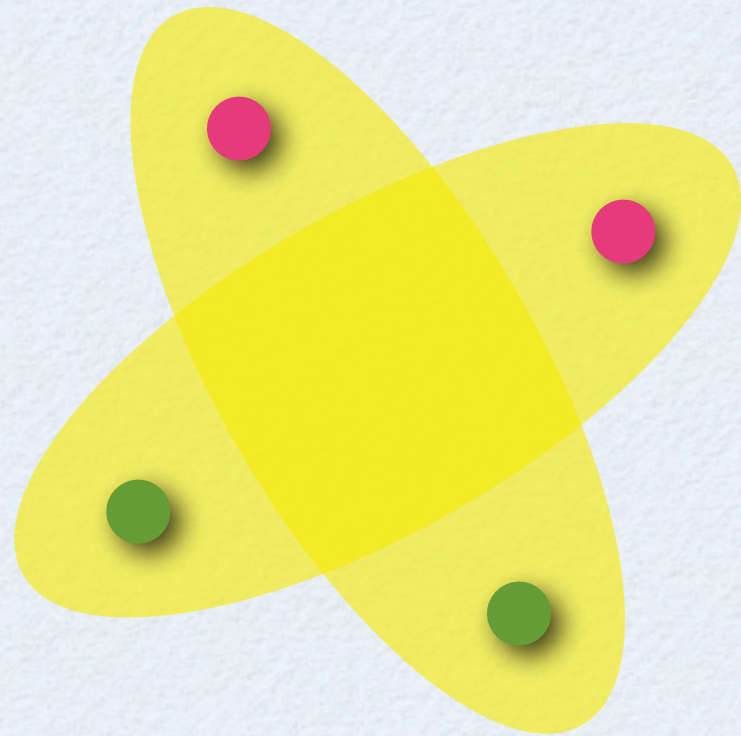
**“Quark-hadron continuity”
in cold atoms**

BCS-BEC crossover

- 2-component Fermi gas

loosely bound Cooper pairs

tightly bound dimers

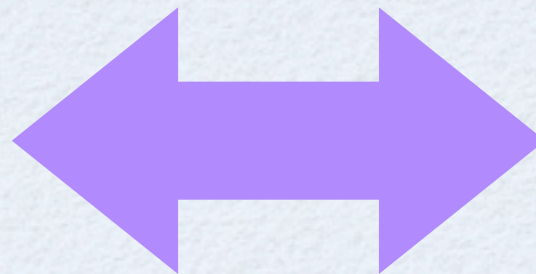
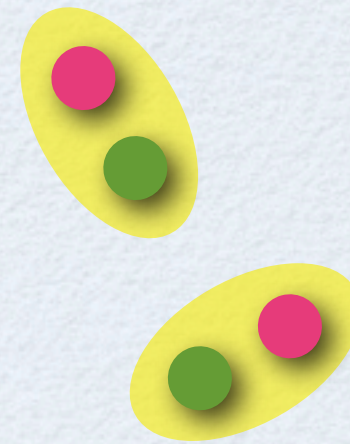
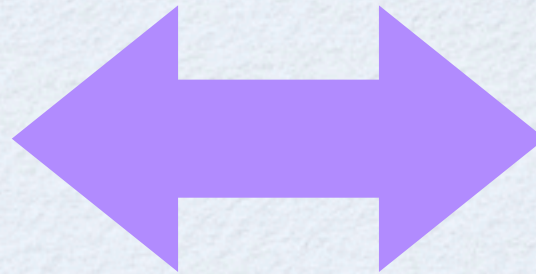
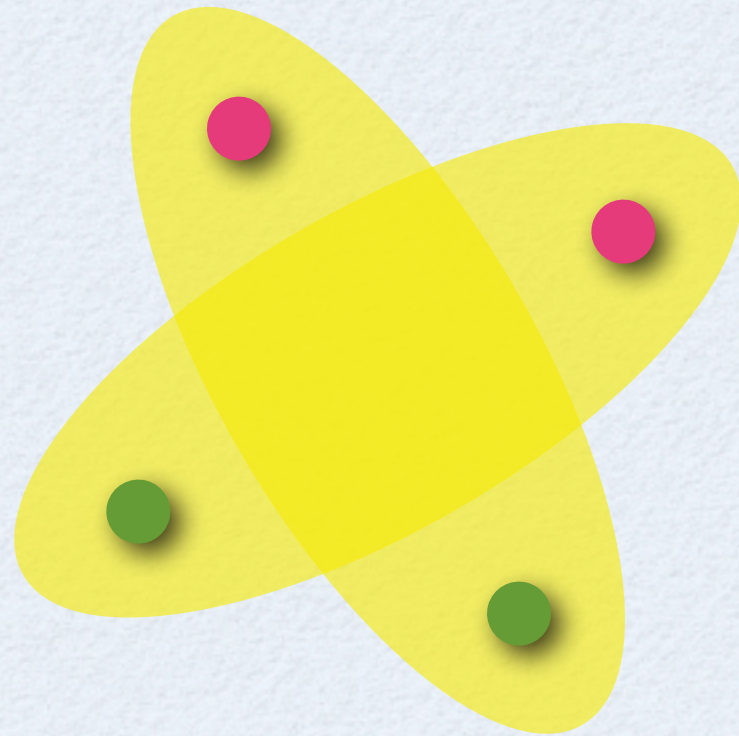


BCS-BEC crossover

- 3-component Fermi gas

loosely bound Cooper pairs

tightly bound dimers



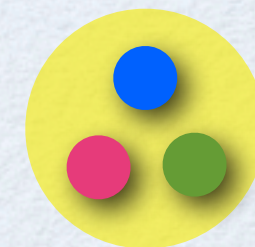
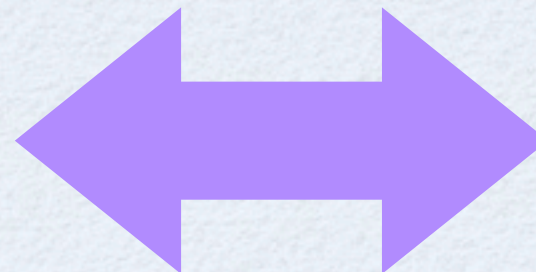
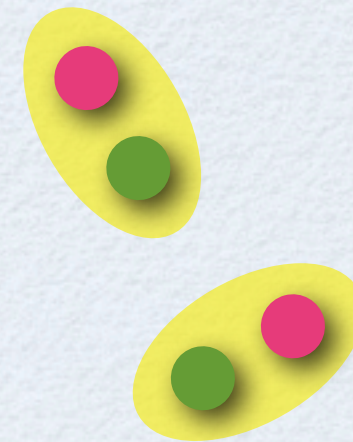
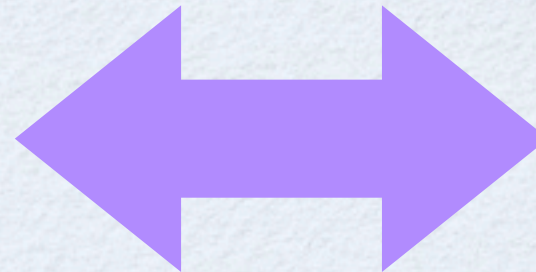
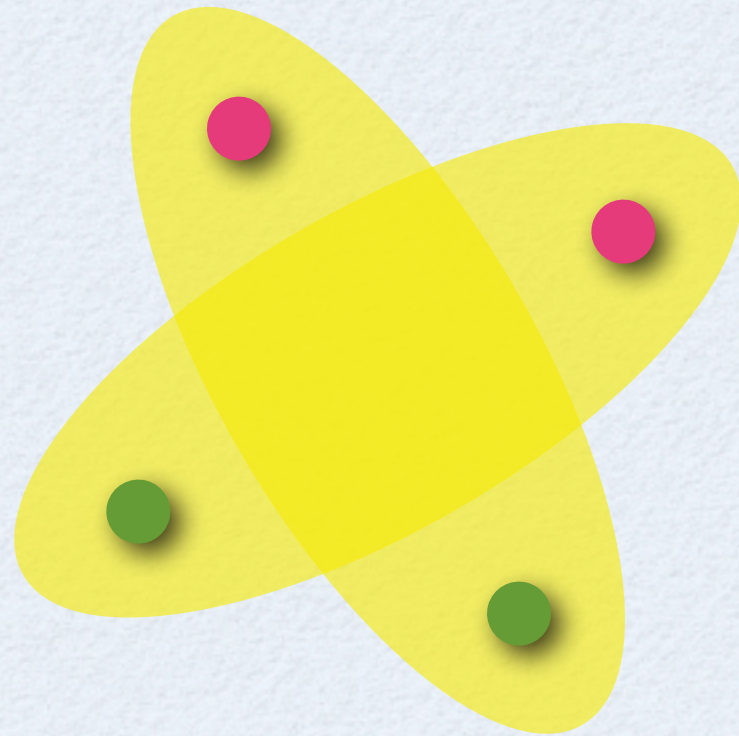
unpaired atoms

BCS-BEC crossover

- 3-component Fermi gas

loosely bound Cooper pairs

tightly bound dimers



unpaired atoms

unpaired trimers

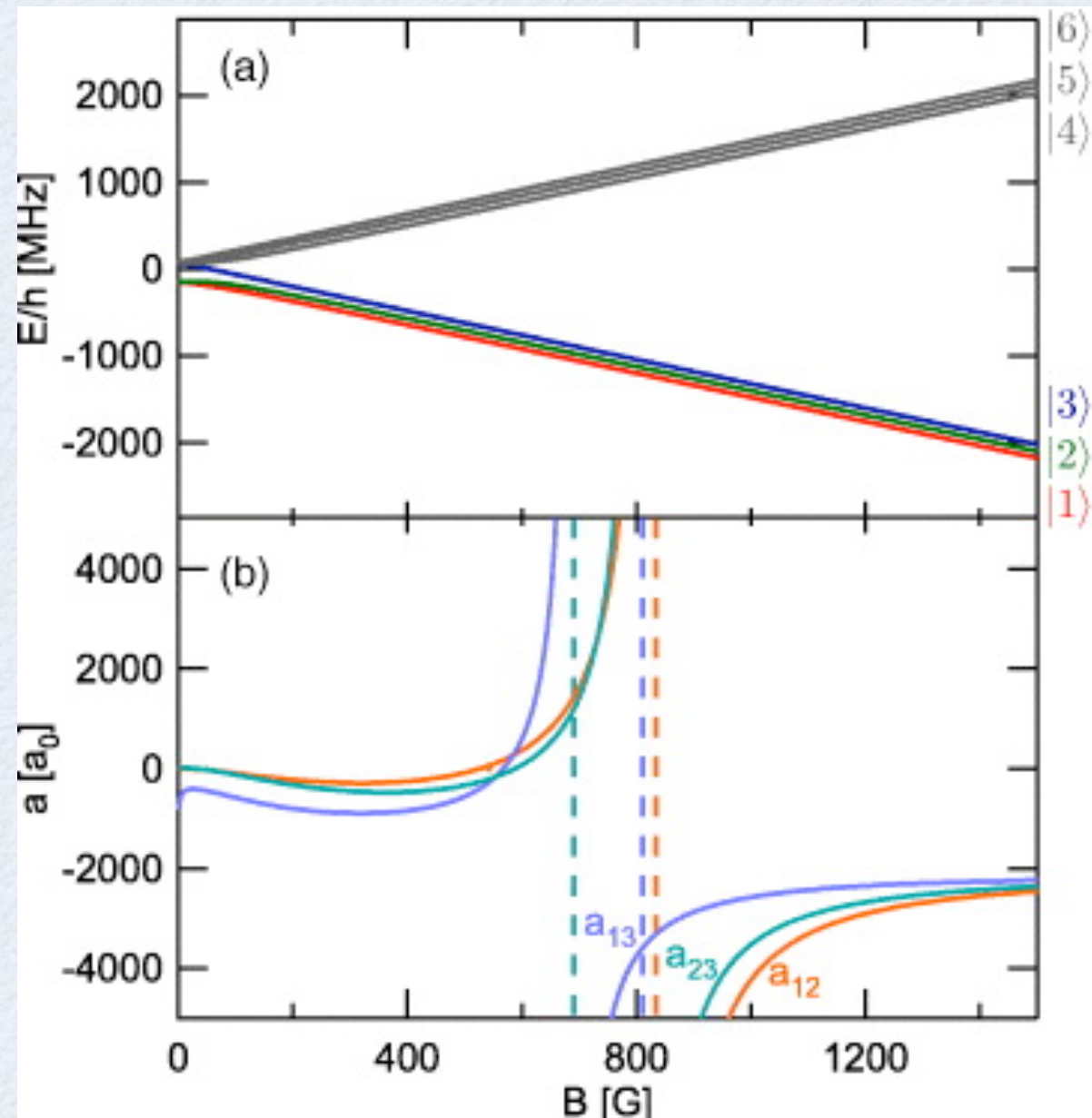
“Atom-trimer continuity” = New crossover physics !

3-component Fermi gas

- 3 spin states ($i=1,2,3$) of ${}^6\text{Li}$ atoms near a Feshbach resonance:

$$f(k) = \frac{-1}{ik + \frac{1}{a}}$$

- $a_{12} = a_{23} = a_{31}$



3-component Fermi gas

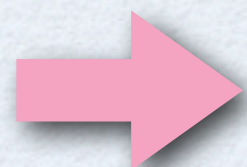
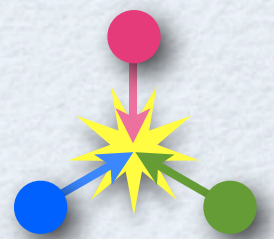
- 3 spin states ($i=1,2,3$) of ${}^6\text{Li}$ atoms near a Feshbach resonance:

$$f(k) = \frac{-1}{ik + \frac{1}{a}}$$

- $a_{12} = a_{23} = a_{31} \Rightarrow \text{SU}(3) \times \text{U}(1)$ invariance

$$\mathcal{L} = \psi_i^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi_i + \frac{g}{2} \psi_i^\dagger \psi_j^\dagger \psi_j \psi_i$$

- **Problem!** 3 fermions form an infinitely deep bound state (**Thomas collapse**)



No many-body ground state :-)

3-component Fermi gas

- 3 spin states ($i=1,2,3$) of ${}^6\text{Li}$ atoms near a “narrow” Feshbach resonance:

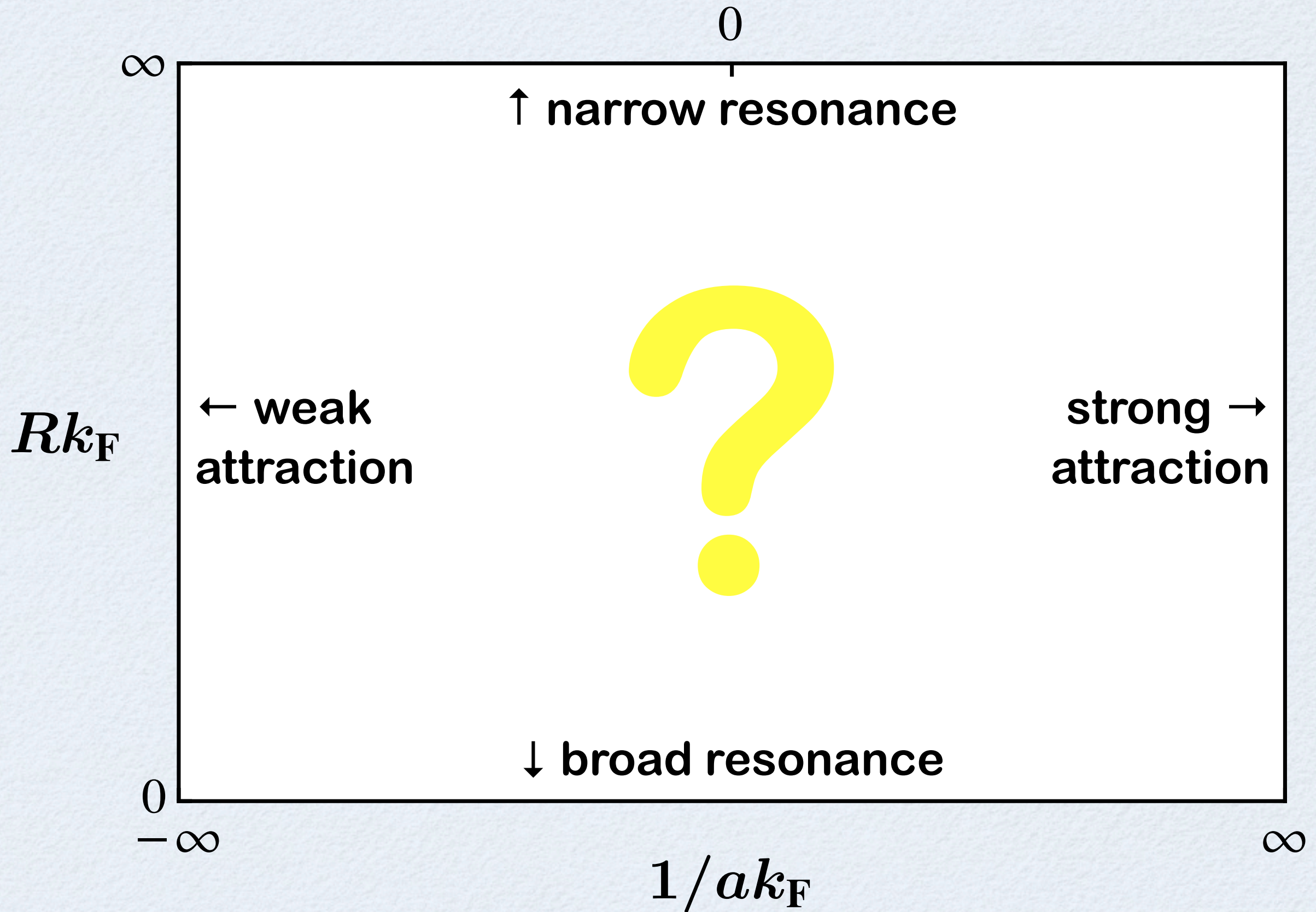
$$f(k) = \frac{-1}{ik + \frac{1}{a}} \quad \longrightarrow \quad f(k) = \frac{-1}{ik + \frac{1}{a} + Rk^2}$$

$r_{\text{eff}} = -2R$ is the effective range

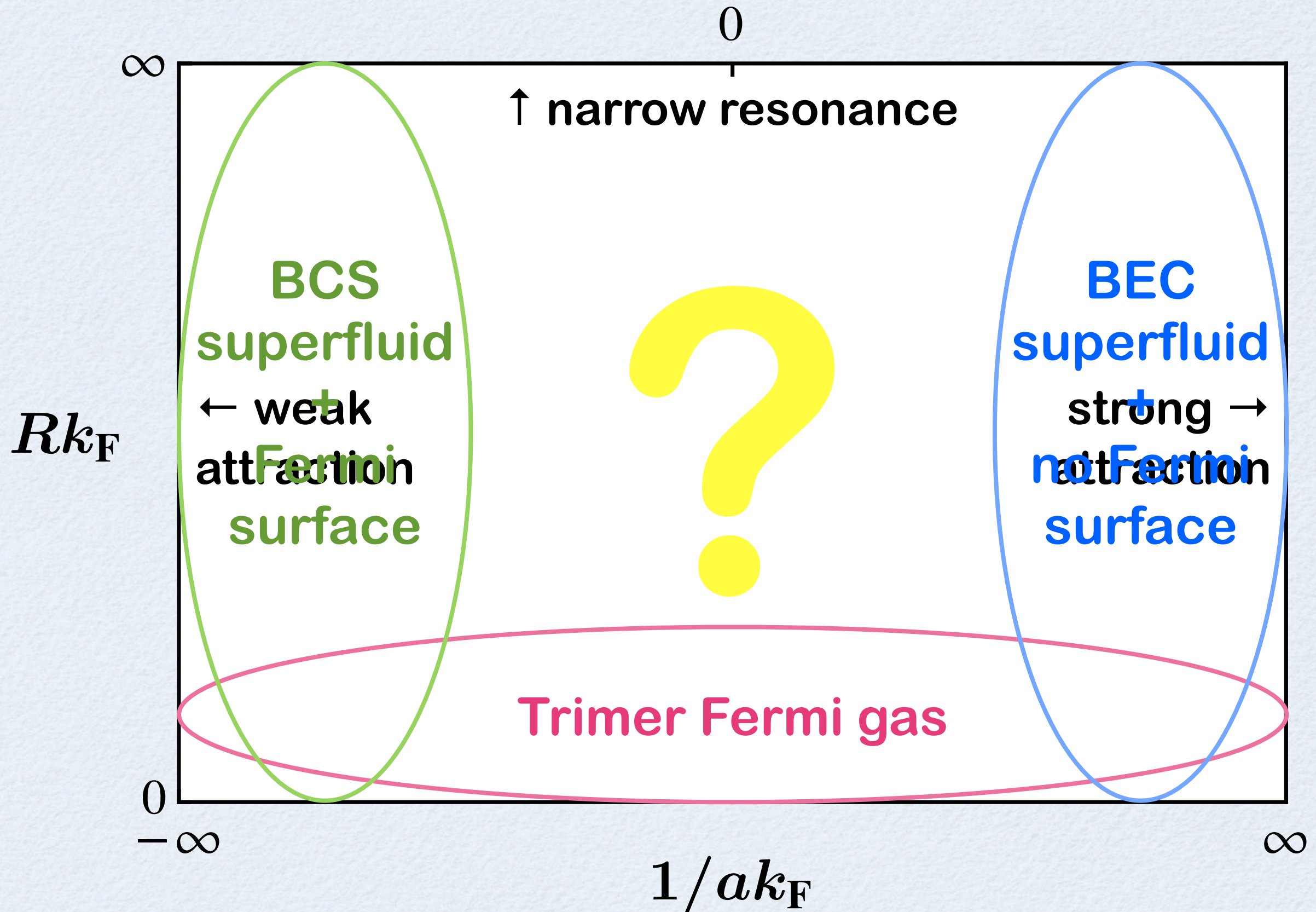
- R regularizes short-distance behaviors
(\Rightarrow no Thomas collapse)

 **Universal many-body ground state**
(depends only on a, R, k_F)

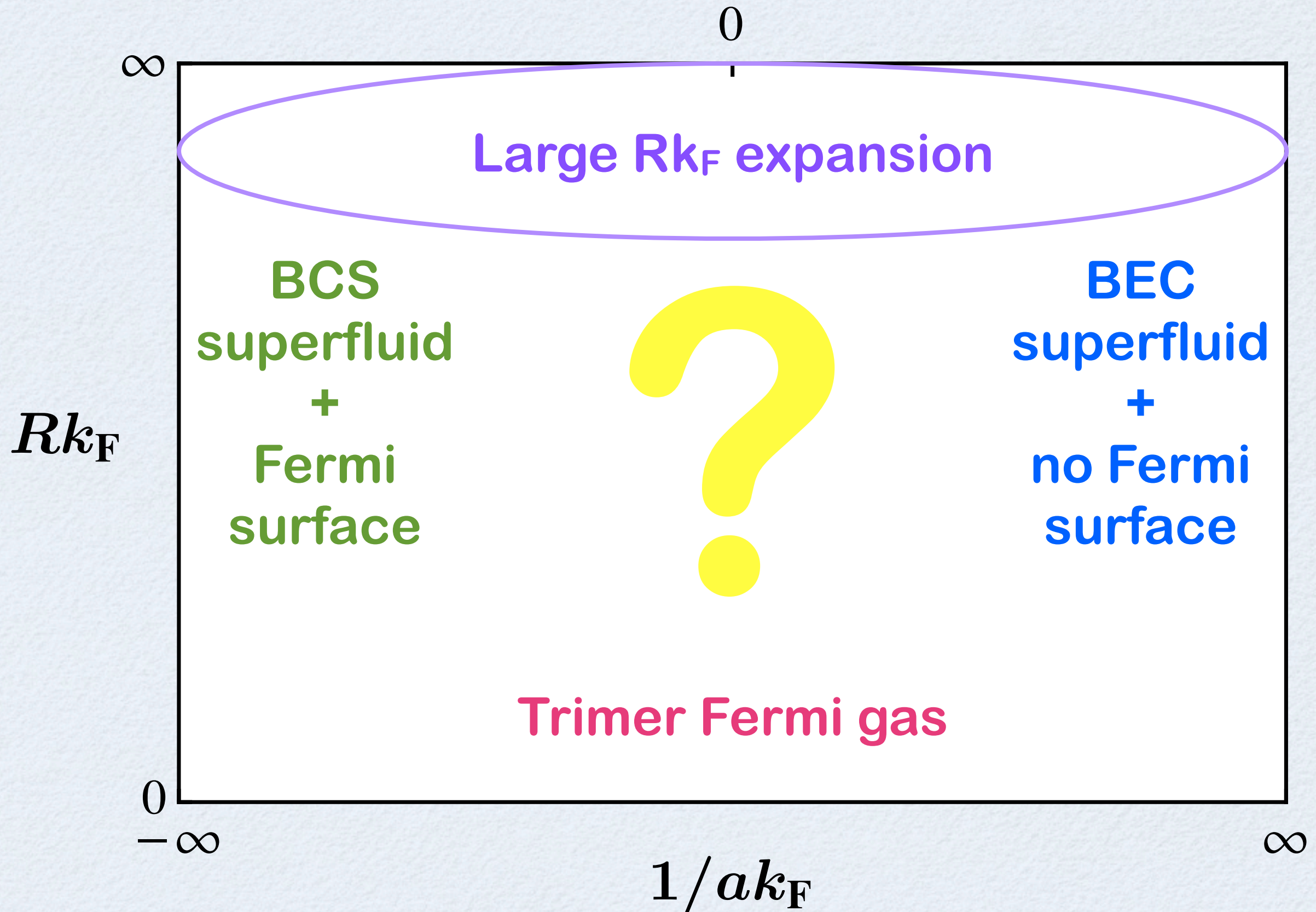
Phase diagram



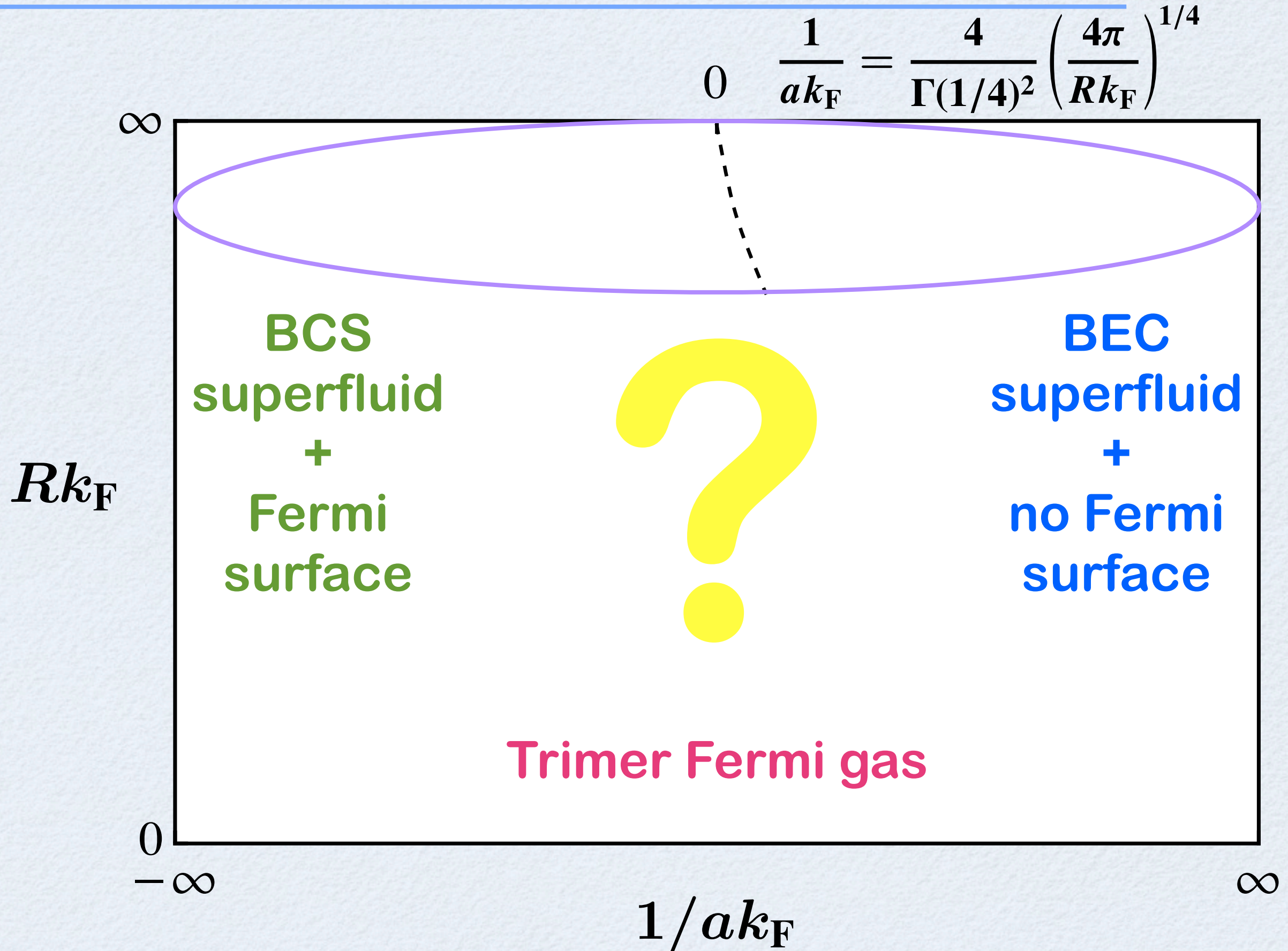
Phase diagram



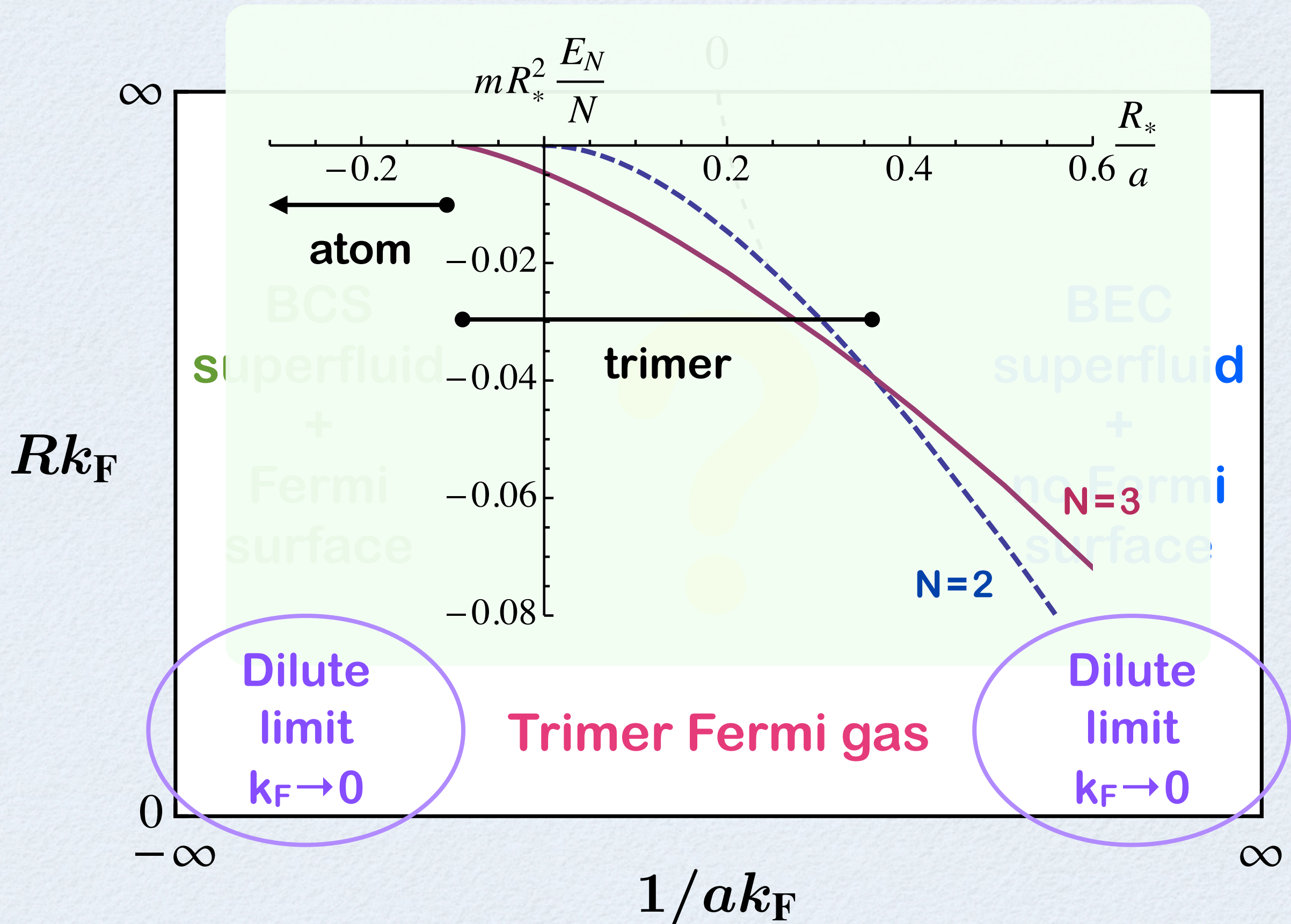
Phase diagram



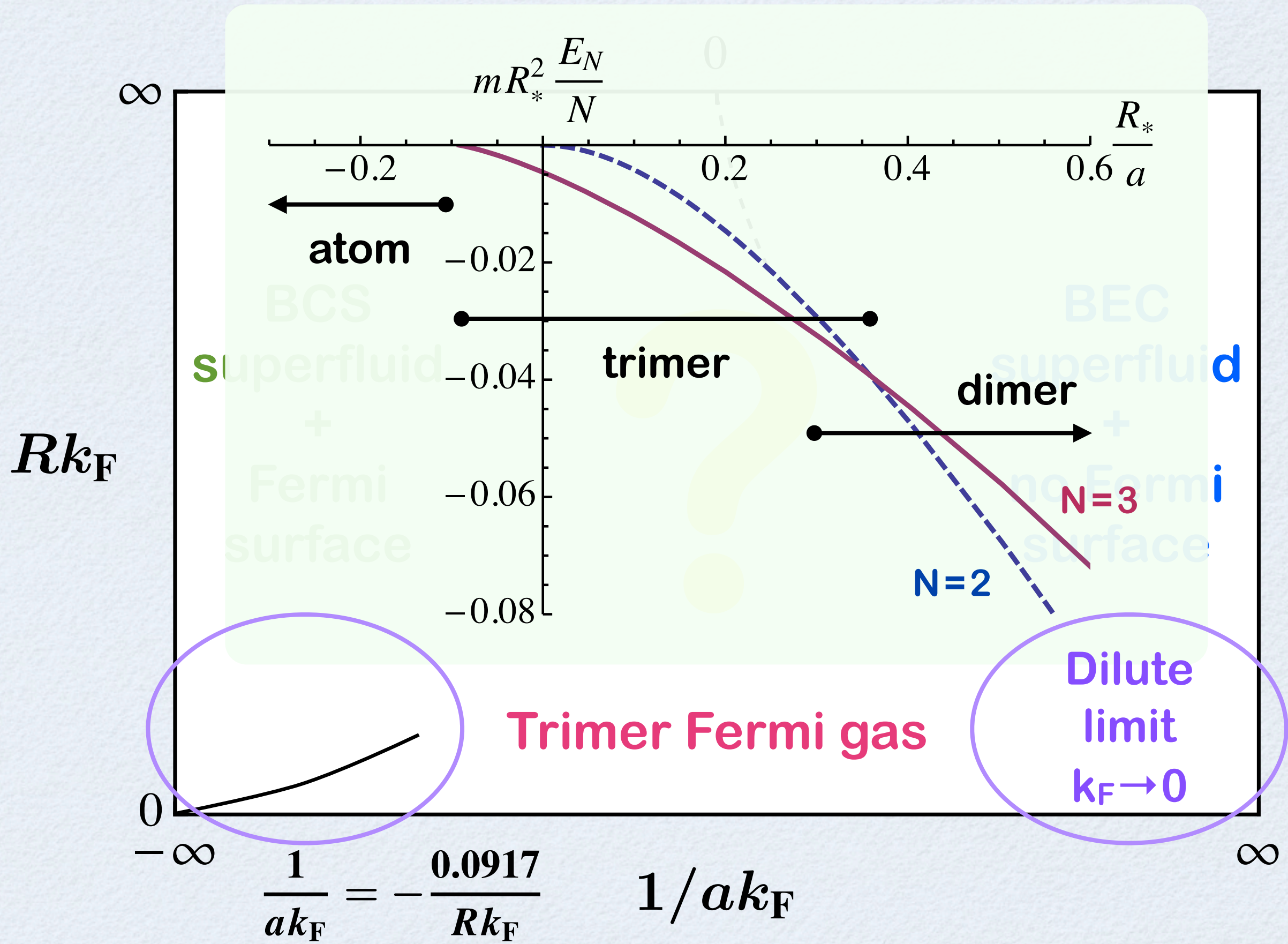
Phase diagram



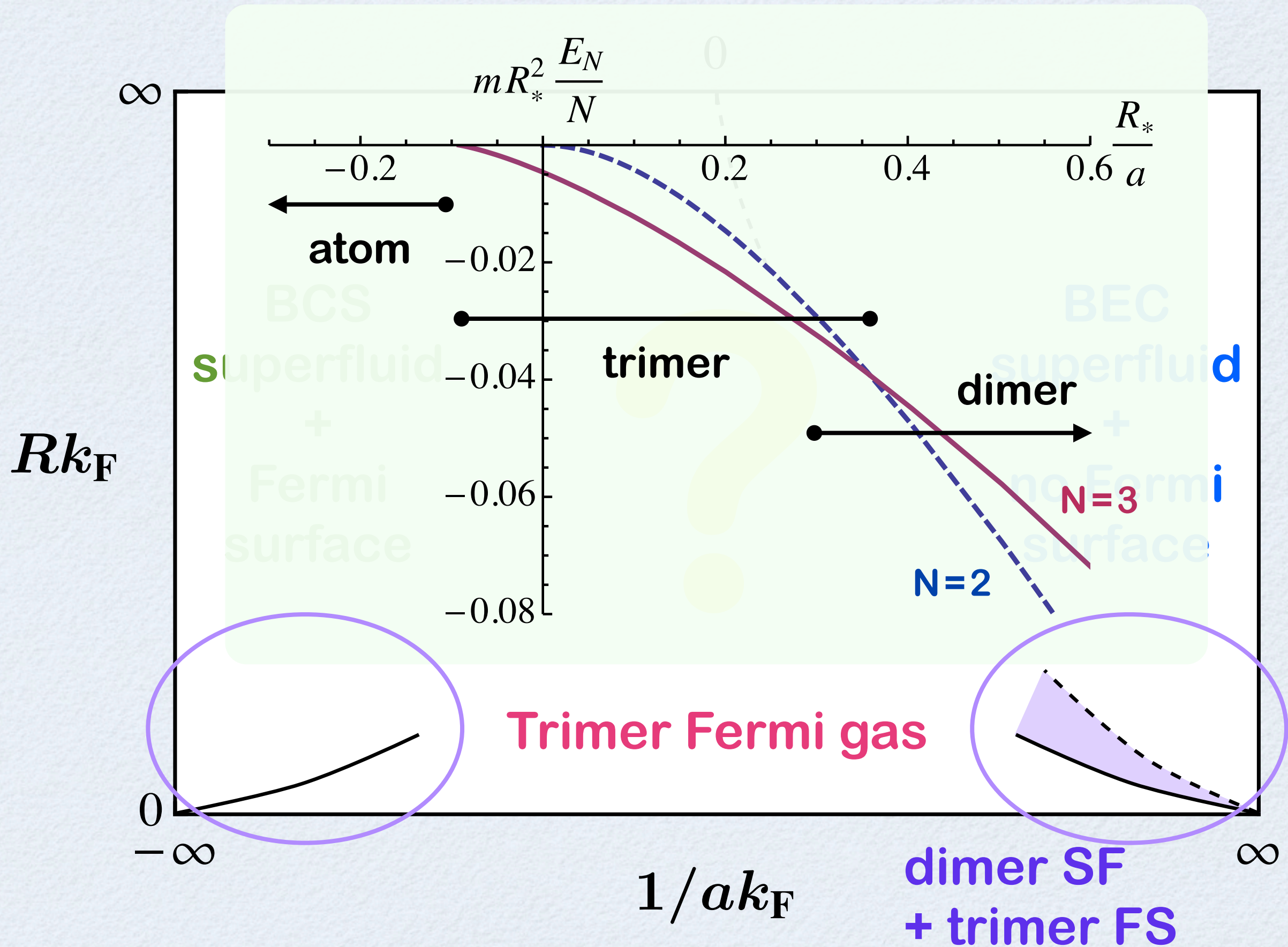
Phase diagram



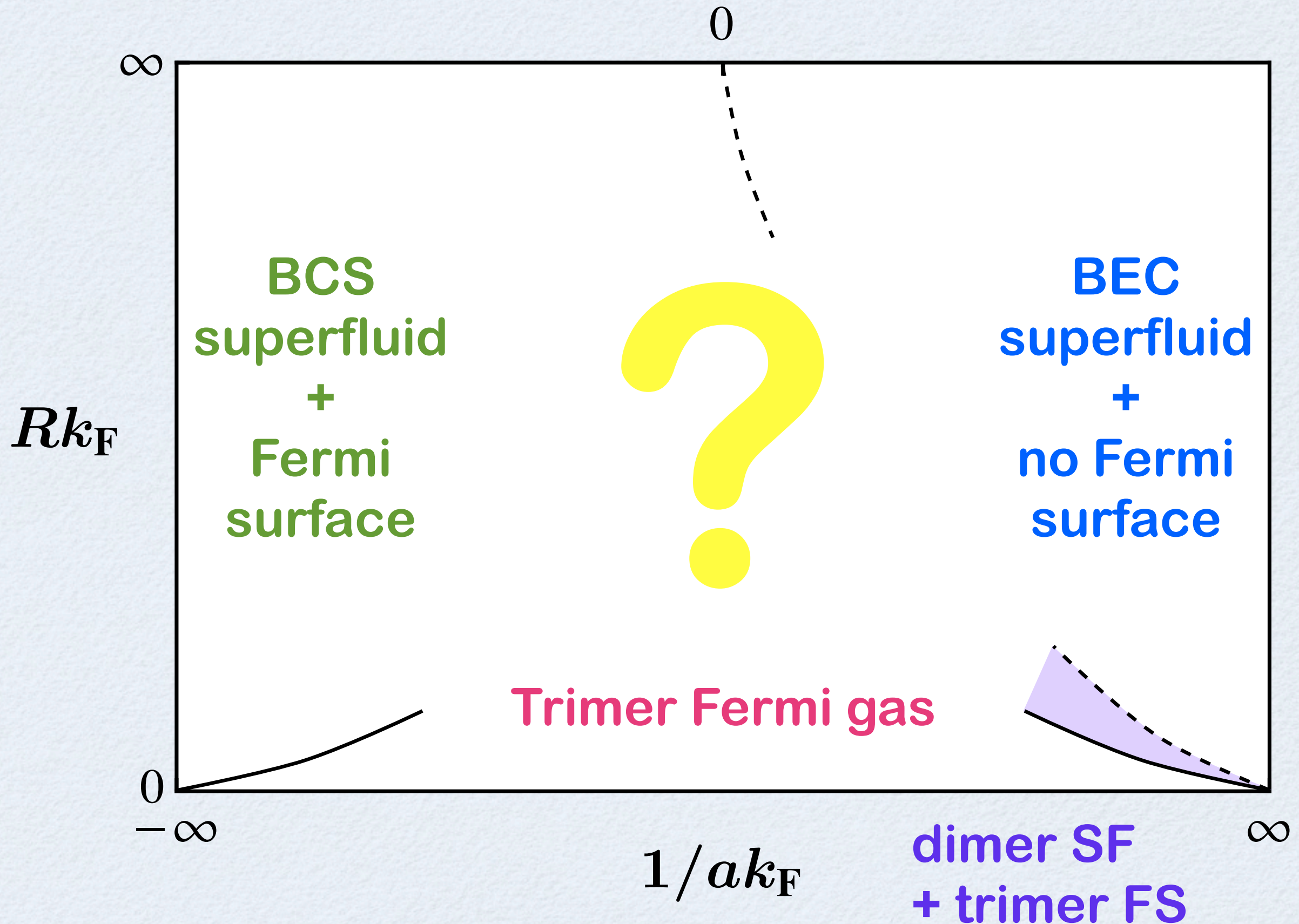
Phase diagram



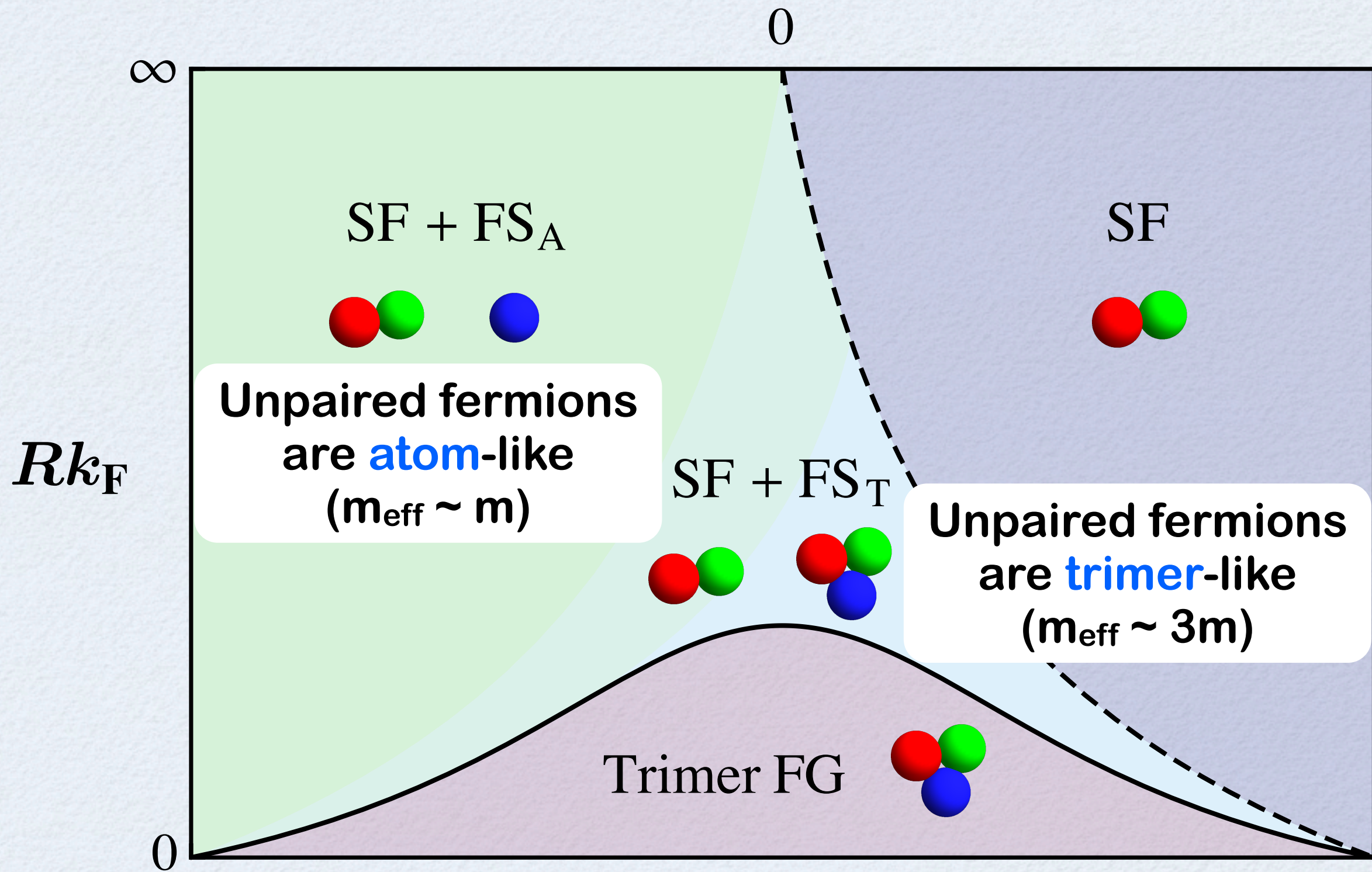
Phase diagram



Phase diagram



Phase diagram



“Atom-trimer continuity” = New crossover physics !

VOLUME 82, NUMBER 20

PHYSICAL REVIEW LETTERS

17 MAY 1999

Continuity of Quark and Hadron Matter

Thomas Schäfer and Frank Wilczek

School of Natural Sciences, Institute for Advanced Study, Princeton, New Jersey 08540

(Received 30 November 1998)

We review, clarify, and extend the notion of color-flavor locking. We present evidence that for three degenerate flavors the qualitative features of the color-flavor locked state, reliably predicted for high density, match the expected features of hadronic matter at low density. This provides, in particular, a controlled, weak-coupling realization of confinement and chiral symmetry breaking in this (slight) idealization of QCD. [S0031-9007(99)09191-7]

PACS numbers: 12.38.Aw

In a recent study [1] of QCD with three degenerate flavors at high density, a new form of ordering was predicted, wherein the color and flavor degrees of freedom become rigidly correlated in the ground state: color-flavor locking. This prediction is based on a weak-coupling analysis using a four-fermion interaction with quantum numbers abstracted from one gluon exchange. One expects that such a weak-coupling analysis is appropriate at high density, for the following reason [2,3]. Tentatively assuming that the quarks start out in a state close to their free quark state, i.e., with large Fermi surfaces, one finds that the relevant interactions, which are scattering the states near the Fermi surface, for the most part involve large momentum trans-

fer quantum numbers, including integral electric charge. Thus, the gluons match the octet of vector mesons, the quark octet matches the baryon octet, and an octet of collective modes associated with chiral symmetry breaking matches the pseudoscalar octet. However, there are also a few apparent discrepancies: there is an extra massless singlet scalar, associated with the spontaneous breaking of baryon number (superfluidity); there are eight rather than nine vector mesons (no singlet); and there are nine rather than eight baryons (extra singlet). We will argue that these “discrepancies” are superficial — or rather that they are features, not bugs.

Let us first briefly recall the fundamental concepts of

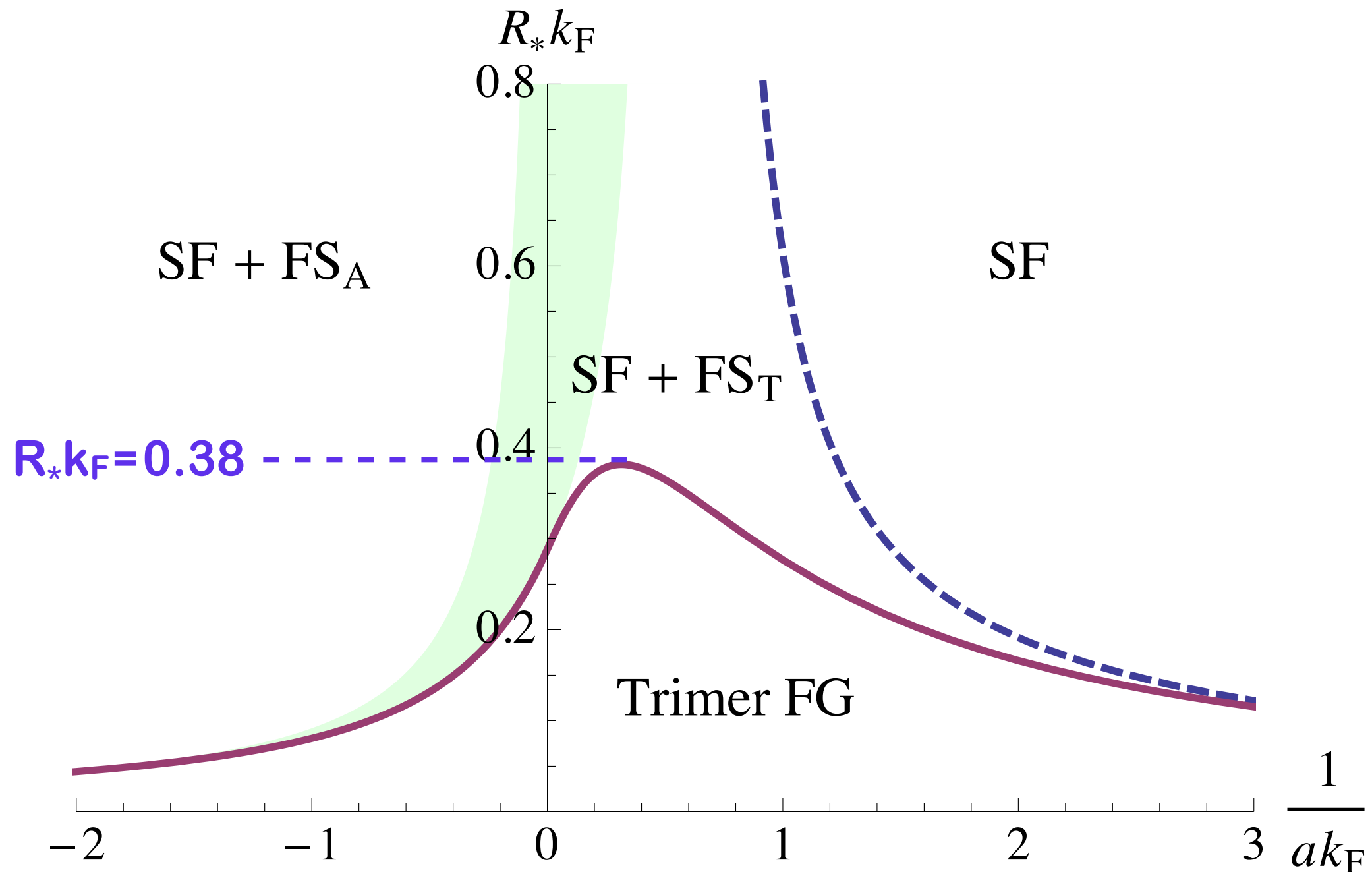
New link between atomic and nuclear systems !

tivity [4], even weak couplings near the Fermi surface can form [1]

Mean-field + trimer model

$$\Omega_{\text{MF+T}} = \Omega_{\text{mean field}} + \Omega_{\text{trimer}}$$

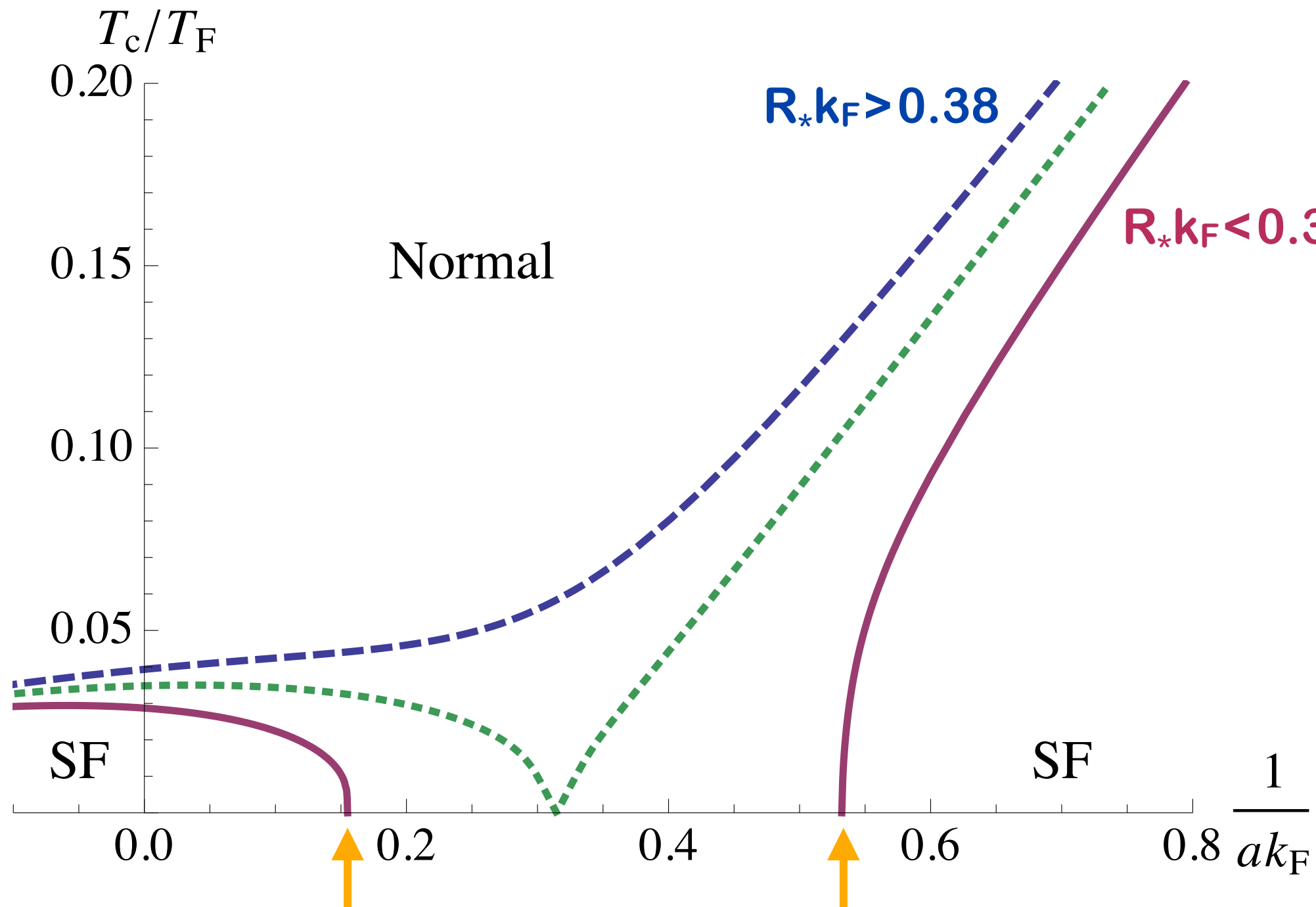
knows correct asymptotic behaviors



Critical temperature

A pair of quantum critical points
(complete depletion of SF) appear for $R_*k_F < 0.38$

See also, S. Floerchinger et al., PRA (2009)



1. “Hard probes” in cold atoms

- Use of energetic atoms to locally probe strongly-interacting atomic gases
- Y.N., Phys. Rev. A (2012) [arXiv:1110.5926]

2. “Quark-hadron continuity” in cold atoms

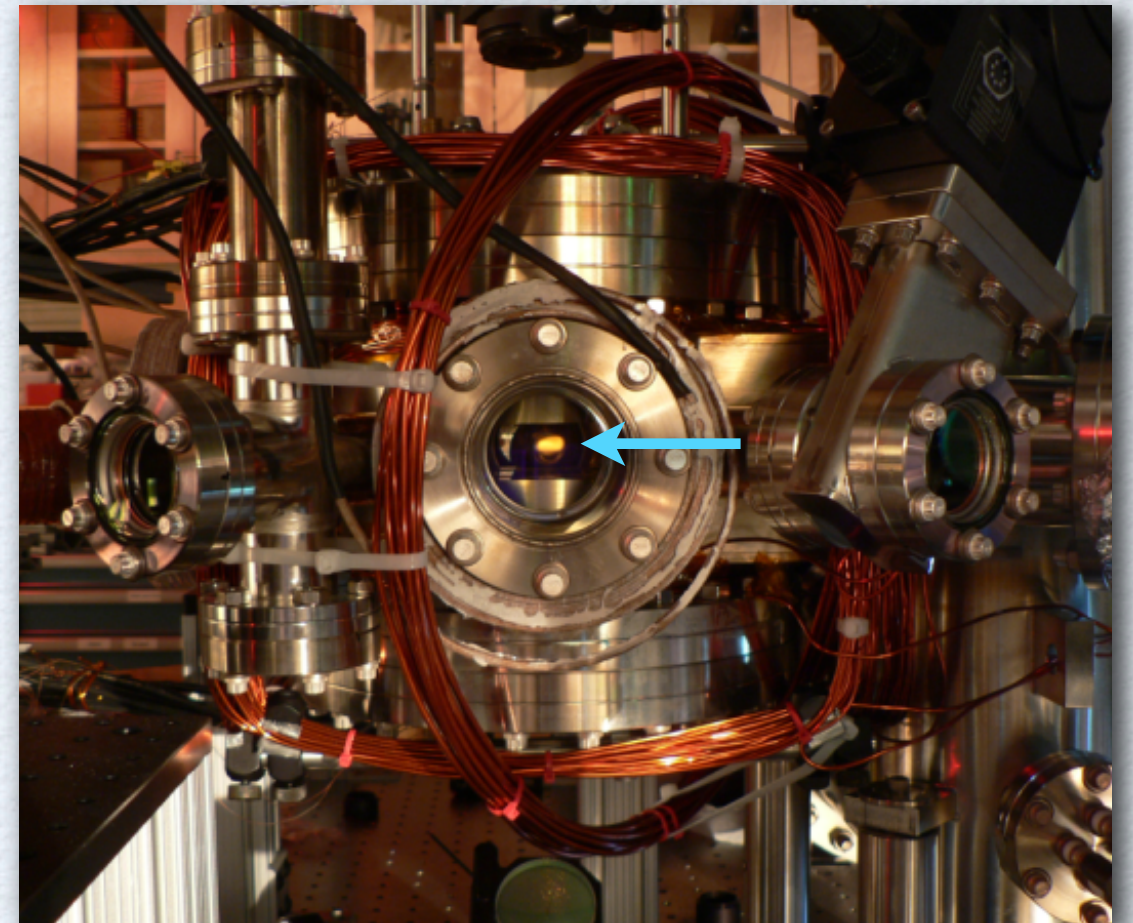
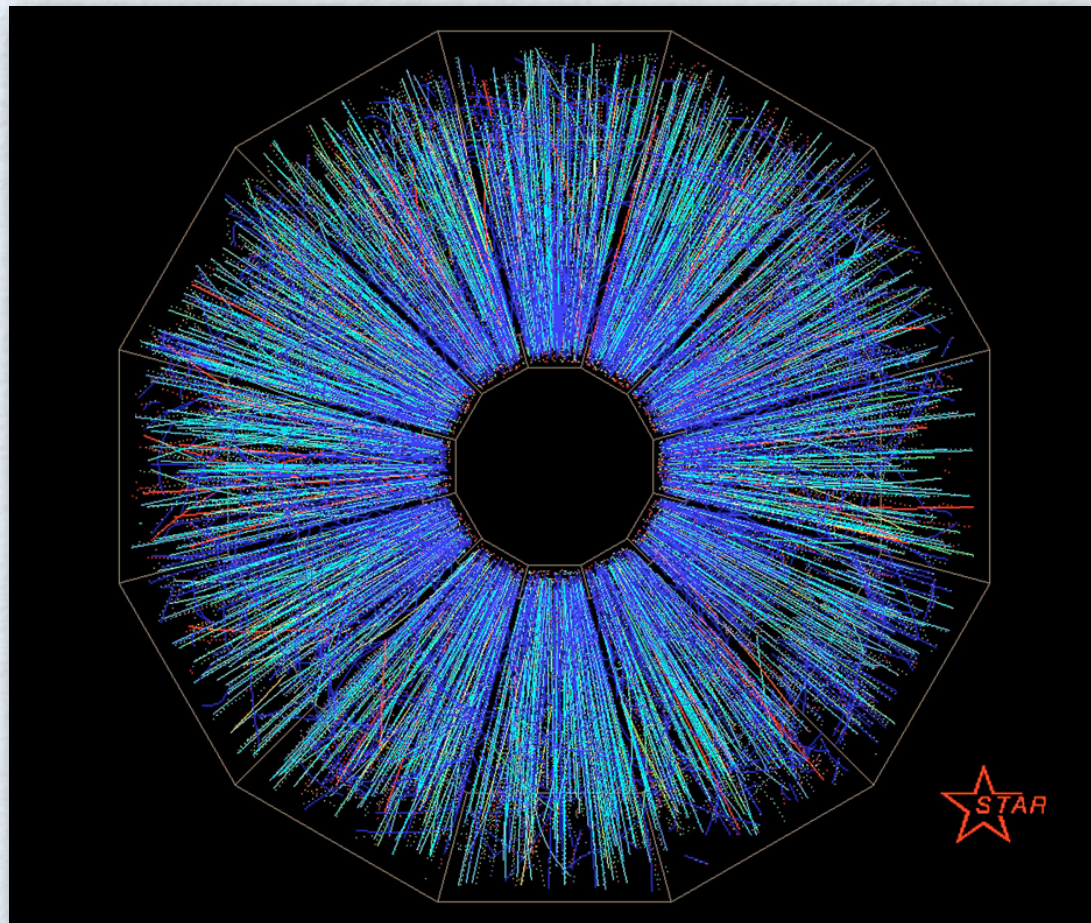
- Smooth crossover from atoms to trimers in 3-component Fermi gases
- Y.N., Phys. Rev. Lett. (2012) [arXiv:1207.6971]

Summary of this talk

Extreme QCD



Ultracold atoms



New ideas wanted !