3フレーバーPNJL模型による
QCD相転移の研究

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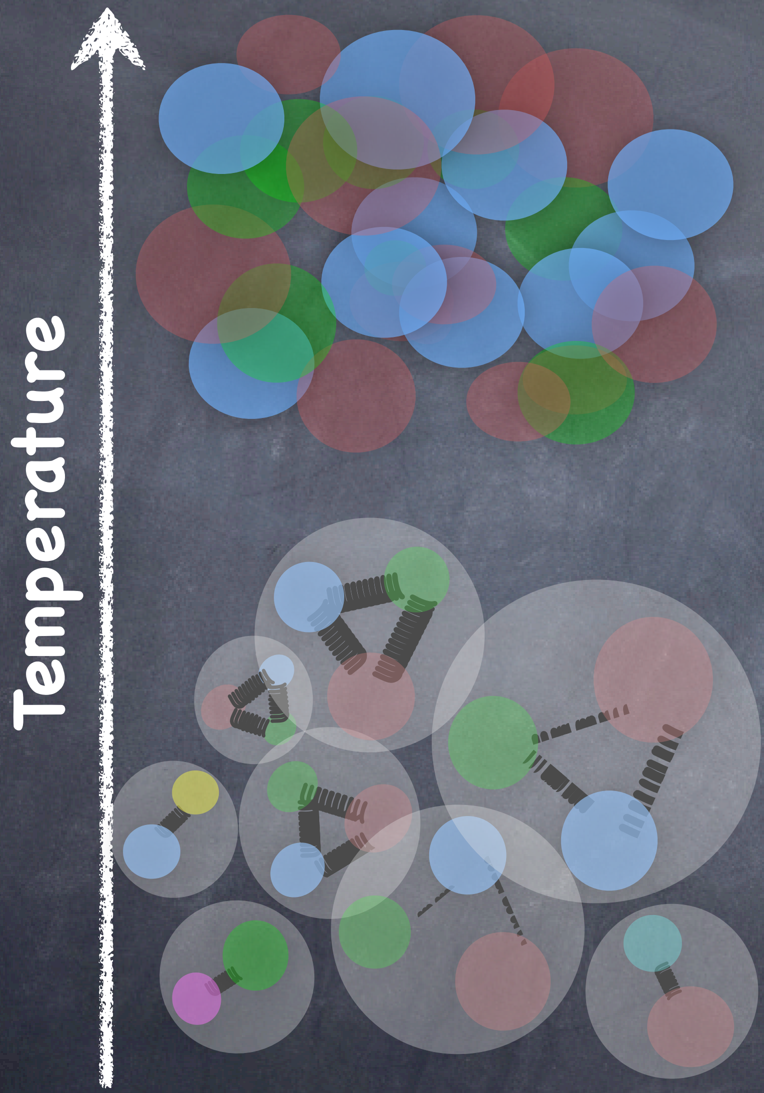
1. クォーク・ハドロン相転移
2. 3フレーバーPNJL模型
3. 擬スカラーメソンとスカラーメソン
4. 状態方程式

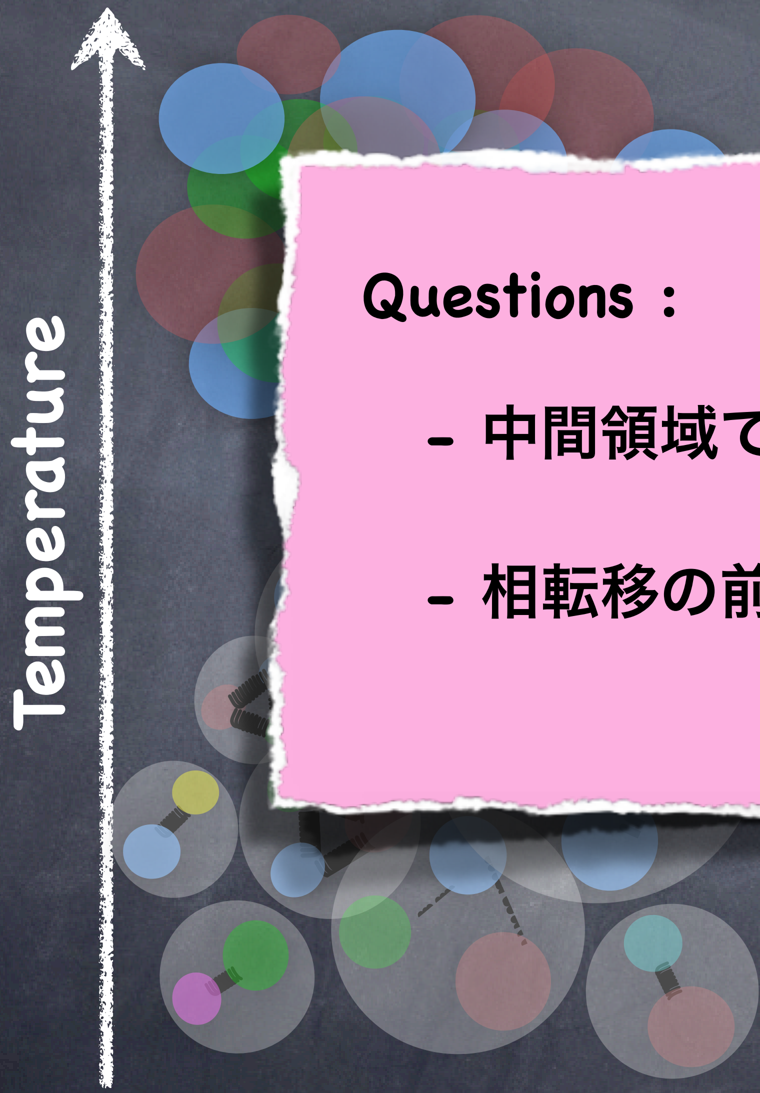
クォーク・ハドロン相転移



- カイラル対称性の**回復**
- 閉じ込めの**解消**

- カイラル対称性の**破れ**
- カラーの**閉じ込め**

クォーク・ハドロン相転移



- カイラル対称性の**回復**

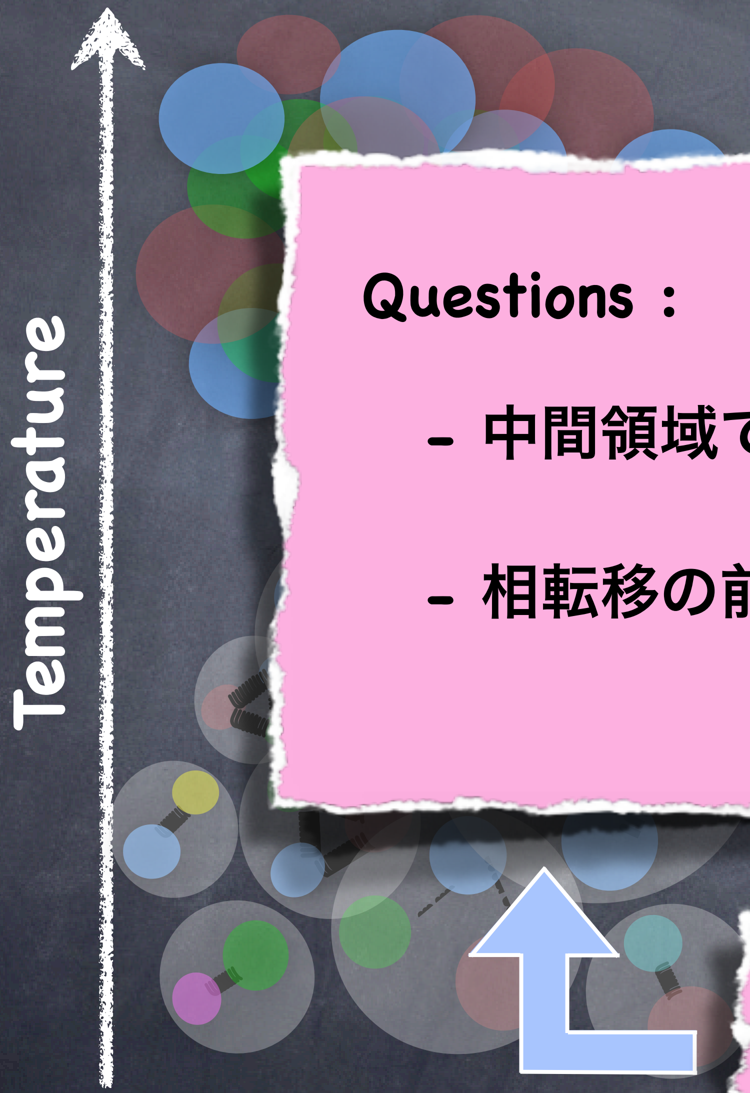
- カラーの**閉じ込め**

Questions :

- 中間領域で何が起こっているか？

- 相転移の前後で自由度はどう変わるか？

クォーク・ハドロン相転移



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状態方程式

Method

- 経路積分法で分配関数を計算

- 有効模型

  - カイラル相転移
    → Nambu-Jona-Lasinio (NJL)
  - 非閉じ込め相転移
    → Polyakov loop

  } PNJL model
  K. Fukushima, 2004

- ボソン化

  - dummy integralを挿入
  - 4点相互作用、6点相互作用 --> ボソン場で書き換える

- 平均場近似 + Mesonic correlations

3フレーバー PNJL模型

分配関数

$$Z(T, A_4) = \int [dq][d\bar{q}] \exp\left[\int_0^\beta d\tau \int d^3x \mathcal{L}_{\mathrm{NJL}}(q, \bar{q}, A_4)\right]$$

$$\mathcal{L}_{NJL} = \sum_{i,j=1}^{3} \bar{q}_i (i\not{D} - \hat{m})_{i,j} q_j + \boxed{\mathcal{L}_4} + \boxed{\mathcal{L}_6}$$

$$D_\mu = \partial_\mu + g \boxed{A_0} \delta_{\mu,0}$$

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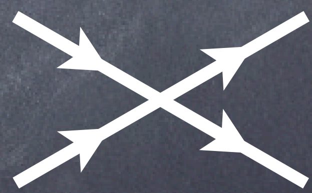
- 4点相互作用

$$\boxed{\mathcal{L}_4 = G \sum_{a=0}^{8} \left[(\bar{q}\lambda^a q)^2 + (\bar{q} i \gamma_5 \lambda^a q)^2 \right]}$$

$a = 0 \sim 8$



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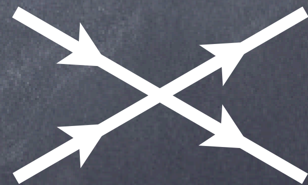
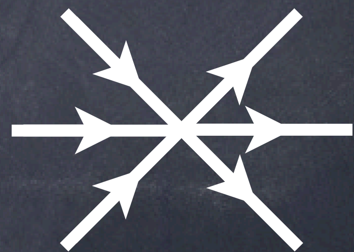


- 6点相互作用

U(1)A breaking

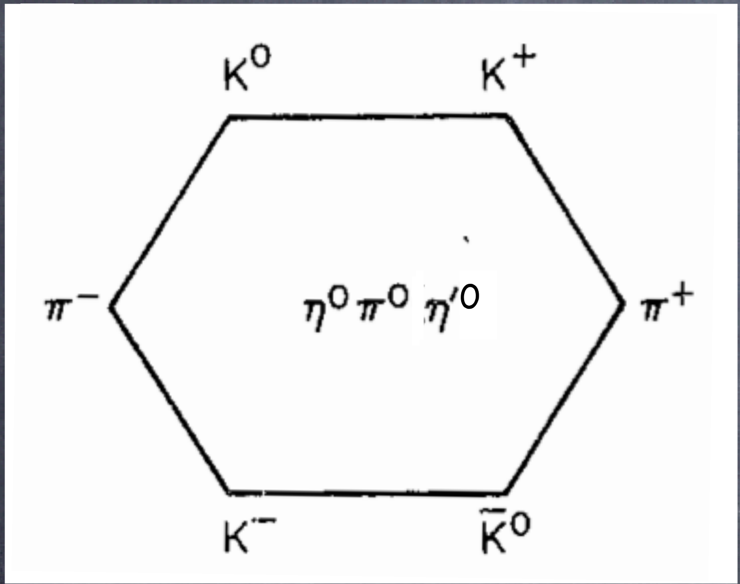
$$\boxed{\mathcal{L}_6 = -K\left[\det \bar{q}(1+\gamma_5)q + \det \bar{q}(1-\gamma_5)q\right]}$$

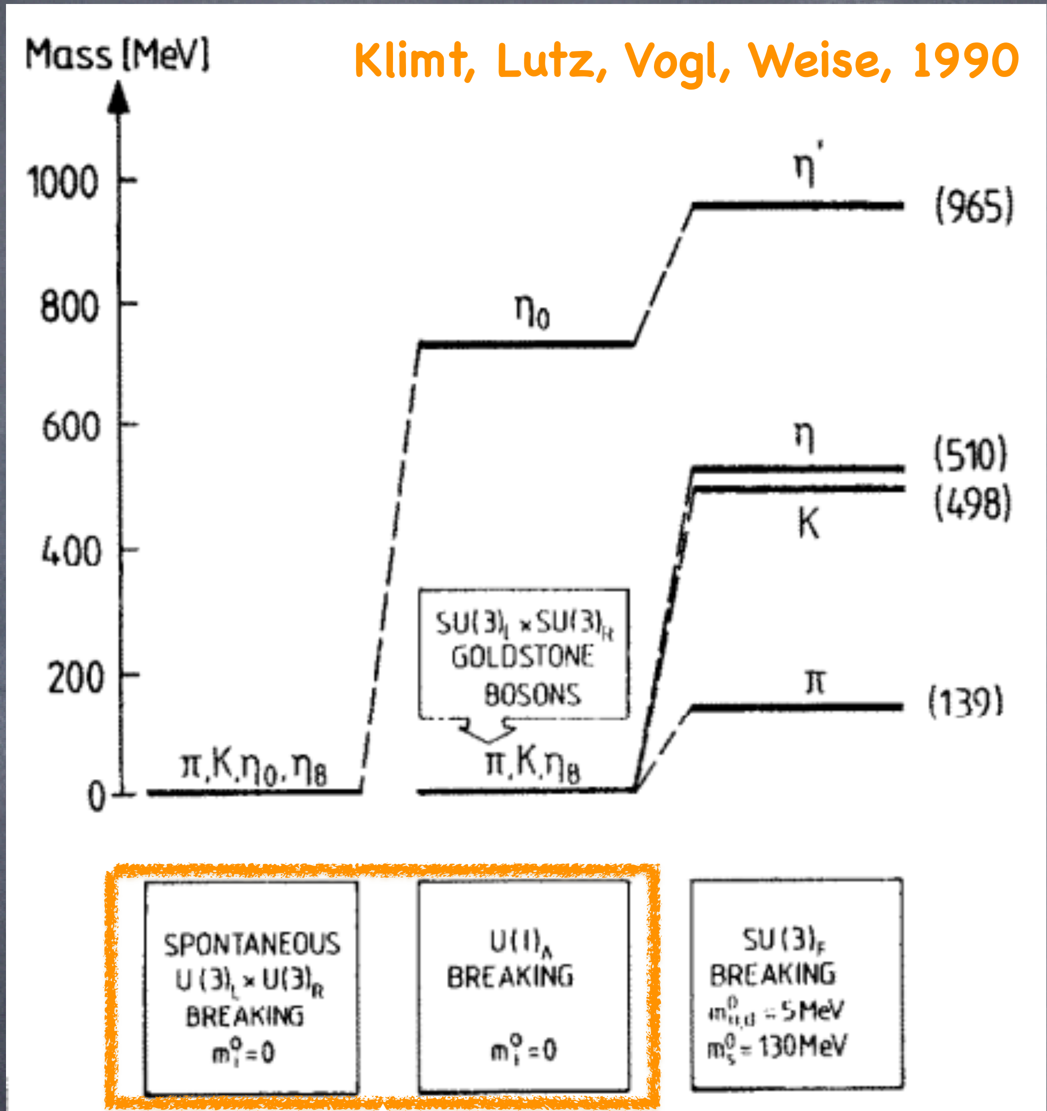


Meson nonets

擬スカラーメソン

π, K, η, η′





$\mathcal{L}_6$: mass splitting を起こす

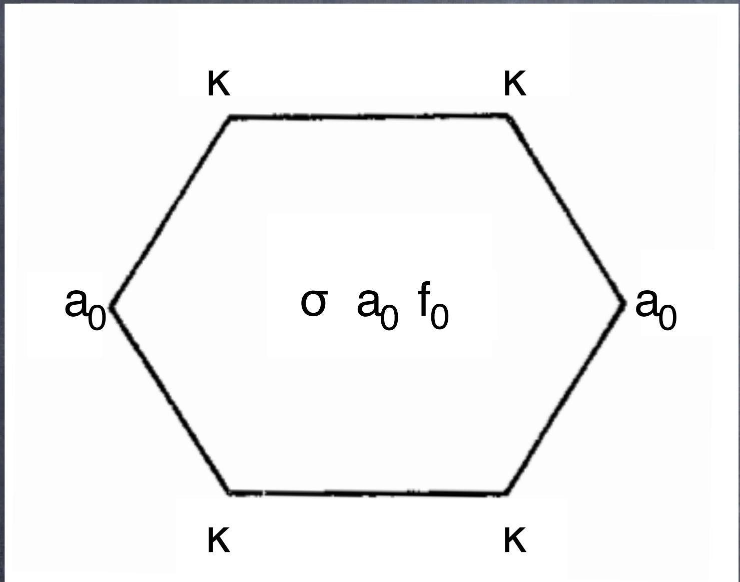
Meson nonets

スカラーメソン

σ, κ, f0, a0

Ishida, 1998

Fariborz, Jora, Schechter, 2009

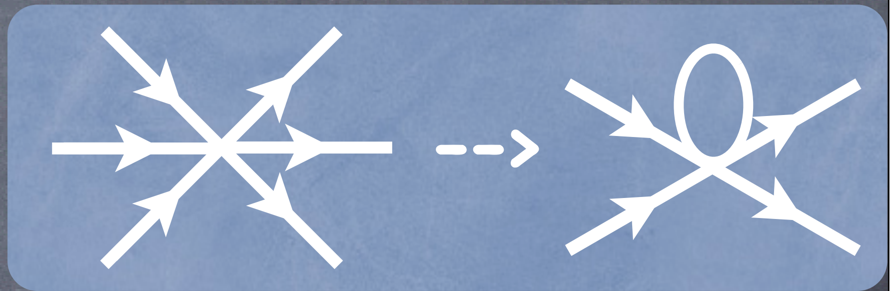


?

	Mass[MeV]	Width[MeV]
σ	～550	400 - 700
κ	～800	
$f_0(980)$	～980	40 - 100
$a_0(980)$	～980	50 - 100

分配関数の計算

- Lagrangian はフェルミオン場の **4次の項** と **6次の項** を含む.
- 6次の項は **condensate** で置き換えることで、有効的に4次に置き換えられる.



- 4点相互作用をボソン場で置き換える： ϕ^a, π^a
- 分配関数は **補助場の関数** として得られる：

$$Z(T, A_4) = \int [d\phi][d\pi] \exp\left[-I_{eff}(\phi^a, \pi^a, A_4)\right]$$

熱力学ポテンシャル

- 有効作用を**安定点**の周りで **2次まで展開**

- 安定点を求める条件：

$$\left.\frac{\delta I}{\delta \phi_a}\right|_{\phi=\phi_0} = 0.$$

- ボソン場について **ガウス積分**

- 熱力学ポテンシャル

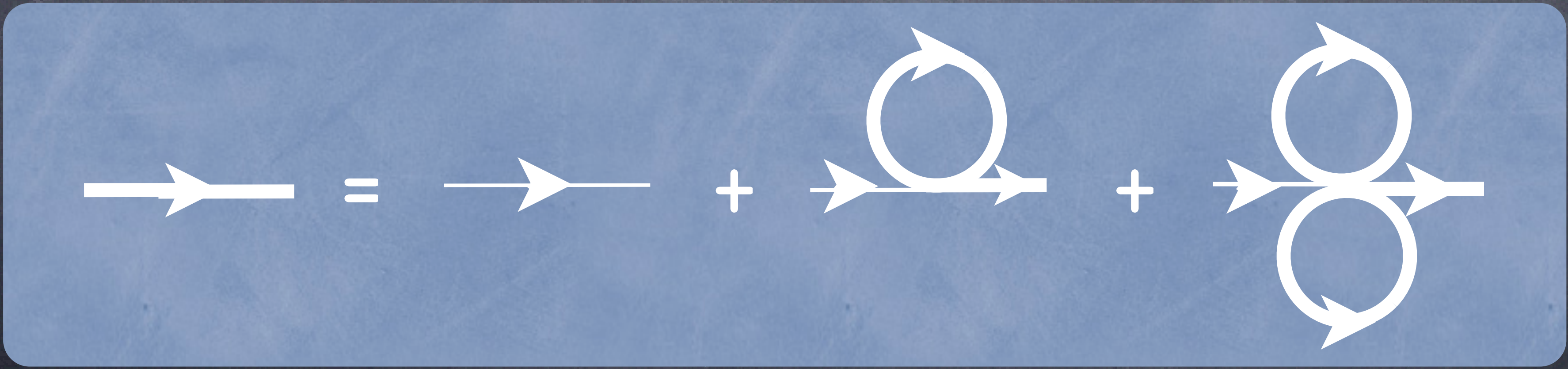
$$\Omega(T, A_4) = T\left(\underbrace{I_0}_{\text{mean field}} + \underbrace{\frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\phi_a \delta\phi_b} + \frac{1}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\pi_a \delta\pi_b}}_{\text{mesonic excitations}}\right)$$

Constituent quark mass

- 圧力は **構成子クォーク質量** に依存

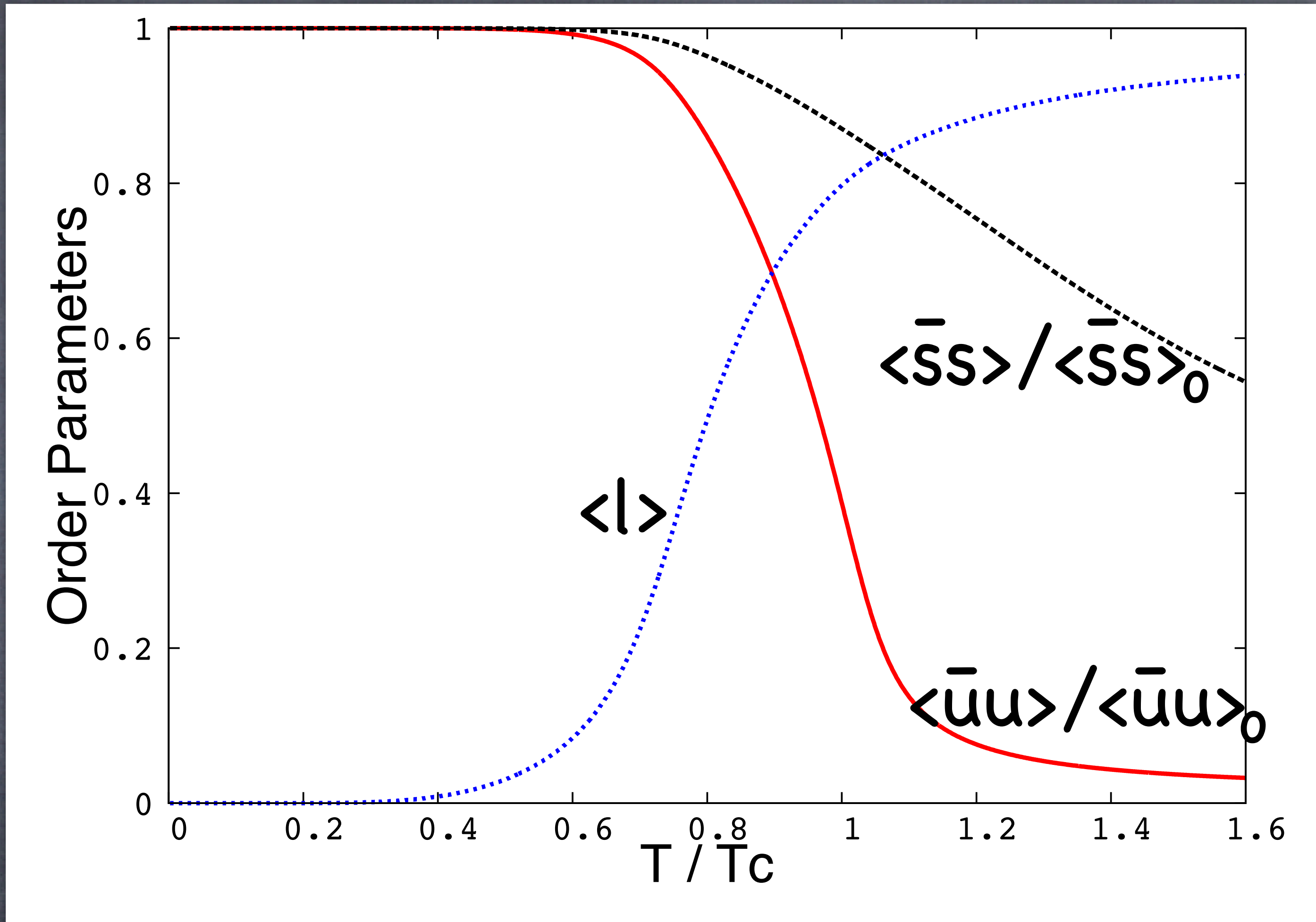
- 構成子クォーク質量は **ギャップ方程式** を解くことで得られる：

$$
\begin{cases}
M_u = m_u - 4G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle \\
M_s = m_s - 4G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle
\end{cases}
$$



- Chiral condensates : $\langle \bar{u}u \rangle (= \langle \bar{d}d \rangle), \langle \bar{s}s \rangle$

Order Parameters



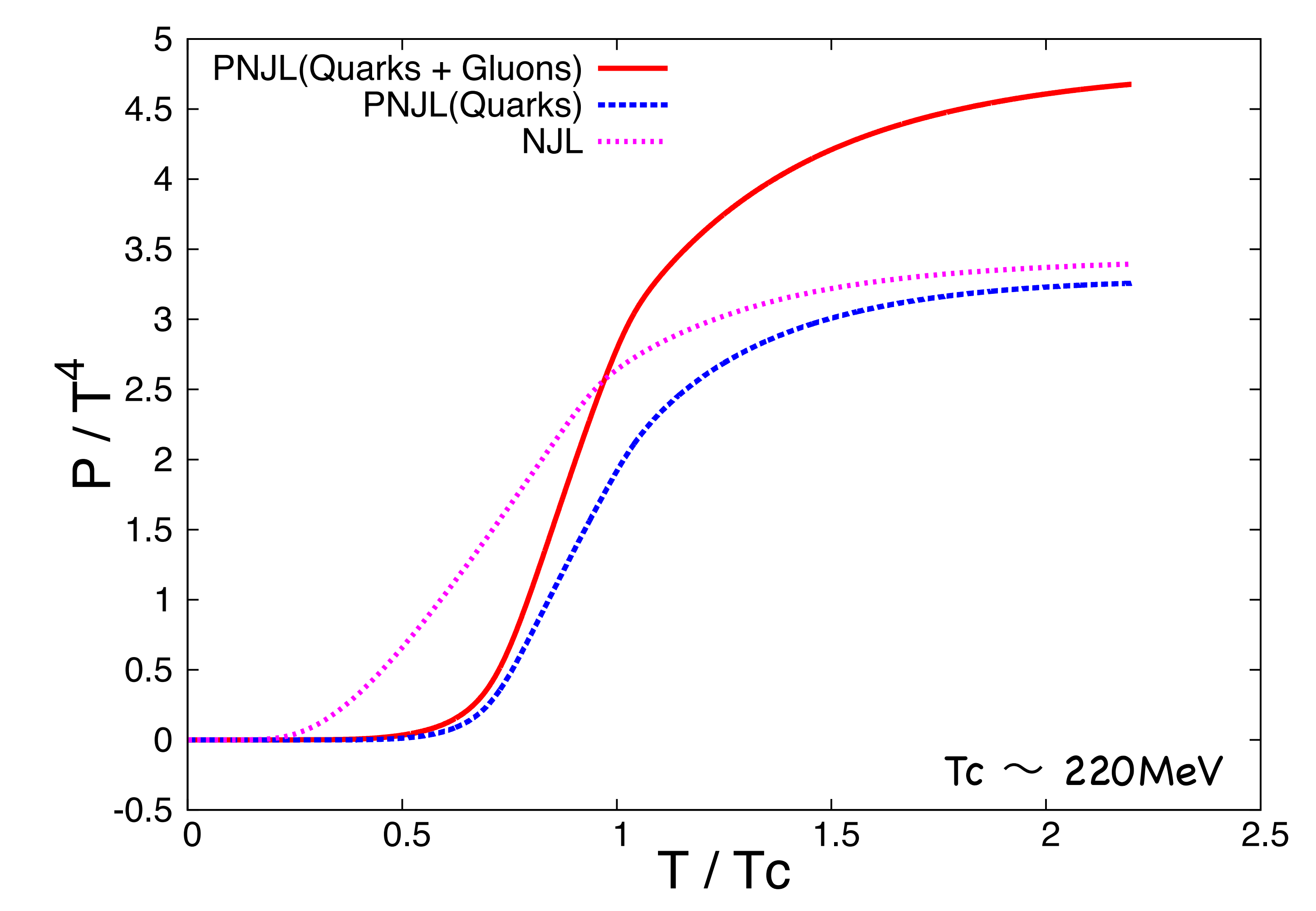
<l> : Expectation value of Polyakov loop

Tc : pseudo critical temperature

Tc ∼ 220MeV

平均場近似のもとでの圧力

$$p_{MF}(T) = \sum_f p^0_{M_f} + 4N_c \sum_f \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_f} f_{\langle l \rangle}(E_f) - \mathcal{U}(T)$$



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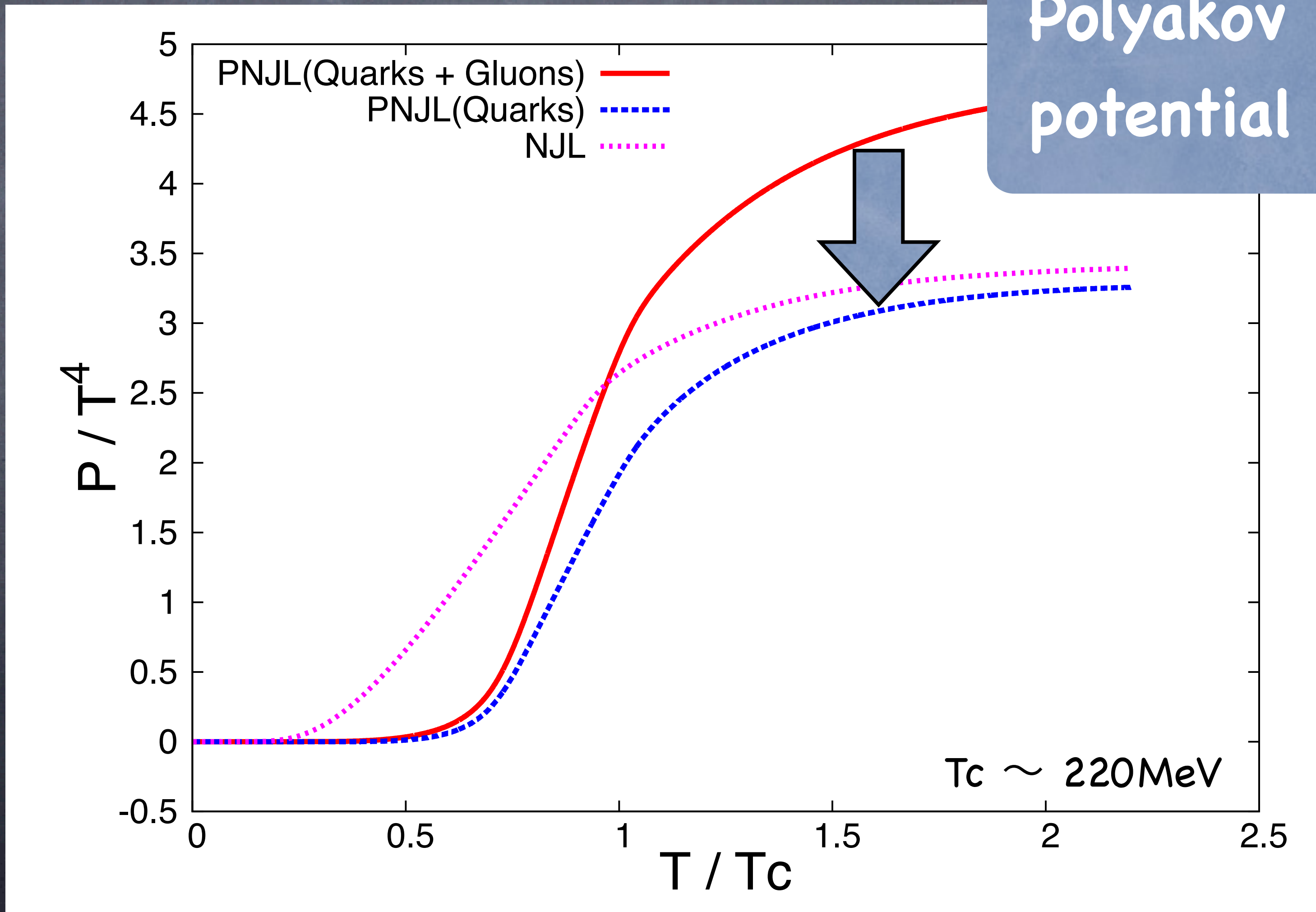

Polyakov loop potential : $\mathcal{U}(T)$

平均場近似のもとでの圧力

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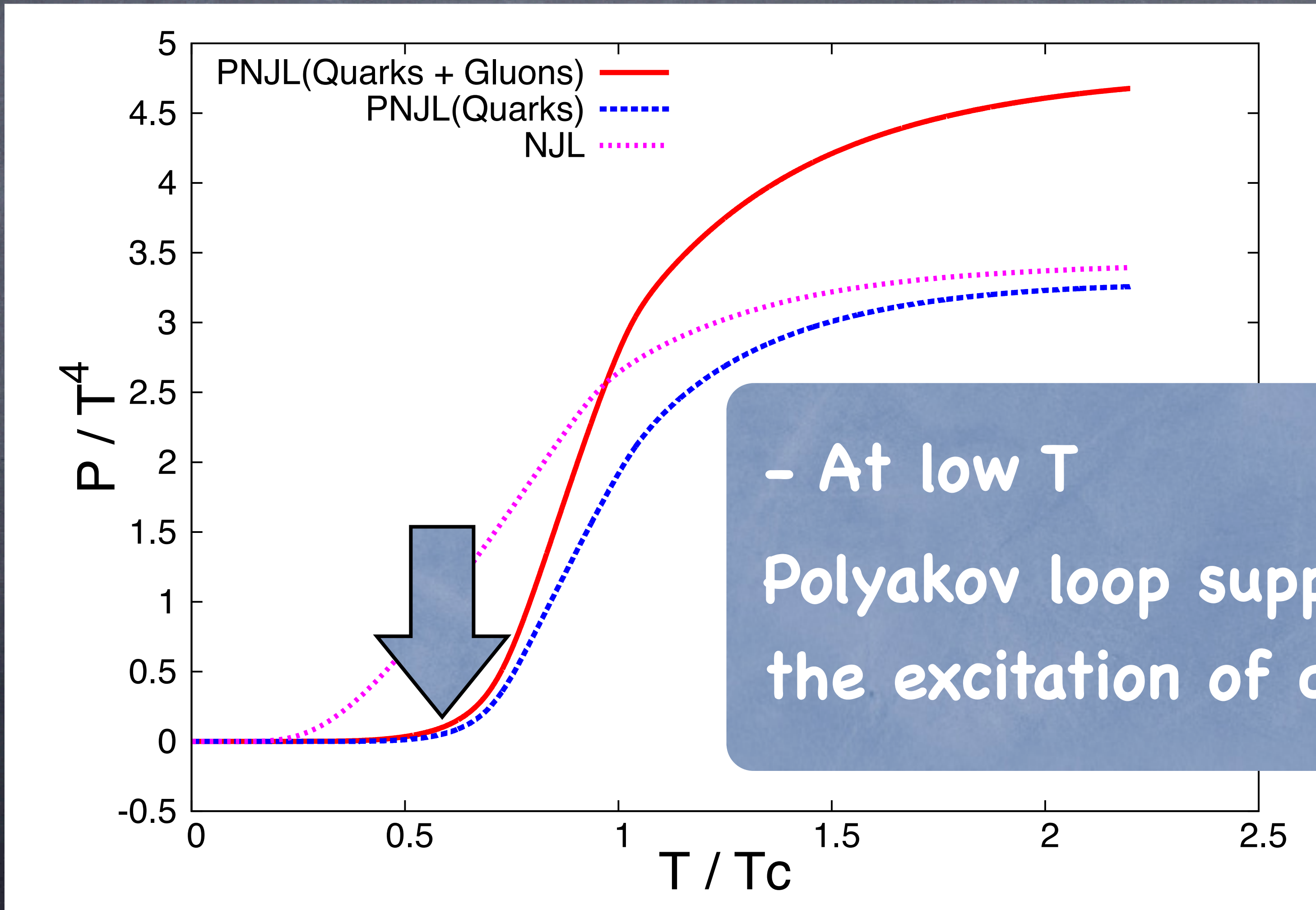

- At low T

Polyakov loop suppresses the excitation of quarks.

平均場近似のもとでの圧力

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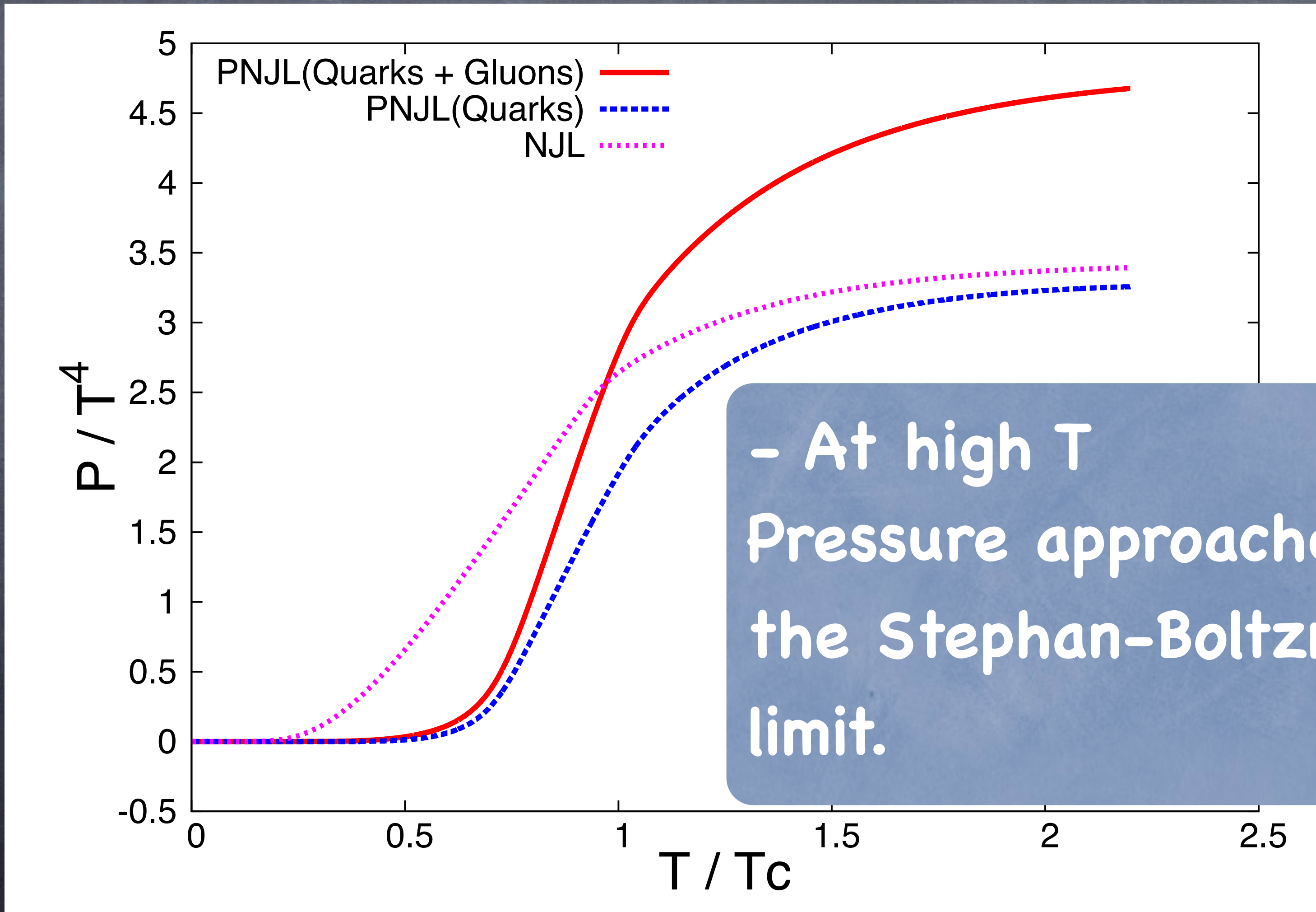


- At high T Pressure approaches the Stephan-Boltzmann limit.

Mesonic Correlations

- mesonic correlation からの圧力への寄与

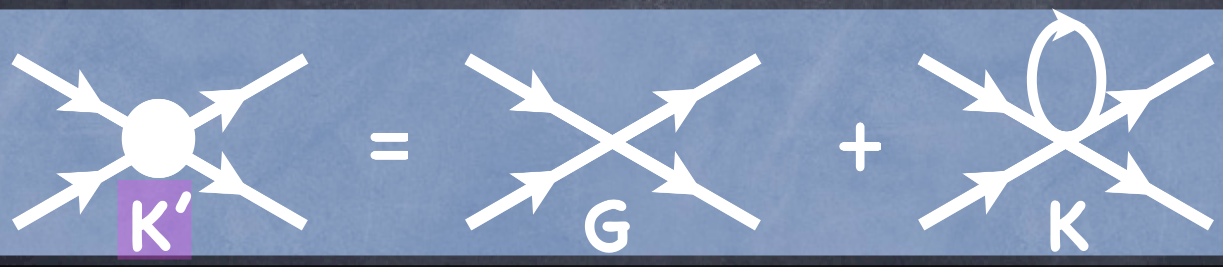
$$p_M = -\sum_n \int \frac{d^3q}{(2\pi)^3}\Big\{3\ln\mathcal{M}_\pi(\omega_n,q) + 4\ln\mathcal{M}_K(\omega_n,q) + \ln\mathcal{M}_\eta(\omega_n,q) + \ln\mathcal{M}_{\eta'}(\omega_n,q)$$

$$+\ln\mathcal{M}_\sigma(\omega_n,q) + 4\ln\mathcal{M}_\kappa(\omega_n,q) + 3\ln\mathcal{M}_{a_0}(\omega_n,q) + \ln\mathcal{M}_{f_0}(\omega_n,q)\Big\}$$

$$\mathcal{M}(\omega_n,q) = \frac{1}{2K'} - \Pi(\omega_n,q)$$

$\Pi(\omega_n,q)$ =

- K' : effective coupling



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Pseudo scalar

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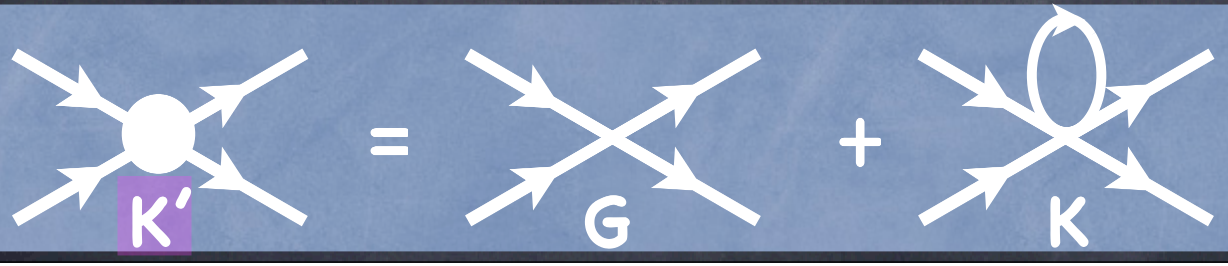

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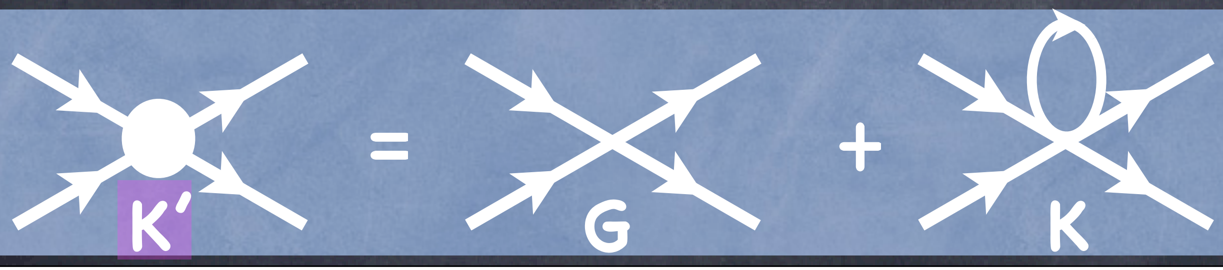

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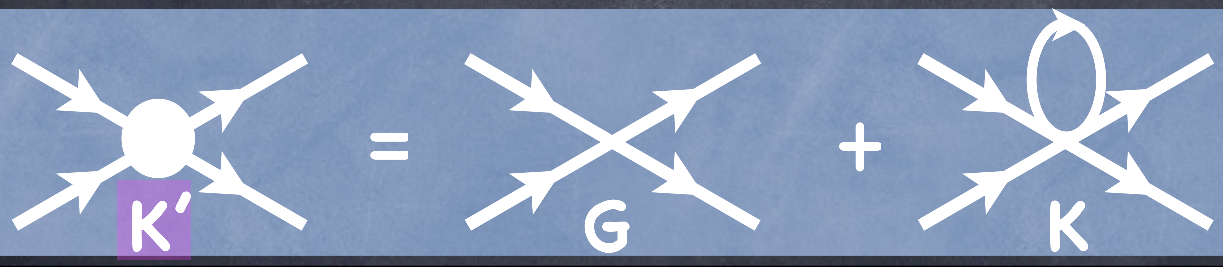
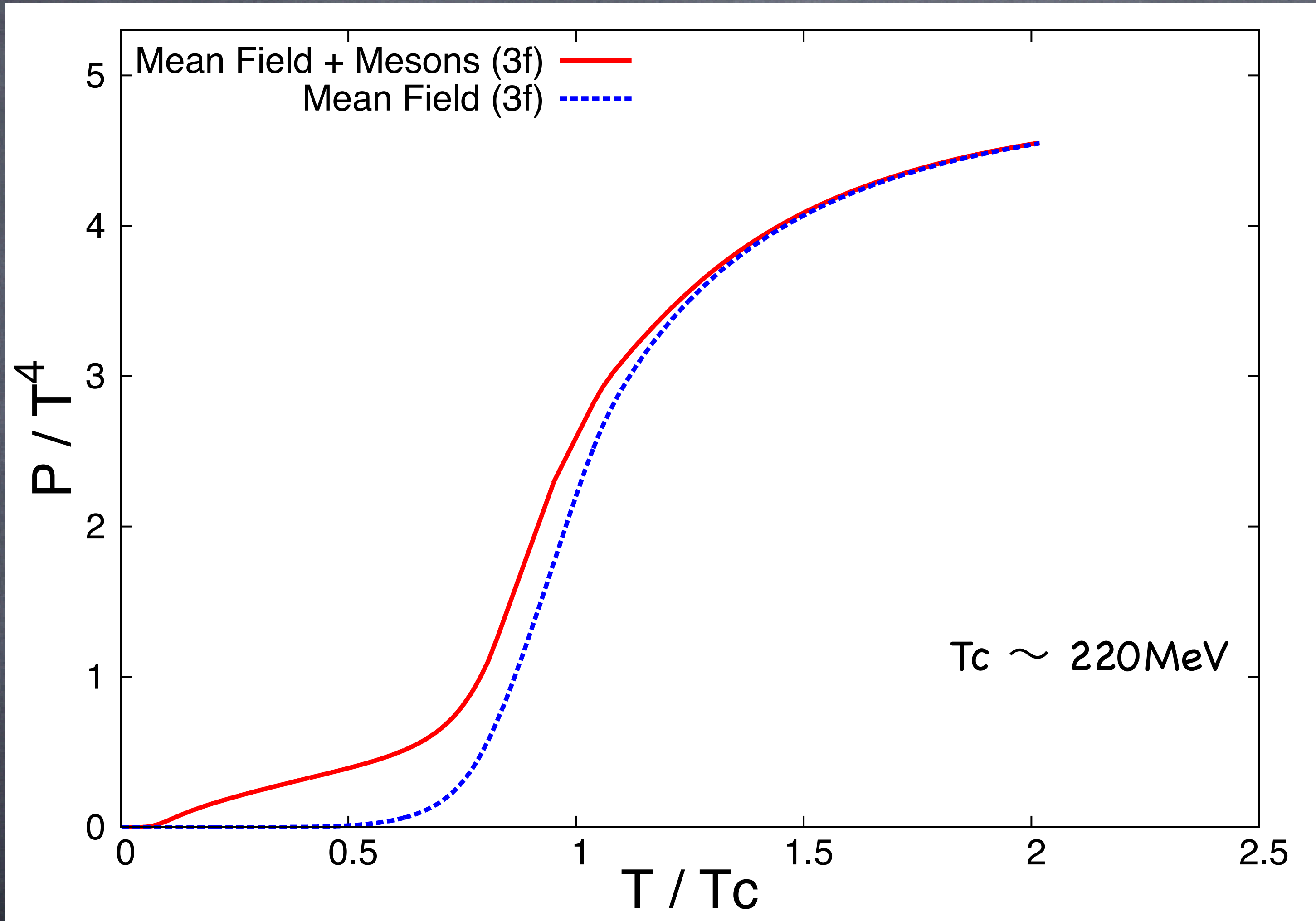
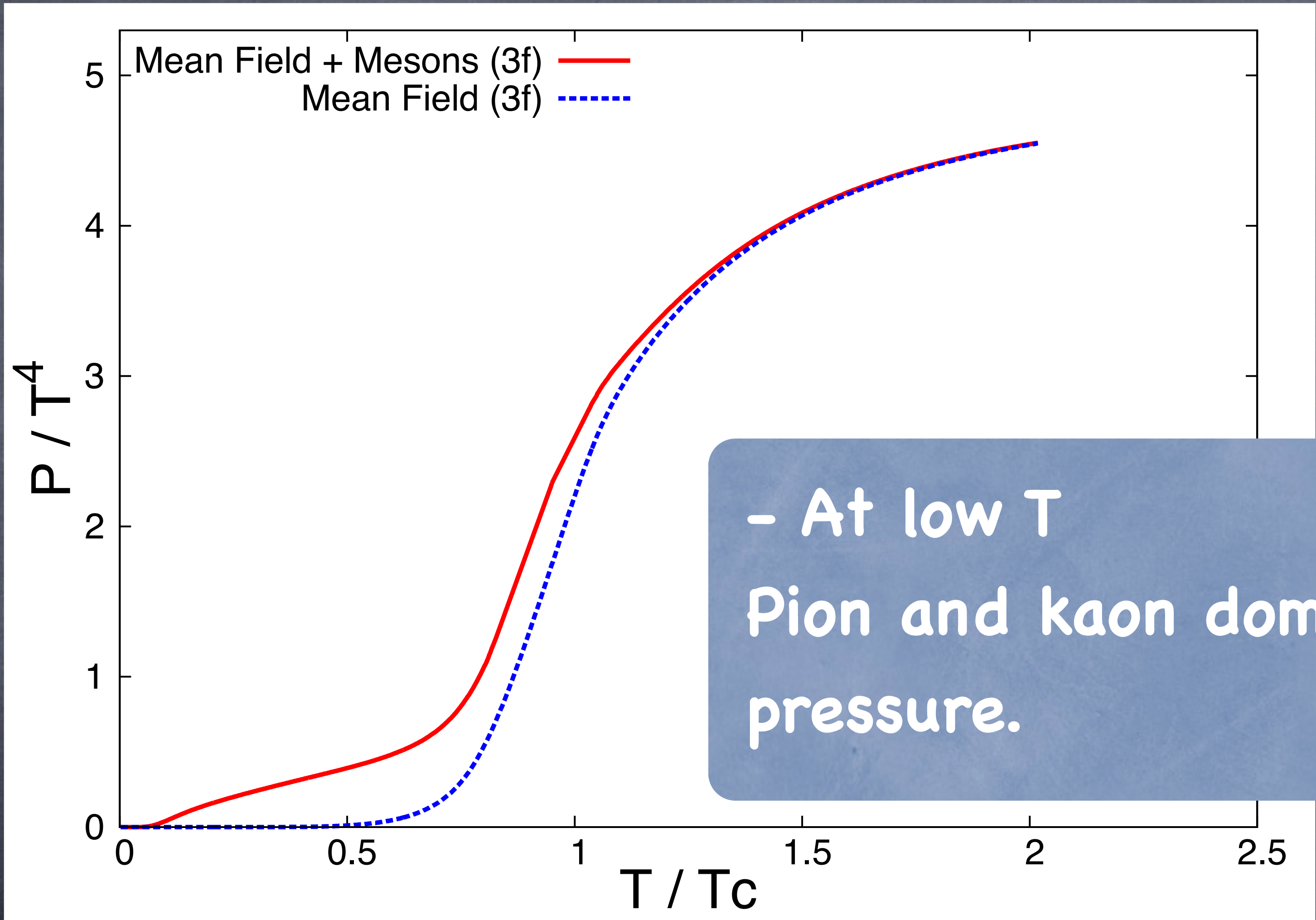


Pressure



π, K and σ are taken into this calculation.

Pressure



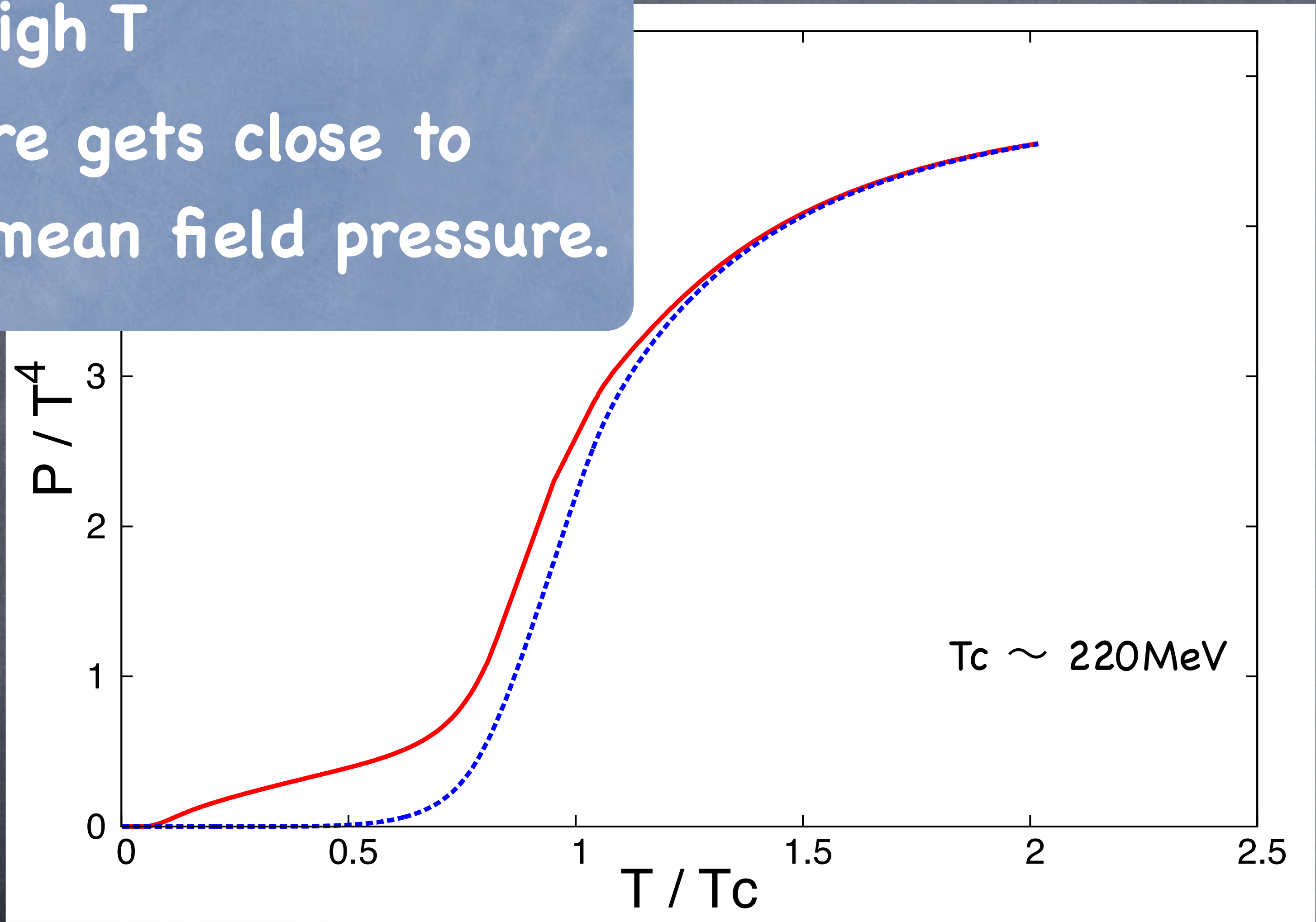
- At low T
Pion and kaon dominate pressure.

π, K and σ are taken into this calculation.

Pressure

- At high T

Pressure gets close to quark mean field pressure.



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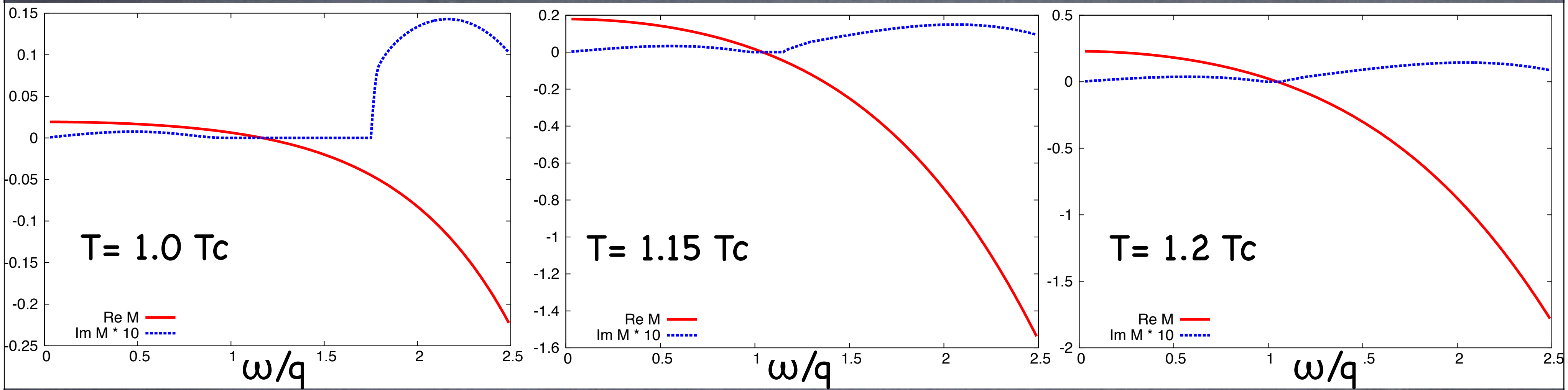
Collective modes

Pion



Real part of M

Imaginary part of M

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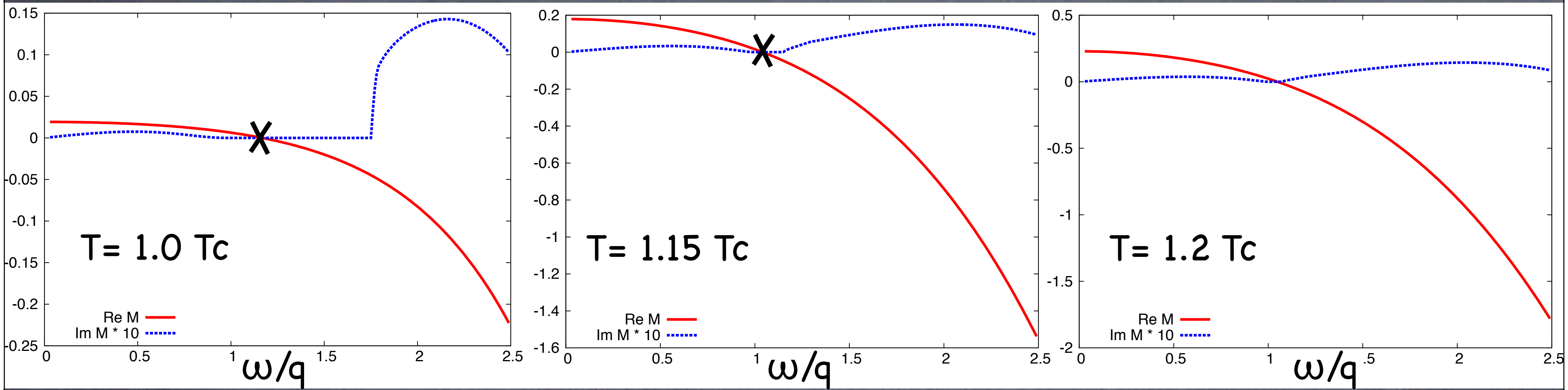
Collective modes

Pion



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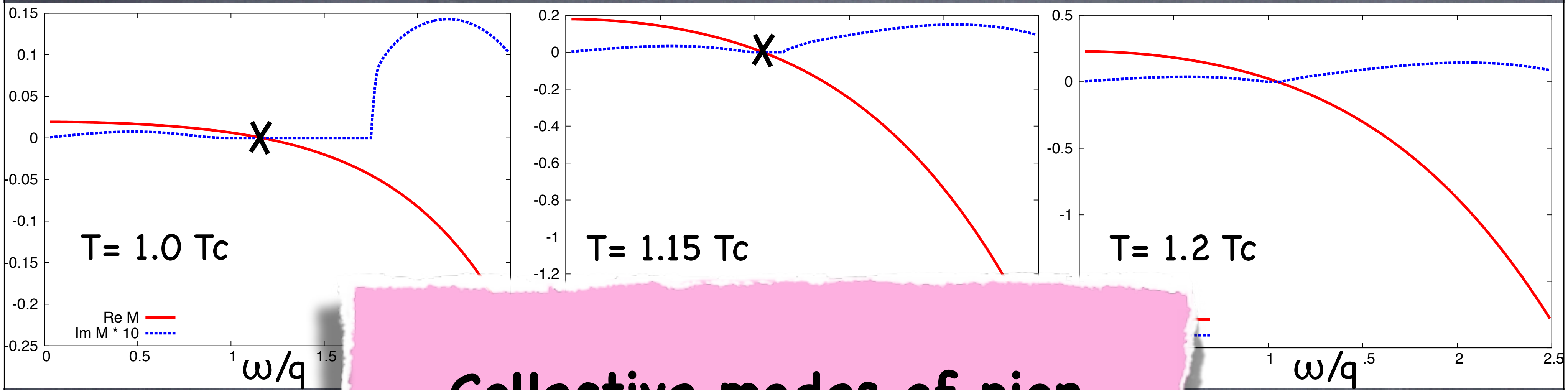
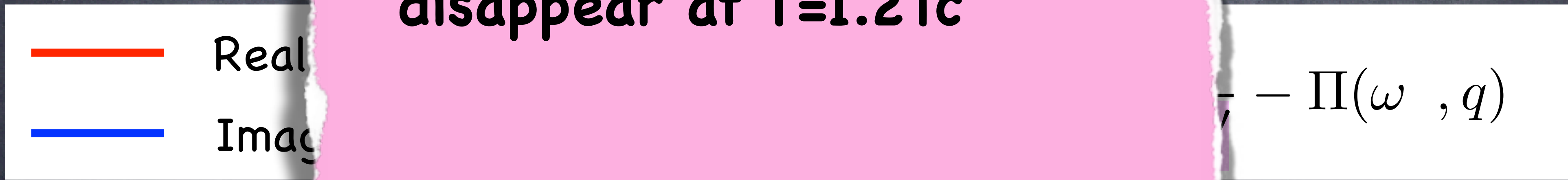
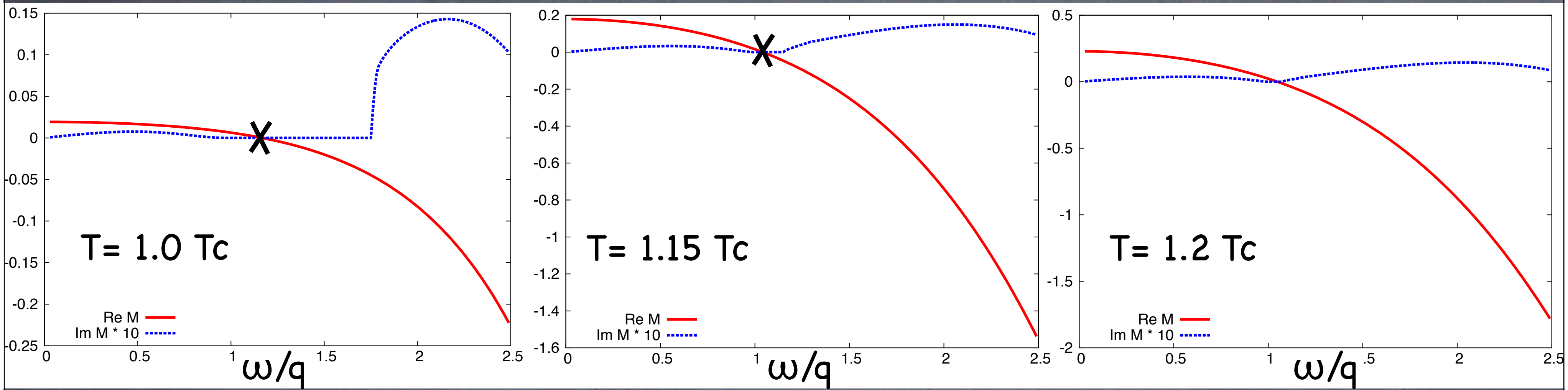




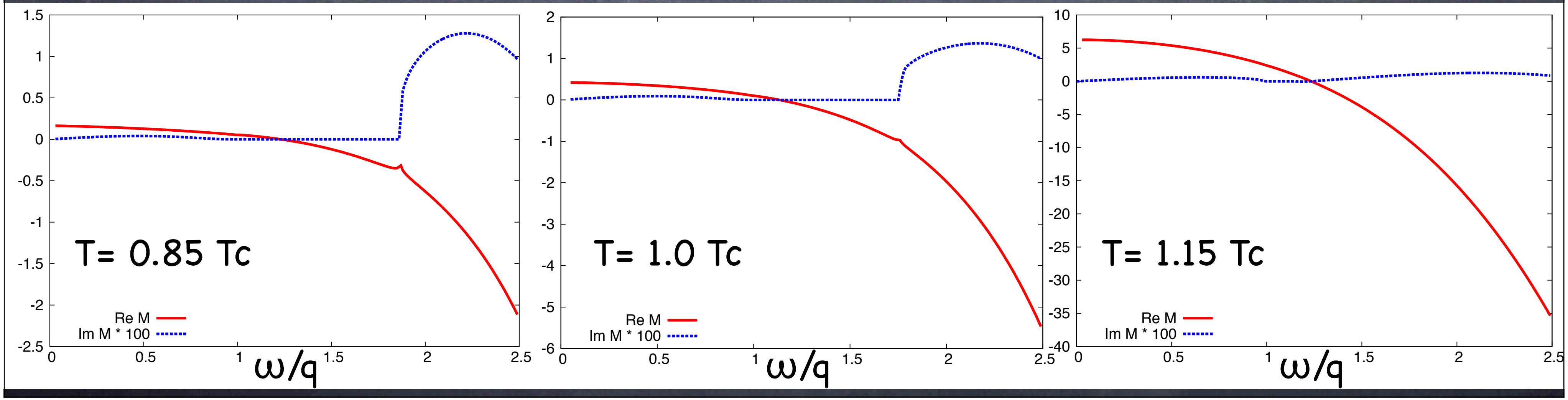
- Collective modes of pion disappear at T=1.2Tc

Collective modes

Pion

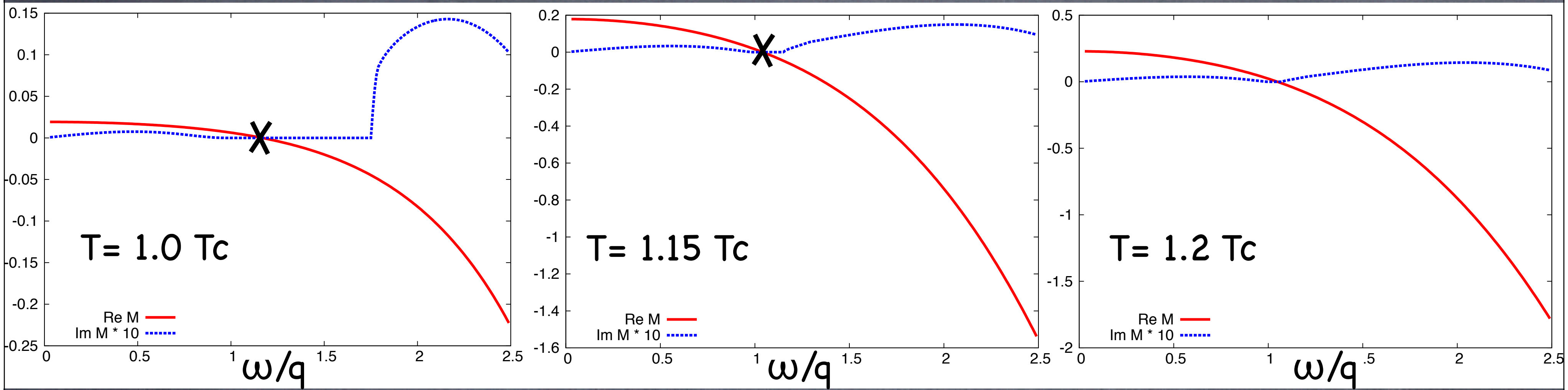


Kaon



Collective modes

Pion



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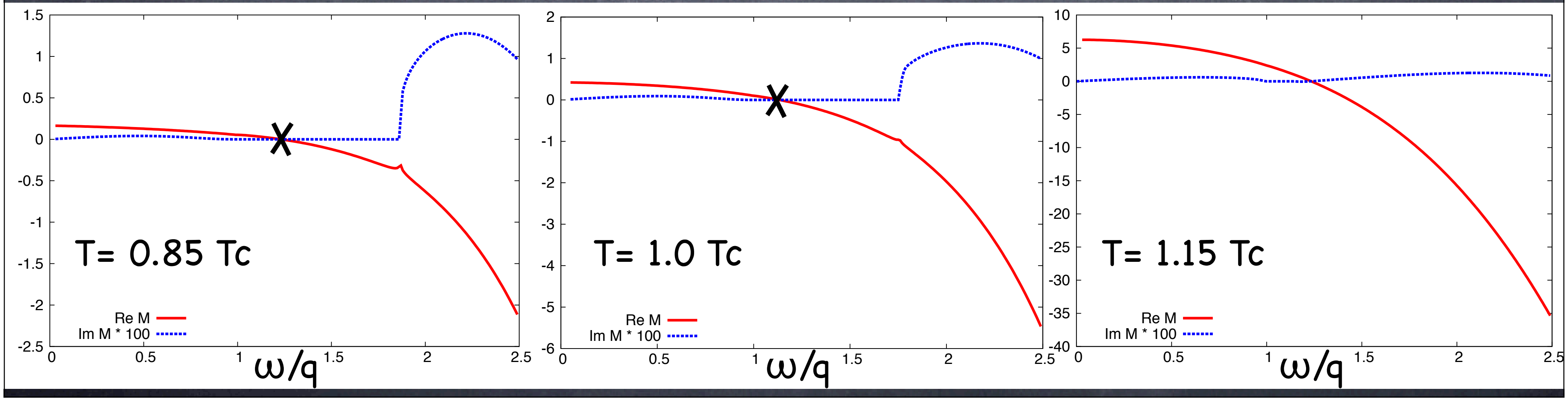
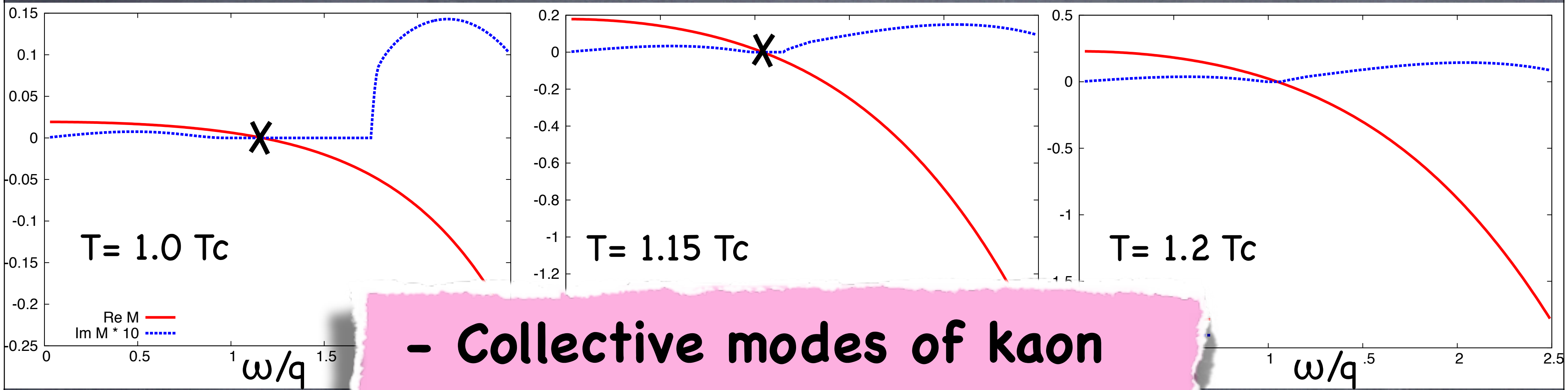


Collective modes

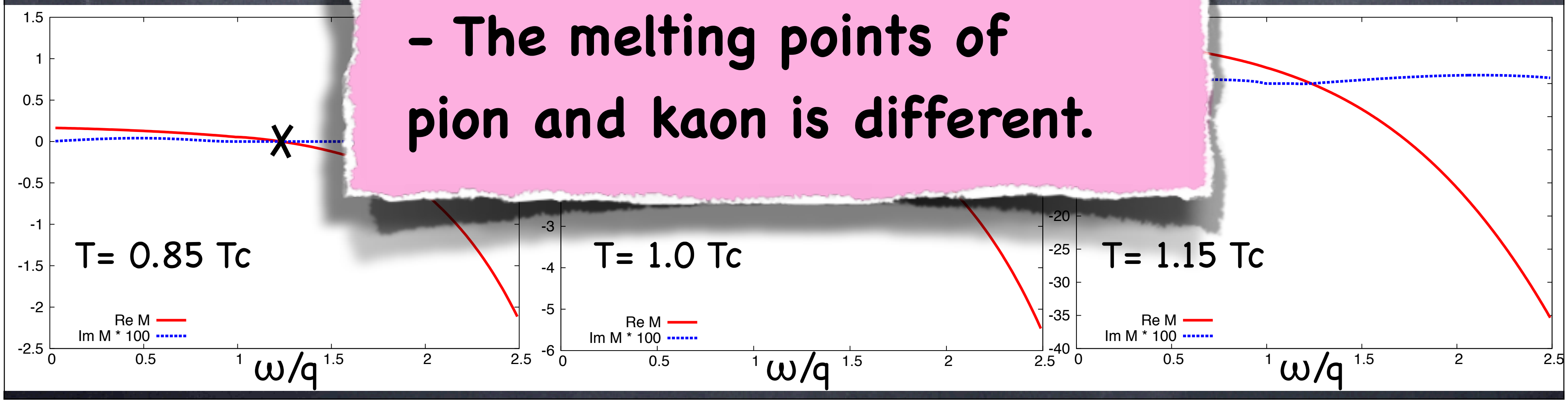
Pion



- Collective modes of kaon disappear at T=1.15 Tc

- The melting points of pion and kaon is different.

Kaon



Summary and Outlook

クォーク・ハドロン相転移　at μ=0

3フレーバーPNJL模型

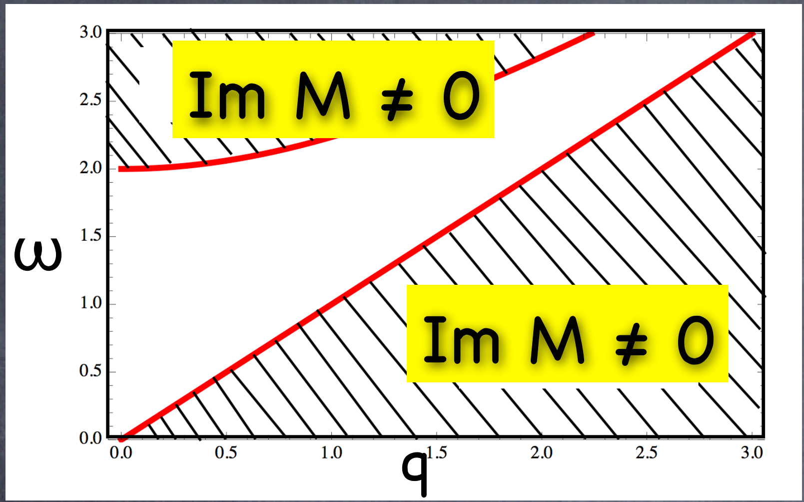
- カイラル凝縮
- Polyakov ループ

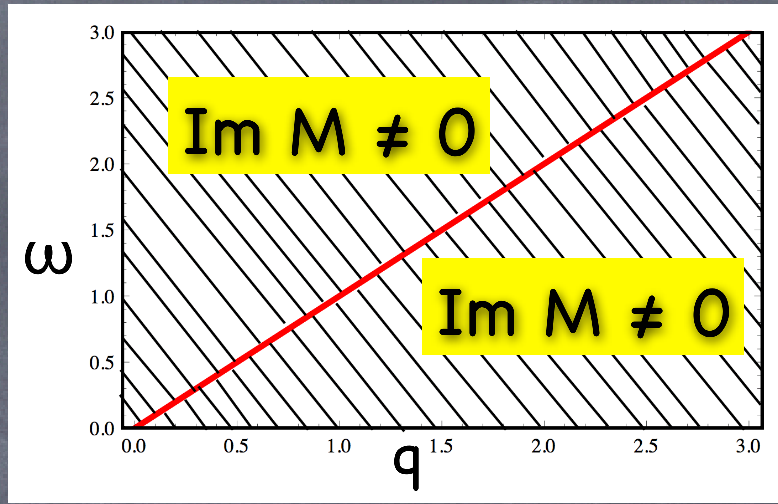
結果

- 低温：π中間子 と K中間子 が圧力を支配
- 高温：クォーク (とグルーオン) が圧力を支配
- 中間領域：K中間子が先に溶ける

今後の課題：バリオン励起を考慮し有限密度に拡張

Difference of melting temperature





Temperature