1 1st order phase transition in Maxwell-Chern-Simon QED

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Outline

1995 Kondo& Maris

3-dimensional QED with Chern-Simon term with 4-component ferimon

θ :topological mass

small θ Parity and Chiral symmetry broken

θ_{cr} $\rightarrow$only Parity broken phase and chiral symmetric Phase

Method:Schwinger-Dyson eq in non-local gauge with massless loop correction

2011 Raya tested by Ladder Schwinger-Dyson eq and found $\theta_{cr} \sim 8 \times 10^{-3} e^2$

○ Our case covariant gauge with Ball-Chiu vertex which satisfy Ward-Takahashi-identity

○Recently Mizutani showed the θ_{cr}? in quenched case removing rolling error in low energy region.

○Advantage of our case

1 order parameter is gauge invariant & low-energy mass is gauge invarint

(on-shell limit only transverse degree of freedom contribute)

2 Z factor is gauge dependent

○ My results

Schwinger-Dyson eq with massless loop without vertex correction is solved in Landau gauge

and find the consistent solution.

infrared problem does not arise: massless loop soften the photon propagator as $1/p$.

with vertex correction:now in propgress

Ward-Takahashi-identity by gauge invariance

S-D(equation of motion)

$$S_F^{-1}(p) = S_F^{(0)-1} - ie^2 \int \frac{d^3k}{(2\pi)^3} \Gamma_\mu(p,k) S_F(k) \gamma_\nu D_F^{\mu\nu}(p-k)$$
$$= A(p) p\cdot\gamma - B(p)$$

$$(p-q)_\nu S'_F(p) \Gamma_\nu(p,q) S'_F(q) = S'_F(p) - S'_F(q)$$

2 Ball-Chiu Ansatz(1980)

Bashir,Pennington(1994)

$$
\begin{aligned}
S_F^{-1}(p) &= A(p)\gamma \cdot p - B(p) \\
\Gamma_\mu(p,q) &= \Gamma_\mu^L(p,q) + \Gamma_\mu^T(p,q) \\
(p-q)^\mu \Gamma_\mu^L(p,q) &= S_F^{-1}(q) - S_F^{-1}(p) \\
(p-q)^\mu \Gamma_\mu^T(p,q) &= 0
\end{aligned}
$$

Transverse vertex does not disturb the W-T identity.

Assume

$$\Gamma_\mu^L(p,q) = a(p,q)\gamma_\mu + b(p,q)(p+q)\cdot\gamma(p+q)_\mu - c(p,q)(p+q)_\mu$$

solution

$$\Gamma^L_\mu(p,q) = \frac{A(p)+A(q)}{2}\gamma_\mu + \frac{A(p)-A(q)}{2(p^2-q^2)}(p+q)\cdot\gamma(p+q)_\mu$$
$$- \frac{B(p)-B(q)}{p^2-q^2}(p+q)_\mu$$

3 Chirality & Parity in (2+1)-dimension

$\gamma_\mu = \{\gamma_0, \gamma_1, \gamma_2\}, \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}, \gamma_3, \gamma_5$ Dirac representaion

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \tag{1}$$

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

chiral transform $\psi \mapsto \exp(i\alpha\gamma_3)\psi, \psi \mapsto \exp(i\beta\gamma_5)\psi$

$\overline{\psi}\psi \mapsto \cos(2\alpha)\overline{\psi}\,(1)\,\psi + i\sin(2\alpha)\overline{\psi}(\gamma_3\text{or }\gamma_5)\psi$

$m_e\overline{\psi}\psi$ is not singlet and violate chiral symmetry.

$\sigma = \overline{\psi}\psi, \pi = \overline{\psi}(\gamma_3 \text{ or } \gamma_5)\psi,$

○ σ plays the role of Higgs and π restores symmetry as in the σ model.

Rotator or symmetric top.

Another mass $m_o\overline{\psi}\tau\psi$(spin density)$, \tau = [\gamma_3, \gamma_5]/2$

$\overline{\psi}\psi$ is singlet under Parity $\psi(x_0, x_1, x_2) = P\psi(x_0, -x_1, x_2),$

$P = \gamma_1\gamma_5$

$\overline{\psi}\tau\psi \mapsto -\overline{\psi}\tau\psi$ but singlet under chiral transform

$m = m_e + \tau m_o$

Solving Dirac eqation $\psi \mapsto \psi_+ + \psi_-$,chiral representation 2-spinor with m_+, m_-

$$m = m_e \begin{pmatrix} I_2 & 0 \\ 0 & I_2 \end{pmatrix} + m_o \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} = \begin{pmatrix} m_+ & 0 \\ 0 & m_- \end{pmatrix}$$

$$\begin{aligned} L = \overline{\psi}_+(x)(i\partial \cdot \gamma - m_+)\psi_+(x) \\ + \overline{\psi}_-(x)(i\partial \cdot \gamma - m_-)\psi_-(x) \end{aligned} \tag{3}$$

Propagator can be decomposed

$$\begin{aligned} S(p) = \frac{-1}{\gamma \cdot pA(p) - B(p)} = \frac{\gamma \cdot pA_+(p) + B_+(p)}{A_+(p)^2p^2 + B_+(p)^2}\chi_+ \\ + \frac{\gamma \cdot pA_-(p) + B_-(p)}{A_-(p)^2p^2 + B_-(p)^2}\chi_- \end{aligned} \tag{4}$$

$\chi_+ = (1 + \tau)/2 = \begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}, \chi_- = (1 - \tau)/2 = \begin{pmatrix} 0 & 0 \\ 0 & I_2 \end{pmatrix}$

S-D equation split into 2 spinor(S_+, S_-) with full $\Gamma_\mu(p,q)$

2-kinds of mass:$m_e = (m_+ + m_-)/2, m_0 = (m_+ - m_-)/2$

4 Maxwell-Chern-Simon QED

$$L = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{\theta}{4}\epsilon_{\mu\nu\rho}F_{\mu\nu}A_\rho + ...$$

$F \times A$:Parity violating

current

$$J_\nu = \partial_\mu F_{\mu\nu} + \frac{\theta}{2}\epsilon_{\alpha\beta\nu}F_{\alpha\beta}$$

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - p_\mu p_\nu/p^2 - i\theta\epsilon_{\mu\nu\rho}p_\rho/p^2}{p^2 - \theta^2 + i\epsilon} + \xi\frac{p_\mu p_\nu}{p^4}$$

θ may be corrected by vacuum polarization

$$\Pi_e = \frac{e^2}{8}\sqrt{k^2}(m=0)$$

massless loop $c = e^2/8$

$$D_e(k) = \frac{k^2 + c\sqrt{k^2}}{(k^2 + c\sqrt{k^2})^2 + \theta^2 k^2}, \tag{5}$$

$$D_O(k) = \frac{-\theta\sqrt{k^2}}{(k^2 + c\sqrt{k^2})^2 + \theta^2 k^2}. \tag{6}$$

∘ use of complex number

$$D_e(k) = \mathrm{Re}(\frac{1}{k^2 + (c + i\theta)\sqrt{k^2}}),$$

$$D_O(k) = \mathrm{Im}(\frac{1}{k^2 + (c + i\theta)\sqrt{k^2}}). \tag{7}$$

is helpful to perform angular integration in S-D equation.S-

D Landau gauge with bare photon mass θ

$$B_{\pm}(p) = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dqq^2}{q^2A_{\pm}(q)^2 + B_{\pm}(q)^2}$$
$$\times[2(B_{\pm}(q)I_0(p,q) \mp \theta A_{\pm}(q)I_2(p,q)_{-})].$$

$$p^2(A_{\pm}(p) - 1) = \frac{e^2}{4\pi^2}\int_0^\infty \frac{dqq^2}{q^2A_{\pm}(q)^2 + B_{\pm}(q)^2}$$
$$\times [2(A_{\pm}(q)I_3(p,q){\mp}\theta B_{\pm}(q)I_2(p,q)_{+})]$$

angular integral replace $\theta \to c + i\theta$

$$I_0(p,q) = \frac{-1}{2pq}\ln(\frac{(p-q)^2+\theta)^2}{(p+q)^2+\theta)^2}) \to \mathrm{Re}(I_0(p,q,c,\theta)$$

$$I_2(p,q)_{\pm} = \frac{-1}{4pq}\ln(\frac{(p-q)^2+\theta)^2}{(p+q)^2+\theta)^2})$$
$$\pm \frac{p^2-q^2}{4\theta^2 pq}\ln(\frac{1+\theta^2/(p-q)^2}{1+\theta^2/(p+q)^2}) \to \mathrm{Im}(I_2(p,q,c,\theta))$$

$$I_3(p,q) = \frac{(p^2-q^2)^2}{8\theta^2 pq}\ln(\frac{1+\theta^2/(p-q)^2}{1+\theta^2/(p+q)^2})$$
$$-\frac{1}{2} - \frac{\theta^2}{8pq}\ln(\frac{(p-q)^2+\theta^2}{(p+q)^2+\theta^2}) \to \mathrm{Re}(I_3(p,q,c,\theta).$$

Add vertex correction terms.

5 Numerical results shown later

$10^{-5} \leq p \leq 10^5, B_\pm(p), A_\pm(p), 50\ Digit$

We searched the value of θ_{cr} where $\left\langle \overline{\psi}\psi \right\rangle \simeq 0$

$\left\langle \overline{\psi}\psi \right\rangle_e \simeq 0 \leftarrow \left\langle \overline{\psi}\psi \right\rangle_e \leq 10^{-5}$, at $\theta = 0$ $\left\langle \overline{\psi}\psi \right\rangle \simeq 3 \times 10^{-3} e^4$,

at $\theta \simeq 1 \times 10^{-2} e^2$, $\left\langle \overline{\psi}\psi \right\rangle_e \simeq 2 \times 10^{-5}$,without vertex correction:Parity broken and Chiral symmetric

θ dependence is slow in case with massless loop.

quenched case it is clear to see

Phase transition is not discussed.We need more dynamics as effective potential.

6 Summary

$\circ\theta = 0$ QED3 gauge invariant & chiral symmetry breaking:Maris et al

$\circ$4-comonent case:small θ $\mapsto$chiral symmetry & Parity broken

$m_e \neq 0$,small $m_o \neq 0$

$\circ$ θ_{cr} $\mapsto$Parity broken& chiral symmetric $m_e = 0, m_+ = -m_-, m_o \neq 0(Kondo\&Maris, 1995)$ non-local gauge

quenched Landau gauge $\theta_{cr} = 8 \times 10^{-3} e^2 (Raya, 2011)$

vertex correction Parity broken,chiral symmetric $(MIZUTANI$ $)$ $\theta_{cr} \sim 8 \times 10^{-3} e^2$

My case massless loop + Landau gauge $\theta_{cr} \sim 10^{-2} e^2$

with vertex correction of BC, $\left\langle \overline{\psi}\psi \right\rangle_e \neq 0$ for large θ ?
Chirality and Parity both broken

Now in progress

infrared behavior is soft by massless loop correction.

Our study has relations to finite density case.

with chemical potential which violates prity,C-S is induced by anomaly.