

~~ハドロン~~  
QCD物質の緩和と  
相対論的流体方程式の  
輸送係数

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# 相対論的な流体方程式

$$\partial^\rho T_{\rho\sigma} = 0$$

時間の方向

空間の方向

$$U^\mu U_\mu = 1, \quad \Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$$

E-frame or Landau frame:  $U^\mu$  エネルギーの流れ

$$\varepsilon U^\mu = T^{\mu\nu} U_\nu, \quad P = \frac{-1}{3} \Delta_{\mu\nu} T^{\mu\nu}$$

固有ベクトル

ストレス・シアーテンソル

$$\pi^{\mu\nu} = \left( \Delta^\mu{}_\rho \Delta^\nu{}_\sigma - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\rho\sigma} \right) T^{\rho\sigma}$$

粒子の流れ

$$n = J^\mu U_\mu, \quad J^\mu = n U^\mu + I^\mu$$

“熱”

流体方程式は拡散型・・・相対論的因果律に問題？

# 双曲型流体方程式 Israel & Stewart

Hyperbolic equation

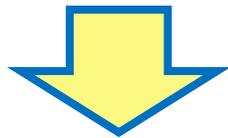
$$\begin{aligned}
 I^\mu &= -\kappa \Delta^{\mu\nu} \left( \frac{1}{T} \partial_\nu T + DU_\nu + \bar{\beta}_1 DI_\nu - \bar{\alpha}_0 \partial_\nu P' - \bar{\alpha}_1 \partial_\alpha \pi_\nu^\alpha \right) \\
 P' &= -\frac{1}{3} \eta_v (\partial_\mu U^\mu + \beta_0 DP' - \bar{\alpha}_0 \partial_\mu I^\mu) \\
 \pi_{\mu\nu} &= -2\eta_s \{ \partial_\mu U_\nu + \beta_2 D\pi_{\mu\nu} - \bar{\alpha}_1 \partial_\mu I_\nu \}
 \end{aligned}$$

where

$$D \equiv U_\mu \partial^\mu, \quad \sigma = \langle P' \rangle - \langle P' \rangle_l, \quad P_s = \langle P' \rangle_l$$

$$\{A_{\mu\nu}\} \equiv \frac{1}{2} \Delta_\mu^\lambda \Delta_\nu^\rho (A_{\lambda\rho} + A_{\rho\lambda} - \frac{2}{3} \Delta_{\lambda\rho} \Delta^{\alpha\beta} A_{\alpha\beta})$$

Israel, Israel-Stewart 双曲型の輸送方程式を考えた



新しい係数が出てくる。  
この係数をどう決めたらいいのか？

流体方程式なので、  
 $\alpha_i$  も  $\beta_i$  も  
熱力学的パラメータ  
Tや $\mu$ の関数であり、  
統計力学で計算できるはず

# 線形応答理論

$$\hat{H}_{tot} = \hat{H}_0 + \hat{A}f(t)$$

$$\hat{\rho} = C e^{-\beta \hat{H}_{tot}}$$



f で展開して1次までとる

$$\langle X \hat{\rho}(t) \rangle \sim \text{tr} \left[ e^{-\beta \hat{H}_0} X \hat{\rho}(t) \hat{A} \right] f(t)$$

外力のない平衡系での期待値

# Non-equilibrium density operator

非平衡密度演算子

Nakajima, Kubo, Zubarev

Local equilibrium part



$\hat{\rho}_l$

$$\hat{\rho} = Q^{-1} \exp \left[ \int dx^\mu \left\{ \beta(\mathbf{x}, t) U^\nu(\mathbf{x}, t) \hat{T}_{\mu\nu}(\mathbf{x}, t) - \beta(\mathbf{x}, t) \mu(\mathbf{x}, t) U^\nu(\mathbf{x}, t) \hat{J}_\nu(\mathbf{x}, t) \right\} \right]$$

$$- \int d^\mu \mathbf{x} \int_{-\infty}^t dx_\mu e^{\varepsilon(t'-t)} \left[ \hat{T}_{\mu\nu}(\mathbf{x}, t') \partial^\mu (\beta U^\nu(\mathbf{x}, t')) + \hat{J}_\nu(\mathbf{x}, t') \partial^\nu (\beta(\mathbf{x}, t') \mu(\mathbf{x}, t')) \right]$$

Thermo-dynamical force part

# Linear response theory for non-equilibrium (Kubo et al; Zubarev)

$$\langle \hat{T}^{\mu\nu}(x) \rangle = \langle \hat{T}^{\mu\nu}(x) \rangle_l + \int d^3x' \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} (\hat{T}^{\mu\nu}(x), \hat{T}_{\rho\sigma}(x')) \underbrace{\partial^\rho F^\sigma(x', t')}_{\text{Changes slowly}}$$

Expand thermo-dynamical forces as,

$$\begin{aligned} & \partial^\rho F^\sigma(x', t') \\ = & \partial^\rho F^\sigma(x, t) + (x^{\mu'} - x^\mu) \partial_\mu \partial^\rho F^\sigma(x, t) + \dots \end{aligned}$$

$$\begin{aligned} \langle \hat{T}^{\mu\nu}(x) \rangle &= \langle \hat{T}^{\mu\nu}(x) \rangle_l \\ &+ \int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{T}^{\mu\nu}(x), \hat{T}_{\rho\sigma}(x')) \partial^\rho F^\sigma(x, t) \\ &+ \int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{T}^{\mu\nu}(x), (x^{\mu'} - x^\mu) \hat{T}_{\rho\sigma}(x')) \partial_\mu \partial^\rho F^\sigma(x, t) \end{aligned}$$

κ, η(v), η(s)

New coefficients, α<sub>i</sub>, β<sub>i</sub>
Local hyperbolic equation

微視的な相関距離・時間 vs 巨視的な変化の様子 ← 流体モデルの診断

## Expectation of operators

$$\langle \hat{T}^{\mu\nu}(x) \rangle_l = (\varepsilon + P)U^\mu U^\nu - P g^{\mu\nu}$$

$$\langle \hat{J}^\mu(x) \rangle_l = n J^\mu$$

緩和の0次

## Correlation of operators

緩和の1次

粘性係数

$$\int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{T}^{\mu\nu}(x), \hat{T}_{\rho\sigma}(x'))$$

$\eta(v), \eta(s)$

$$\int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{J}^\mu(x), \hat{J}_\rho(x'))$$

$\kappa$

熱伝導率

## Moment-Correlation

緩和の2次

$$\int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{T}^{\mu\nu}(x), U^\lambda(x'_\lambda - x_\lambda) \hat{T}_{\rho\sigma}(x'))$$

$$\int d^3x' \int_{-\infty}^t dt' e^{(t-t')} (\hat{T}^{\mu\nu}(x), (x'_\lambda - x_\lambda) \hat{J}_\rho(x'))$$

Coefficients  
Of 2nd order  
Hydro.

**New coefficients,  $\alpha_i, \beta_i$**

Israel-Stewart Relativistic Causal hydrodynamics

$\alpha_i$  や  $\beta_i$  はカレントを重みとした  
カノニカル相関時間やカノニカル相関距離を  
計算すればよい arXiv:1211.7173

$$\beta_0 \sim \langle \pi | \mathbf{t} | \pi \rangle$$

$$\beta_1 \sim \langle I | \mathbf{t} | I \rangle$$

$$\beta_2 \sim \langle P | \mathbf{t} | P' \rangle$$

$$\alpha_1 \sim \langle I | \mathbf{x} | \pi \rangle$$

$$\alpha_0 \sim \langle P | \mathbf{x} | I \rangle$$

$$-\alpha_0 \sim \langle I | \mathbf{x} | P' \rangle$$

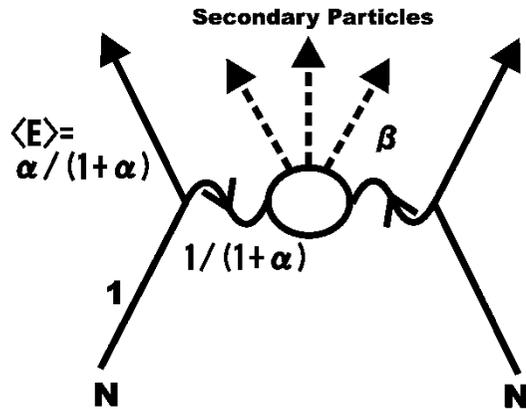
どうやって？

Lattice QCD ??

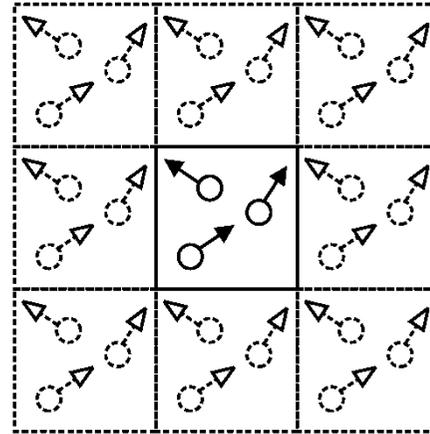
# Hadro-molecular dynamic calculation

# URASiMA 1.

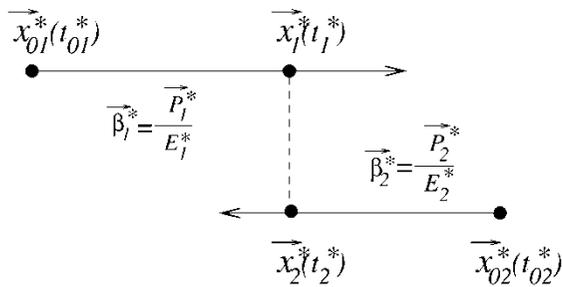
(Ultra-Relativistic AA collision Simulator based on Multiple Scattering Algorithm)



hadronic picture

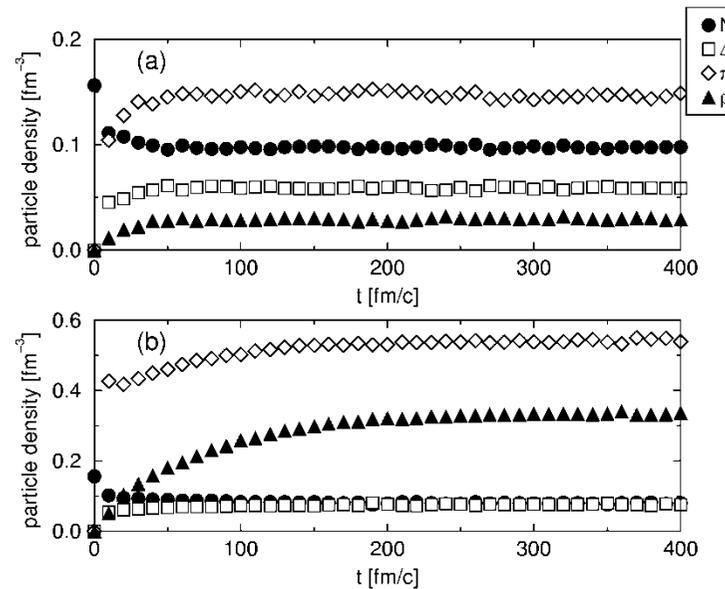


box with periodic boundary condition



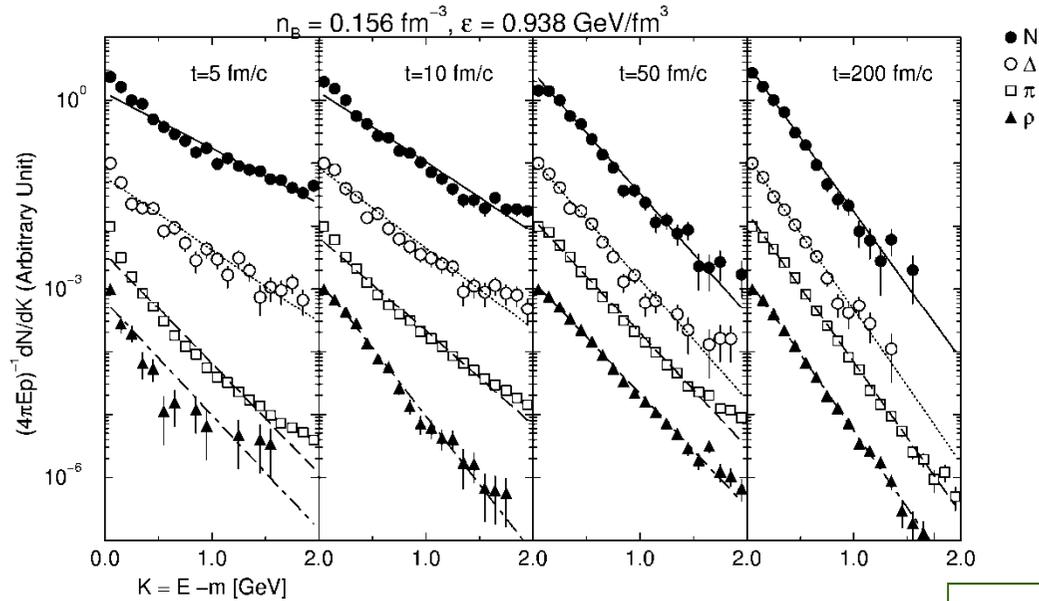
where  $\vec{P}_1^* = -\vec{P}_2^*$

relativistic



stationary state

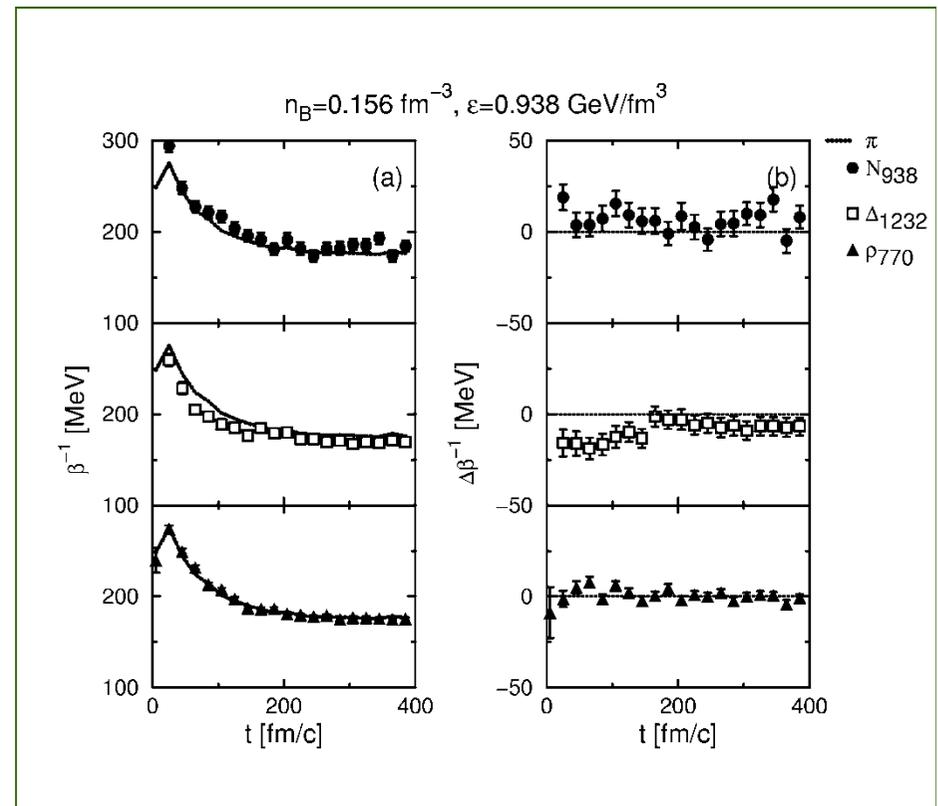
# URASiMA 2.



**common** slope parameter  
 for the distribution  
 function of  
 different particles



“temperature”



# 2-flavor low energy version

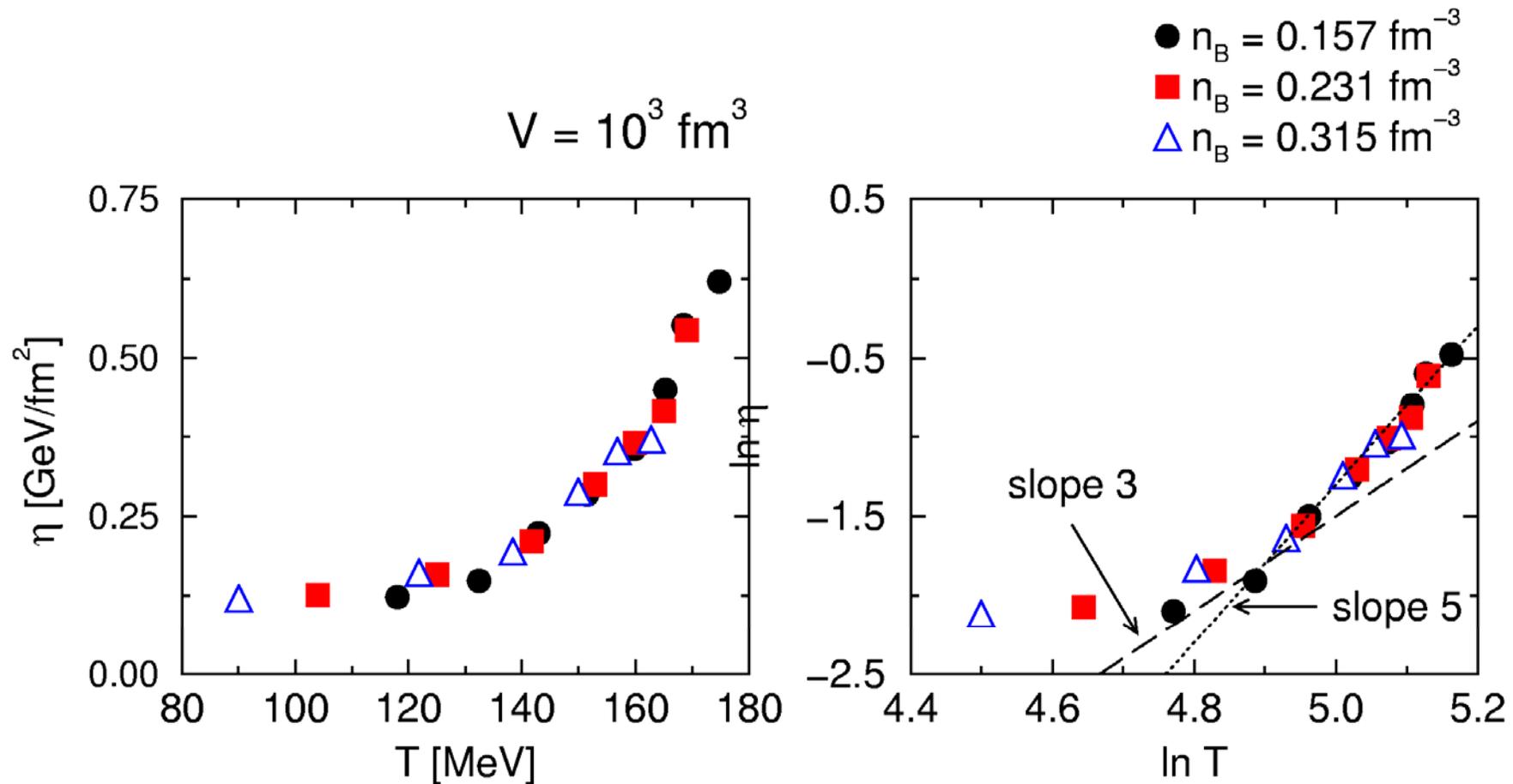
Baryons, mesons and their resonances  
included in the present URASiMA.

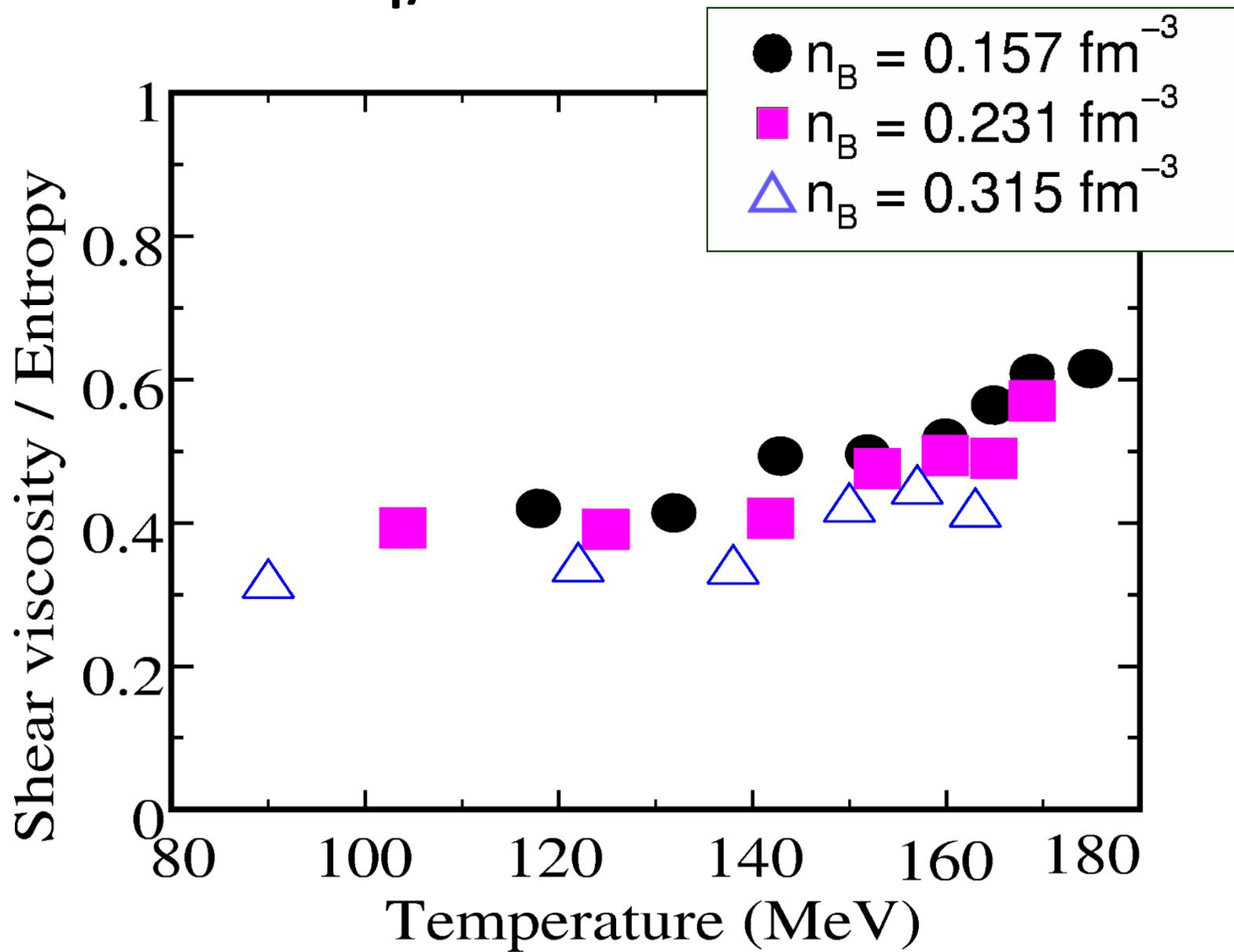
nucleon	$N_{938}$	$N_{1440}$	$N_{1520}$	$N_{1535}$
	$N_{1650}$	$N_{1675}$	$N_{1680}$	$N_{1720}$
$\Delta$	$\Delta_{1232}$	$\Delta_{1600}$	$\Delta_{1620}$	$\Delta_{1700}$
	$\Delta_{1905}$	$\Delta_{1910}$	$\Delta_{1950}$	
meson	$\pi$	$\eta_s$	$\sigma_{800}$	$\rho_{770}$

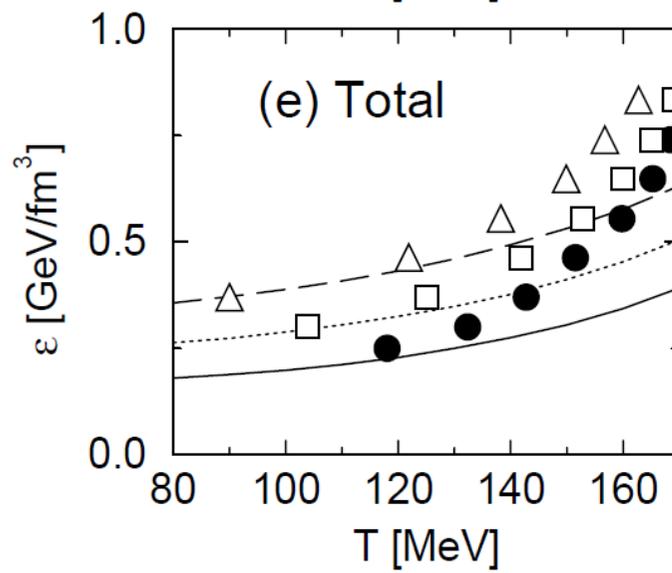
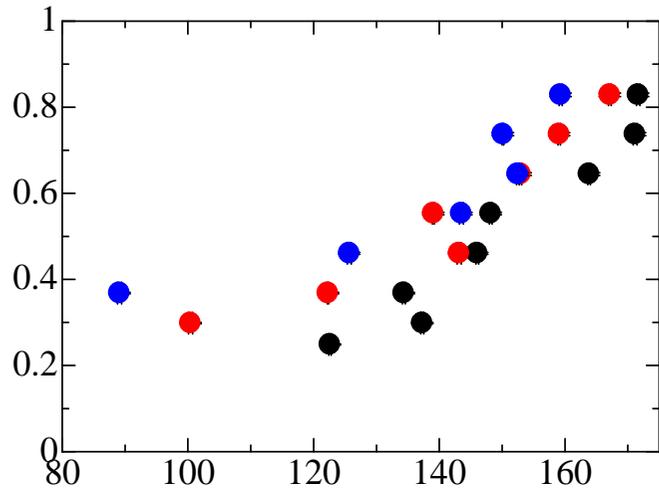
Strangeness and anti-baryon is neglected

# transport coefficients of hadron gas

## Shear Viscosity



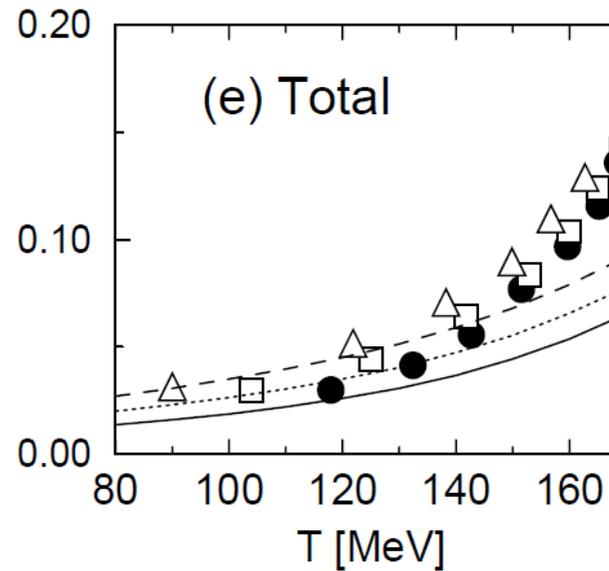
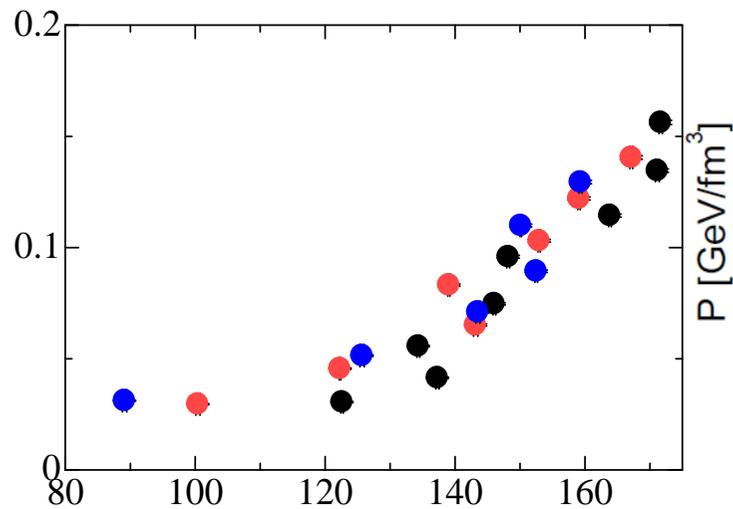
$\eta/s$ 



- $n_B = 0.157 \text{ [fm}^{-3}\text{]}$
- $n_B = 0.231 \text{ [fm}^{-3}\text{]}$
- △  $n_B = 0.315 \text{ [fm}^{-3}\text{]}$

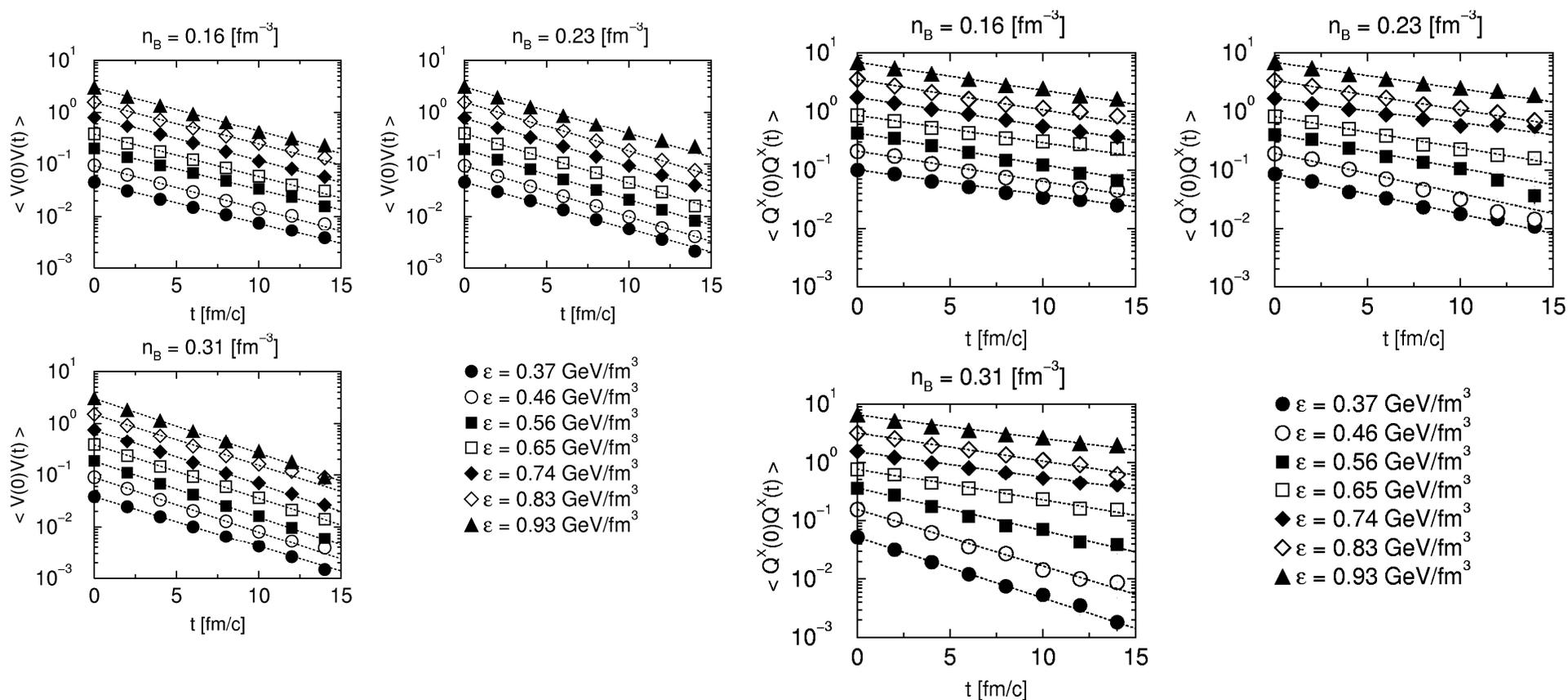
10年ぶりの計算は  
統計精度がまだ不十分

Nobuo-Sasaki-PTP-106-783 (2001)



- $n_B = 0.157 \text{ [fm}^{-3}\text{]}$
- $n_B = 0.231 \text{ [fm}^{-3}\text{]}$
- △  $n_B = 0.315 \text{ [fm}^{-3}\text{]}$

# Relaxation of the system(1)



velocity correlation of  
the Baryons

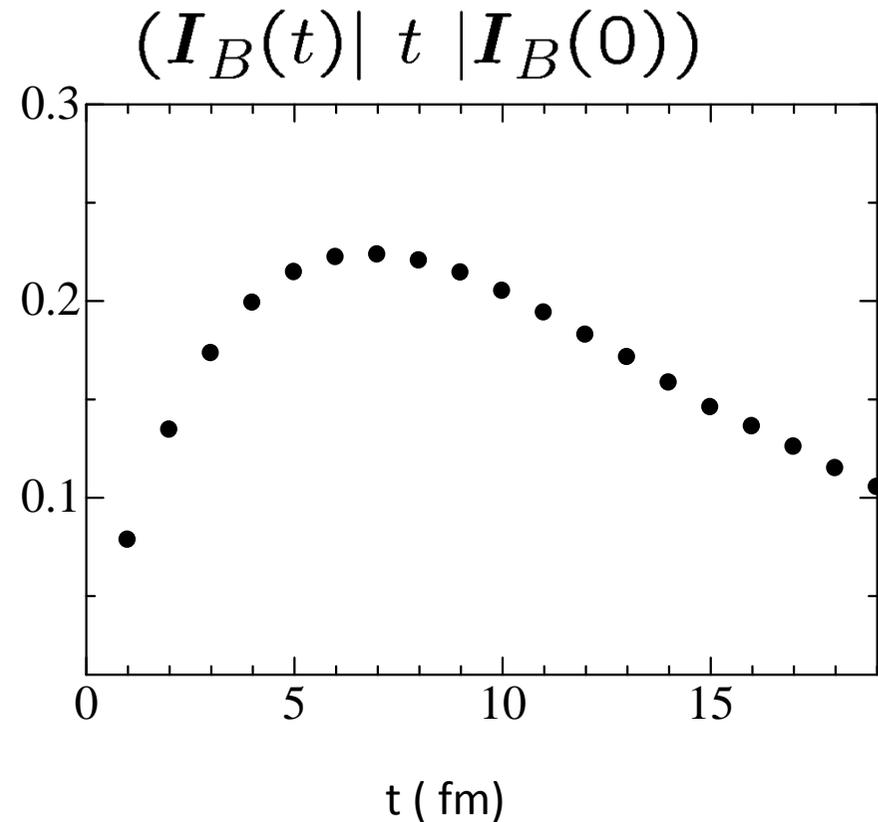
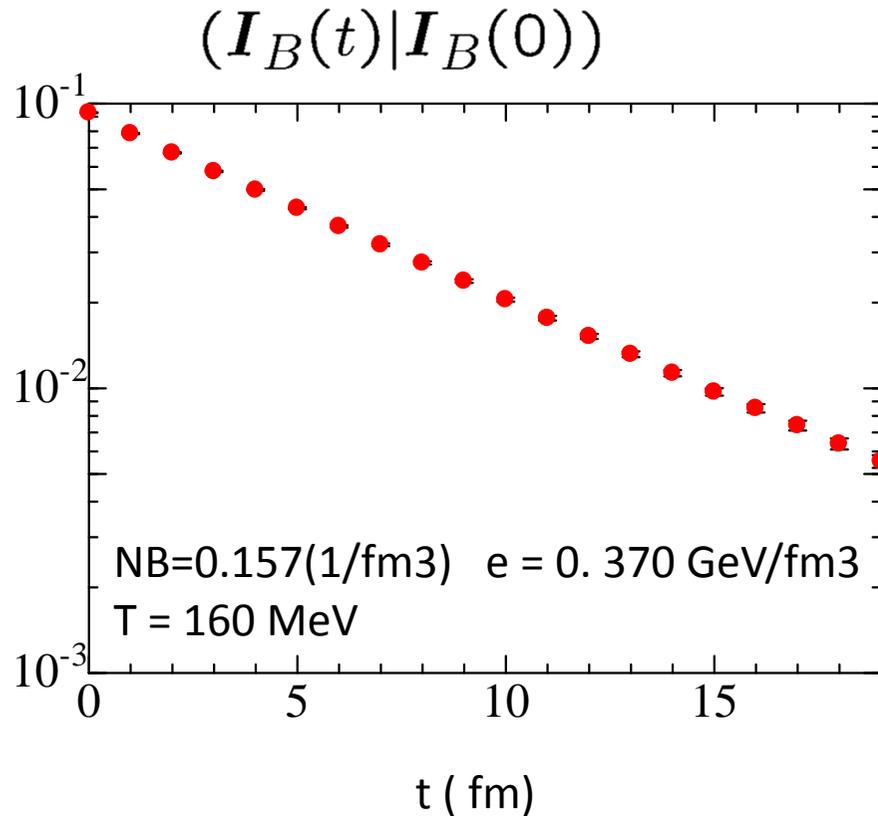
velocity correlation of  
the charged particles

$$\langle\langle O_1|O_2\rangle\rangle = \lim_{\zeta \rightarrow 0^+} \int_{-\infty}^t dt' e^{-\zeta(t-t')} \int d^3x' (\hat{O}_1(\mathbf{x}, t), \hat{O}_2(\mathbf{x}', t')),$$

相関の“緩和関数”

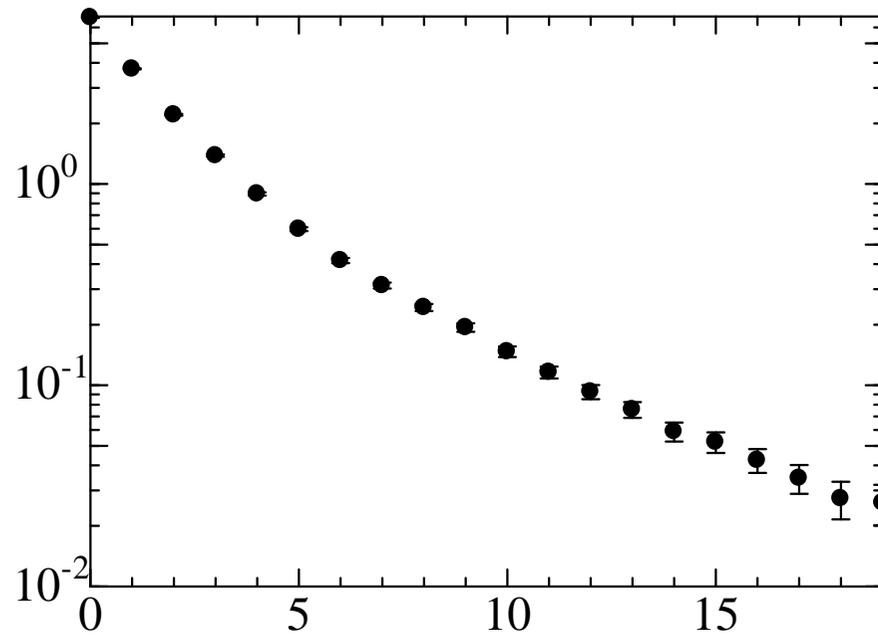
$$(\hat{O}_1(t)|\hat{O}_2(t'))$$

$\beta_i$  は相関時間



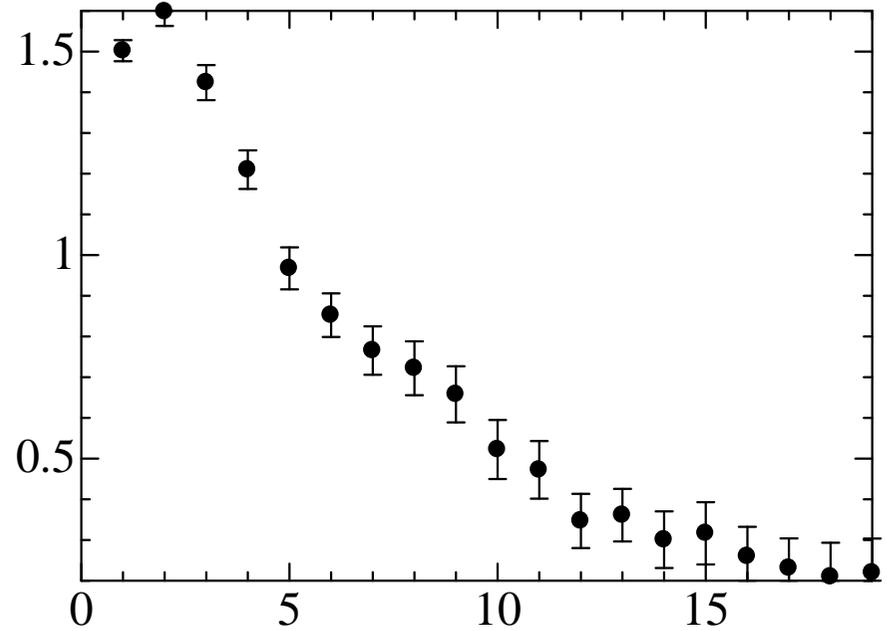
# $\beta_i$ は相関時間

$$\left( \pi^{ij}(t) | \pi^{ij}(0) \right)$$



t ( fm)

$$\left( \pi^{ij}(t) | t | \pi^{ij}(0) \right)$$



t ( fm)

NB=0.157(1/fm<sup>3</sup>) e = 0.370 GeV/fm<sup>3</sup>

T = 160 MeV

$$\alpha_1 \sim \langle I_B | \mathbf{x} | \pi \rangle$$

## 異なったカレントの相関

分子動力学では

$$I_B \sim Q_B \frac{\mathbf{p}}{E} \quad \mathbf{x} \sim \frac{\mathbf{p}}{E} t \quad T^{ij} = \frac{p^i p^j}{E}$$

$$\alpha_1 \sim \langle Q_B \frac{p^i}{E} \frac{p^j}{E} | t | \frac{p^i p^j}{E} \rangle$$



Net charge

対生成は効かない

$I_B$   
 $T^{ij}$  } 中間ぐらいの緩和時間のはず

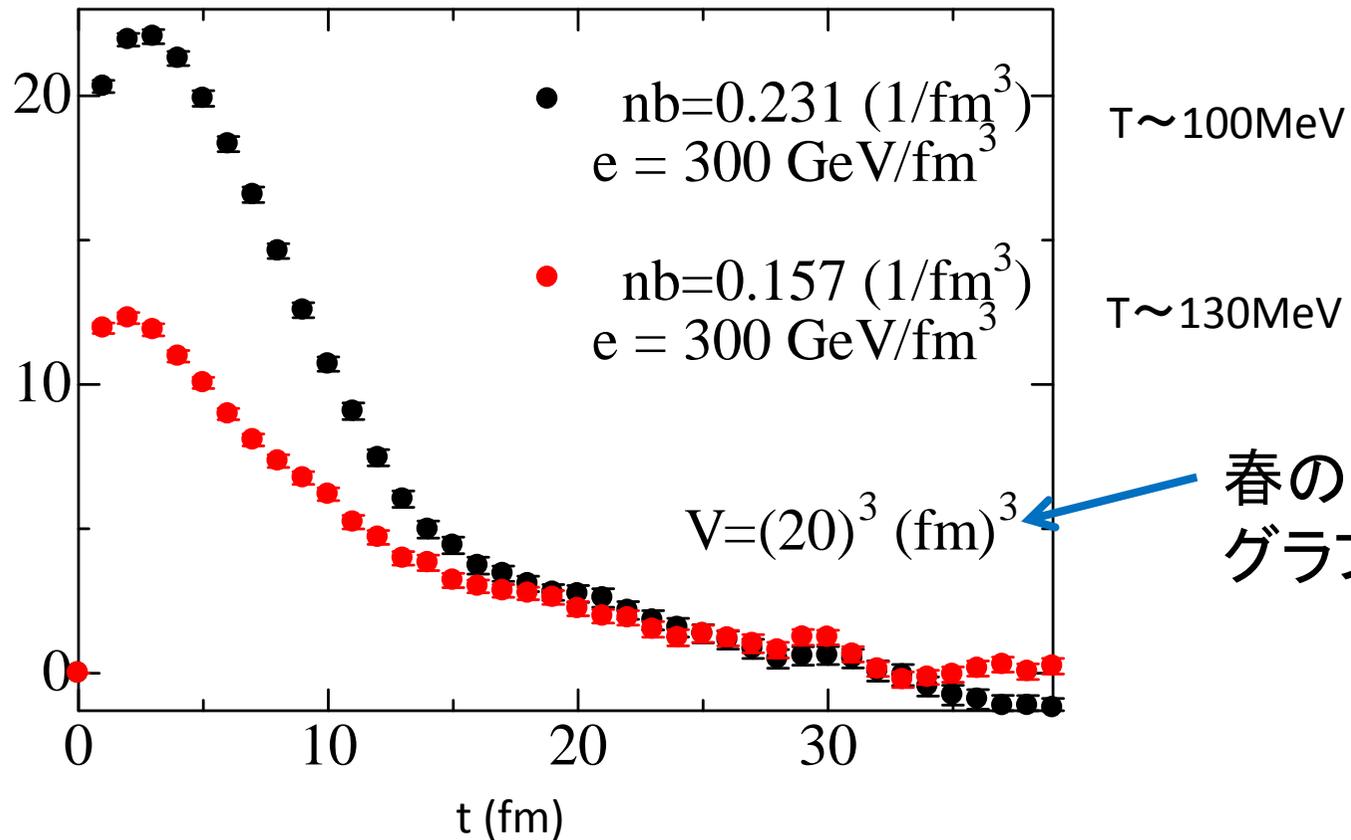
電荷伝導率は  $(I_B I_B)$  で  
 $\sim \langle Q_B Q_B \rangle$

$$\langle\langle O_1|O_2\rangle\rangle = \lim_{\zeta \rightarrow 0^+} \int_{-\infty}^t dt' e^{-\zeta(t-t')} \int d^3x' (\hat{O}_1(\mathbf{x}, t), \hat{O}_2(\mathbf{x}', t')),$$

相関の“緩和関数”

$$(\hat{O}_1(t)|\hat{O}_2(t'))$$

$\langle I_B | \mathbf{x} | \pi \rangle$  の緩和関数



## 現状のまとめ

パソコンハードの進歩に期待して

Hadro-Molecular dynamic 統計力学復活

双曲型流体の輸送係数

$\beta$  も  $\alpha$  も計算可能

$\alpha$  は相関”時間”で近似も可能

もっと体積と統計が必要