Brane-Antibrane at Finite Temperature in the Framework of Thermo Field Dynamics

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Previously we have investigated the thermodynamical properties of brane-antibrane pairs in the framework of Matsubara formalism. In this talk, we show the calculation of the thermal vacuum state and the partition function for a single open string on a brane-antibrane pair in the framework of thermo field dynamics. From this we can obtain free energy of multiple strings, and it agrees with that in the framework of Matsubara formalism.

1 Brane-Antibrane at Finite Temperature in the Framework of Matsubara Formalism

It is well known that ideal string gas has a characteristic temperature called the Hagedorn temperature. We can compute the one-loop free energy of strings by using path integral in Matsubara method. The one-loop free energy of strings diverges above this temperature. This Hagedorn temperature is limiting temperature for perturbative strings.

We have previously discussed the behavior of brane-antibrane pairs at finite temperature in the constant tachyon background [1]. At zero temperature, the spectrum of open strings on these unstable branes contains a complex scalar tachyon field T. In the brane–antibrane configuration, we have $T = 0$, and the potential of this tachyon field has a local maximum at $T = 0$. The tachyon potential has a non-trivial minimum, and the tachyon falls into it at zero temperature. The potential height of the tachyon potential exactly cancels the tension of the original brane-antibrane pairs. This implies that these unstable brane systems disappear at the end of the tachyon condensation.

We have calculated the finite temperature effective potential of open strings on these branes based on boundary string field theory. In this case we are confronted with the problem of the choice of Weyl factors in the two boundaries of the one-loop worldsheet, because the conformal invariance is broken by boundary action. Our calculation is based on the one-loop amplitude of open strings, which has been proposed by Andreev and Oft [2]. Their choice of Weyl factors is natural in the sense that both sides of the cylinder worldsheet are treated on an equal footing. Although brane-antibrane pairs are unstable at zero temperature, there are the cases that they become stable at finite temperature. For the D9-brane–$\overline{\text{D9}}$-brane pairs, a phase transition occurs at slightly below the Hagedorn temperature and the D9-brane–$\overline{\text{D9}}$-brane pairs become stable above this temperature. On the other hand, for the Dp-brane–$\overline{\text{D}p}$-brane pairs with $p \leq 8$, such a phase transition does not occur. We thus concluded that not a lower dimensional brane-antibrane pairs but D9-brane–$\overline{\text{D9}}$-brane pairs are created near the Hagedorn temperature.

2 Brane-Antibrane at Finite Temperature in the Framework of Thermo Field Dynamics

Let us turn to consider the finite temperature system of Brane-antibrane pair in the framework of thermo field dynamics. The thermal vacuum state for bosonic D-brane has been calculated by Vancea [3]. We treat an open string on these branes as first quantized string, since second quantized string field theory is quite difficult. Using the light-cone coordinates, we can express partition function for a single string as

$$Z_1(\beta) = \mathrm{Tr}\ \exp\left(-\ \frac{1}{2}\ \beta p^0\right) = \mathrm{Tr}\ \exp\left[-\ \frac{1}{2}\ \beta\left(p^+ + \frac{|\boldsymbol{p}|^2 + M^2}{p^+}\right)\right]. \tag{1}$$

Here we explain the calculation for Neveu-Schwarz string, which represents spacetime boson, and show only the results for Ramond string, which represents spacetime fermion. The mass spectra of Neveu-Schwarz open string is expressed as

$$M_b{}^2 = \frac{1}{\alpha'}\left(\sum_{l=1}^{\infty}\sum_{I=2}^{9}\alpha_{-l}^I\alpha_l^I + \sum_{r=\frac{1}{2}}^{\infty}\sum_{I=2}^{9} r b_{-r}^I b_r^I + 2|T|^2 - \ \frac{1}{2}\right), \tag{2}$$

where α_l (α_{-l}) are annihilation (creation) operators for bosonic oscillators, and b_r (b_{-r}) annihilation (creation) operators for fermionic oscillators. α' is the slope parameter, which is the only dimensionful parameter string theory has.

In the framework of thermo field dynamics, the thermal vacuum state is obtained from the Bogoliubov transformation of the vacuum state. In our case the generator of Bogoliubov transformation is given by

$$G_b \ = \ i\sum_{l=1}^{\infty}\frac{1}{l}\ \theta_l\left(\alpha_{-l}\cdot\tilde{\alpha}_{-l} - \tilde{\alpha}_l\cdot\alpha_l\right) + i\sum_{r=\frac{1}{2}}^{\infty}\ \theta_r\left(b_{-r}\cdot\tilde{b}_{-r} - \tilde{b}_r\cdot b_r\right), \tag{3}$$

where the dot denotes inner product on transverse space. The thermal vacuum state is calculated as

$$\begin{aligned} |0_{1b}(\theta)\rangle\rangle \ &= \ \prod_{l=1}^{\infty}\left\{\left(\frac{1}{\cosh(\theta_l)}\right)^8 \exp\left[\frac{1}{l}\ \tanh(\theta_l)\alpha_{-l}\cdot\tilde{\alpha}_{-l}\right]\right\} \\ &\quad\times\prod_{r=\frac{1}{2}}^{\infty}\left\{(\cos(\theta_r))^8\exp\left[\ \tan(\theta_r)b_{-r}\cdot\tilde{b}_{-r}\right]\right\}|0\rangle\rangle\,|p^+\rangle\,|\boldsymbol{p}\rangle\,. \end{aligned} \tag{4}$$

Following the recipe of thermo field dynamics, we can compute the free energy for a single string from this thermal vacuum state, Hamiltonian operator and an appropriate entropy operator, although we omit to show explicit calculation. The partition function for a single Neveu-Schwarz string on a Dp-brane–anti-Dp-brane pair can be obtained by exponentiating this free energy as

$$Z_{1b}(\beta) = \frac{16\pi^4 \beta \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau}\, \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \left\{ \frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right\}^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau} \right), \tag{5}$$

where β_H denotes the inverse of the Hagedorn temperature, and $\mathcal{V}_p$ the volume of the system. Similarly, partition function for a single Ramond string can be calculated as

$$Z_{1f}(\beta) = \frac{16\pi^4 \beta \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau}\, \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \left\{ \frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right\}^4 \exp\left(-\frac{\pi\beta^2}{{\beta_H}^2\tau} \right). \tag{6}$$

From these single string partition functions, free energy of multiple strings can be obtained from the following equation.

$$F(\beta) = -\sum_{w=1}^{\infty} \frac{1}{\beta w} \left\{ Z_{1b}(\beta w) - (-1)^w Z_{1f}(\beta w) \right\}. \tag{7}$$

We finally obtain

$$\begin{aligned} F(\beta) \;=\; & -\frac{16\pi^4 \mathcal{V}_p}{{\beta_H}^{p+1}} \int_0^\infty \frac{d\tau}{\tau}\, \tau^{-\frac{p+1}{2}} e^{-4\pi|T|^2\tau} \left[\left(\frac{\vartheta_3(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left\{ \vartheta_3\left(0 \left| \frac{i\beta^2}{{\beta_H}^2\tau} \right.\right) - 1 \right\} \right. \\ & \left. - \left(\frac{\vartheta_2(0|i\tau)}{{\vartheta_1}'(0|i\tau)} \right)^4 \left\{ \vartheta_4\left(0 \left| \frac{i\beta^2}{{\beta_H}^2\tau} \right.\right) - 1 \right\} \right]. \end{aligned} \tag{8}$$

This equals to the free energy based on Matsubara formalism. It should be noted that we have not been confronted with the problem of the choice of Weyl factors like in the case of Matsubara formalism. This implies that our choice of Weyl factors in that case is quite natural. We need to use second quantized string field theory in order to compute the free energy for multiple strings directly from their thermal vacuum state. We leave this calculation for future work.

References

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