Magnetic QCD Critical Point

Naoki Yamamoto
Maryland Center for Fundamental Physics, University of Maryland

1 Introduction

Quantum chromodynamics (QCD) is known to have rich phase structures at finite temperature T and baryon chemical potential μ_B. Understanding the QCD phase diagram is particularly relevant to the physics of heavy ion collisions, neutron stars, and the early Universe. However, these systems are often subject to a strong magnetic field B. It is thus important to unravel the QCD phase diagram in the presence of B in addition to T and/or μ_B.

In this work, we present a general argument for the possible existence of a new critical point associated with the deconfinement transition in the (T, B) plane. For the detail, see Ref. [1]. In the following, we assume the homogeneous magnetic field with the magnitude B in the $\hat{z}$ direction.

2 QCD in a strong magnetic field

We first consider the regime at $eB \gg \Lambda_{\rm QCD}^2$, where the strong coupling constant is sufficeintly small, $\alpha_s \ll 1$, and the analysis is under theoretical control. If we turn off the interactions, the energy levels induced by the magnetic field (Landau levels) are given by

$$E_n^2 = p_z^2 + (2n+1)e_q B - 2e_q B S_z, \tag{1}$$

where n is nonnegative integer, e_q are the charges of the quarks, $(e_u, e_d, \cdots) = \left(\frac{2}{3}, -\frac{1}{3}, \cdots\right) e$, and S_z is the spin in the $\hat{z}$ direction. For the low-energy physics well below $\sqrt{eB}$, quarks in the higher Landau levels ($n \geq 1$) are irrelevant, and we can concentrate on the quarks in the lowest Landau level (LLL) with $n = 0$. Now turning on the interactions, the quarks in the LLL acqure the mass gap, which is given by

$$M_{\rm dyn} = C(\alpha_s)\sqrt{|e_q B|}, \tag{2}$$

where the explicit form of $C(\alpha_s)$ is found by solving the self-consistent gap equation [2]. Here we just note that $M_{\rm dyn} \to \infty$ for $B \to \infty$ instead of giving the detailed expression for $C(\alpha_s)$. Then, quarks in the LLL also decouple from the low-energy dynamics well below $\sqrt{eB}$ at sufficiently large B.

That all the quarks decouple from the low-energy physics at large B means that the low-energy effective theory there is pure gluodynamics. In the presence of the magnetic field, the rotational invariance is explicitly broken. Thus, the lowest-order effective theory for low-energy dynamics is described by an anisotropic pure SU(3) gauge theory of the form

$$\mathcal{L}_{\rm eff}^0 = -\frac{1}{4} F_{\mu\nu}^a \Gamma^{\mu\nu}_{\ \ \alpha\beta} F^{a\,\alpha\beta}, \qquad \Gamma^{\mu\nu}_{\ \ \alpha\beta} \equiv g^\mu_\alpha g^\nu_\beta + (\epsilon_{zz} - 1)(\delta^{\mu 3}\delta^{\nu 0}\delta_{\alpha 3}\delta_{\beta 0} + \delta^{\mu 0}\delta^{\nu 3}\delta_{\alpha 0}\delta_{\beta 3}), \tag{3}$$

where $\epsilon_{zz} \gg 1$ [2]. Higher order terms are suppressed by factors of $p/M_{\rm dyn}$ at low energy, where p is a characteristic momentum.

3 Magnetic critical point

One immediately notes that the low-energy effective theory in Eq. (3) has an emergent center symmetry. For the isotropic pure SU(3) gauge theory, it is well known that the center symmetry is

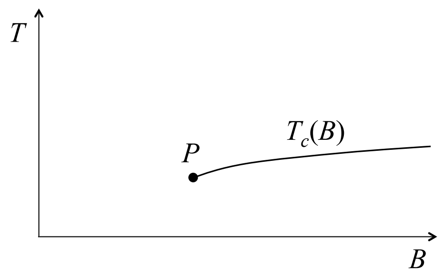

Figure 1: Putative phase diagram in the (T, B) plane. $T_c(B)$ denotes the critical temperature of the first-order deconfinement transition as a function of B that ends at the critical point P.

unbroken at $T = 0$ while it is broken at higher T, and the deconfinement phase transition between the two is first order [3, 4]. We assume it is also the case for our anisotropic effective theory in Eq. (3). Note that this assumption can be directly checked in the lattice studies relatively straightforwardly as it does not require the calculation of a fermion determinant. Then, it follows that the underlying theory, namely, QCD at sufficiently large B, must have a first-order deconfinement transition as a function of T too, because small corrections of the effective theory cannot smear the transition due to the existence of a nonzero latent heat.

On the other hand at $B = 0$, it has been well established from lattice QCD calculations that the deconfinement regime emerges as a result of a crossover at finite T [5]. Therefore, the line of first-order deconfinement transitions at large B above has to terminate at some point. The most natural way for this to occur is for it to terminate at a critical point in the T-B plane—(T_c, B_c) as shown in Fig. 1. Although other scenarios are also possible logically, we suspect that they are unlikely to be realized in QCD [1]. As QCD with B does not have a sign problem unlike QCD with μ_B, practical lattice QCD studies can distinguish between these scenarios and determine the location of the magnetic critical point.

4 Experimental signatures

What is the possible experimental signatures of the magnetic critical point P in heavy ion collisions? The magnetic critical point is characterized by the vanishing screening mass of the glueball [6]. Due to the mixing between the glueball and the flavor-singlet meson $\bar{q}q$, the divergence of the Polyakov loop susceptibility at the point P is reflected in that of the chiral susceptibility,

$$\chi = \int d^3\boldsymbol{x} \langle \bar{q}q(\boldsymbol{x})\bar{q}q(\boldsymbol{0})\rangle_c \sim \xi^{2-\eta}, \tag{4}$$

where "c" denotes the connected part of the static correlator, ξ is the correlation length, and η is the anomalous dimension.

In the presence of nonzero quark masses and small μ_B, as is the realistic case in the heavy ion collisions, the symmetry does not forbid a mixing between these order parameters and the baryon number density n_B. Then, unless the point P disappears by a small perturbation of μ_B, the singular behavior is also reflected in the baryon number susceptibility. Therefore, physical observables of the magnetic critical point are similar to the conventional QCD critical point.

5 Cosmological implications

Figure 1 also points to an interesting possibility in the cosmology. If a large magnetic field of order (or above) $\Lambda_{\rm QCD}$ existed at the first 10^{-5}–10^{-4} second of the early Universe, the QCD phase transition at that time would be strengthened to be first order by the magnetic field; this would then result in the hadronic bubbles when the quark-gluon plasma supercools, implying the inhomogeneous big bang nucleosysthesis (see, e.g., Ref. [7] and references therein). The first-order deconfinement transition may also generate some relics, e.g., the strange quark matter as dark matter. This should be contrasted with the scenario without (or with weak) magnetic fields, where the QCD is known not to have phase transitions from the first principle QCD calculations [5] and nothing remarkable happens.

Conversely, if the astrophysical constraints could ever disfavor the first-order deconfinement transition in the early Universe, the position of the magnetic critical point (B_c) would in turn give the upper bound of the strength of the magnetic field at the time of the hadronization.

6 Conclusion

In this work, we have argued a possible new critical point in the phase diagram on the (T, B) plane. We have also discussed its possible experimental signatures in heavy ion collisions and cosmological implications. Among others, it would be most interesting to determine the location of this critical point in the lattice simulations without suffering from the sign problem. Whether the magnetic critical point persists at large μ_B would also be an interesting question to be explored in the future.

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