Lepton flavor structure from point interactions in an extra dimension

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The origin of generations, the fermion's mass hierarchy, and the flavor structure are key ingredients of the standard model. In the context of five-dimensional gauge theories on a circle with point interactions, we can produce the three generations, the mass hierarchy, the flavor structure and the CP phase from a single generation fermions with a gauge singlet scalar field. Tiny neutrino masses and the large mixing structure of the leptons are connected with each other.[1]

We consider a five-dimensional (5d) gauge theory on a circle with point interactions. The action for a fermion is given by [2]

$$S = \int d^4x \int_0^L dy\, \bar{\Psi}(x,y)(i\Gamma^M \partial_M + M_F)\Psi(x,y). \tag{1}$$

By introducing three point interactions, which are additions boundary points, we can impose boundary conditions for the fermion.

$$\Psi_L(x,y) = 0 \qquad \text{at positions of the point interactions} \tag{2}$$

or

$$\Psi_R(x,y) = 0 \qquad \text{at positions of the point interactions} \tag{3}$$

Under the boundary conditions, we can obtain three chiral zero modes,

$$\Psi(x,y) = \sum_{i=1}^{3} \psi_R^{(i)}(x) f_0^{(i)}(y) + \text{ (KK-particles)}, \qquad \text{for } \Psi_L = 0, \tag{4}$$

$$\Psi(x,y) = \sum_{i=1}^{3} \psi_L^{(i)}(x) g_0^{(i)}(y) + \text{ (KK-particles)}, \qquad \text{for } \Psi_R = 0. \tag{5}$$

The schematic figure of the mode functions $\{f_0^{(i)}(y)\}$ and $\{g_0^{(i)}(y)\}$ are depicted in Fig.1. Mode functions live in the different segment of the extra dimension.

[1] The collaborators of this work are Kenji Nisiwaki (Harish-Chandra Inst.), Makoto Sakamoto (Kobe Univ.) and Ryo Takahashi (Shimane Univ.).

[2] We adopt $\eta_{MN} = \mathrm{diag}(-1,1,1,1,1)$, $\Gamma^y \equiv -i\gamma_5$ and $\{\Gamma^M, \Gamma^N\} = -2\eta^{MN} 1_4$.

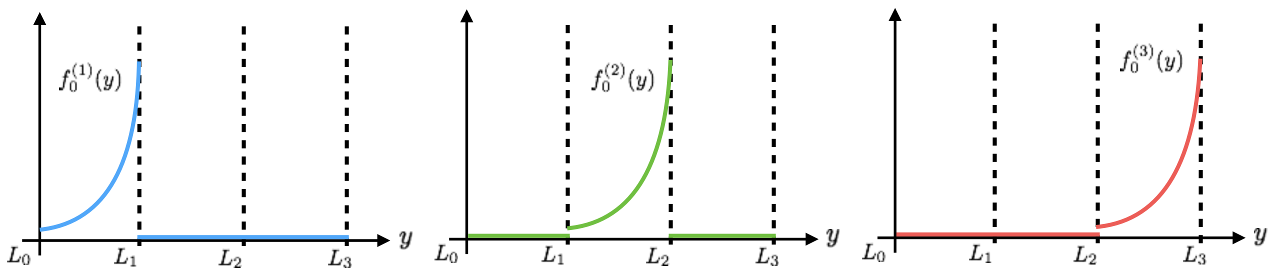

Figure 1: The schematic figure of $f_0^{(i)}(y)$ with $M_F > 0$.

In general, we can change the position of point interactions with respect to fermions, and then off diagonal components appears in a mass matrix. The mass matrix elements including off diagonal components can be expressed as

$$m_{ij} = \lambda^{(5)} \int_0^L dy \langle H(y)\rangle \langle \Phi(y)\rangle \mathcal{F}_0^{(i)}(y) f_0^{(j)}(y), \tag{6}$$

where $\langle H(y)\rangle, \langle \Phi(y)\rangle$ are the vacuum expectation value (VEV) of the Higgs doublet and the gauge singlet, and $\mathcal{F}_0^{(i)}(y)$ and $f_0^{(i)}$ are mode functions of SU(2) doublet fermion and singlet fermion, respectively. It comes from the geometry of the extra dimension that the large value of the bulk mass M_F makes diagonal elements small and off diagonal elements large. This implies that tiny neutrino masses (large quark masses) may be realized with a large (small) flavor mixing. We should note that in this set up, the system consists of single-generation fermions so that we need a new source of a CP phase. It was also found that under the twisted boundary conditions, the Higgs VEV possesses an extra-dimension coordinate-dependent phase,

$$\langle H(y)\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} e^{i\frac{\theta}{L}y}, \tag{7}$$

which provide us a CP phase[1]. The Robin boundary condition for the gauge singlet

$$\begin{cases} \Phi(0) + L_+ \partial_y \Phi(0) = 0, \\ \Phi(L) - L_- \partial_y \Phi(L) = 0, \end{cases} \qquad (-\infty \le L_\pm \le +\infty), \tag{8}$$

will lead a extra-dimensional coordinate-dependent VEV[2, 3],

$$\langle \Phi(y)\rangle \sim e^{\alpha(y-L)}, \tag{9}$$

which may provide a large mass hierarchy to us through the overlap integral eq.(6).

References

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