Dynamics of $\mathcal{N}=1$ gauge theories and M5-branes

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Introduction

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An important point is the relation with M-theory or 6d (2,0) theory:
a class of N=2 theories, so-called **class S theories**, is obtained by M5-branes on

$$\mathbb{R}^{1,3} \times$$

Riemann surface C

in $\mathbb{R}^{1,3} \times T^*C \times \mathbb{R}^3$

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- **enlarges** drastically the N=2 theory to a large class by associating the theory with the Riemann surface C

- **explains/proposes** S-duality of class S theories as a symmetry of the Riemann surface C

- **leads** to a remarkable relation between 4d N=2 theories and 2d CFT on the Riemann surface C.

[Alday-Gaiotto-Tachikawa]

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This theory is determined by the Seiberg-Witten curve, which is in this picture a curve in $(x, t) \in T^*C$:

$$x^N + \sum_k \phi_k(t) x^{N-k} = 0$$

where ϕ_k is k-th differential on C.

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elucidate dynamics of

***N*=1 gauge theories**

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➢ **how do we describe it in the M-theory?**

The M-theory picture proposes dualities of a wide variety of N=2 theories.

➢ ***N*=1 duality?** (e.g. Seiberg duality)

Confining phase

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At the same time, we have another curve, so-called **Dijkgraaf-Vafa curve**, which determines the gaugino condensate. [Dijkgraaf-Vafa 2002, Cachazo-Douglas-Seiberg-Witten]

Field theoretical observation

Naively, we may have two equations to describe the confining phase:

the Seiberg-Witten curve

another curve

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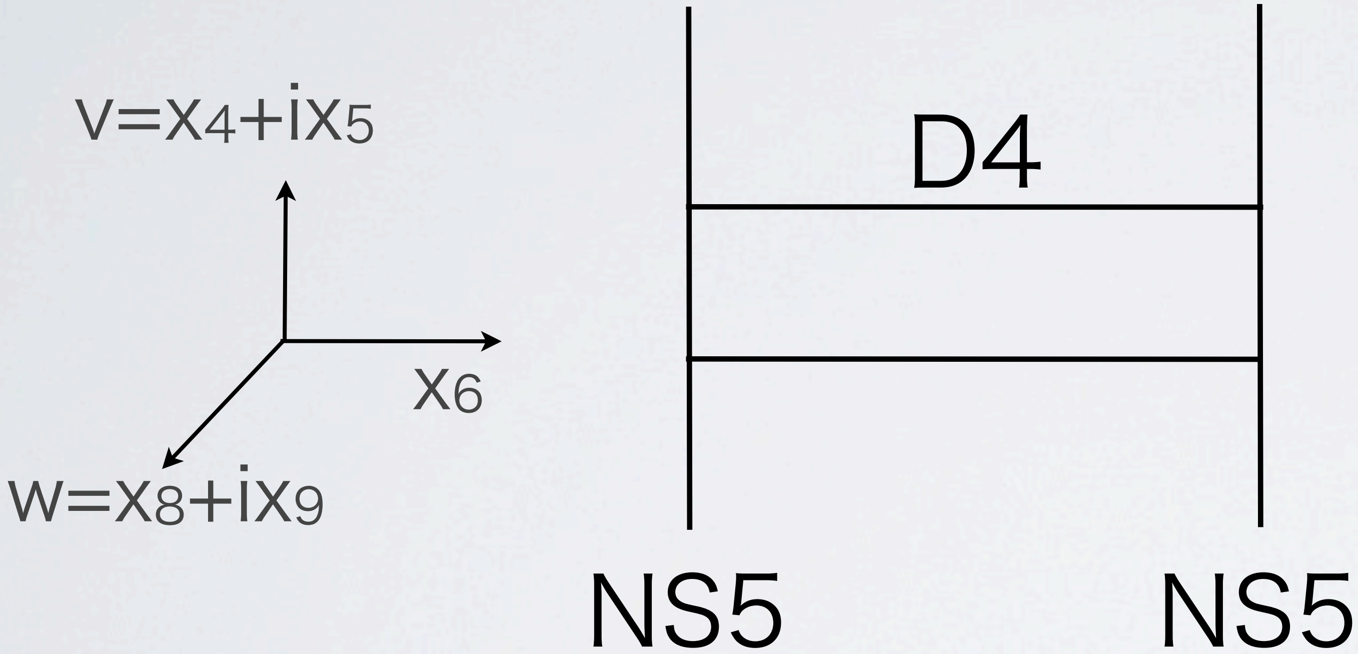
string theoretical observation



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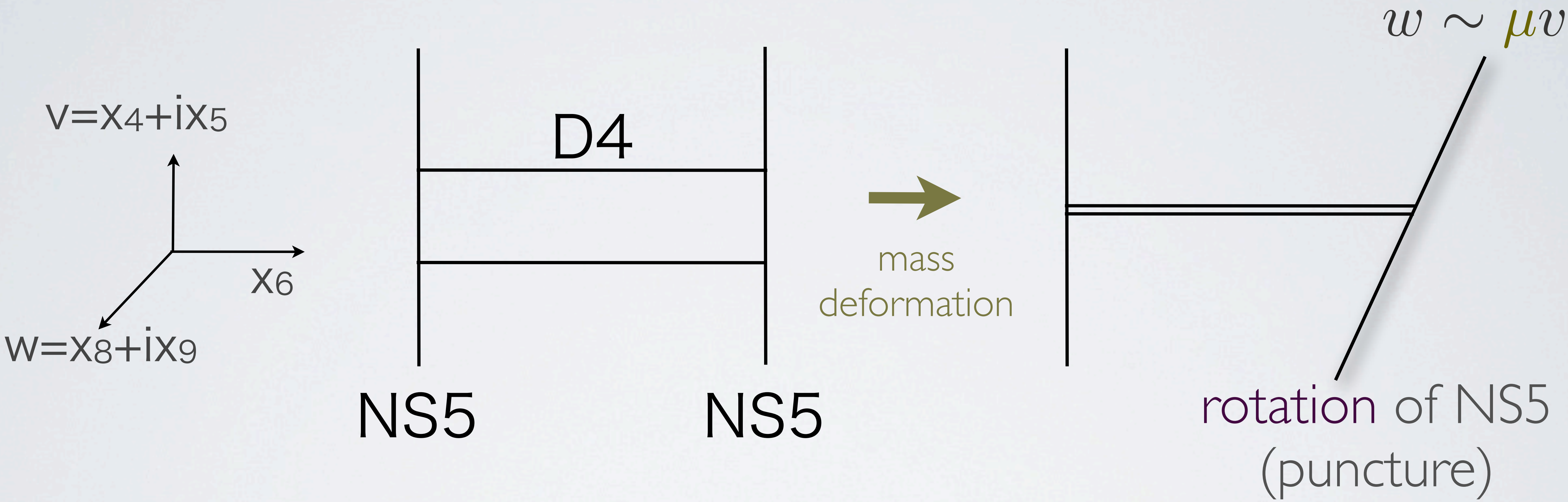

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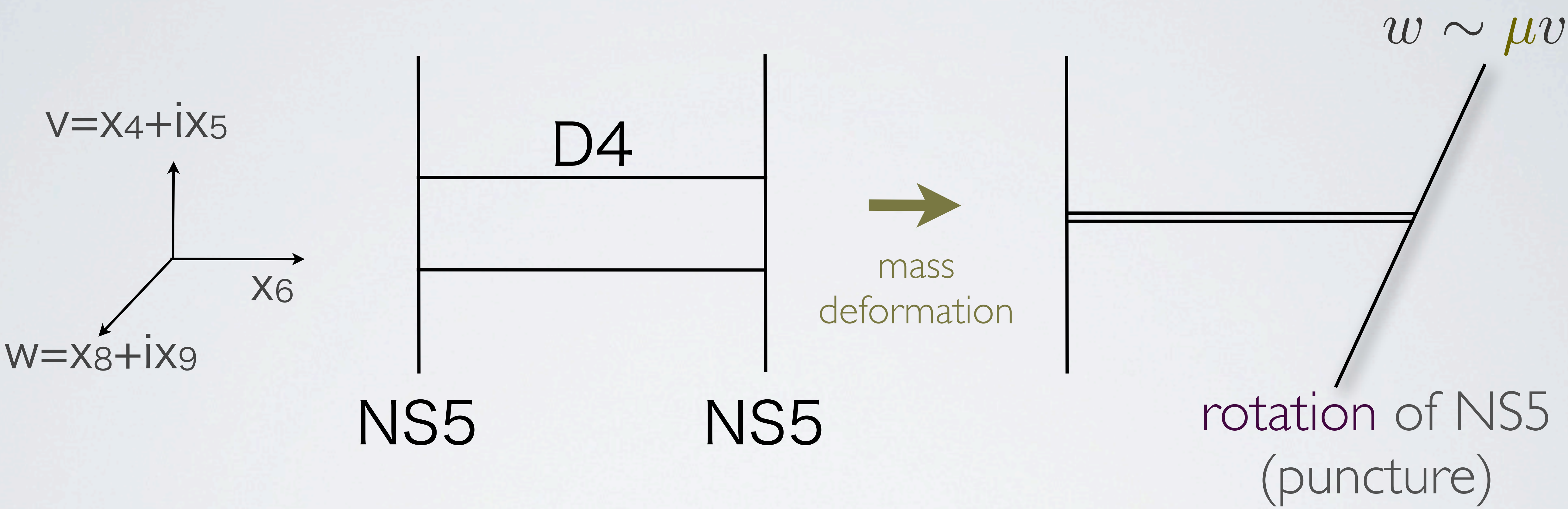

w-direction should enter **the second equation** describing the vacua of *N*=1 theory

[Hori-Ooguri-Oz ,Witten, de Boer-Oz]

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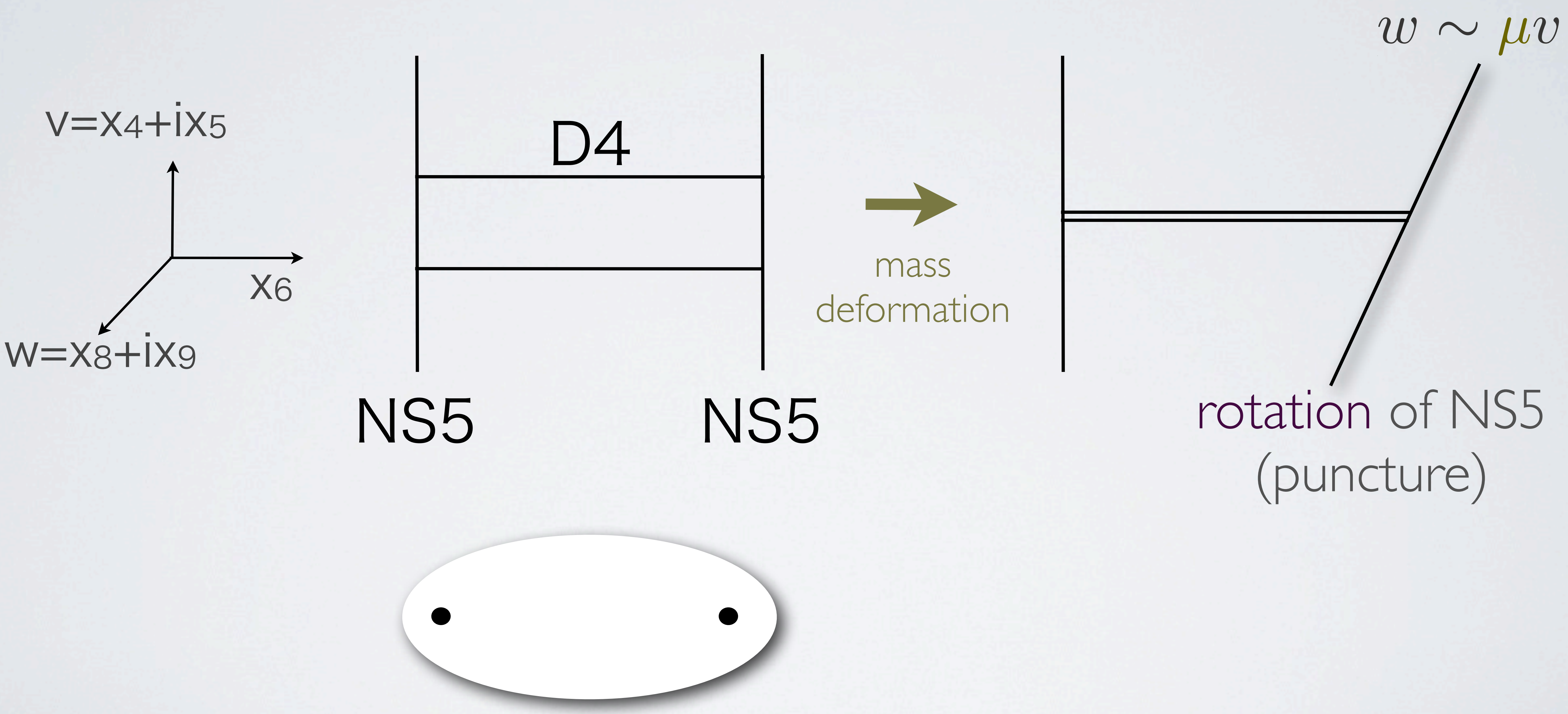

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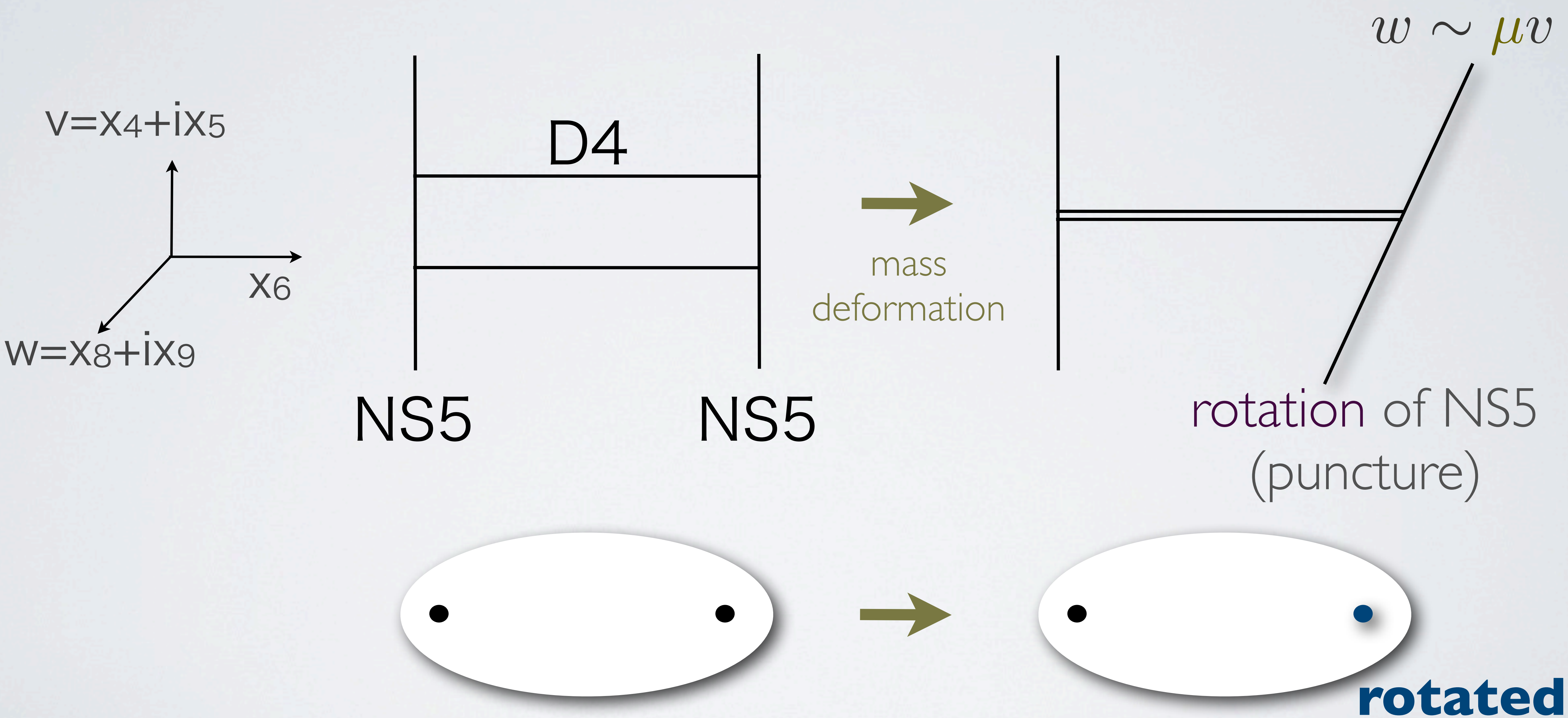

N=1 curve

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A combination of these equations gives **the Dijkgraaf-Vafa curve**, a curve in (v, w). $(v = xt)$

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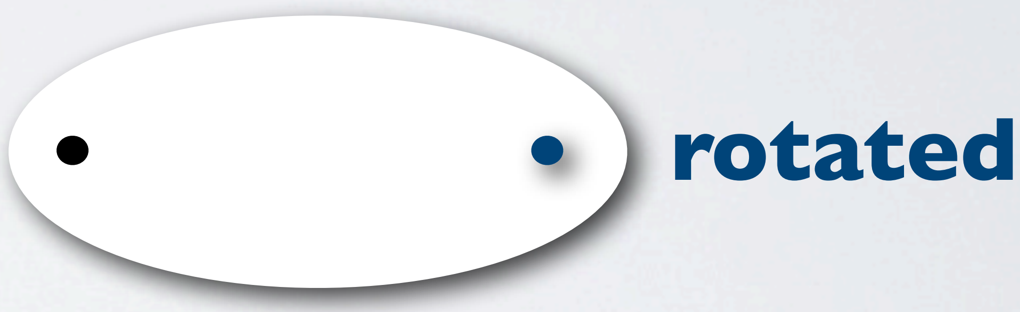

where Φ and φ are su(N)-valued differential and scalar on C, having prescribed singularity at the puncture.

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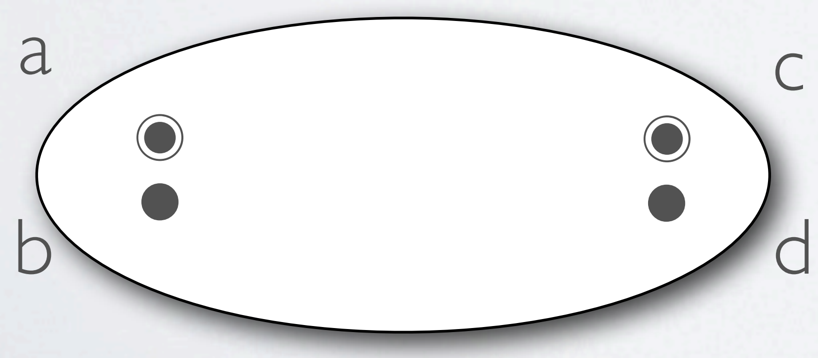

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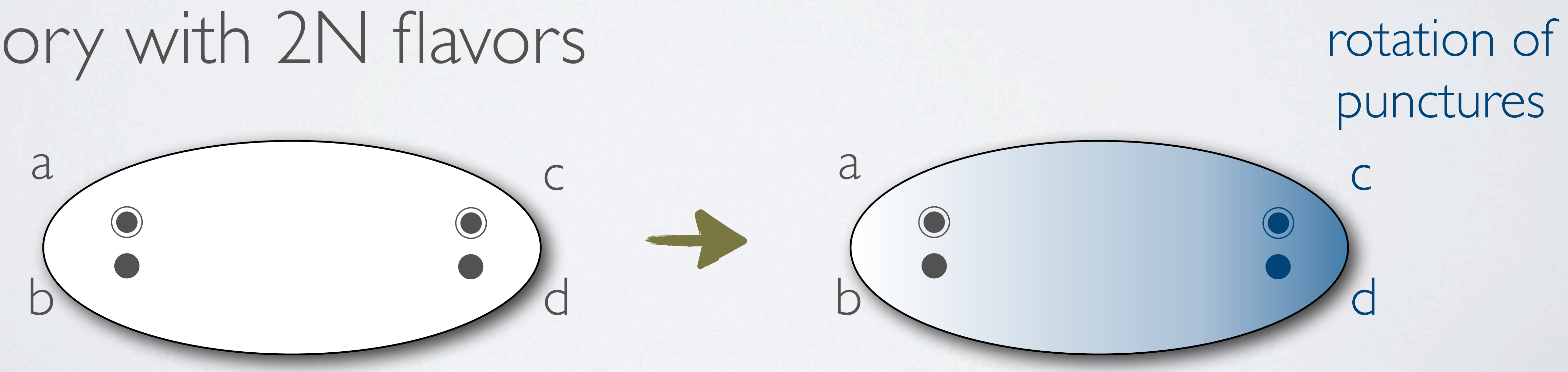
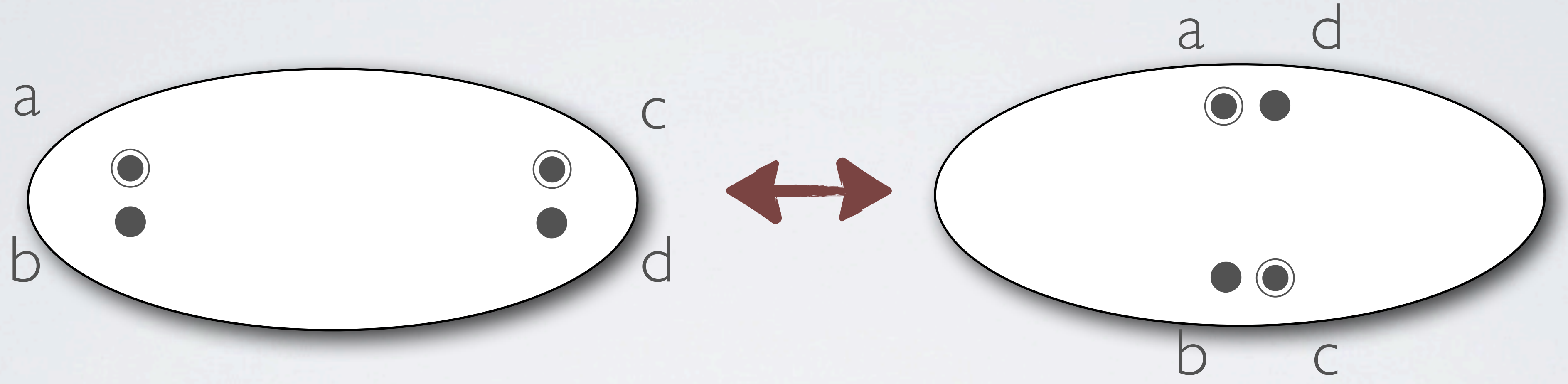


N=1 duality from *N*=2 duality

In *N*=2 theory, the S-duality is denoted by



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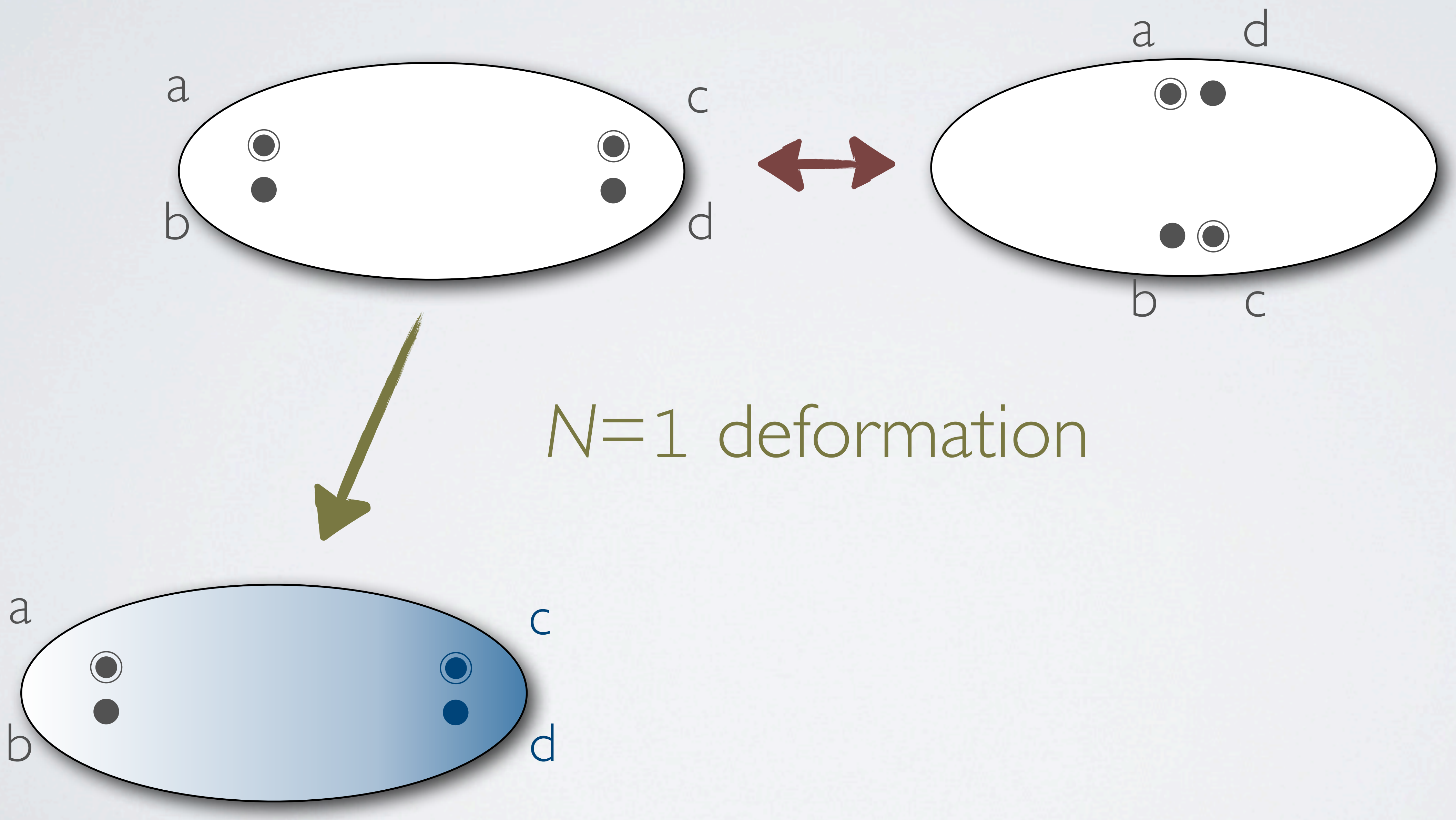

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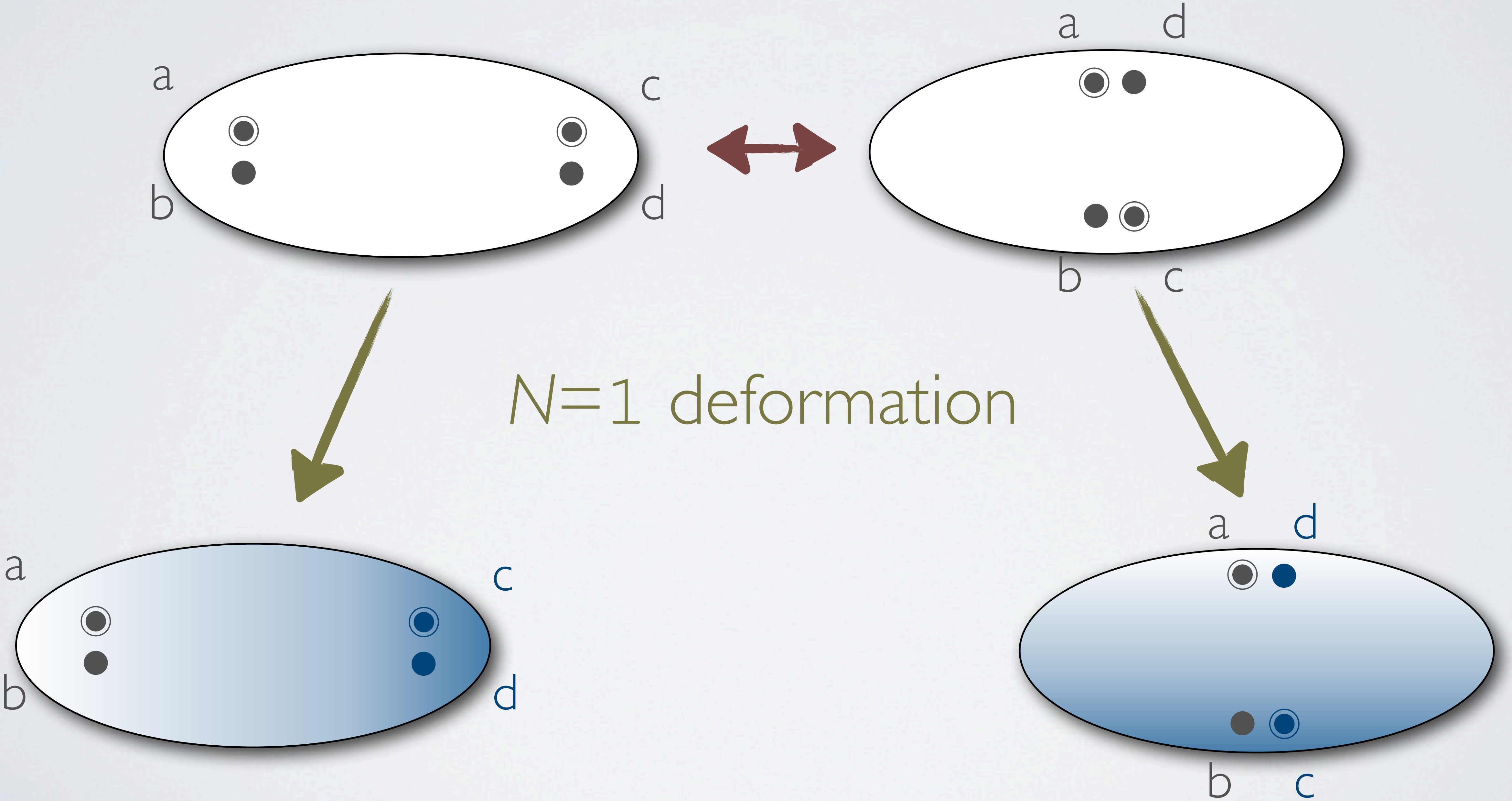

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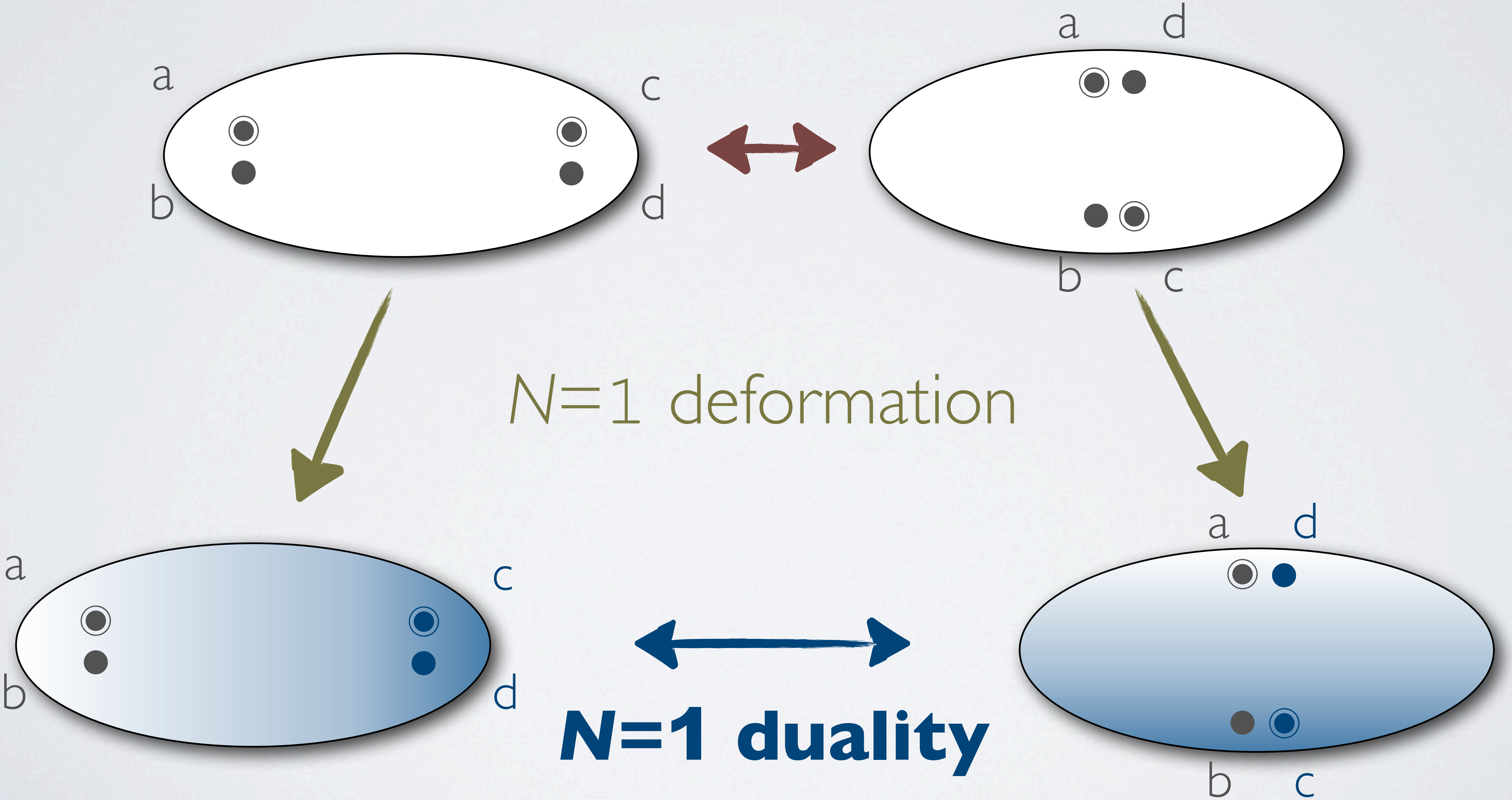

PLAN

- Review of $\mathcal{N}=2$ gauge theories in class S
- $\mathcal{N}=1$ theories in confining phase
- Superconformal phase and $\mathcal{N}=1$ dualities

N=2 gauge theories

in class S

N=2 theory from M-theory

Consider M-theory geometry

cotangent bundle on Riemann surface C

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We call the obtained N=2 theory as class S which is classified by N and C with punctures:

[Gaiotto, Gaiotto-Moore-Neitzke]

Physical meaning of the Riemann surface

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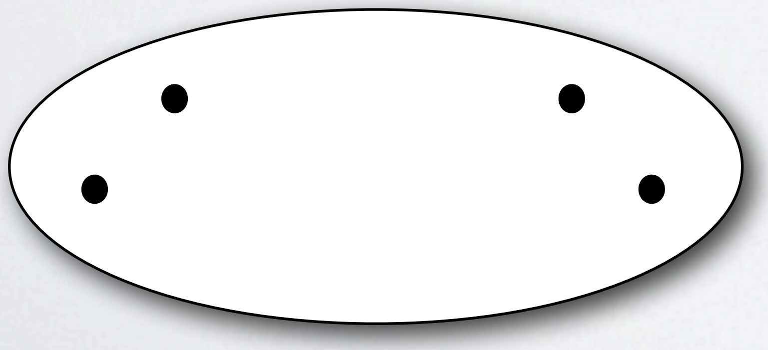
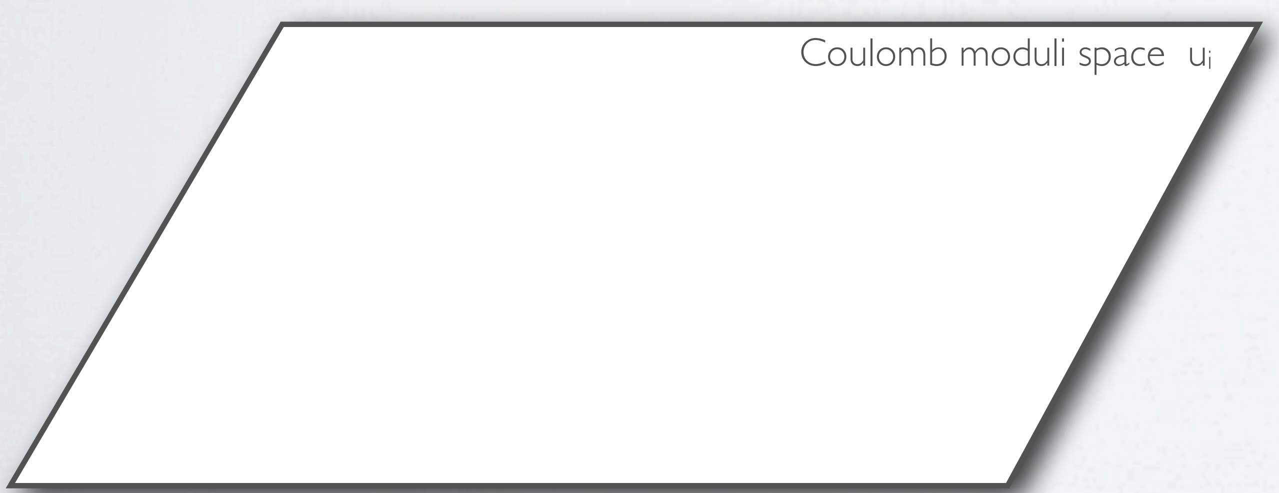


2 M5-branes on four-punctured sphere is **SU(2) theory with 4 flavors** with UV coupling constant q

Seiberg-Witten theory

The low energy effective theory on the Coulomb branch parametrized by $u_i = \langle \mathrm{tr} \phi^i \rangle$ can be solved by the Seiberg-Witten curve:



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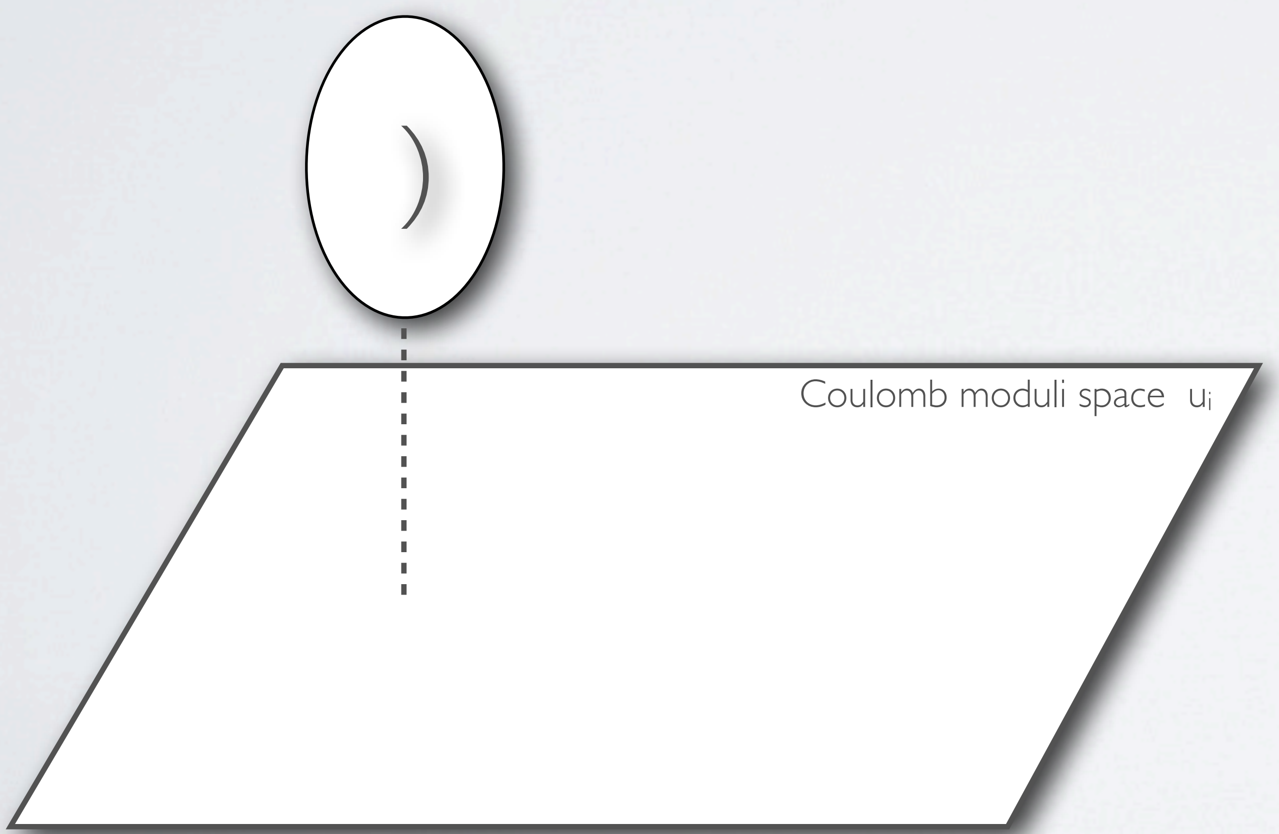

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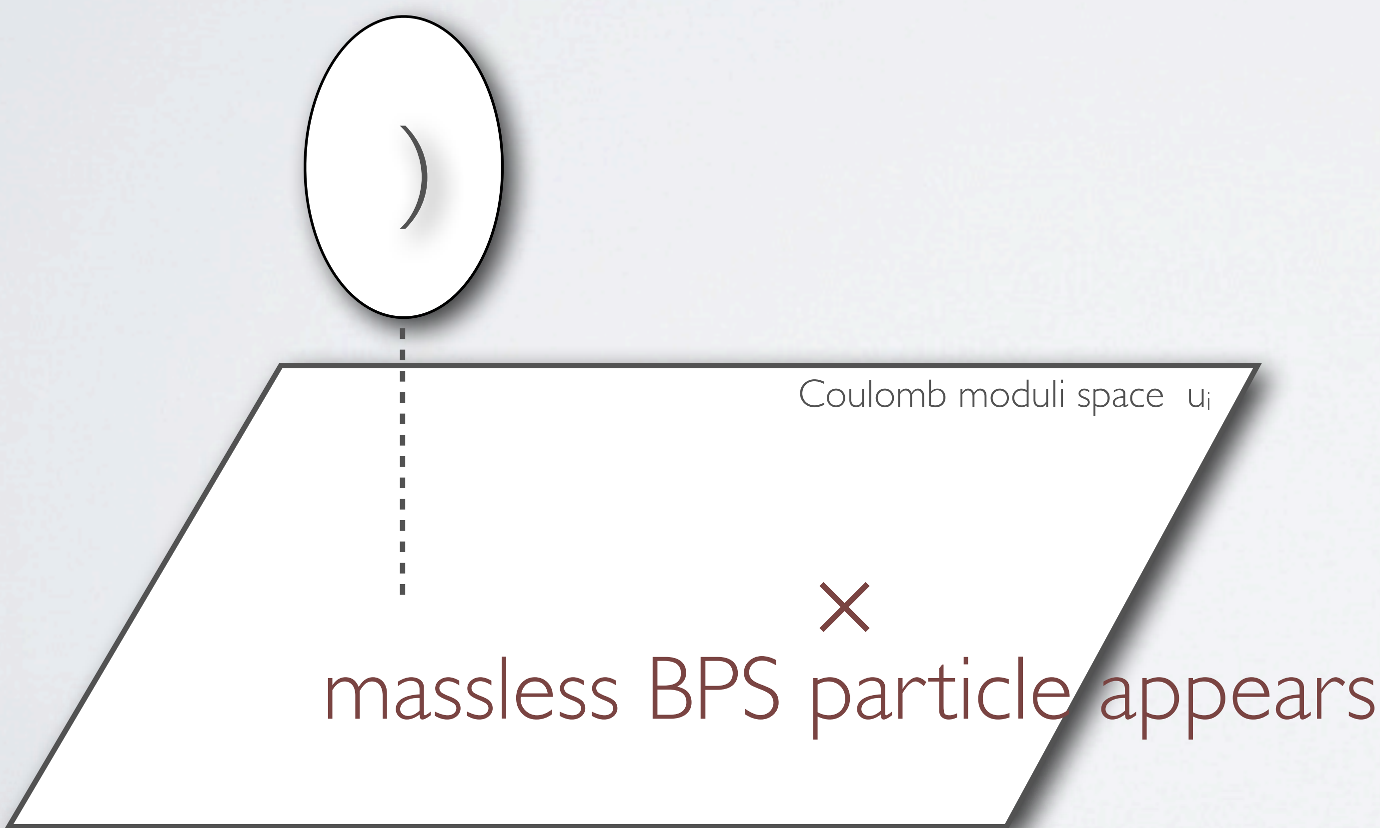

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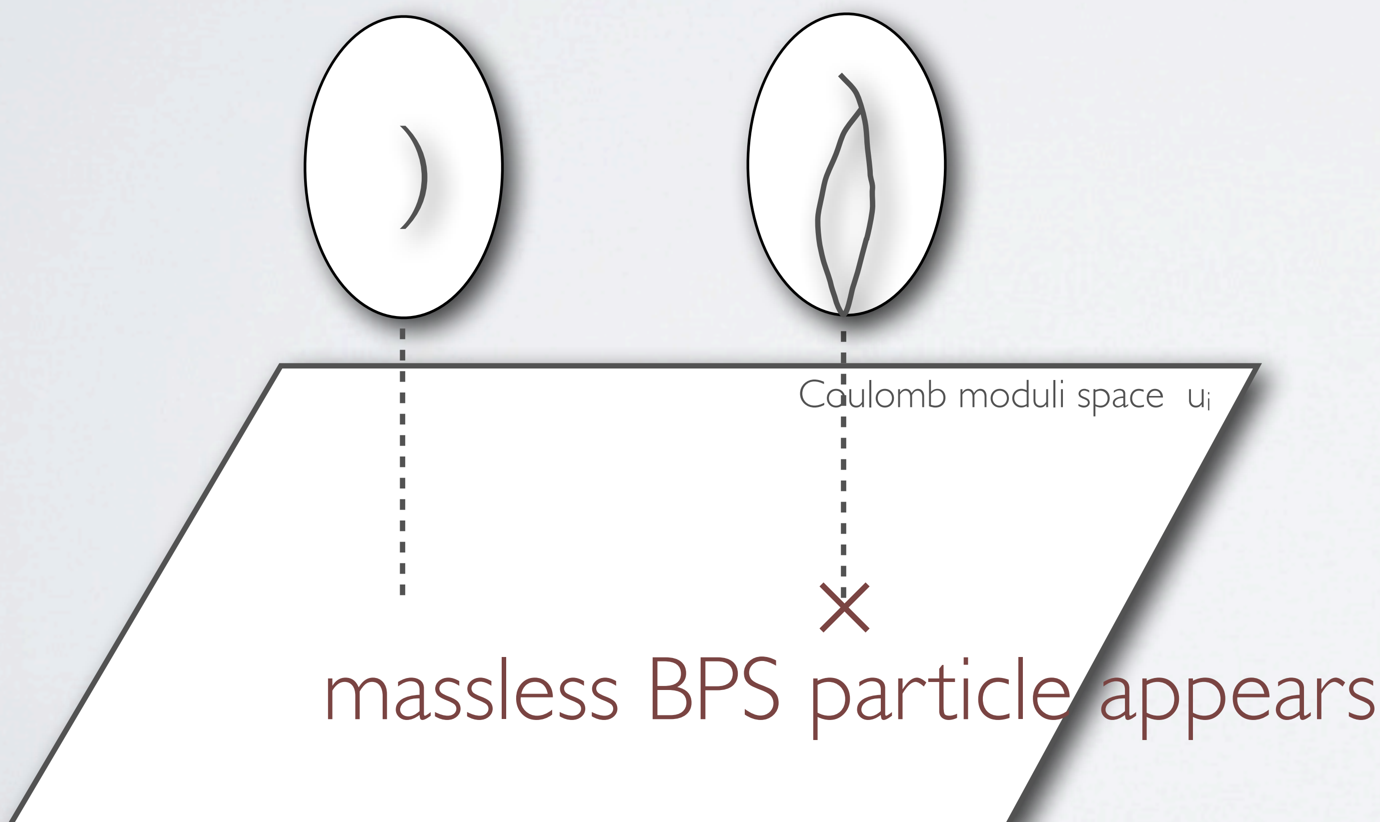

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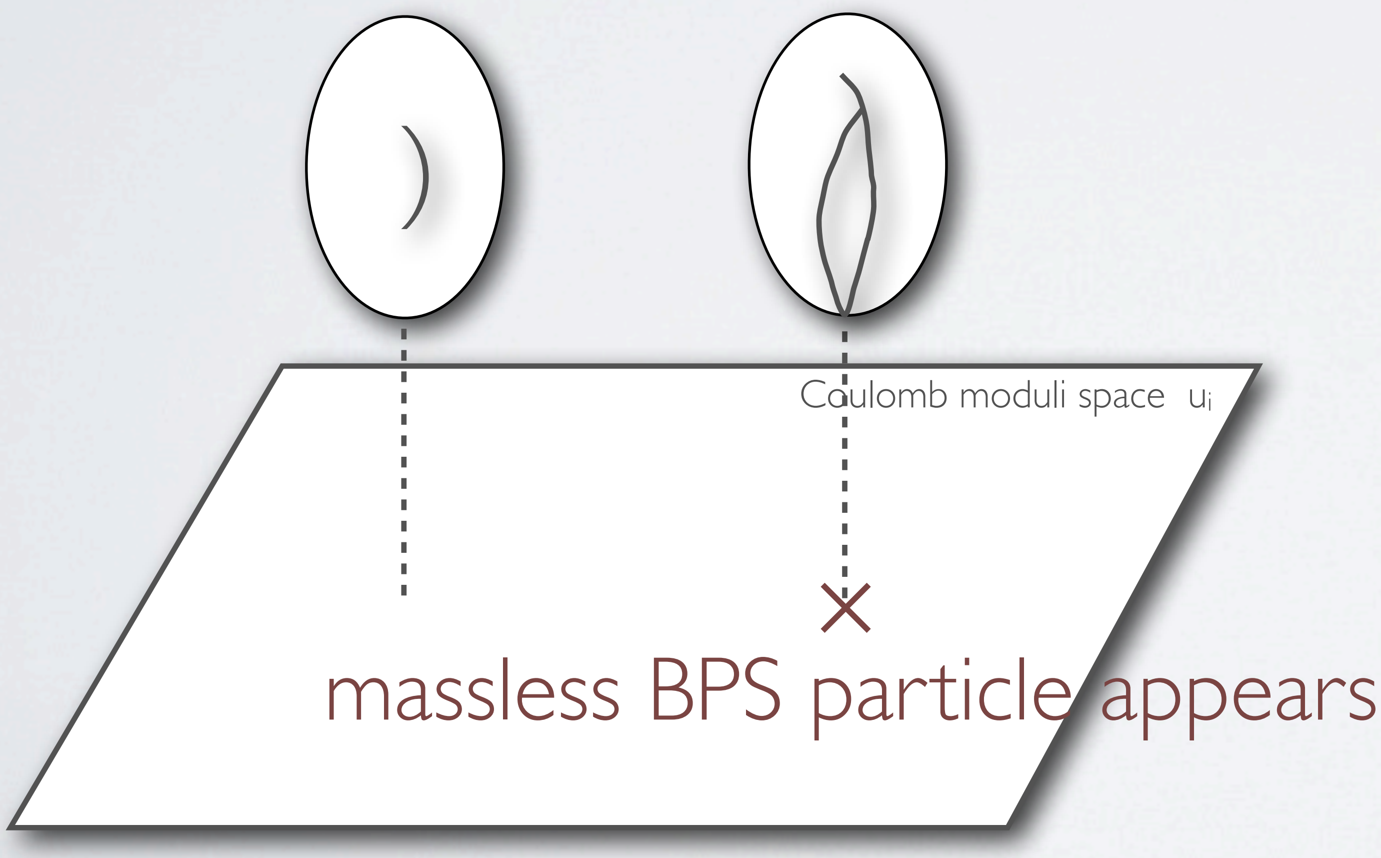

$$a_i = \int_{A_i} \lambda_{\mathrm{SW}}(u)$$

$$\frac{\partial \mathcal{F}}{\partial a_i} = \int_{B_i} \lambda_{\mathrm{SW}}(u)$$

Seiberg-Witten curve:
N-sheeted cover of C

The Seiberg-Witten curve is

$$x^N + \sum_{k=2}^{N} x^{N-k} \phi_k(t) = 0$$

ϕ_k is k-th meromorphic differential with poles at $t = t_a$ and has moduli which are identified with the Coulomb moduli.

with the differential: $\lambda_{\mathrm{SW}} = x dt$

Regular singularity and UV SCFT

Focus on the N=2 case the Seiberg-Witten curve is

$$x^2 = \phi_2(t)$$

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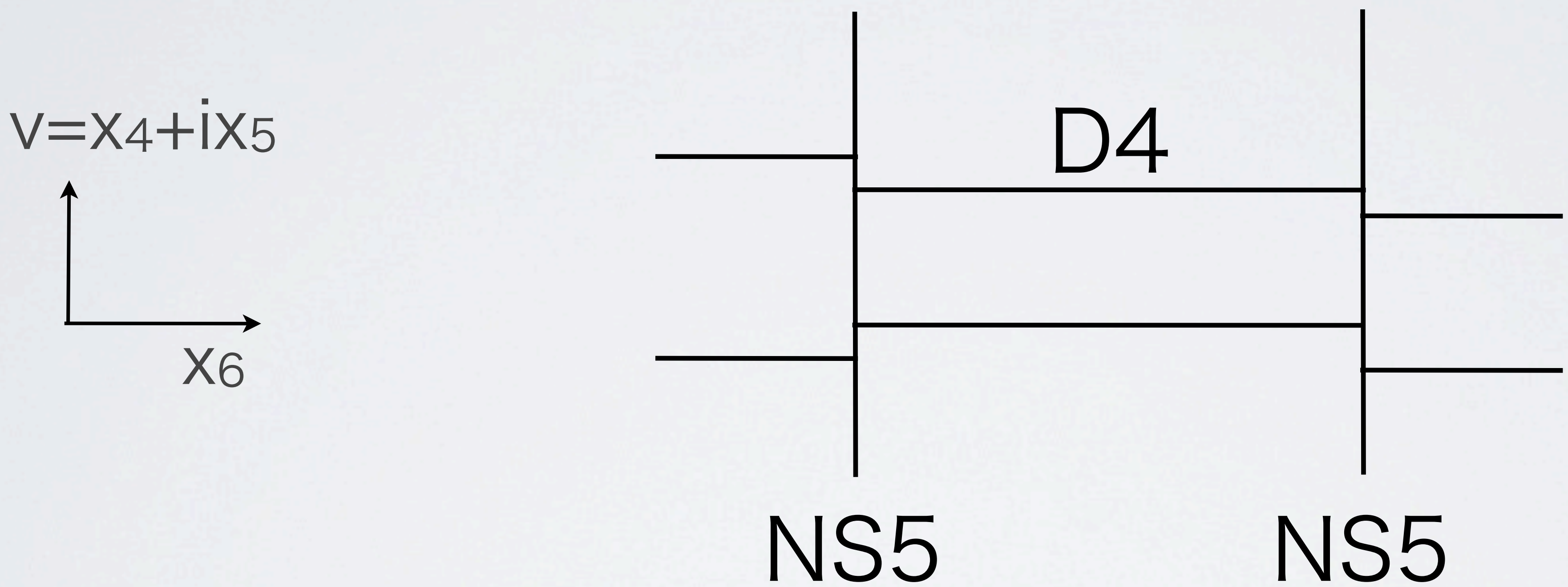
REGULAR puncture:

$$\phi_2(t) \sim \frac{m_a^2}{(t - t_a)^2} \quad \rightarrow \quad \lambda_{\rm SW} \sim \pm \frac{m_a}{t - t_a}$$

: SU(2) flavor symmetry

SU(2) w/ 4 flavors

Let us consider Type IIA construction of this theory



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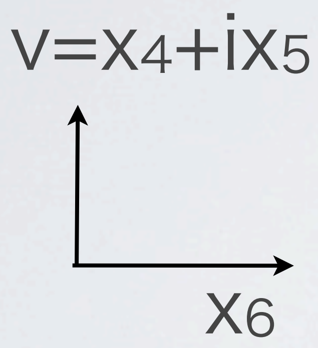

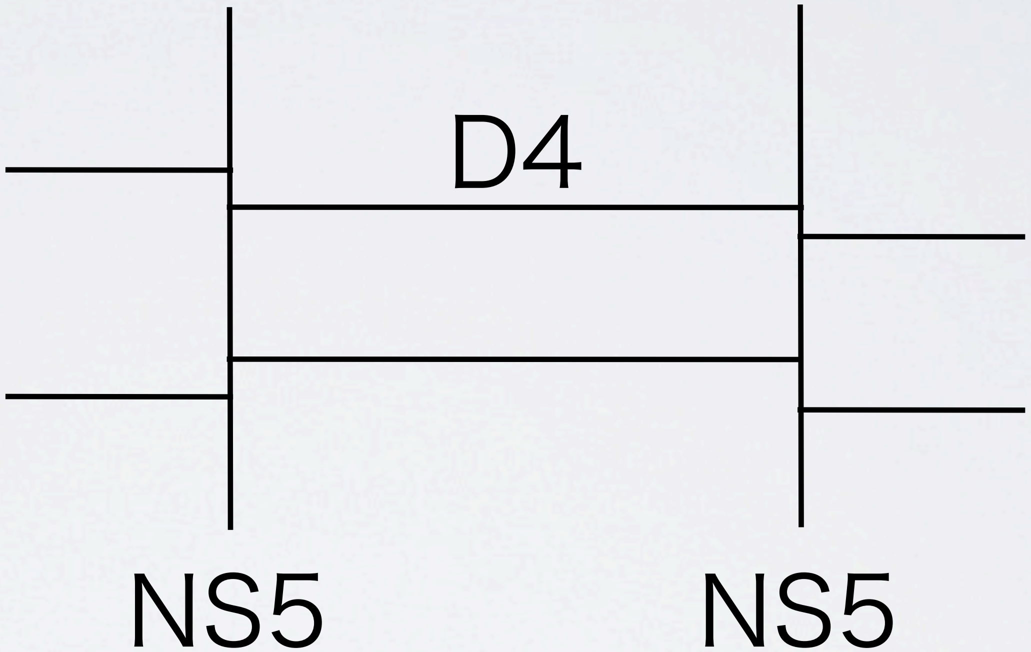

M-theory up-lift: add an S^1-direction parametrized by x_{10}

cylinder: x_6+ix_{10} → $t=e^{-(x6+ix10)}$

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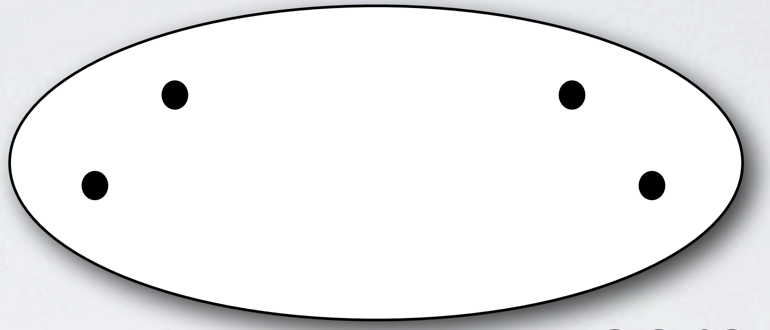
2 M5-branes on 

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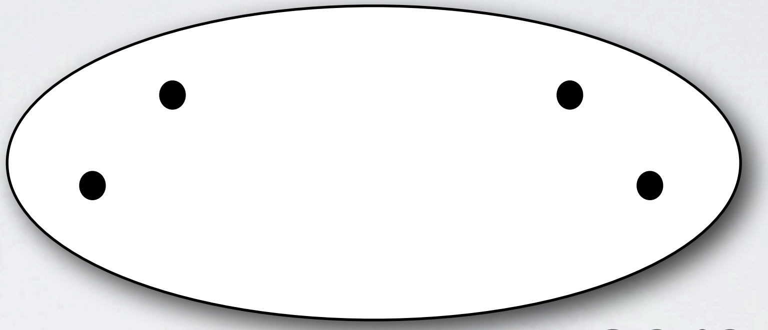
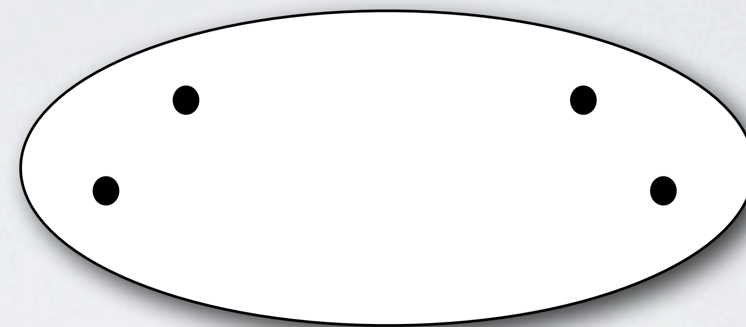


parametrized by t

The M-theory curve is

$$(v - m_1)(v - m_2)t^2 - (1 + q)P_2(v) + q(v - m_3)(v - m_4) = 0$$

SU(2) w/ 4 flavors

2 M5-branes on 

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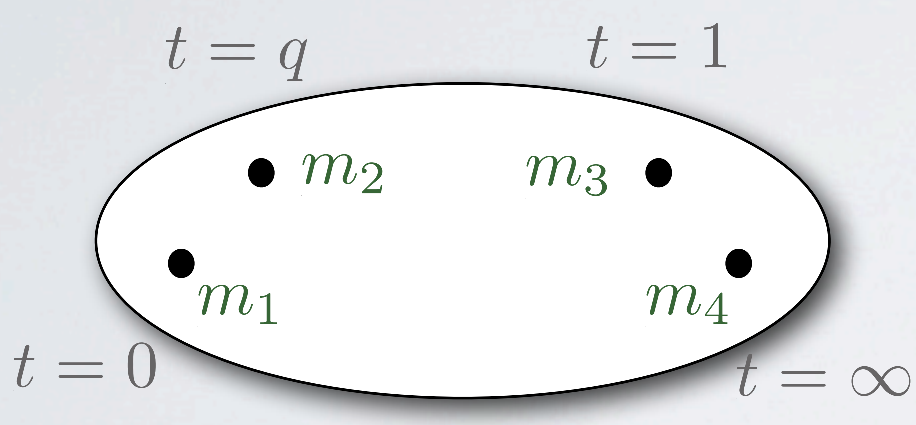
$$\longrightarrow \quad x^2 + \phi_2(t) = 0, \qquad \phi_2 \sim \frac{m_\alpha^2}{t - t_\alpha}$$

$$(x = v/t) \qquad\qquad t_\alpha = 0, q, 1, \infty$$

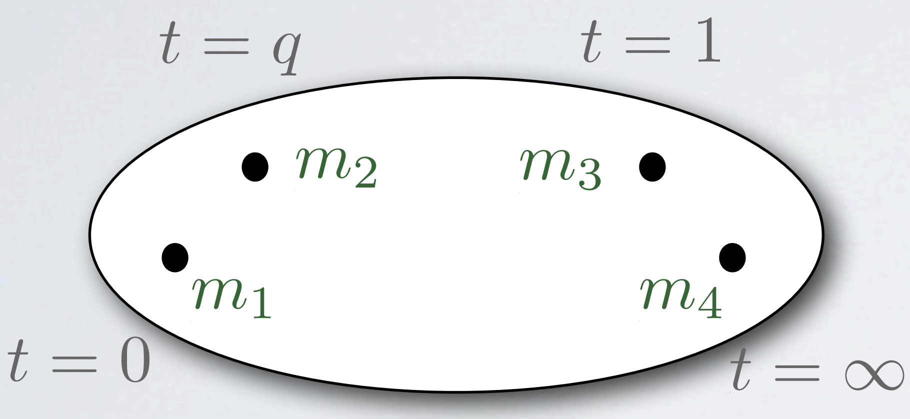
SU(2) w/ 4 flavors



q: gauge coupling constant of SU(2) $q = e^{2\pi i \tau}$

flavor symmetry: $\mathrm{SU}(2)^4 \subset \mathrm{SO}(8)$

SU(2) w/ 4 flavors



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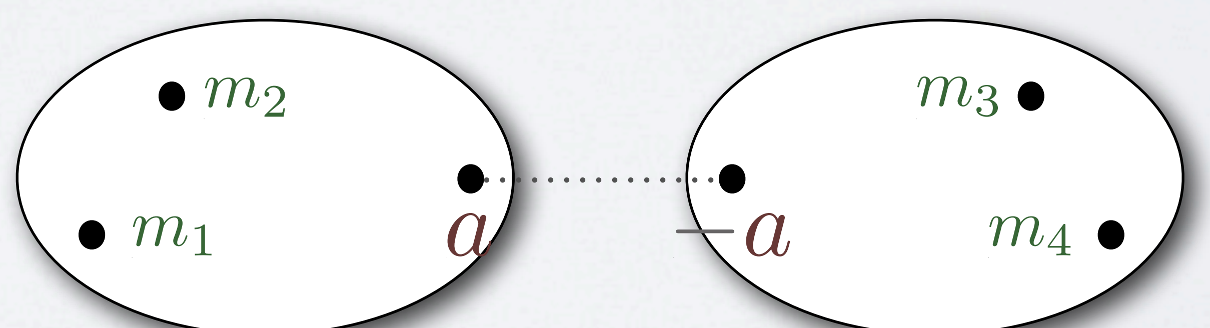
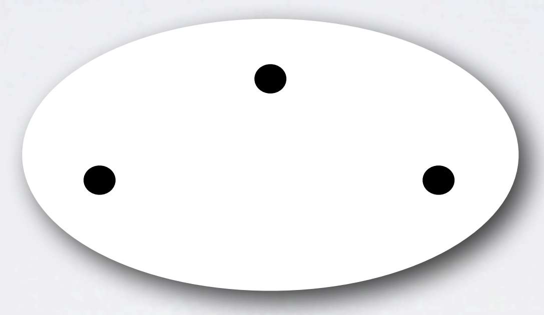
decoupling SU(2) $(\,q \to 0\,)$



Building block

The three-punctured sphere is a building block for the *N*=2 theories:



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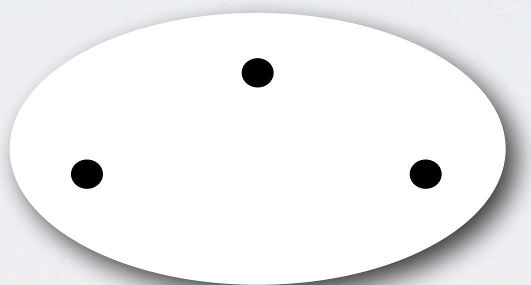

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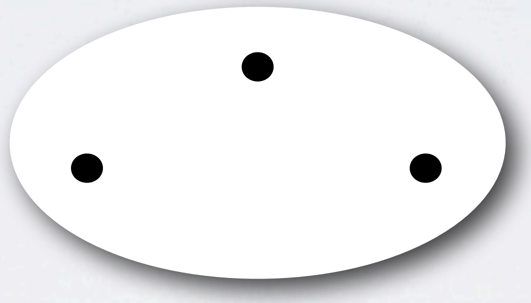
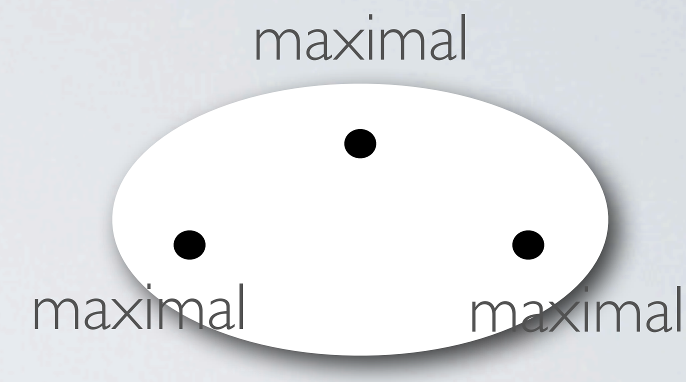


SU(2): **free** hypermultiplets in tri-fundamental representation of $SU(2)^3$

SU(N): **non-trivial SCFTs** with flavor symmetry associated with the punctures

T_N theory



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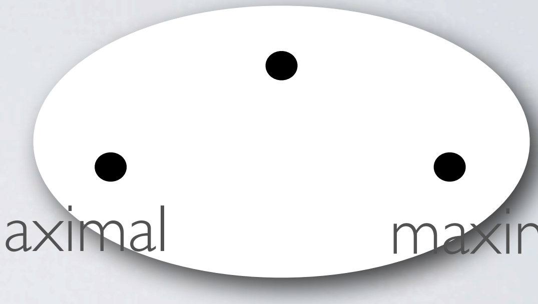

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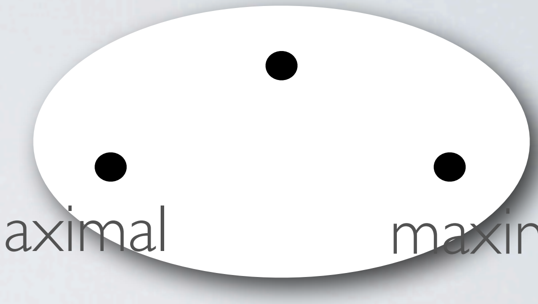

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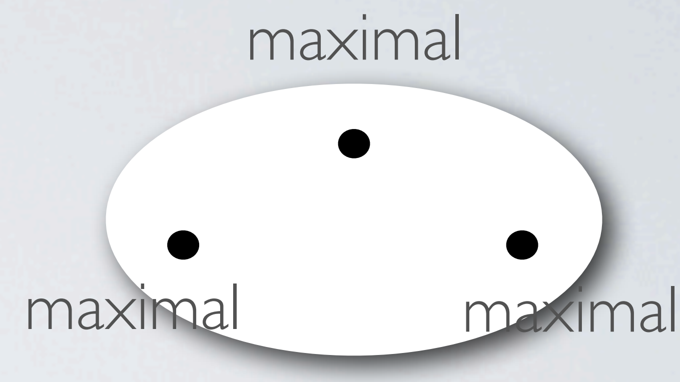
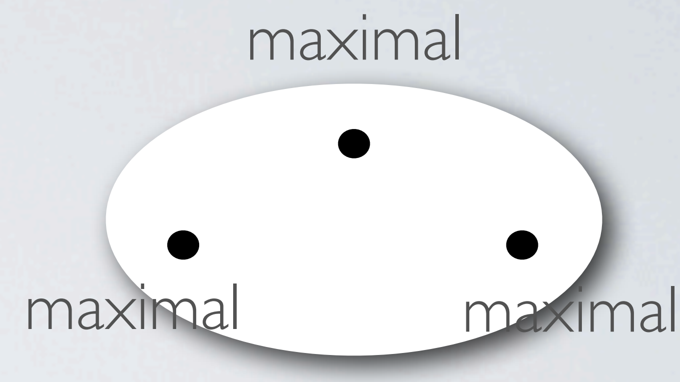


T_N theory is a 4d N=2 SCFT

- obtained by 6d (2,0) theory on a sphere with three maximal punctures
- with $SU(N)^3$ flavor symmetry
- we know (a part of) spectrum of chiral operators

[Gaiotto-Maldacena, Gadde-Lastelli-Razamat-Yan, KM-Tachikawa-Yonekura-Yan]

T_N theory



T_N theory is a 4d N=2 SCFT

- obtained by 6d (2,0) theory on a sphere with three maximal punctures
- with $SU(N)^3$ flavor symmetry
- we know (a part of) spectrum of chiral operators

  [Gaiotto-Maldacena, Gadde-Lastelli-Razamat-Yan, KM-Tachikawa-Yonekura-Yan]
- **we do not know Lagrangian** except for the N=2 case

Asymptotically free theory

The N=2 theory with asymptotically-free gauge group is obtained by considering **irregular puncture**

[Gaiotto-Moore-Neitzke]

Asymptotically free theory

The *N*=2 theory with asymptotically-free gauge group is obtained by considering **irregular puncture**

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For the N=2 case,

$$\phi_2(t) \sim \frac{c_a}{(t-t_a)^n} \qquad (n>2)$$

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[Gaiotto-Moore-Neitzke]

For the N=2 case,

$$\phi_2(t) \sim \frac{c_a}{(t-t_a)^n} \qquad (n>2)$$

n=3; trivial,
n=4; free hypermultiplets in the doublet of SU(2)
n>4; nontrivial SCFTs of Argyres-Douglas type

[Cecotti-Neitzke-Vafa, Cecotti-Vafa, Bonelli-KM-Tanzini]

SU(2) SYM theory

As an example, let us consider *N*=2 SU(2) SYM theory.
The Seiberg-Witten curve is

$$x^2 = \phi_2, \qquad \phi_2 = \frac{\Lambda^2}{t} + \frac{u}{t^2} + \frac{\Lambda^2}{t^3}$$

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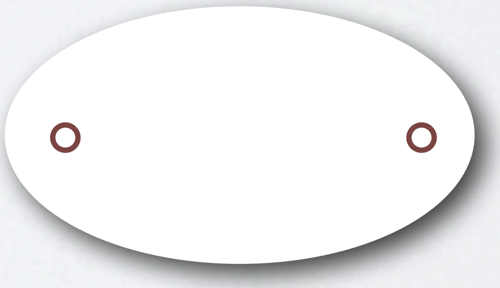
This has punctures at t=0 and ∞ with **irregular** behavior:

$$\phi_2 \sim \frac{1}{t^3}$$



$\mathcal{N}$=1 theories

in confining phase

N=1 deformation

We want to consider *N*=1 deformations of class S theories by adding the adjoint chiral mass terms

$$\sum_i \mu_i \mathrm{Tr}\phi_i^2$$

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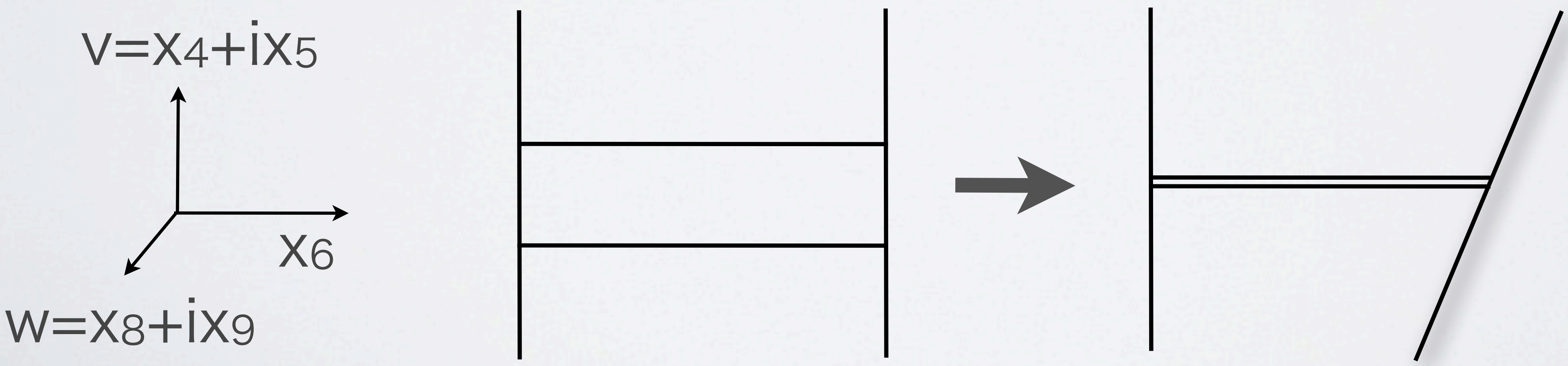
In introduction, we saw that N=1 deformation corresponds to the rotation of an NS5-brane:



N=1 deformation

=

rotation of puncture

to w-direction

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=

rotation of puncture

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$$x^N + \sum_k \phi_k(t) x^{N-k} = 0$$

N=1 deformation

=

rotation of puncture

to w-direction

$$x^N + \sum_k \phi_k(t) x^{N-k} = 0$$

$$w^N + \sum_k V_k(t) w^{N-k} = 0$$

N=1 deformation

=

rotation of puncture

to w-direction

$$x^N + \sum_k \phi_k(t) x^{N-k} = 0$$

$$w^N + \sum_k V_k(t) w^{N-k} = 0$$

At punctures, V_k has singularity determined by $w \sim \mu_i x t$

Proposal

A generalization of the Hitchin system with two commuting su(N)-valued fields Φ and φ .

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The spectral curve consists of

$$\det(x \cdot \mathbf{1} - \Phi) = 0$$
$$\det(w \cdot \mathbf{1} - \varphi) = 0$$
$$\det(xw \cdot \mathbf{1} - \Phi\varphi) = 0$$

where Φ and φ have prescribed singularities at the punctures of the Riemann surface.

Rank one case

Let us consider the case with **two** M5-branes wrapped on C. The curve is simply

$$x^2 = \phi_2(t), \qquad \phi_2\text{: meromorphic differential}$$

$$w^2 = V_2(t), \qquad V_2\text{: meromorphic function}$$

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$$\longrightarrow \quad x^2 = F(t)^2 w^2$$

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Let us consider the N=1 deformation of N=2 SU(2) SYM theory (V, ϕ) by

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At energy below the mass scale |μ|, the theory is *N*=1 pure SYM theory describing gluino condensation in the IR

$$\langle \lambda_\alpha \lambda^\alpha \rangle = \Lambda^3_{\mathcal{N}=1}$$

SU(2) SYM theory

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No other singularity of (meromorphic function) V_2

$$w^2 = \frac{\mu^2 \Lambda^2}{t} + a$$

a: unknown constant

SU(2) SYM theory

The condition from the commuting fields reads

$$\frac{w^2}{v^2} = F^2(t) = \frac{at + (\mu\Lambda)^2}{\Lambda^2 t^2 + ut + \Lambda^2}$$

SU(2) SYM theory

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$$\frac{w^2}{v^2} = F^2(t) = \frac{at + (\mu\Lambda)^2}{\Lambda^2 t^2 + ut + \Lambda^2}$$

→ This gives $a = 0$ and $u = \pm 2\Lambda^2$

Namely, **the *N*=1 curve** is simply

$$w^2 = \frac{\mu^2\Lambda^2}{t}$$

SU(2) SYM theory

These are indeed right values as follows:

❶ $u = \pm 2\Lambda^2$ are the loci on the Coulomb branch where the massless monopole or dyon appears.

By the mass deformation, the supersymmetric vacua are only these points.

[Seiberg-Witten]

SU(2) SYM theory

❷ by eliminating t, we get from the two equations:

$$w^2 - W'(v)w \pm \mu^2\Lambda^2 = 0 \qquad (W'(v) = \mu v)$$

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This is the curve obtained from **the matrix model** [Dijkgraaf-Vafa 2002], or from **the Konishi anomaly equation** [Cachazo-Douglas-Seiberg-Witten]

$$w^2 - W'(v)w + \mu S = 0$$

with $S \equiv \langle \lambda_\alpha \lambda^\alpha \rangle = \pm\mu\Lambda^2 = \pm\Lambda^3_{\mathcal{N}=1}$

In summary,

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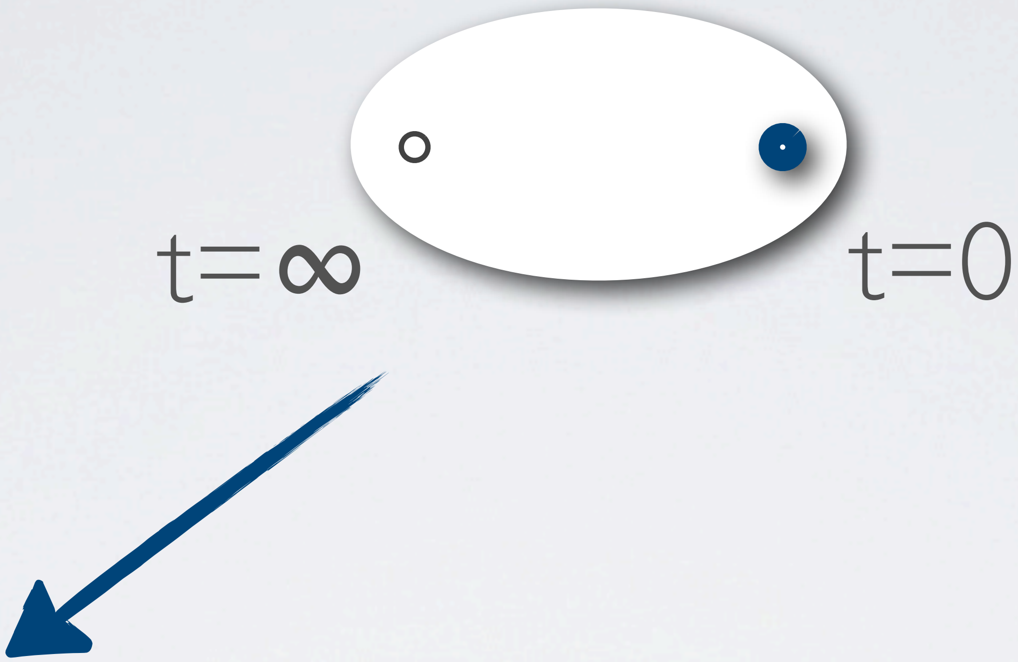

Seiberg-Witten curve

$$x^2 = \phi_2(t),$$

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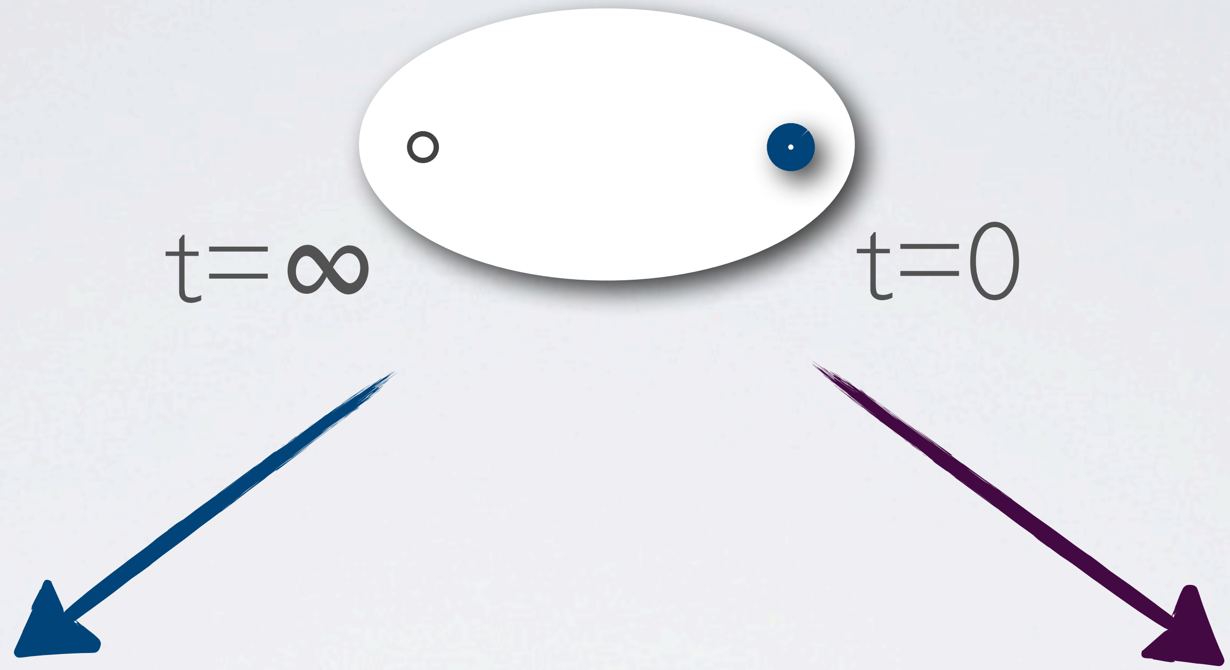

Seiberg-Witten curve

$$x^2 = \phi_2(t),$$

the quadratic differential ϕ_2 is singular at t=0 and ∞

***N*=1 curve**

$$w^2 = V_2(t),$$

the meromorphic function V_2 is singular only at t=0

Application to other cases

This method can be applied to other SU(2) gauge theories, e.g. SU(2)xSU(2) quiver gauge theory etc.

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For higher rank theory (with SU(N) gauge group), there is no systematic way to solve the model, because there is no easy expression denoting the two commuting fields.

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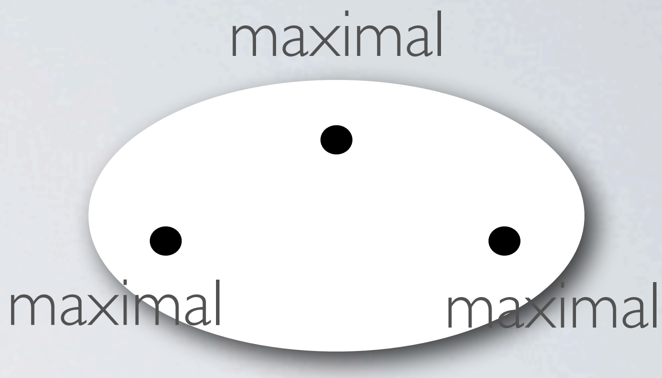
For higher rank theory (with SU(N) gauge group), there is no systematic way to solve the model, because there is no easy expression denoting the two commuting fields.

But still we can solve case by case, e.g. SU(N) SYM theory, the T_N theory coupled to SU(N) gauge group etc.

Superconformal phase and $\mathcal{N}$=1 dualities

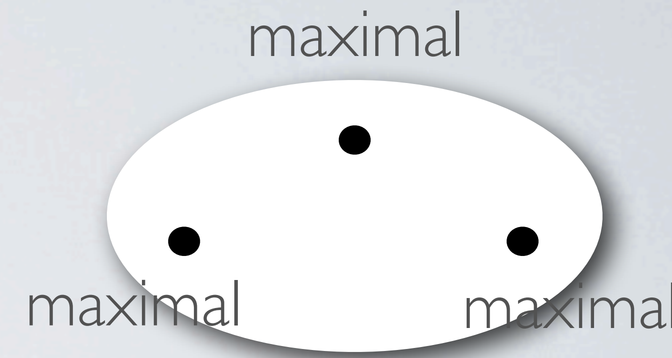
T_N theory



Global symmetry: $SU(N)^3 \times U(1)_R \times U(1)_J$

$$R = R_{\mathcal{N}=2}/2 + I_3, \quad J = R_{\mathcal{N}=2} - 2\, I_3, \qquad I_3 \subset SU(2)$$

T_N theory



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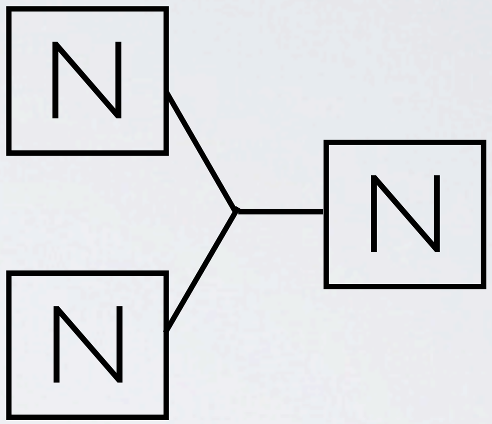
Chiral primary operators:

	$SU(N)_A$	$SU(N)_B$	$SU(N)_C$	$U(1)_R$	$U(1)_J$
μ_A	adj			1	-2
μ_B		adj		1	-2
μ_C			adj	1	-2

$$\mathrm{tr}\mu_A^k = \mathrm{tr}\mu_B^k = \mathrm{tr}\mu_C^k$$

T_N theories coupled to $\mathcal{N}=1$ vector multiplet

Let us denote the T_N theory as



T_N theories coupled to N=1 vector multiplet

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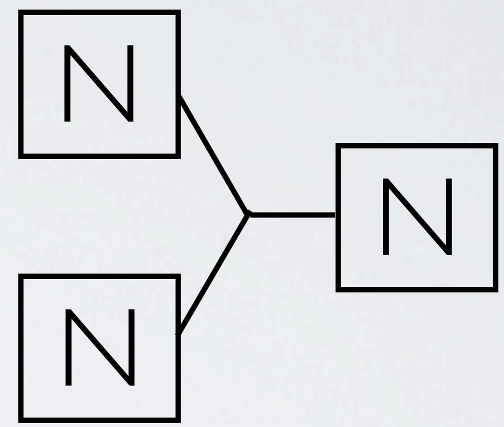
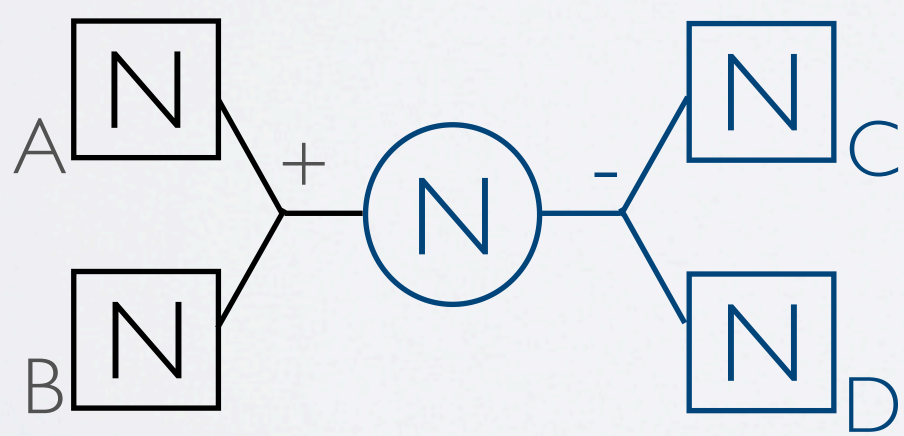


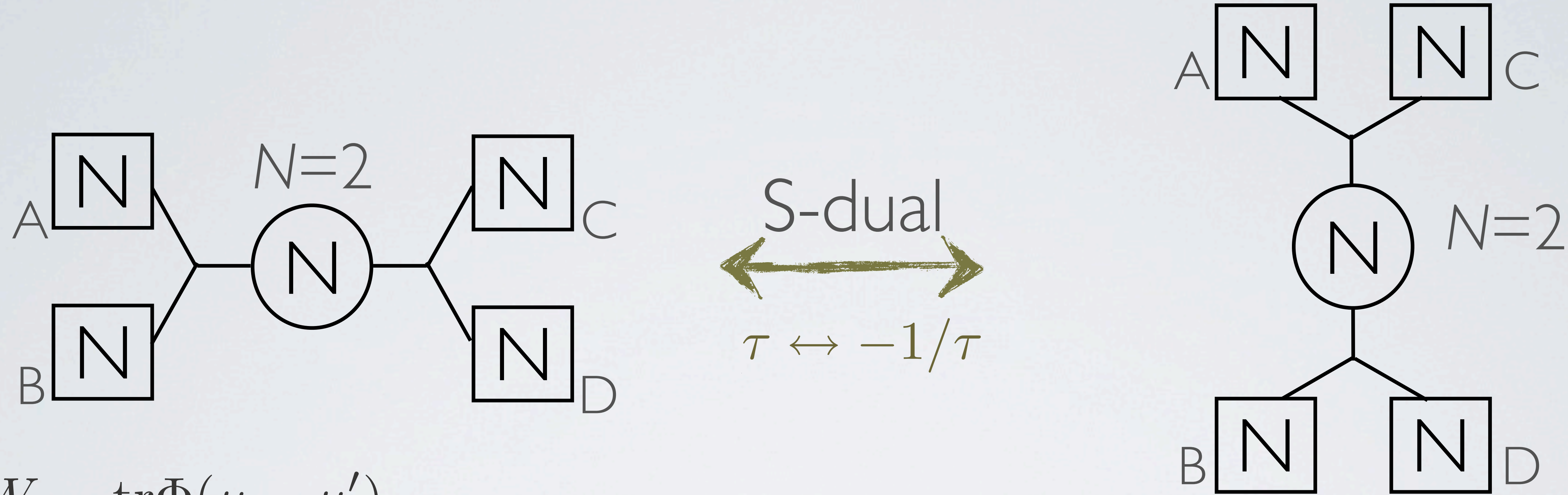
We couple a pair of the T_N theories to N=1 vector multiplet:

[Benini-Tachikawa-Wecht]



$$\mathcal{F} = J_1 - J_2$$

N=1 duality



$$W = \mathrm{tr}\Phi(\mu - \mu')$$

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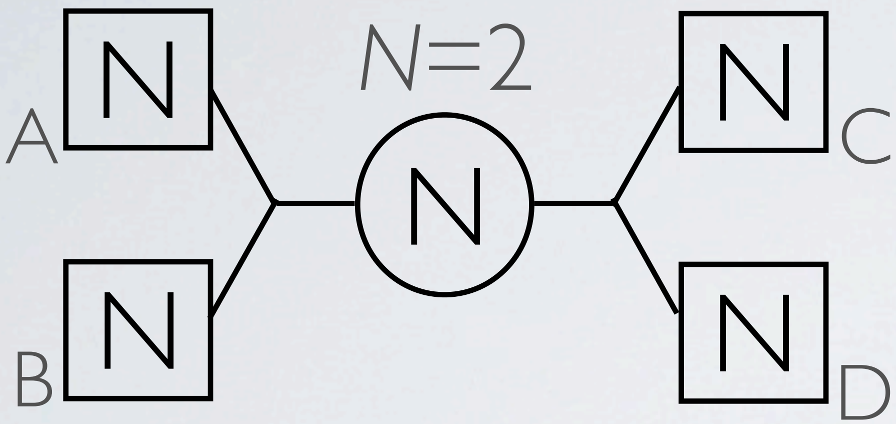
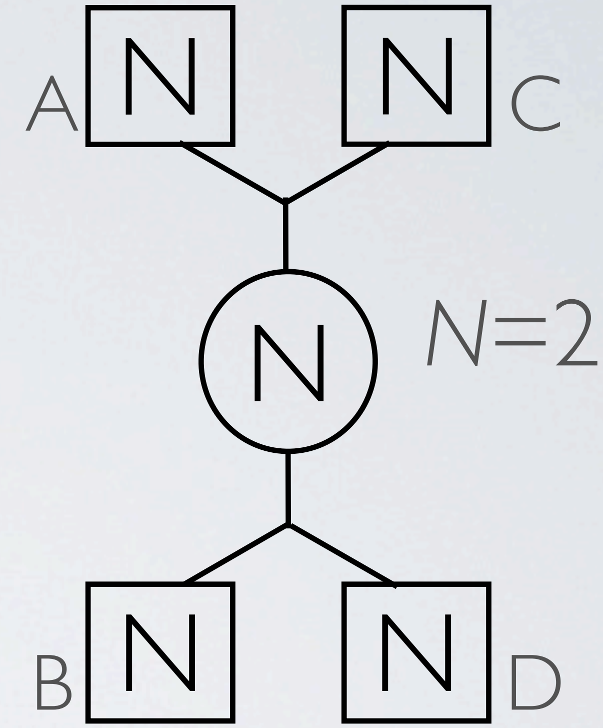


S-dual

$\tau \leftrightarrow -1/\tau$

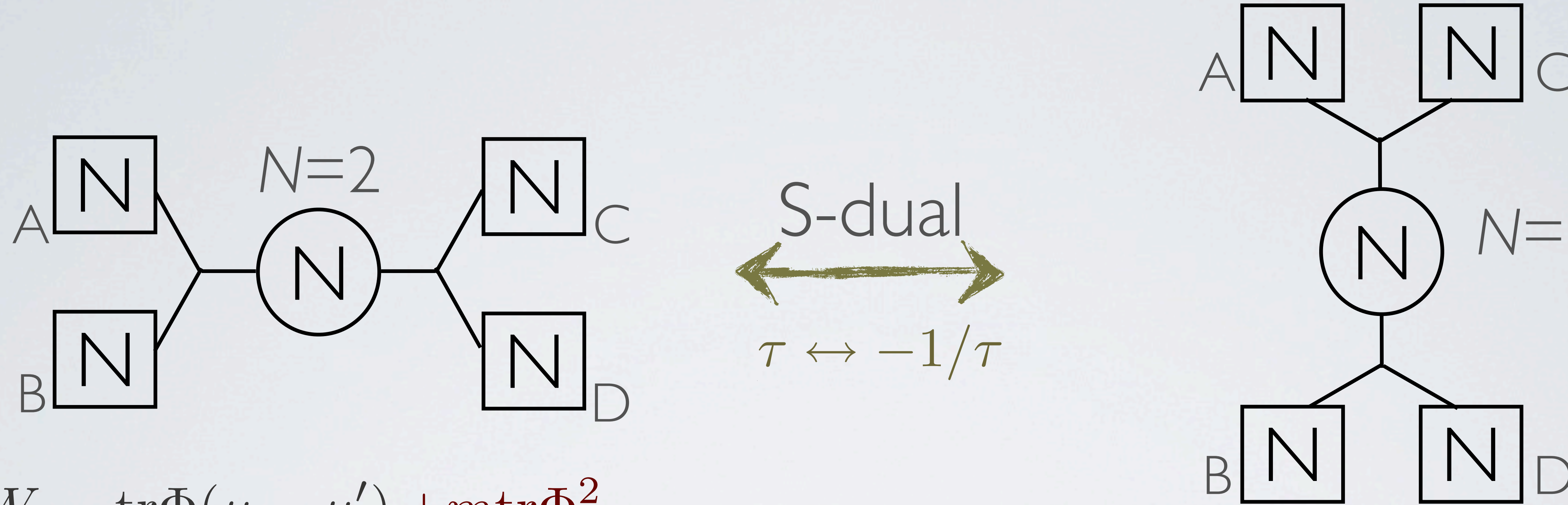


$$W = \mathrm{tr}\Phi(\mu - \mu') + m\,\mathrm{tr}\Phi^2$$

$$W = c(2\mathrm{tr}\mu\mu' - \mathrm{tr}\mu^2 - \mathrm{tr}\mu'^2)$$

$c \sim 1/m$

N=1 duality



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$$W = \hat{c}\,\mathrm{tr}(2\hat{\mu}\hat{\mu}' - \hat{\mu}^2 - \hat{\mu}'^2)$$

$$= 2\hat{c}\,\mathrm{tr}\hat{\mu}\hat{\mu}' + \frac{1}{4\hat{c}}\mathrm{tr}M_B^2 + \mathrm{tr}M_B\hat{\mu}_B$$

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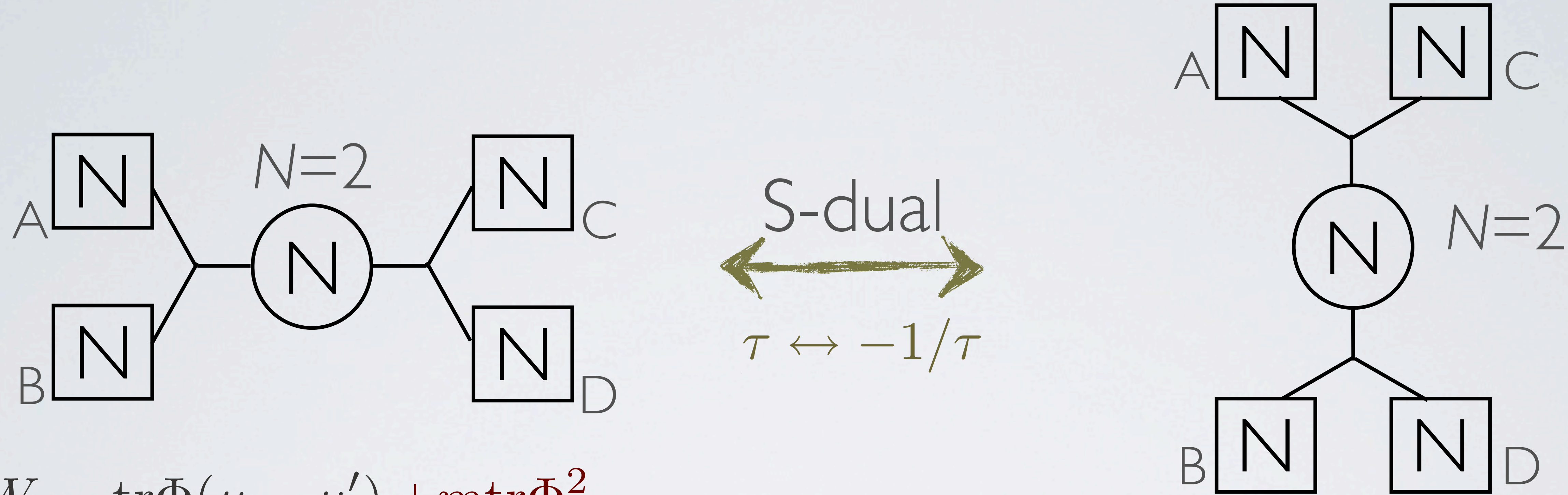

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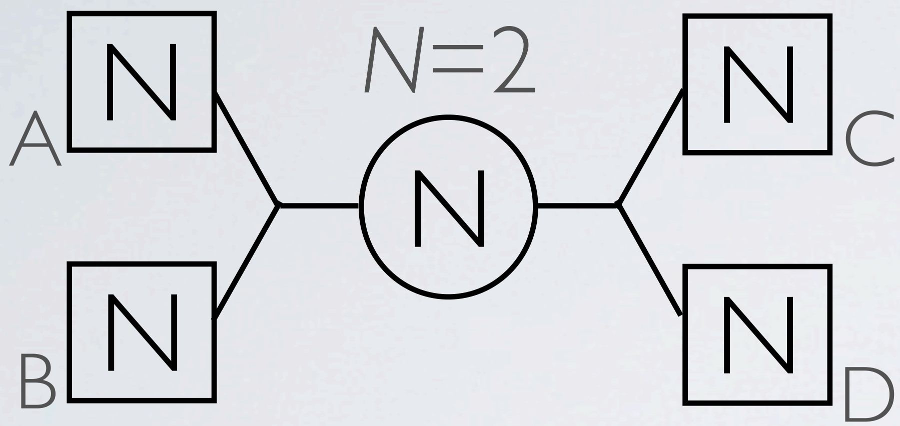

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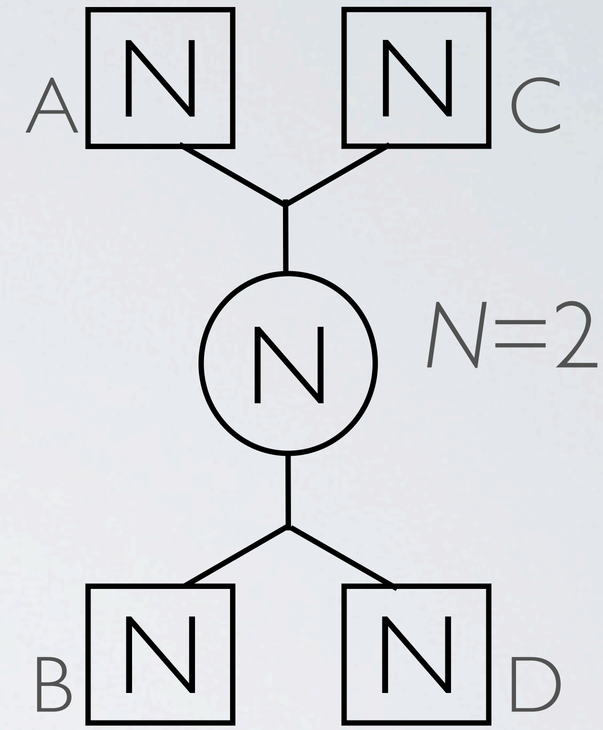

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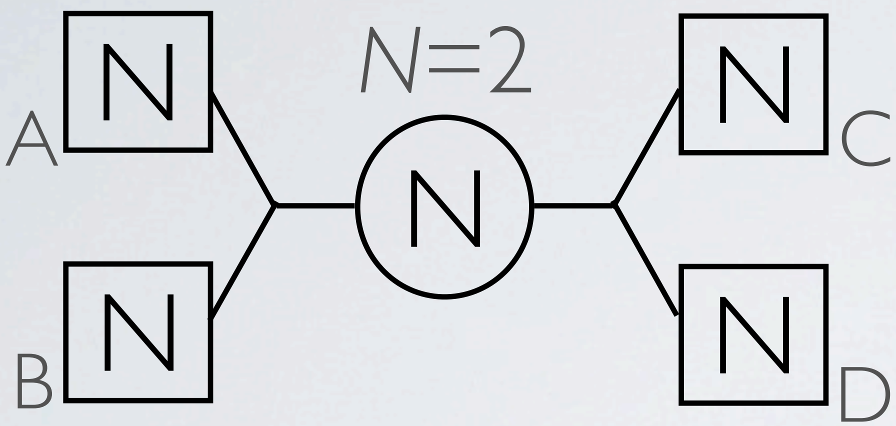

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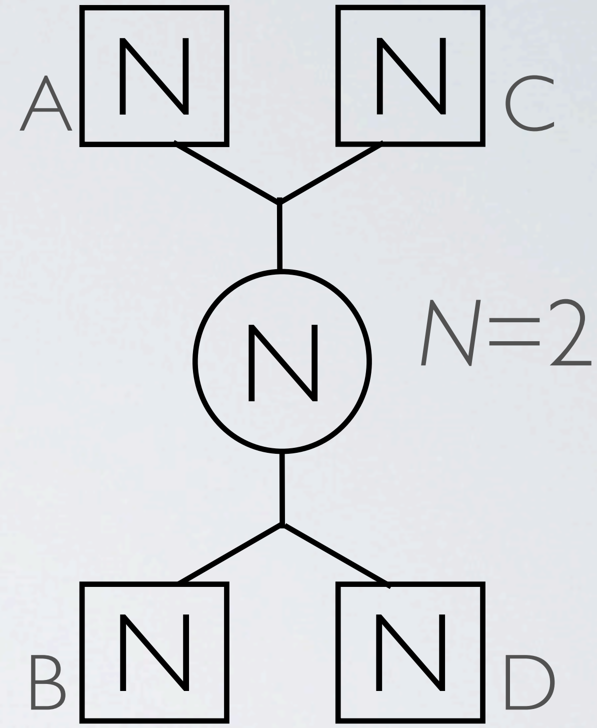

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independent
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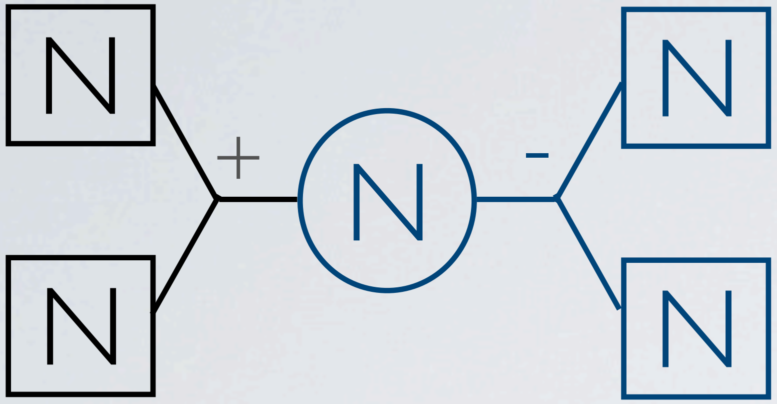

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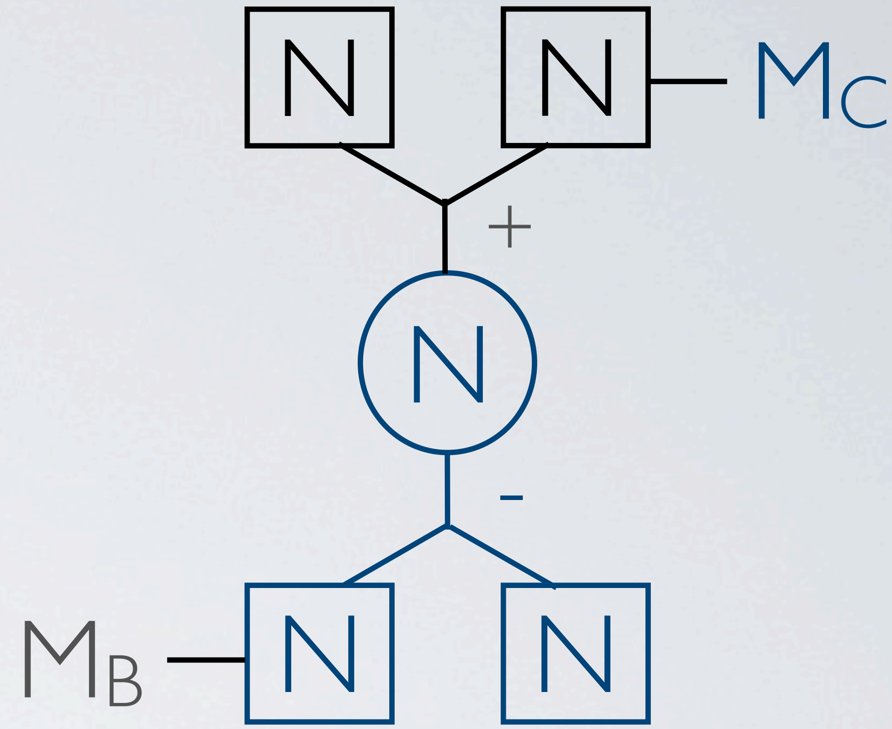

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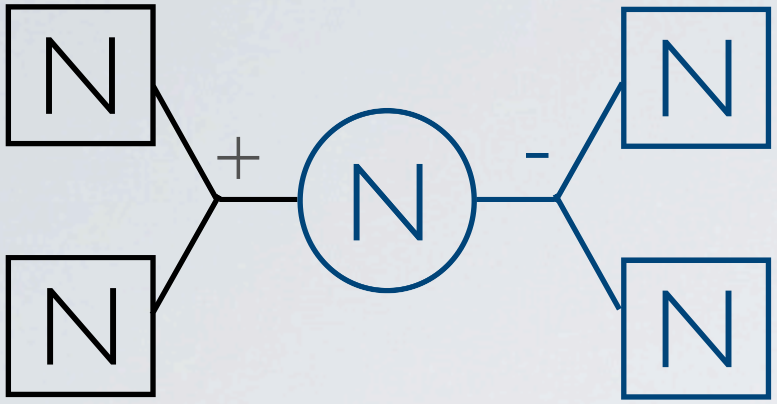

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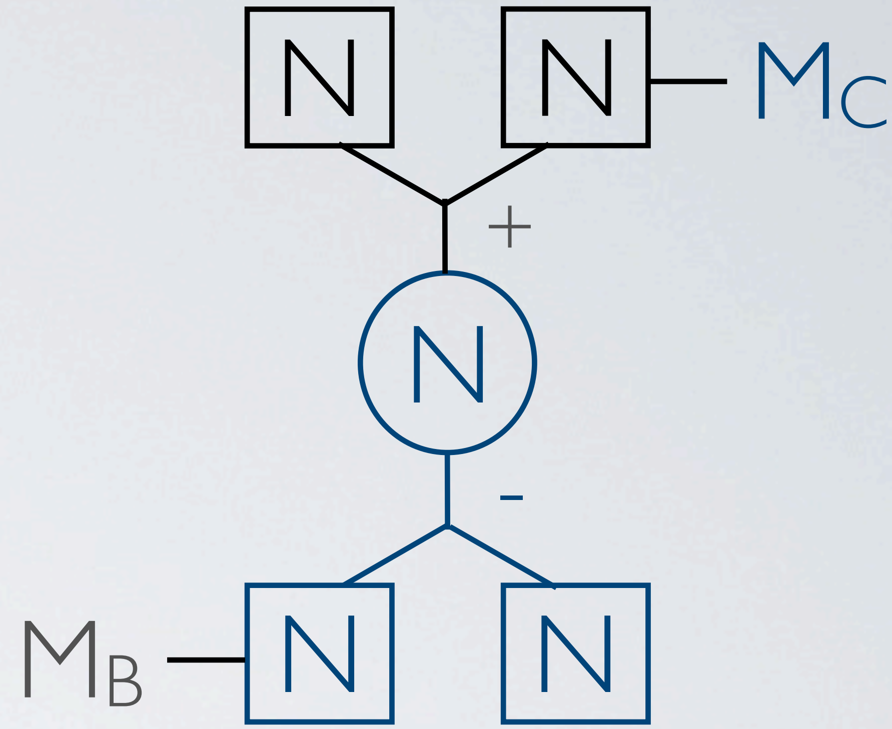

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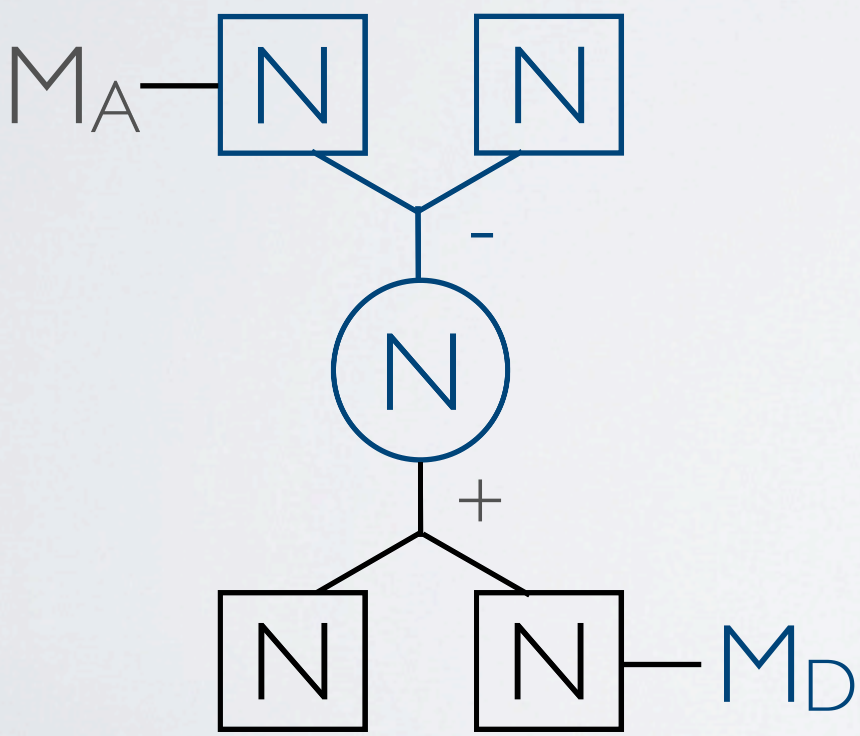
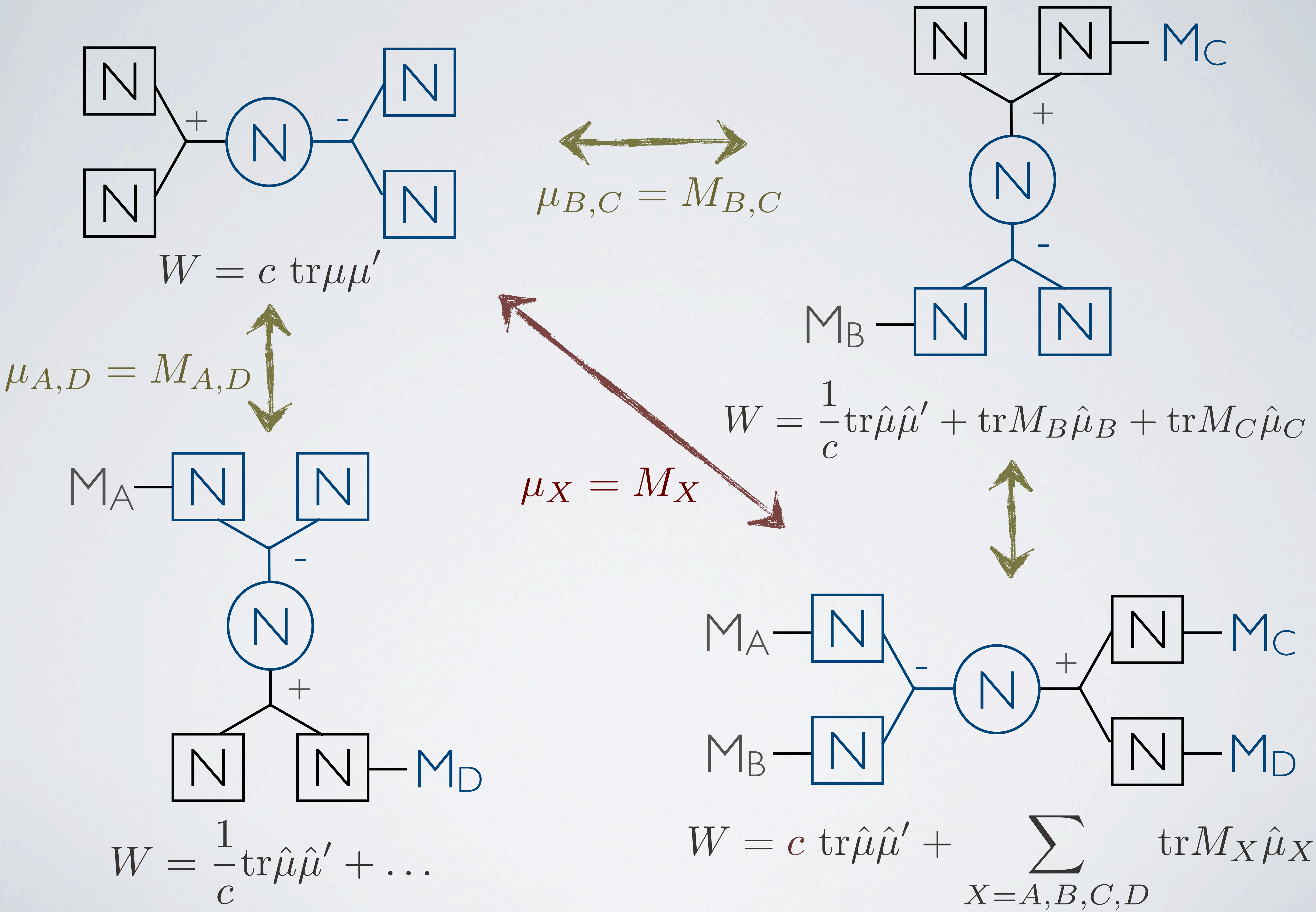


$$W = \frac{1}{c}\mathrm{tr}\hat{\mu}\hat{\mu}' + \ldots$$



N
+
-
$\mu_{B,C} = M_{B,C}$
M_C
M_B
$W = c\, \mathrm{tr} \mu \mu'$
$\mu_{A,D} = M_{A,D}$
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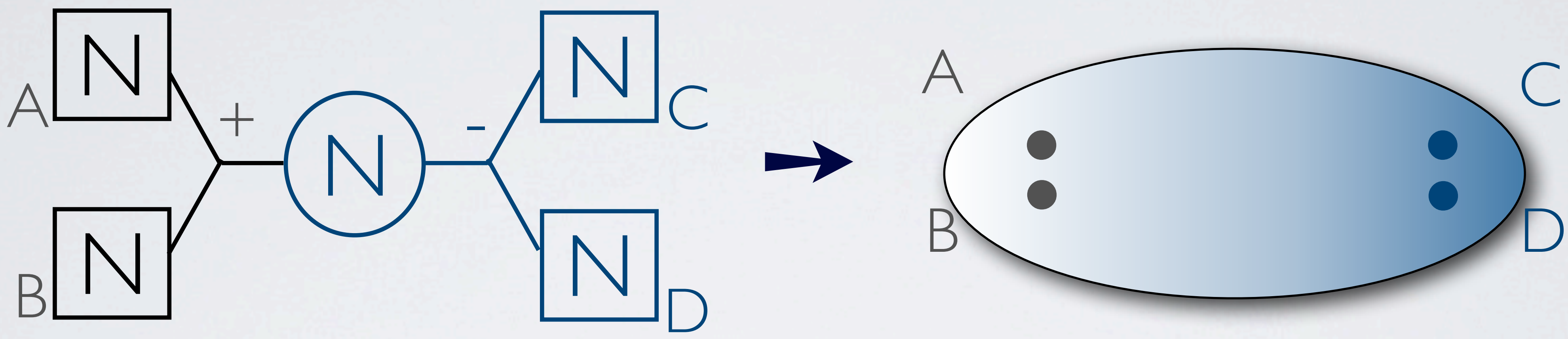
$\mu_X = M_X$
M_A
M_D
$W = \frac{1}{c} \mathrm{tr} \hat{\mu} \hat{\mu}' + \ldots$
$W = c\, \mathrm{tr} \hat{\mu} \hat{\mu}' + \sum_{X=A,B,C,D} \mathrm{tr} M_X \hat{\mu}_X$

'colored' puncture



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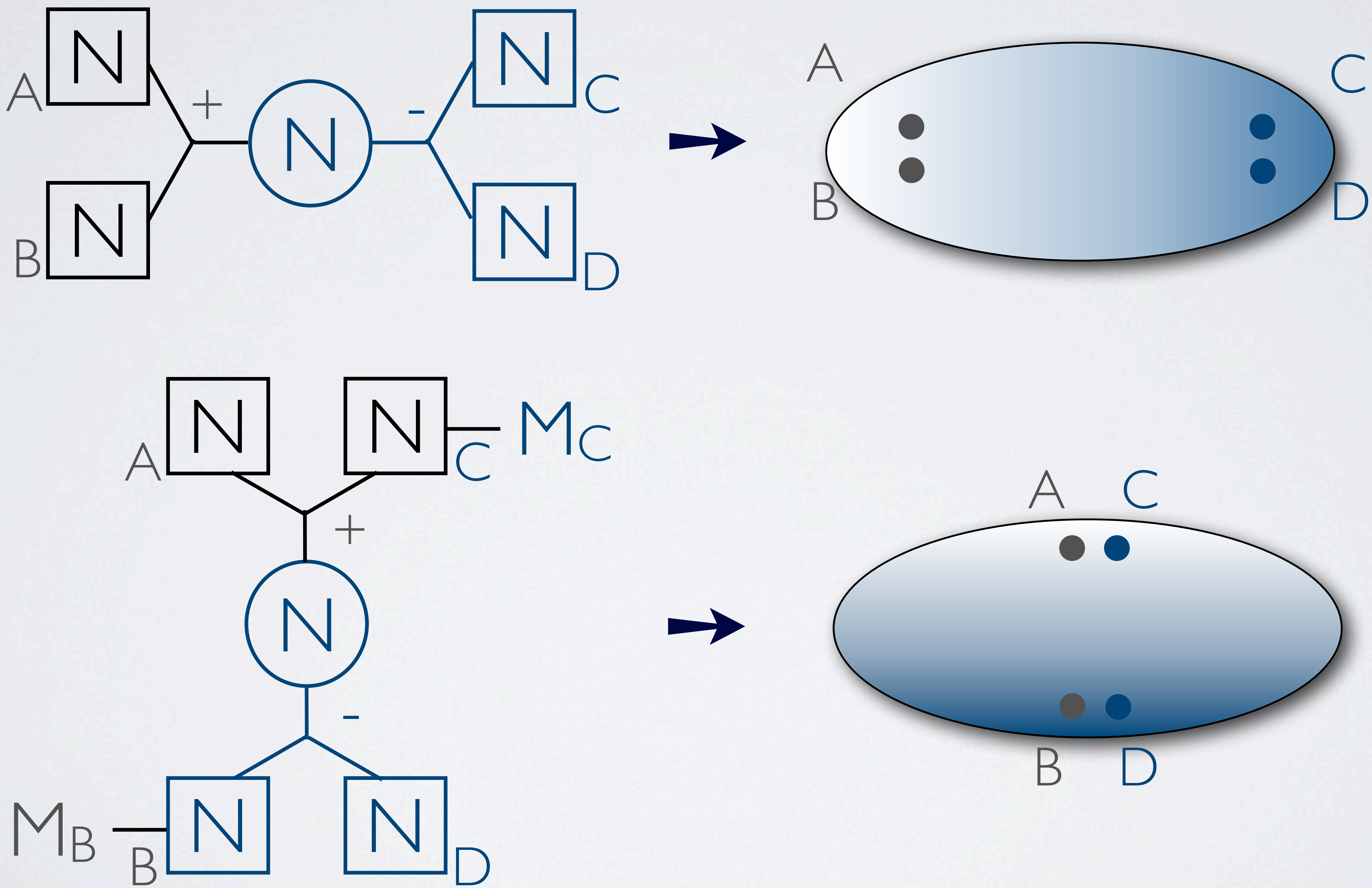

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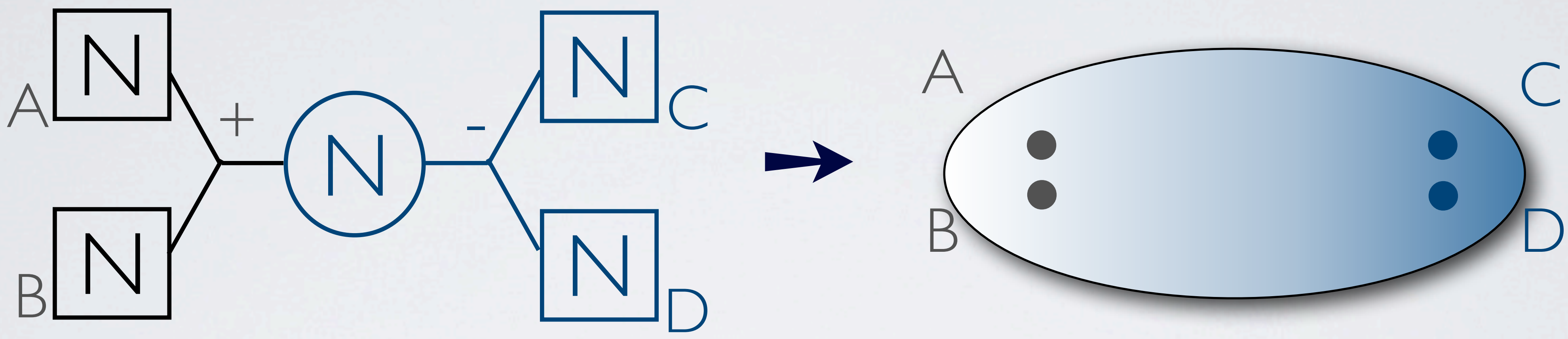

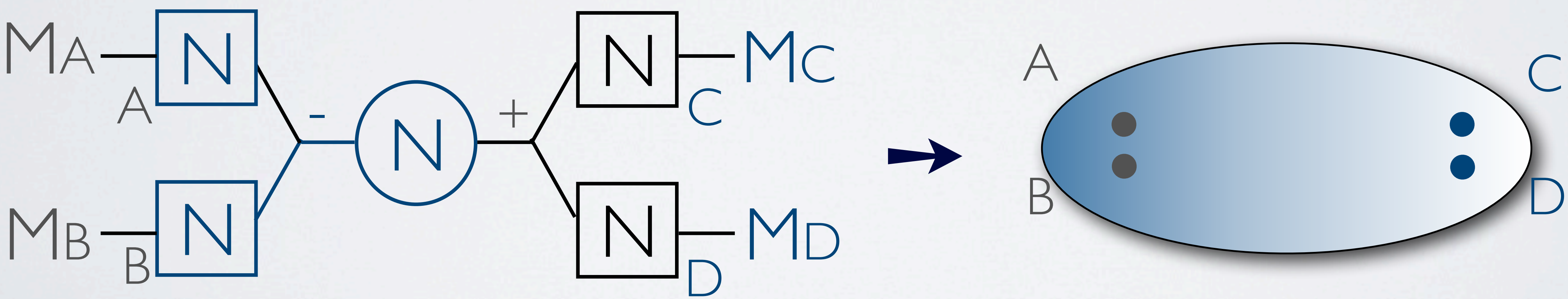

M-theory interpretation?

The $\mathcal{N}$=1 dualities may be able to understand as a symmetry of **colored**-punctured Riemann surface.

no puncture case [Bah-Wecht, Bah-Beem-Bobev-Wecht]

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no puncture case [Bah-Wecht, Bah-Beem-Bobev-Wecht]

But **the derivation of the adjoint fields** from the M-theory point of view is lacking.

cf. generalized Hitchin system viewpoint [Xie]

M-theory interpretation?

The N=1 dualities may be able to understand as a symmetry of **colored**-punctured Riemann surface.

no puncture case [Bah-Wecht, Bah-Beem-Bobev-Wecht]

But **the derivation of the adjoint fields** from the M-theory point of view is lacking.

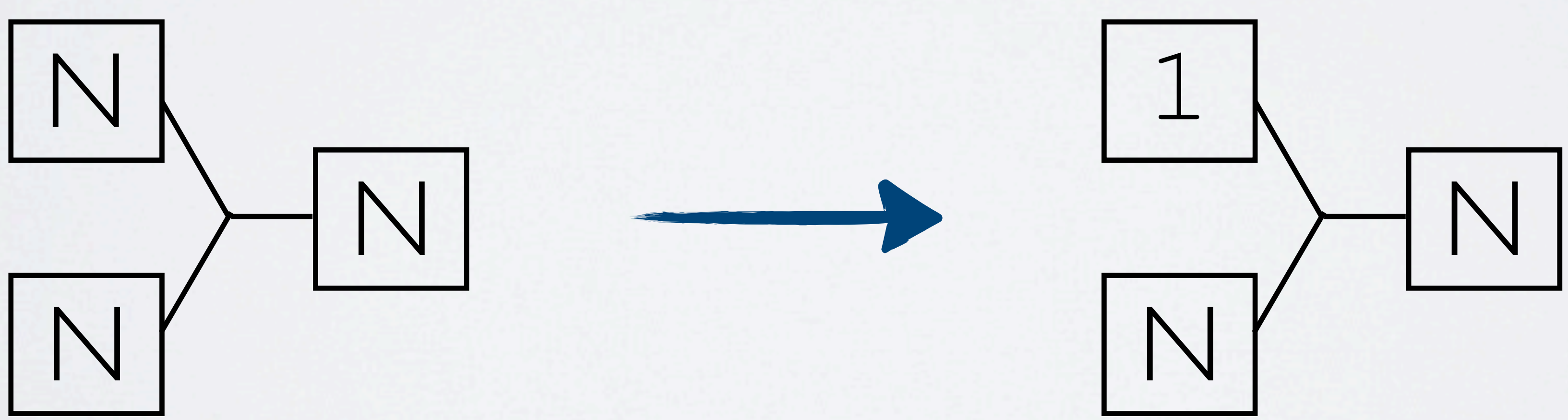
cf. generalized Hitchin system viewpoint [Xie]

Furthermore, the meaning of the Riemann surface is not so clear in the N=1 set-up, compared to the N=2 one: just can be used to read off the UV N=1 theory.

Higgsing

We give a nilpotent vev $\langle \mu \rangle = \rho(\sigma^{+})$, where **ρ** specifies the embedding **ρ**: SU(2) → SU(N) and characterized by a partition Λ.

Focus on the embedding corresponding to Λ=(N-1,1):

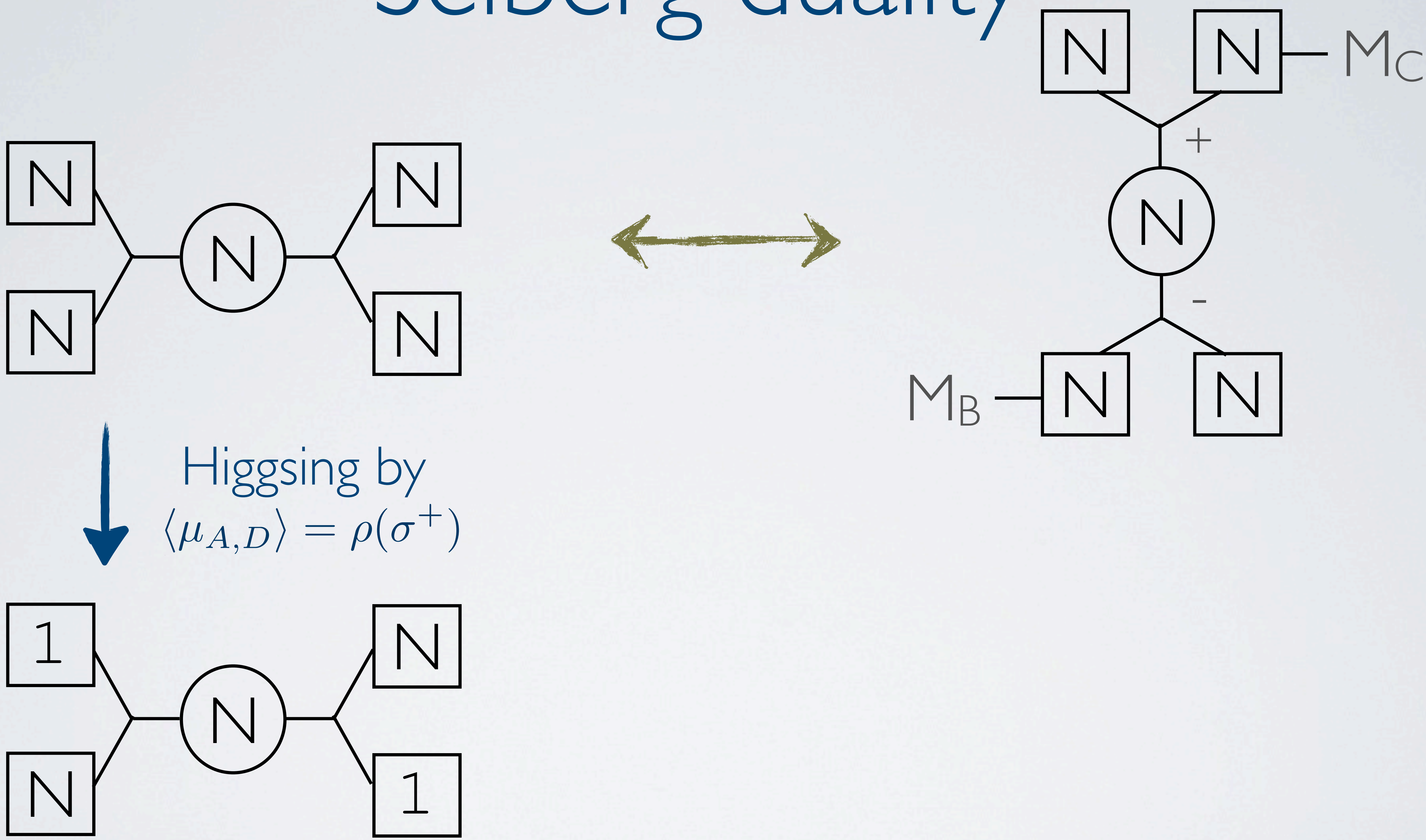


T_N

free bifundamental hypermultiplet

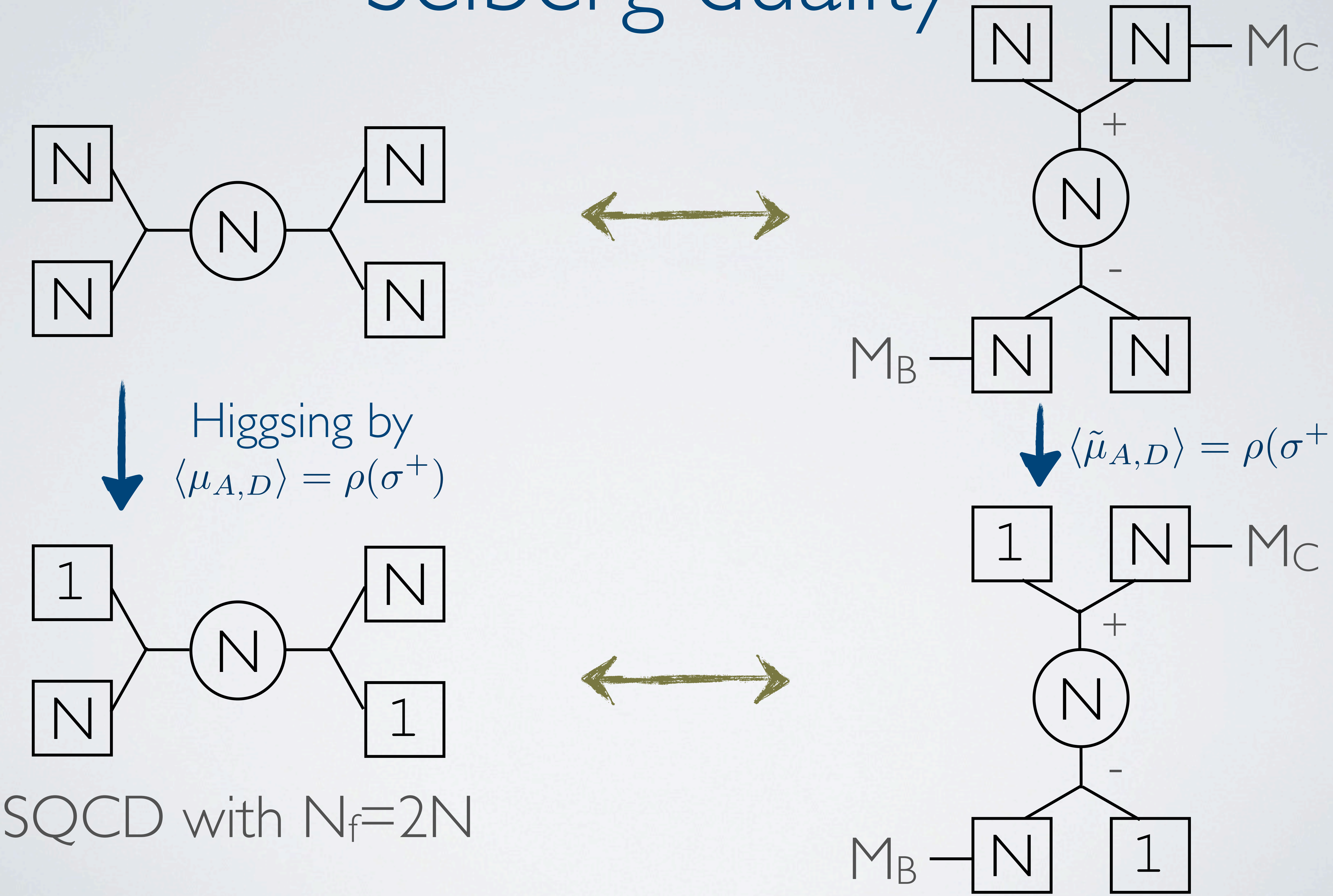
Seiberg duality

Seiberg duality

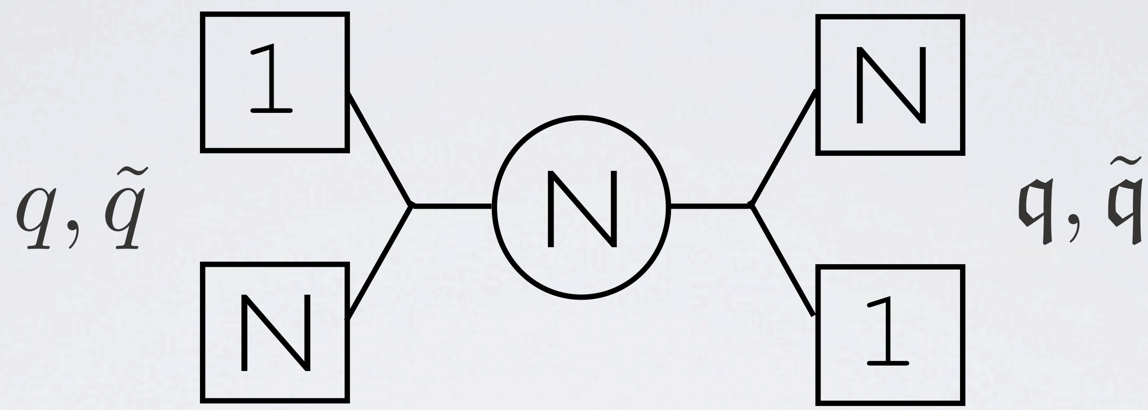


SQCD with N_f=2N

Seiberg duality



Seiberg duality

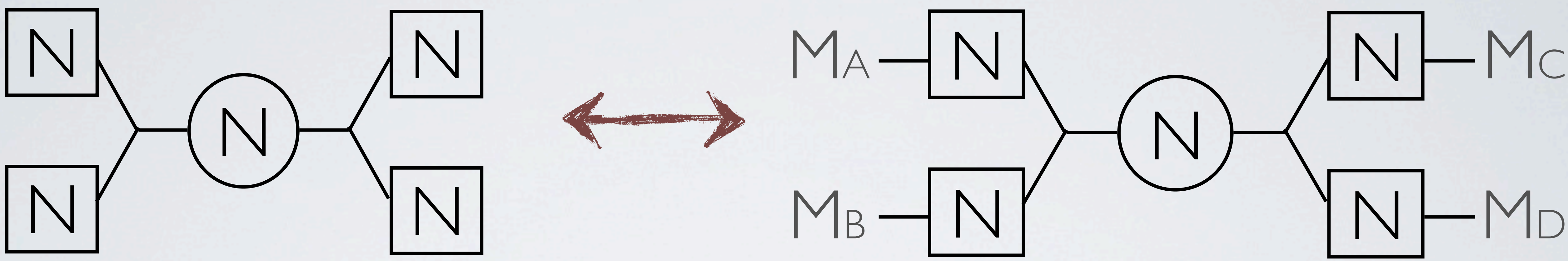


$$
\begin{aligned}
W &= c\,\mathrm{tr}(q\tilde{q})_g(\mathfrak{q}\tilde{\mathfrak{q}})_g \\
&= c\left[(q_{i\alpha}\tilde{\mathfrak{q}}^{k\alpha})(\tilde{q}^{i\beta}\mathfrak{q}_{k\beta}) - \frac{1}{N}(q_{i\gamma}\tilde{q}^{i\gamma})(\mathfrak{q}_{k\gamma}\tilde{\mathfrak{q}}^{k\gamma})\right]
\end{aligned}
$$

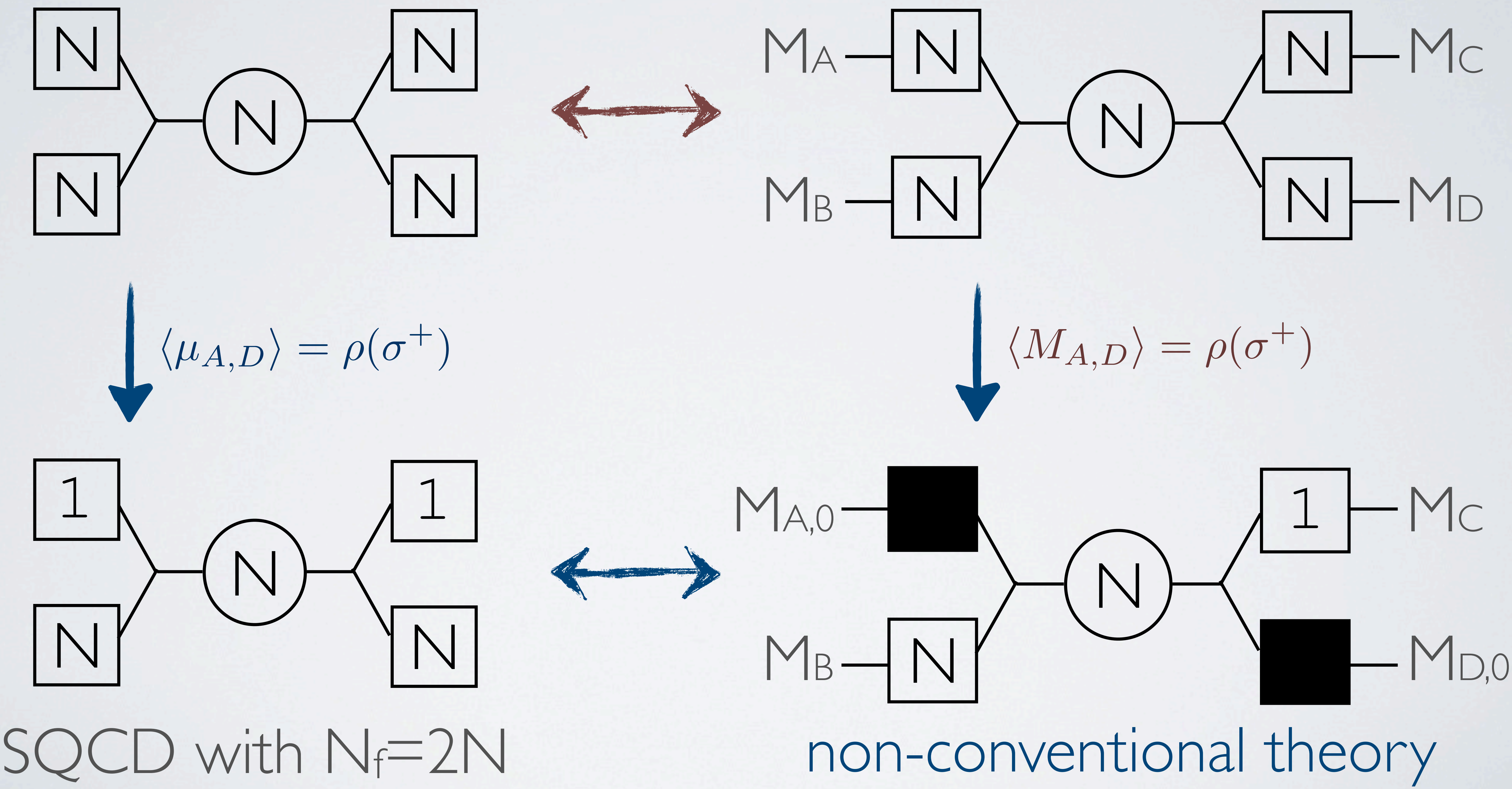
In the Seiberg dual, there are $(2N)^2$ mesons.
The above superpotential is mass terms of $2(N^2 + 1)$ mesons

→ $2(N^2 - 1)$ mesons

New duality of SQCD



New duality of SQCD



Conclusion

We have considered N=1 dynamics of gauge theories.

- ✧ generalized Hitchin system describes the dynamics of N=1 theories in confining phase.
- ✧ dualities of N=1 theories via N=2 S-dualities

Future directions

- ✪ Higher rank theory in confining phase
- ✪ M-theoretical interpretation of N=1 dualities
- ✪ Duality of asymptotically free theories
- ✪ Other phases: N=1 Coulomb phase from M-theory
- ✪ Some relation with 2d theory on C???

Thank you very much for your attention!