An Entropy Formula for Higher Spin Black Holes via Conical Singularities

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Based on Work with Per Kraus (UCLA)

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Introduction

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Each higher spin field has it's own gauge symmetry.

Interesting because

1. Provide a toy model of string theory in AdS
2. Holographic duals are proposed [Gaberdiel, Gopakumar]
3. Higher spin black holes do not have event horizon

Higher spin black holes

Existence of a event horizon is gauge dependent. Gauge symmetry of a higher spin field change the metric of the spacetime.

Seemingly pathological. However, they have natural interpretation as The saddle point of the partition function of dual field theory.

A gauge invariant way to impose thermodynamics was proposed. [Kraus et al] Derived free energy matched with that of dual field theory.

No explicit entropy formula (Wald like formula) had not been known.

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→ Our work!!

Plan of the Talk

1.Brief introduction of higher spin theory

2. HS gravity in 3 dim as Chern Simons theory

3.Black hole thermodynamics from connections

4. Higher spin black holes and entropy formula

Introduction of higher spin

・Spin S field: $\varphi_{\mu_1 \cdots \mu_s}$ fully symmetric rank s tensor.

・Linearized EOM: $\Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1|} \partial^{\lambda} \varphi_{\mu_2 \cdots \mu_{s-1})\lambda} + \partial_{(\mu_1} \partial_{\mu_2} \varphi^{\lambda}_{\mu_3 \cdots \mu_{s-2})\lambda} = 0$

・Gauge symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_{s-1})}$

Can be regarded as a direct generalization of spin 2 fluctuation (linearized Einstein equation)

・ Nonlinear EOM for higher spin fields are known.[Vasiliev]
In 3 dimension, The Lagrangians are given by Chern Simons form with gauge group G $\supset sl(2,R)$

Einstein gravity as CS theory [Witten et al]

$SL(2,R)\times SL(2,R)$ Chern Simons theory is Einstein gravity with Λ<0

$$I[A,\bar{A}] = I_{CS}[A] - I_{CS}[\bar{A}] \qquad I_{CS}[A] = \frac{k}{4\pi}\int A\wedge dA + \frac{2}{3}A\wedge A\wedge A \qquad k = \frac{1}{4G}$$

If we use

$$A = \omega + e, \quad \bar{A} = \omega - e$$

EOM :

$$d\omega + \omega\wedge\omega = e\wedge e \qquad de + \omega\wedge e = 0$$

Einstein eq in AdS — Torsion free condition

Metric: $g_{\mu\nu} = \frac{1}{2}\mathrm{tr}[\mathrm{e}_{(\mu}\mathrm{e}_{\nu)}]$

$SL(N,R)\times SL(N,R)$ Chern Simons theory describes a higher spin theory where metric and spin < N+1 fields couples . [Blencowe]

$$\varphi_{\mu_1\cdots\mu_s} = \mathrm{Tr} e_{(\mu_1}\cdots e_{\mu_s)}$$

BTZ black hole and Thermodynamics

BTZ black holes are solution of Einstein equation with $\Lambda < 0$ in 3 dim.

$$ds^2 = -\left(e^{\rho} - \mathcal{L}e^{-\rho}\right)^2 dt^2 + d\rho^2 + \left(e^{\rho} + \mathcal{L}e^{-\rho}\right)^2 d\theta^2$$

In CS formulation, their connections are given by

$$A = \left(e^{\rho} L_1 - \mathcal{L}e^{-\rho} L_{-1}\right) dx^+ + L_0 d\rho \qquad \bar{A} = -\left(e^{\rho} L_{-1} - \mathcal{L}e^{-\rho} L_{+1}\right) dx^- - L_0 d\rho$$

$\{L_i\}$ are the generators of $sl(2,R)$

Thermodynamics
- Temperature: Smoothness of metric $ds^2 \sim \mathcal{L}r^2 d\tau^2 + d\rho^2$

  → $T = \frac{\sqrt{\mathcal{L}}}{2\pi}$
- Entropy : Bekenstein Hawking formula $S = \frac{A}{4G} = \frac{4k\pi^2}{\beta}$

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Problem: The prescription fully depend on metric. Not applicable to HS theory.

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Can we derive these quantities from the connections?

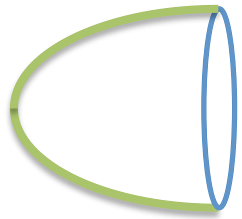
Holonomy Condition and Temperature

The timelike cycle of Euclidian black hole is contractible .

This means the holonomy around the cycle has to be trivial. [Kraus et al]

$$\exp\left[i\int_0^\beta A_t dt\right] = 1$$

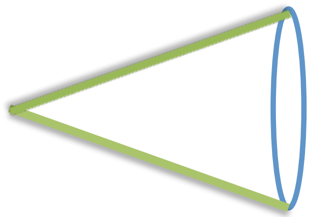
$$\beta = \frac{1}{T} = \frac{2\pi}{\sqrt{\mathcal{L}}}$$



One can indirectly compute entropy from the relation. $\frac{\partial \log Z}{\partial \beta} = \mathcal{L}$

Connection which does not satisfy the holonomy condition around contractible cycle has conical singularity.

$$F = dA + A \wedge A \sim \delta(r)$$



Entropy from Conical Singularity

Entropy of a black hole is given by

$$S(\beta) = \left(B\frac{\partial}{\partial B} - 1 \right) \Big|_{B=\beta} I_c[B, g_c(B)] \qquad \frac{\delta I_c}{\delta g}[B, g]\Big|_{g=g_c(\beta)} = 0$$

$$\left(B\frac{\partial}{\partial B} - 1 \right) \Big|_{B=\beta} I_c[B, g_c(\beta)]$$

Black hole entropy is derived by evaluating the action of conical singularity

- This is a way to derive the Wald formula. [Fursaev Solodukhin]

We generalized the prescription for Chern Simons theory. The result is [Kraus ,UT]

$$S = -ik\mathrm{Tr}(a_+\omega) - ik\mathrm{Tr}(\bar{a}_-\bar{\omega}) \qquad \omega = \bar{\omega} = \begin{pmatrix} 2\pi i & 0 & 0 \\ 0 & -2\pi i & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The formula is applicable for higher spin black holes.

Higher spin black hole

The connection of HSBH with spin 3 charge is

$$A = \left(e^{\rho} L_1 - \frac{2\pi\mathcal{L}}{k} L_{-1} - \frac{\pi}{2k}\mathcal{W} e^{-2\rho} W_{-2} \right) dx^{+}$$
$$+ \mu \left(e^{2\rho} W_2 - 4\pi\mathcal{L} k W_0 + \frac{4\pi^2\mathcal{L}^2}{k^2} e^{-2\rho} W_{-2} + \frac{4\pi\mathcal{W}}{k} e^{-\rho} L_{-1} \right) dx^{-} + L_0 d\rho$$

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Spin3 chemical potential

Mass

Spin 3 charge

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Spin3 chemical potential

Mass

Spin 3 charge

Resulting metric does not have event horizon.

Describe the saddle point of the partition function of dual 2d CFT

$$Z = \mathrm{Tr}e^{-\beta(\mathcal{L}-\mu\mathcal{W})}$$

Thermodynamics of spin 3 black hole

We can apply the holonomy prescription for spin 3 black hole.

$$\exp\left[i \int_0^\beta A_t dt \right] = 1 \quad \Longrightarrow \quad \mathcal{L} = \mathcal{L}(\beta, \mu) \qquad \mathcal{W} = \mathcal{W}(\beta, \mu)$$

Satisfy integrability condition: $\frac{\partial \mathcal{L}}{\partial \mu} = \frac{\partial \mathcal{W}}{\partial \beta}$ ⟹ Existence of free energy

Free energy is determined from $\frac{\partial \log Z}{\partial \beta} = \mathcal{L}(\beta, \mu)$ (1st law)

Entropy is Legendre transformation of the free energy

⟹ Very indirect. No geometrical meaning. Any explicit formula?

⟹ Our formula!

1 st law form our formula

Our entropy formula satisfy a correct first law.

$$S = -ik\mathrm{Tr}(a_+\omega) - ik\mathrm{Tr}(\bar{a}_-\bar{\omega})$$

⇒

$$\delta S = \beta\delta\mathcal{L} + \mu\delta\mathcal{W}$$

Thus our entropy formula reproduces the previous result.

Conclusion

1. higher spin gravities in 3 dim is described by Chern Simons theory.

2. They contains black hole solution which carry higher spin charges (higher spin black holes) Existence of event horizon is gauge dependent.

3. We find a entropy formula of general higher spin black holes in 3 dim. The entropy computed from the formula agree with CFT result.