クイーバーゲージ理論と特異トーリックカラビ・ヤウ多様体上のD4-D2-D0束縛状態の数え上げ

吉田豊(KEK)

based on
Takahiro Nishinaka, Satoshi Yamaguchi and Y. Y
arXiv:1304.6724[hep-th]

type-IIA superstringをtoric CY3-foldでコンパクト化

4次元には8-SUSYが残る。
(central chargeがある)

BPS粒子は準安定であるが、モジュライの特殊な
領域(wall)で崩壊するので全領域で決定するのは難しい

またCY 3-foldのサイクルに巻きつくBPS D-ブレーンの
束縛状態は4次元方向ではBPS particleとして見える。

BPS D-ブレーンの束縛状態を数え上げることは
BPS particleの状態数を数え上げることに対応する

Counting BPS D-brane bound states on toric CY 3-fold

D6-D2-D0 bound states

Wall-crossing

(Jafferis-Moore:arXiv:0810.4909[hep-th])

Melting crystal picture (singular limit)

3-dimensional complex space:
(Iqbal-Nekrasov-Okounkov-Vafa: hep-th/0312022)
conifold:
(Szendroi:arXiv:0705.3419[math.AG])
toric CY 3-fold:
(Mozgovy-Reineke:arXiv:0809.0117 [math.AG])
(Ooguri-Yamazaki:arXiv:0811.2801[hep-th])

D4-D2-D0 bound states

Wall-crossing

(Nishinaka-Yamaguchi: arXiv:1007.2731[hep-th], arXiv:1107.4762[hep-th]
Nishinaka: arXiv:1010.6002[hep-th])

Melting crystal picture (singular limit, interpretation)

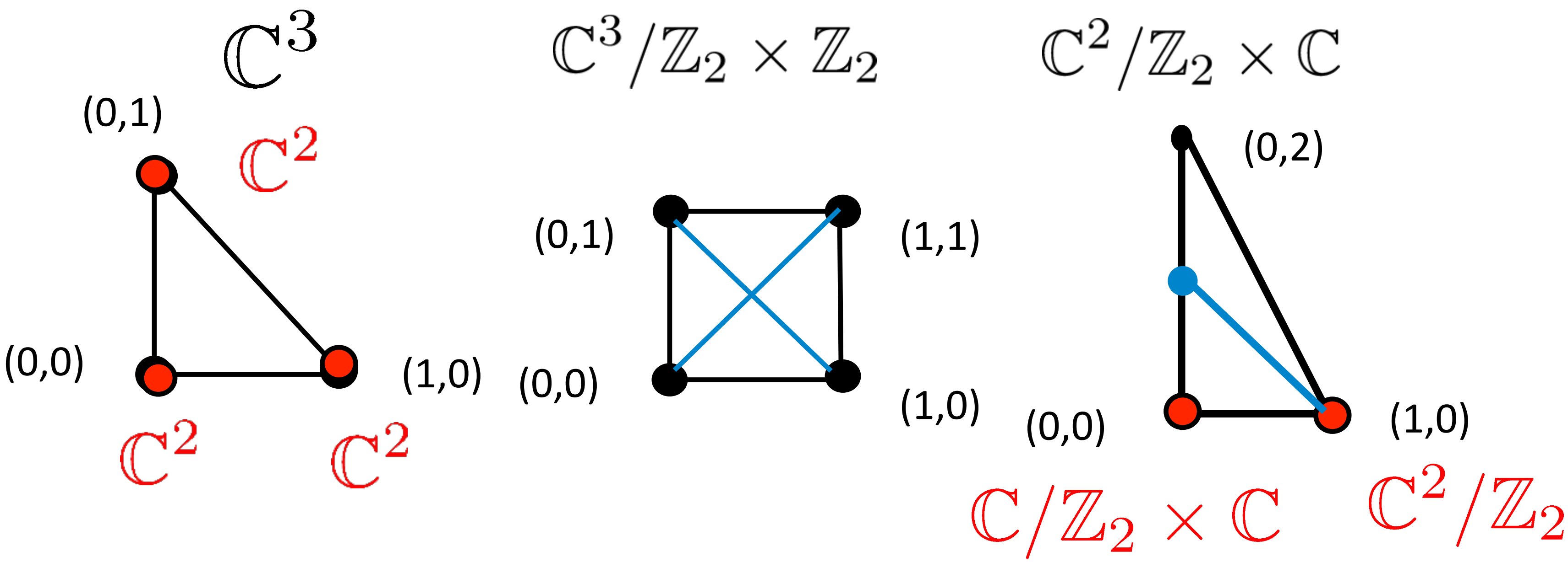
Conifold:
Nishinaka-Yamaguchi:arXiv:1102.2992[hep-th
Generalized conifold:
Nishinaka-Y.Y arXiv:1108.4326[hep-th]

今日の話:singular limitでのD4-D2-D0束縛状態数を与える統計模型の一般的な構成法

content

- Toric diagram
- BPS D6-D2-D0 case(Review)
- BPS D4-D2-D0 case
- まとめ

Toric diagram=integral lattice polytope



Vertex: non–compact 4–cycle (toric divisor)

Edge: non–compact 2–cycle

Triangulation: Blow–up

D6-D2-D0 case

$$Z^{\text{D6-D2-D0}} = \sum_{n,m^I} \Omega_{m^I,n} q^n \prod_I Q_I^{m_I}$$

A D6-brane: CY 3-fold全体に巻きついている
D2-branes: compact 2-cyclesに巻きついている
D0-branes: CY 3-foldでは点状に存在している

q:Boltzmann weight for D0-charge

Q_I:Boltzmann weight for I-th D2-charge

n:D0-brane charge

m_I:I-th D2-brane charge

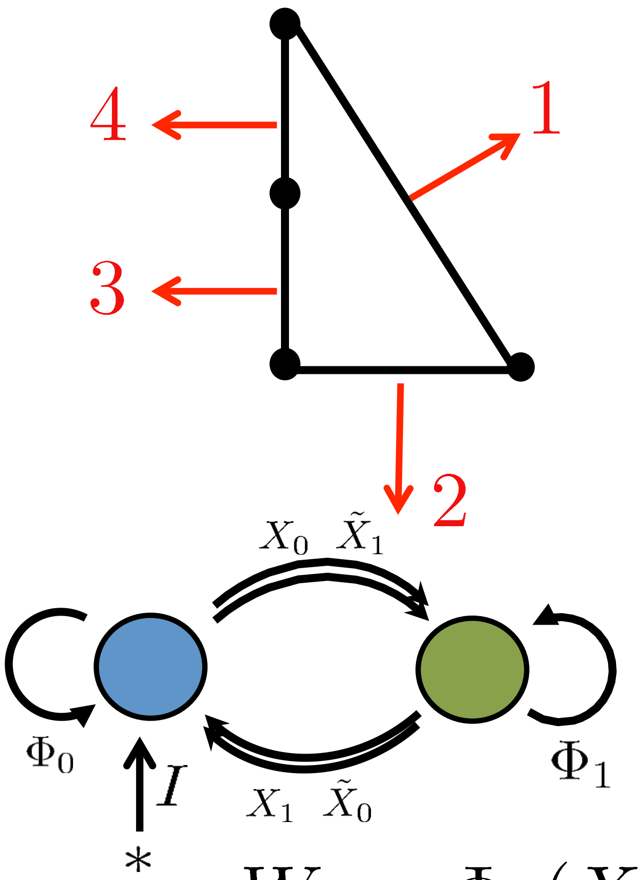
#(BPS bound states)=Witten indices of quiver gauge theory

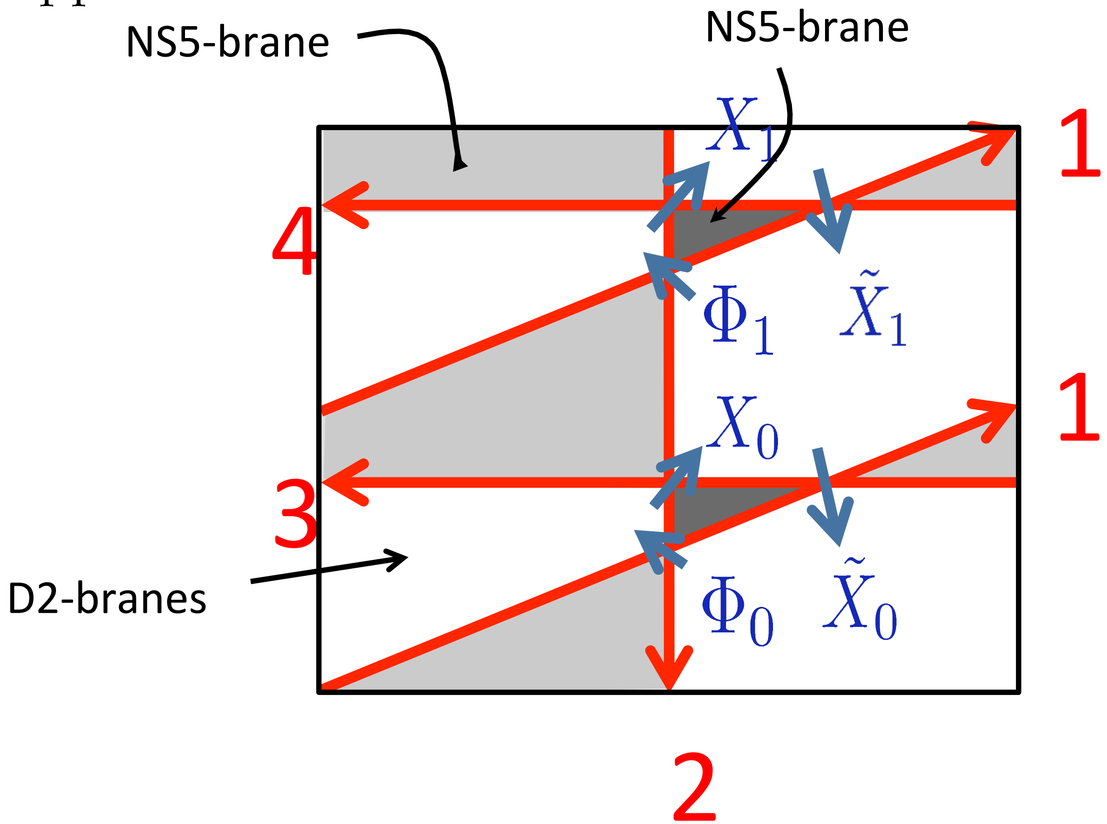
Toric CY 3-fold $\sim T^2 \times \mathbb{R}$ fibration on $\mathbb{R}^3$.

Taking T-dual along the the T^2-directions, (Hanany and Vegh 2005)

CY 3-fold is mapped to NS5-brane system

D0,D2-branes are mapped to D2-branes





$$W_0 = \Phi_0(X_1\tilde{X}_1 - \tilde{X}_0X_0) + \Phi_1(X_0\tilde{X}_0 - \tilde{X}_1X_1)$$

#(BPS bound states)=Witten indices of quiver gauge theory

Witten indices=the number of $U(1)^3$-fixed point of the Vacua.

The fixed points
=(The paths in the quiver diagram)/(The F-term conditions)

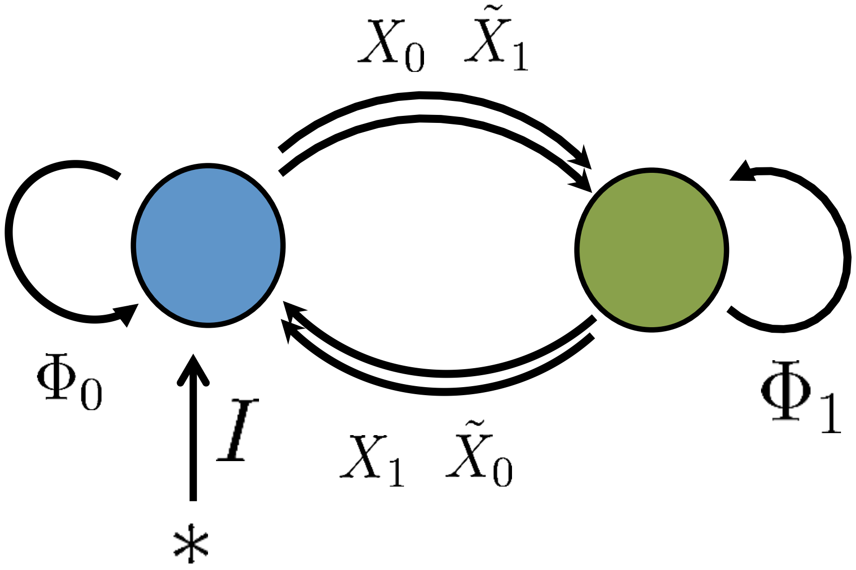
例

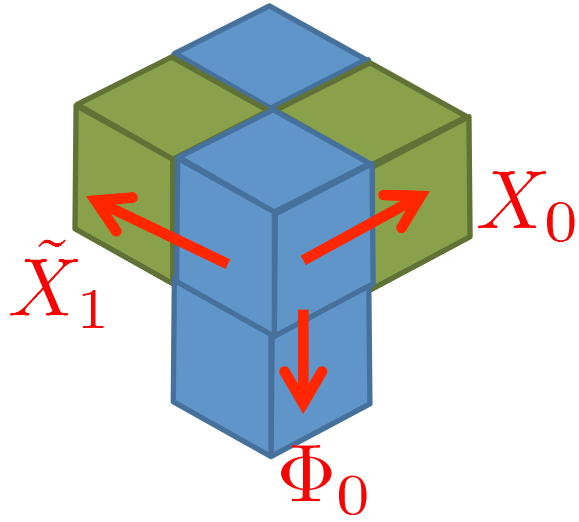
$$Z^{\mathbb{C}^2/\mathbb{Z}_2\times\mathbb{C}}_{\text{D6-D2-D0}} = \sum_{m,n} \Omega_{n,m} q^n Q^m$$

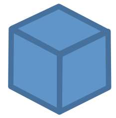
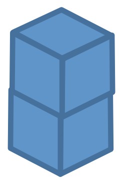
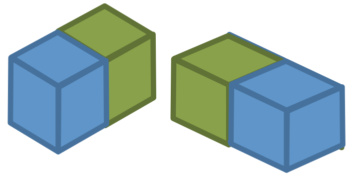
Q: fugacity of D2-brane charge

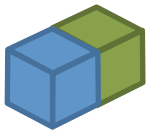
q: fugacity of D0-brane charge

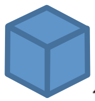
$\Omega_{m,n}$:Witten index





$Z^{\mathbb{C}^2/\mathbb{Z}_2\times\mathbb{C}}_{\text{D6-D2-D0}} = \cdot +$  $+$  $+$  $+$

$q^n = q^{\left(\#\right.}$  $\left.\right)$

$Q^m = Q^{\left(\#\right.}$  $\left.\right)$

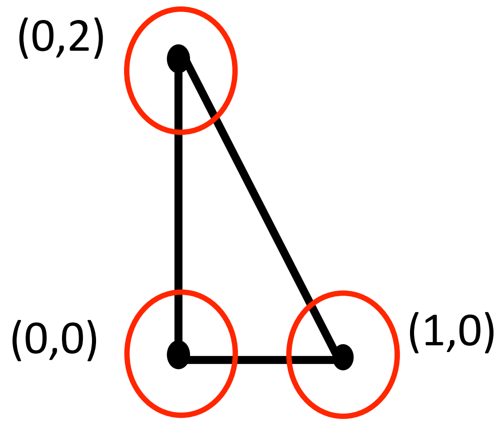
$$= 1 + Q + Q^2 + 2q + \cdots$$

$$= \prod_{n=1}^{\infty} \left(\frac{1}{1-q^n}\right)^{2n} \left(\frac{1}{1-Qq^n}\right)^{n} \left(\frac{1}{1-Q^{-1}q^n}\right)^{n}$$

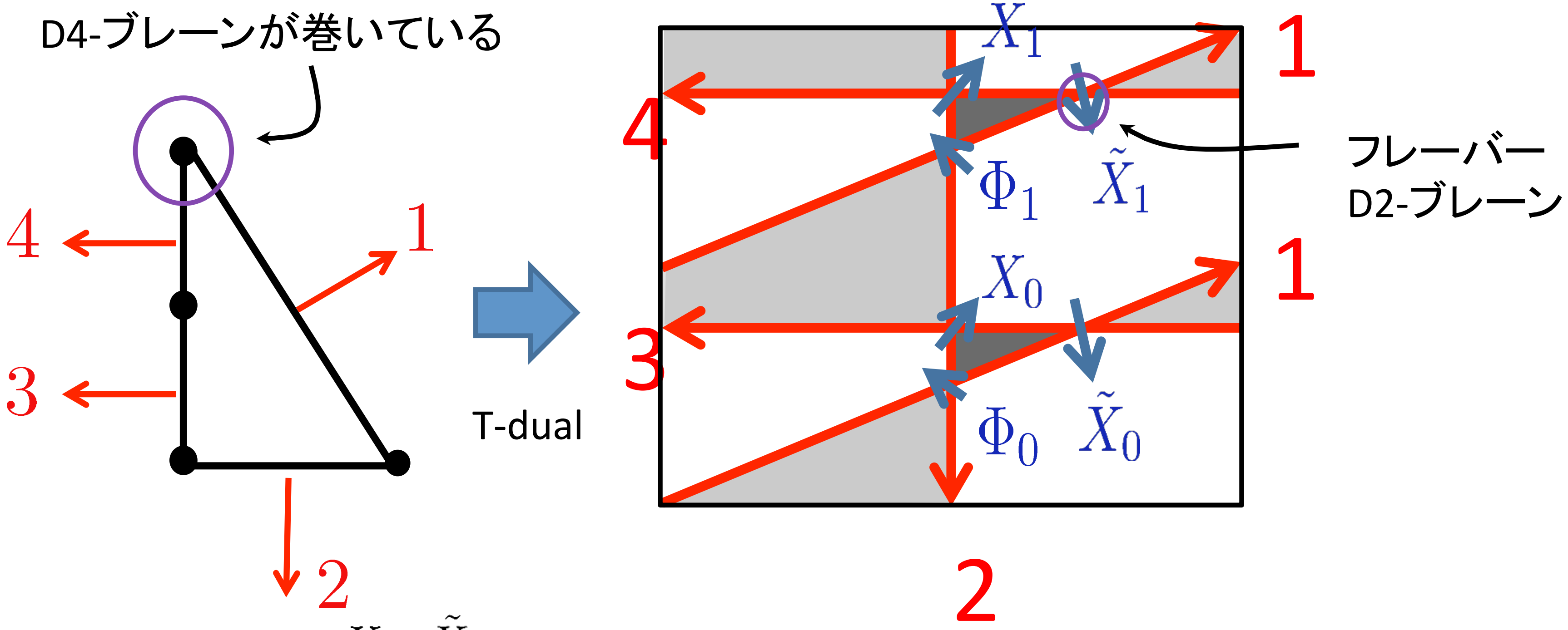
(Young-Bryan: arXiv:0802.3948[math.CO])

BPS D4-D2-D0 case

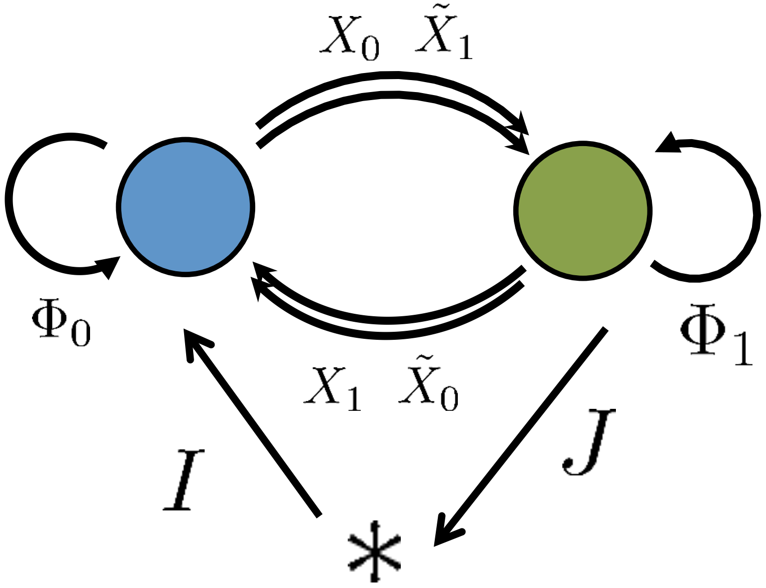
A D4-brane :wrapping on a toric divisor
D2-branes: wrapping on compact 2-cycles
D0-branes: point like on the CY 3-fold



The D4-brane is mapped is mapped to a D2-brane by T-dual.

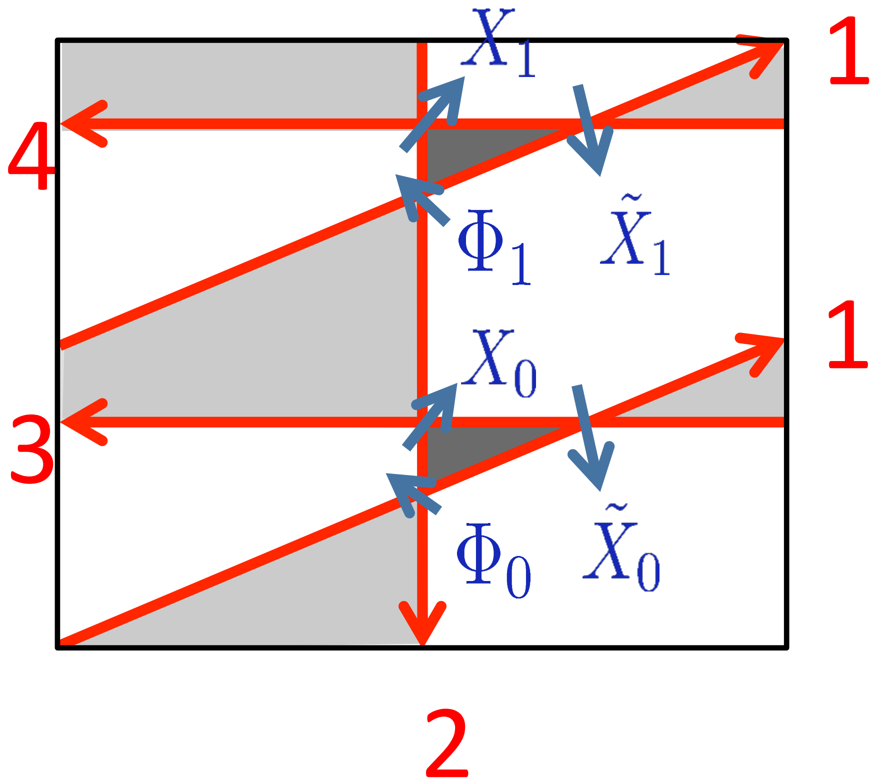


Franco and Uranga (arXiv:hep-th/0604136)

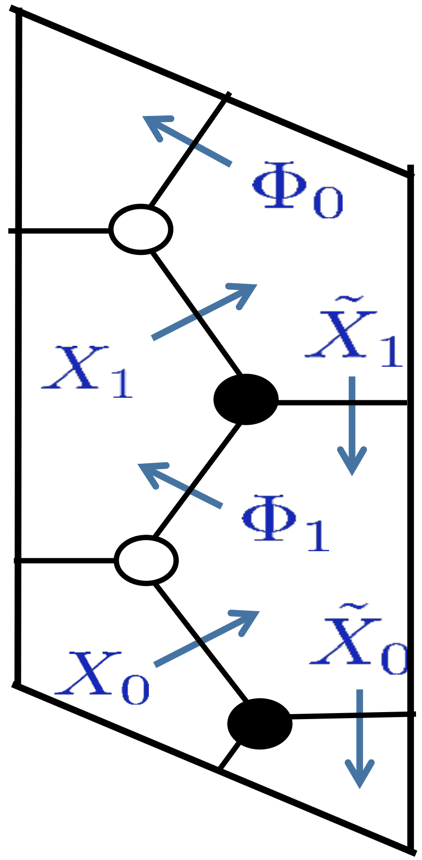


$$W = W_0 + J\tilde{X}_1 I$$

brane-tiling



Bipartite graph

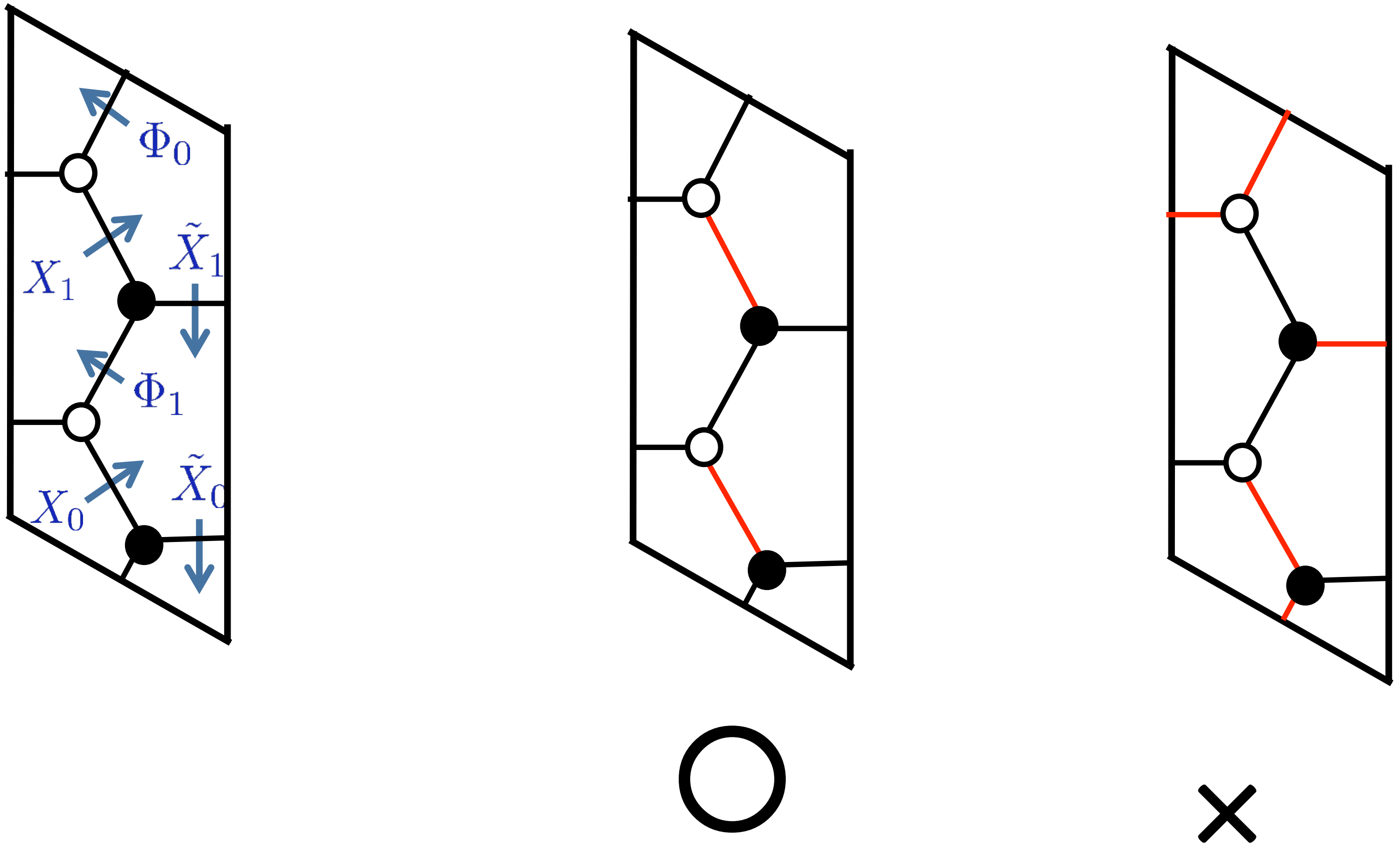


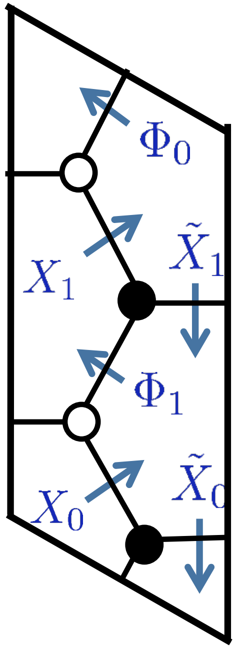
superpotential:face(gray region)
chiral field: intesection points
vector multiplet: face (White region)

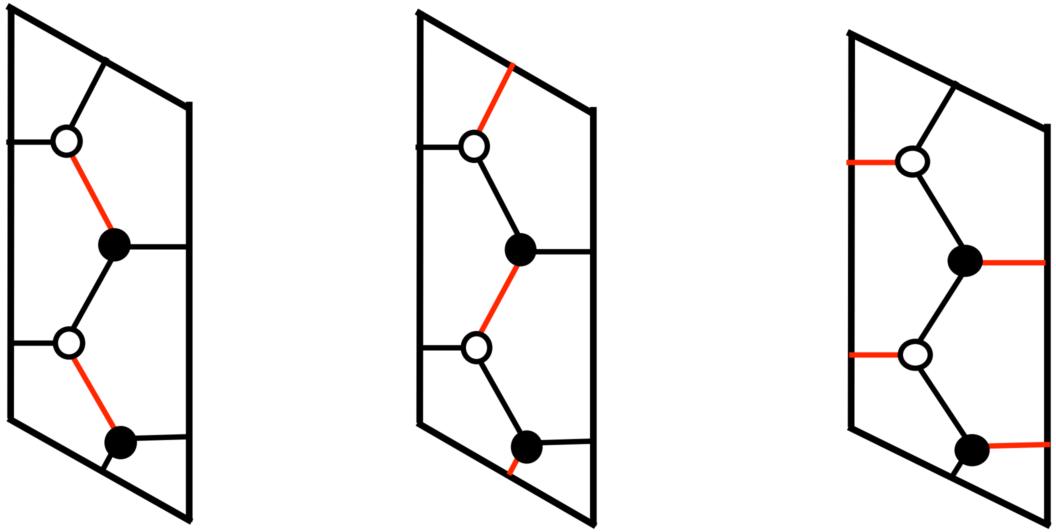
superpotential:vertex
chiral field: edge
vector multiplet: face

Perfect matching condition

perfect matching condition:**bipartite graphの****edgeたちの指定の仕方**
ただし、各vertexは指定されたedgeに一回だけ繋がっている

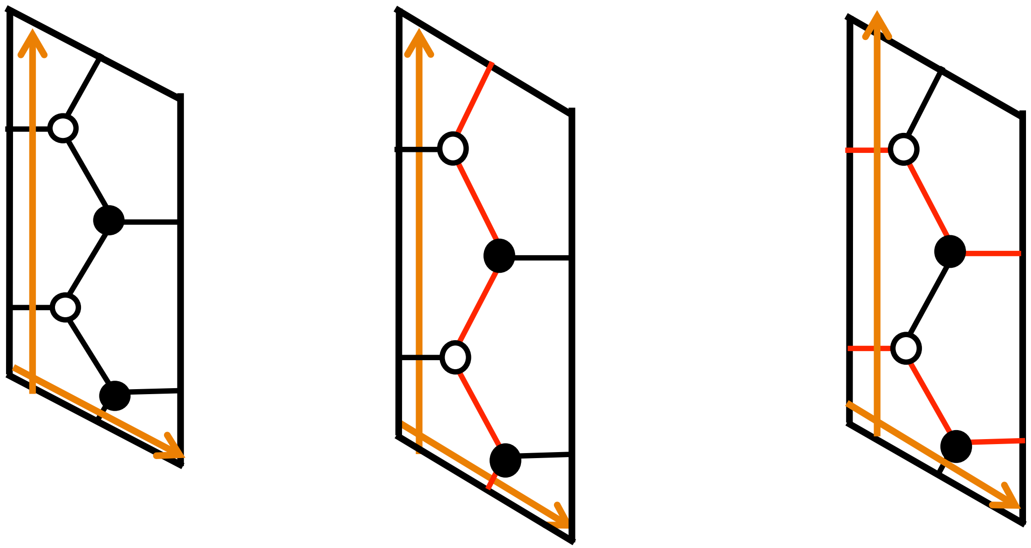






m_0 m_1 m_2

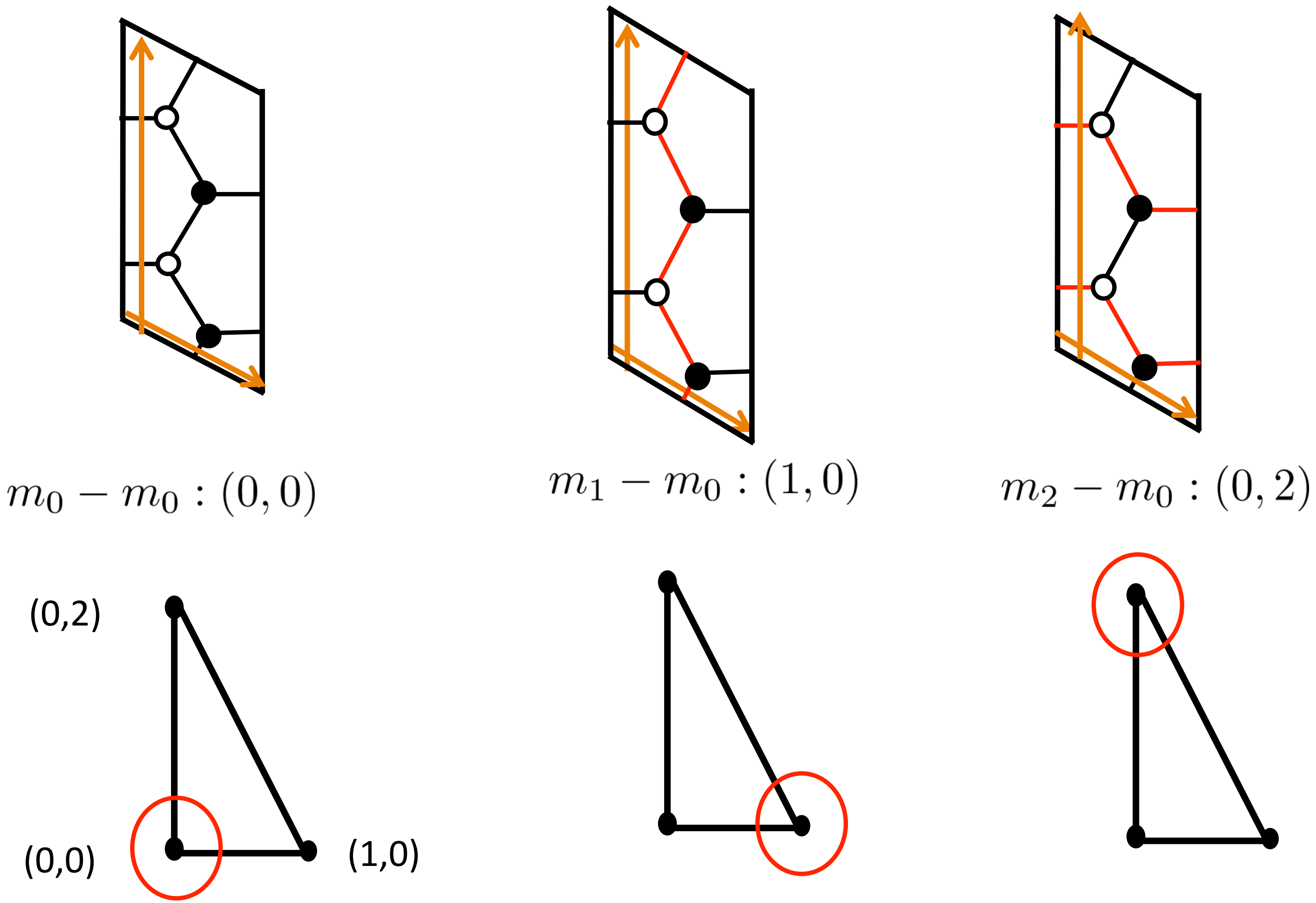
$m_i - m_0$ とトーラスの
基本サイクルとの交点数は



$m_0 - m_0 : (0,0)$ $m_1 - m_0 : (1,0)$ $m_2 - m_0 : (0,2)$

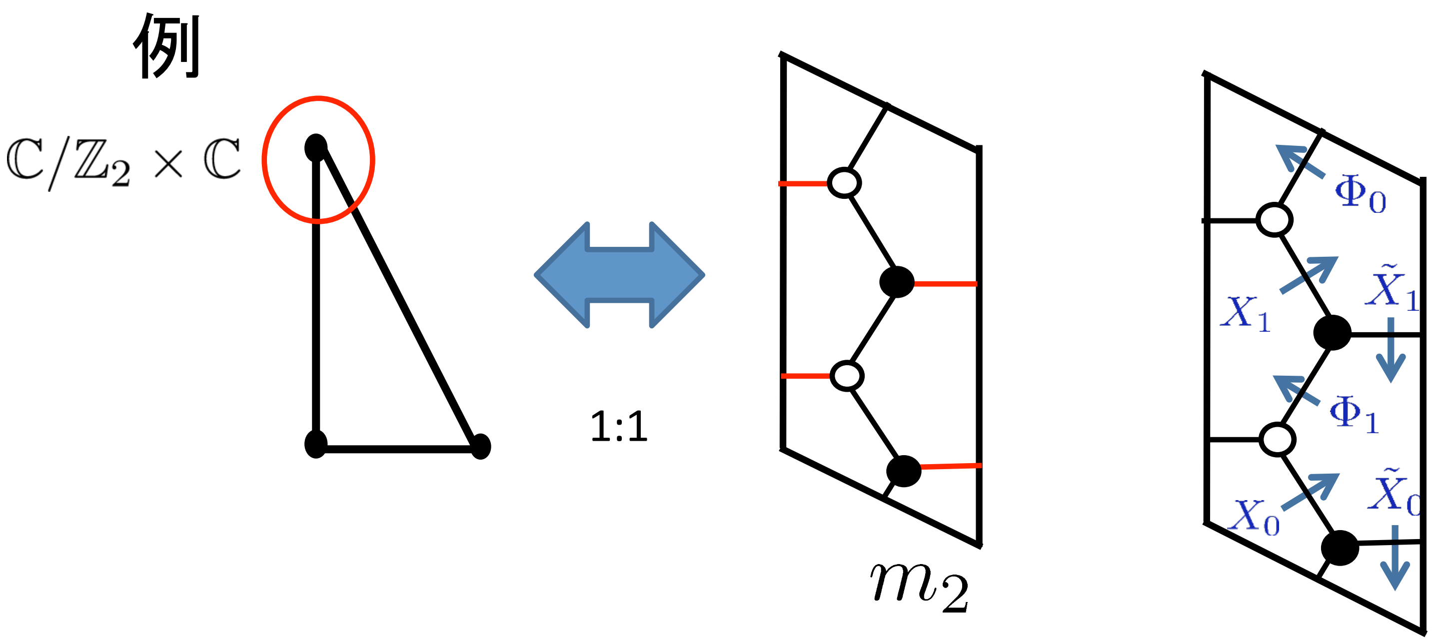
toric diagramの角にあるvertexは次の意味で
perfect matching conditionと一対一対応にある

(Broomhead arXiv:0901.4662[math.AG])

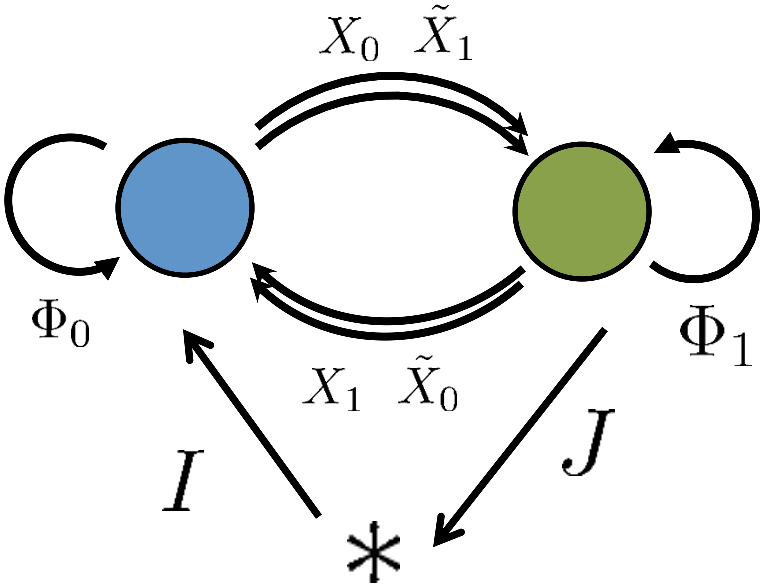


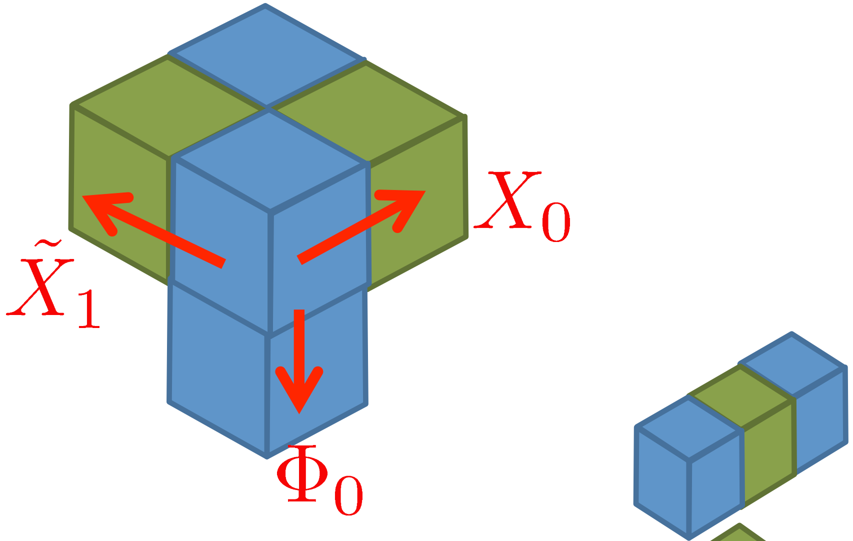
D4-D2-D0 stateに付随するクイーバーゲージ理論のSUSY vacuaでは
D4-ブレーンが巻きつくtoric divisorに1:1対応するperfect matching に
含まれるカイラル場はゼロ。またJ=0 である。
D4-D2-D0 stateの状態数はD6-D2-D0 stateを与える結晶模型に上の条件
を課したもので与えられる。(T.Nishinaka, S.Yamaguchi and Y.Y)

例

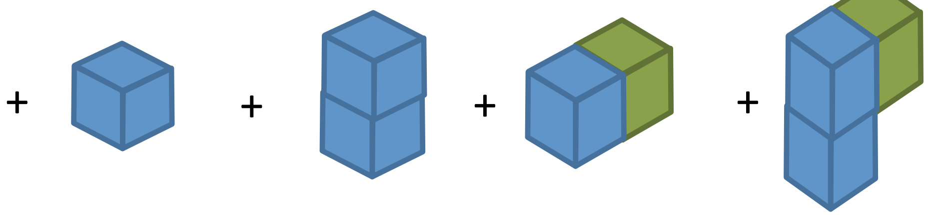


$$\tilde{X}_0 = 0, \quad \tilde{X}_1 = 0$$





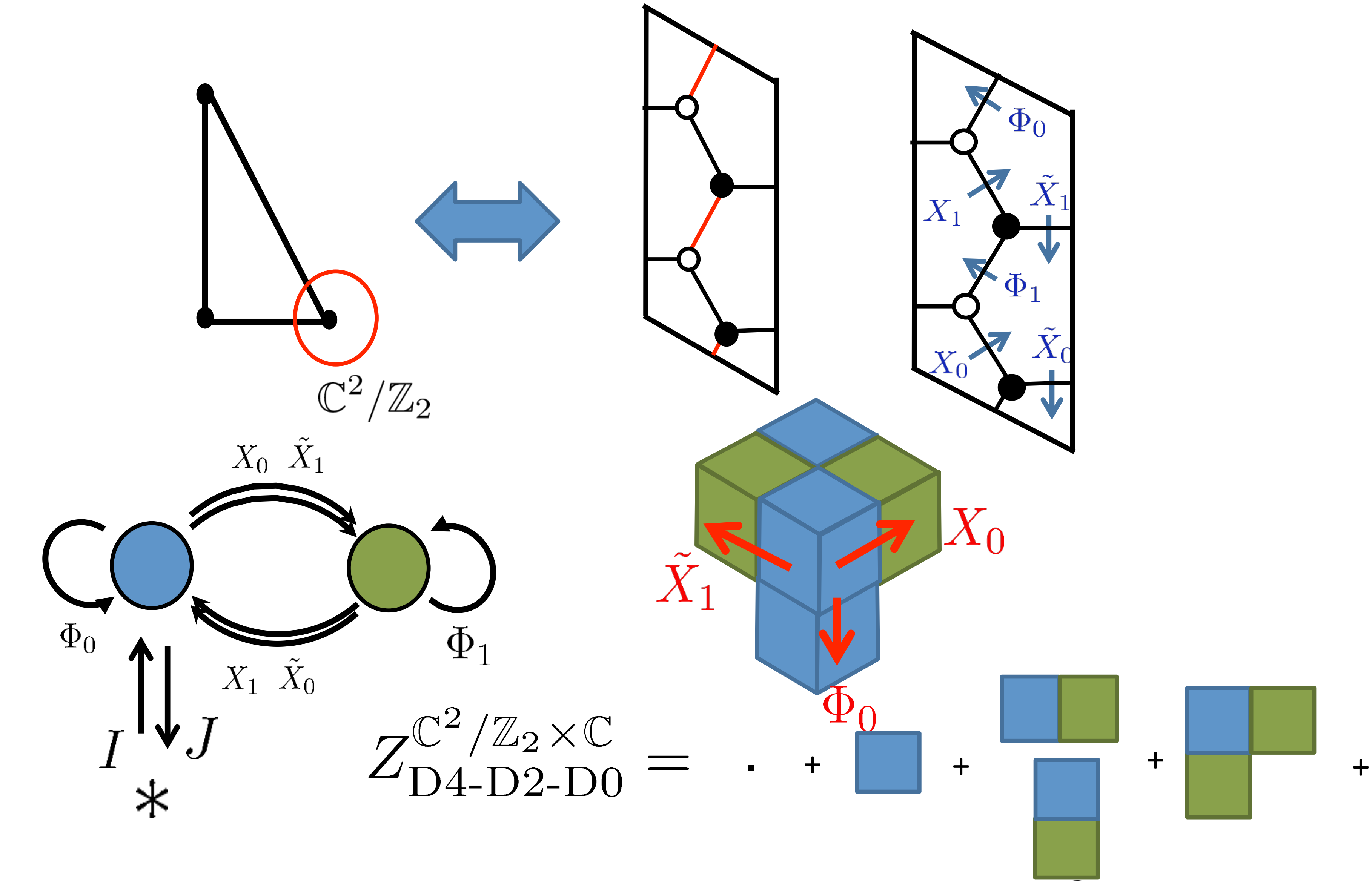
$$Z^{\mathbb{C}^2/\mathbb{Z}_2\times\mathbb{C}}_{\text{D4-D2-D0}} = \cdot + $$



$$= 1 + Q + Q^2 + q + 2qQ + \cdots$$

$\mathbb{C}/\mathbb{Z}_2 \times \mathbb{C}$ 上のU(1) インスタントン
分配関数に一致

$$= \prod_{n=1}^{\infty}\left(\frac{1}{1-q^n}\right)\prod_{m=0}^{\infty}\left(\frac{1}{1-Qq^m}\right)$$



$\mathcal{N}=4$, U(1)-instanton on A_1-ALE $\rightarrow$ $\displaystyle = \prod_{n=1}^{\infty}\left(\frac{1}{1-q^n}\right)^2 \sum_{l\in\mathbb{Z}} q^{l^2} Q^l$

まとめ

コンパクト4-cycleを持たない特異toric CY 3-fold上における D4-D2-D0束縛状態を数え上げる方法を確立した。

D4-D2-D0束縛状態はD6-D2-D0束縛状態の数え上げを与える3次元結晶の2次元部分結晶として与えられる。

我々の与えた結果はWall-Crossing formulaあるいはVafa-Witten理論から計算された結果と一致する。