2d CDT is 2d Hořava-Lifshitz quantum gravity

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August 19th @ YITP workshop

J. Ambjørn, L. Glaser, Y. S. and Y. Watabiki, Physics Letters B 722, 2013

Causal Dynamical Triangulations (CDT)

→ A non-perturbative way to quantise Einstein gravity

lattice

?

Small scale structure regularized by CDT

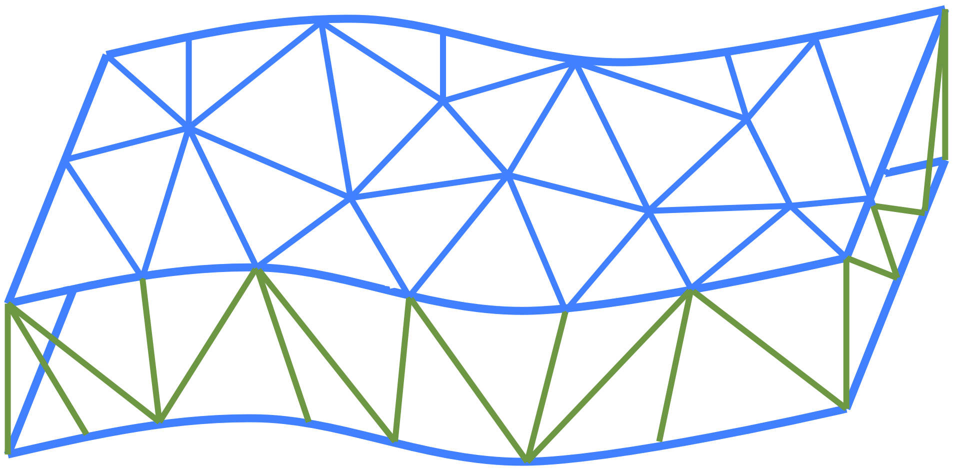
Large scale structure described by field theory

UV - - - - - - - - - → **IR**

Outcome:

1. 4d de Sitter Universe pops up
   (J. Ambjørn, Jurkiewicz and R. Loll, 2004)
2. 2nd-order phase transition
   (J. Ambjørn, A. Gorlich, S. Jordan, Jurkiewicz and R. Loll, 2010)

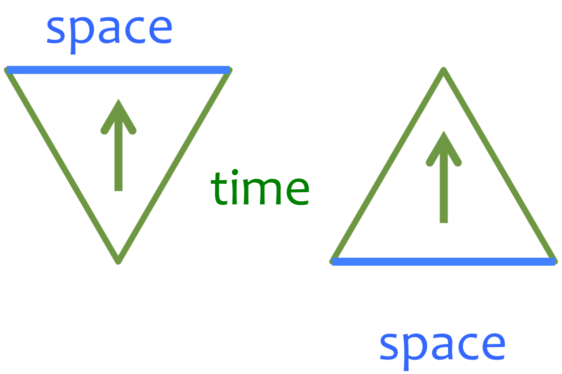
Causal Dynamical Triangulation

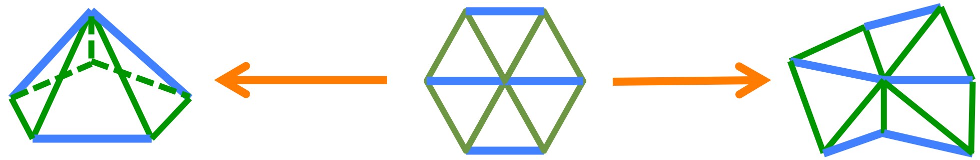


1. 2D CDT

J. Ambjørn, R. Loll, 1998

Lattice (UV cutoff):





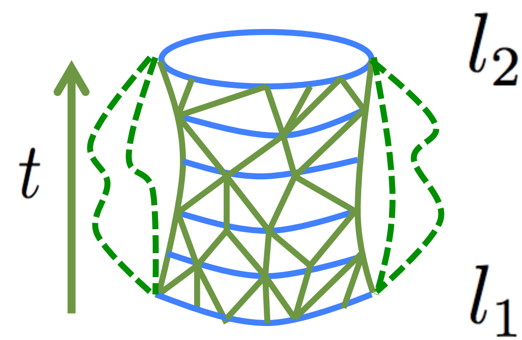
lattice spacing (ε) → fixed

triangulations (T) → dynamical

(= how to divide geometry by triangles)

Metric path-integral → sum over triangulations (T): $\int \mathcal{D}g \to \sum_T$

$$G(l_2, l_1; t) = \sum_{T(l_1,l_2)} e^{-\lambda n(T)}$$



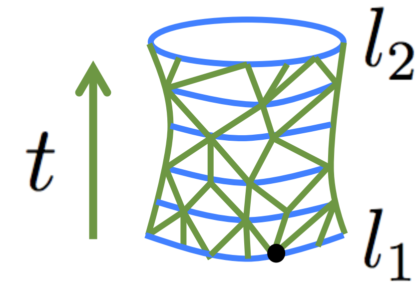
λ : cosmological constant

(△) = n

1. 2D CDT

J. Ambjørn, R. Loll, 1998

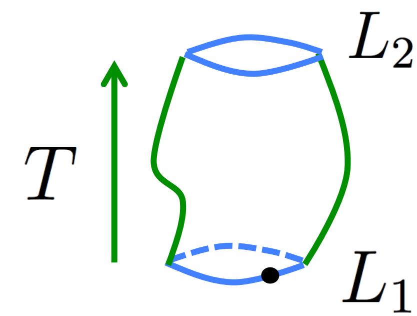
$$G(l_2, l_1; t) = \sum_l G(l_2, l; 1) G(l, l_1; t-1)$$



Continuum limit

$$\lambda \to \lambda_* \quad \& \quad \varepsilon \to 0$$

$$\frac{\partial}{\partial T} G(L_2, L_1; T) = -\hat{H}(L_1) G(L_2, L_1; T)$$



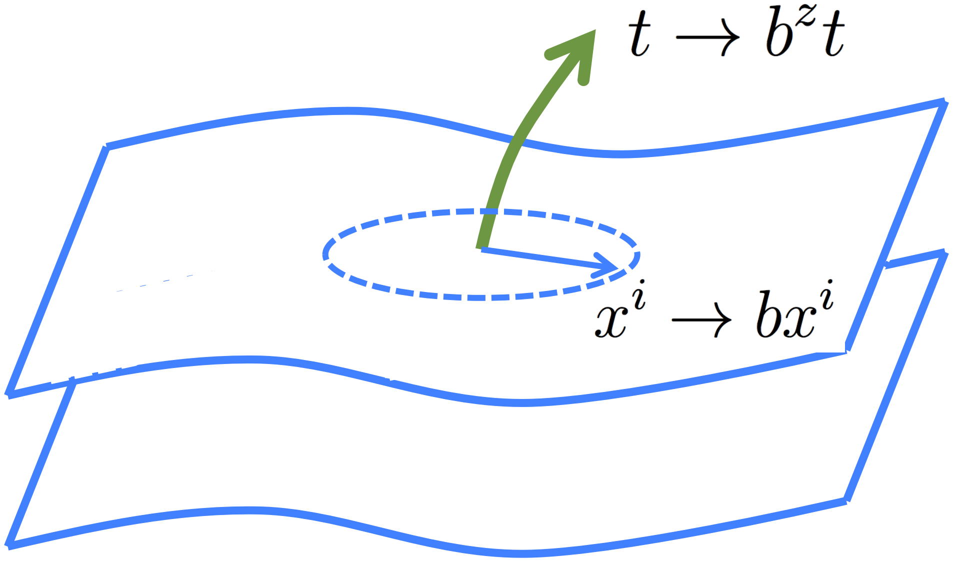
Quantum Hamiltonian

$$\hat{H}(L_1) = -L_1 \frac{\partial^2}{\partial L_1^2} + \Lambda L_1$$

$$\lambda - \lambda_* \sim \varepsilon^2 \Lambda$$

$$L_1 := \varepsilon l_1 \quad L_2 := \varepsilon l_2, \quad T := \varepsilon t \qquad G(L_2, L_1; T) = \lim_{\varepsilon \to 0} \varepsilon^{-1} G(l_2, l_1; t)$$

Horava-Lifshitz quantum gravity



2. 2D HL

J. Ambjørn, L. Glaser, YS, Y. Watabiki 2013

2D **projectable** Horava-Lifshitz gravity (HL):

$$S_{\rm HL} = \int dt\, dx\, N\gamma \left[(1-\lambda)K^2 - 2\Lambda\right]$$

isotropic limit

$$\lambda \to 1$$

projectable lapse

$$N = N(t)$$

where $\gamma := \sqrt{h}$ & $K = \frac{1}{N}\left(\frac{1}{\gamma}\partial_0\gamma - \frac{1}{\gamma^2}\partial_1 N_1 + \frac{N_1}{\gamma^3}\partial_1\gamma\right)$

$$\{\gamma(x,t), \pi^\gamma(y,t)\} = \delta(x-y)$$

$$H = \int dx\, [N\mathcal{H} + N_1\mathcal{H}^1]$$

"Hamiltonian constr." **momentum constr.**

$$\mathcal{H} = \gamma\frac{(\pi^\gamma)^2}{4(1-\lambda)} + 2\Lambda\gamma, \quad \mathcal{H}^1 = -\frac{\partial_1\pi^\gamma}{\gamma}$$

2. 2D HL

J. Ambjørn, L. Glaser, YS, Y. Watabiki 2013

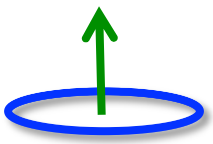
Solve momentum constraint → System reduces to be of 1 dimension

$$H = \int dx \, [N\mathcal{H} + N_1 \mathcal{H}^1]$$

Gauge fixing spatial Diff

$$\mathcal{H}^1 = 0 \quad \text{i.e.} \quad \pi^\gamma(x,t) = \pi^\gamma(t)$$

$$H = N(t) \left(L(t) \frac{(\pi^\gamma(t))^2}{4(1-\lambda)} + 2\Lambda L(t) \right), \qquad L(t) := \int dx \, \gamma(x,t)$$



Quantise the 1D system based on the following action:

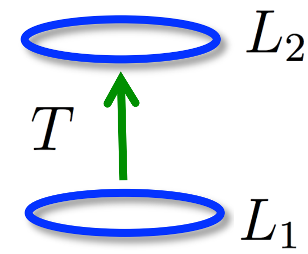
$$S = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} - \tilde{\Lambda} N(t) L(t) \right), \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

(from Hamiltonian constr., $\Lambda > 0 \quad \lambda < 1$)

2. 2D HL

J. Ambjørn, L. Glaser, YS, Y. Watabiki 2013

Quantum amplitude (after a rotation to Euclidean signature):



$$G(L_2, L_1; T) = \int \frac{\mathcal{D}N(t)}{\mathrm{Diff}[0,1]} \int \mathcal{D}L(t)\, \mathrm{e}^{-S_E[N(t),L(t)]}$$

where

$$S_E = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} + \tilde{\Lambda} N(t) L(t) \right)$$

$$\int_0^1 dt\, N(t) = T;$$

$$G(L_2, L_1; T) = \langle L_2 | e^{-T\hat{H}} | L_1 \rangle$$

$$= \int [dL] \boxed{\langle L_2 | e^{-\varepsilon \hat{H}} | L \rangle} G(L, L_1; T - \varepsilon)$$

integrate

$$\boxed{\exp\left(-\frac{(L_2 - L)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)}$$

completeness:

$$\int [dL_1] |L_1\rangle\langle L_1| = 1$$

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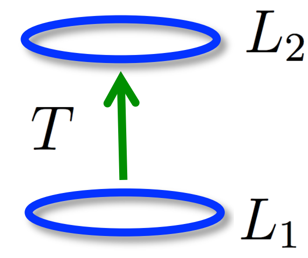

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$$= \int [dL] \langle L_2 | e^{-\varepsilon \hat{H}} | L \rangle G(L, L_1; T - \varepsilon)$$

integrate

compare

$$= G(L_2, L_1; T - \varepsilon) - \varepsilon \hat{H}(L_2) G(L_2, L_1; T) + \cdots$$

expand

$$\exp\left(-\frac{(L_2 - L)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)$$

completeness:

$$\int [dL_1] | L_1 \rangle \langle L_1 | = 1$$

2. 2D HL

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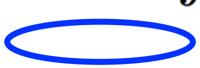
Quantum Hamiltonian for HL

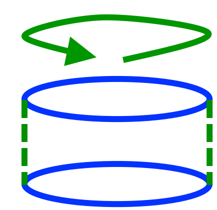
$$\hat{H} = -L\frac{\partial^2}{\partial L^2} - 2a\frac{\partial}{\partial L} - \frac{a(a-1)}{L} + \tilde{\Lambda}L \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

What is a ?

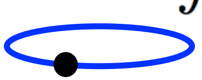
Completeness: $$\int [dL]|L\rangle\langle L| = \int L^a dL|L\rangle\langle L| = 1$$

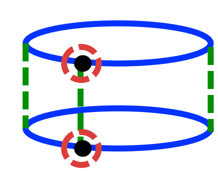
a = 1 $$\int LdL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \frac{1}{L_1}\delta(L_1 - L_2)$$





a = 0 $$\int dL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \delta(L_1 - L_2)$$





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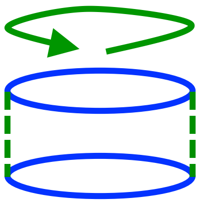
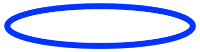
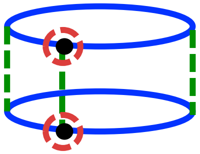
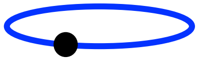




→ CDT Hamiltonian for an **unmarked** loop

a = 0 $$\int dL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \delta(L_1 - L_2)$$





→ CDT Hamiltonian for **a marked** loop

3. SUMMARY & CONJECTURE

2D CDT turns out to be the 2D projectable Horava-Lifshitz quantum gravity:

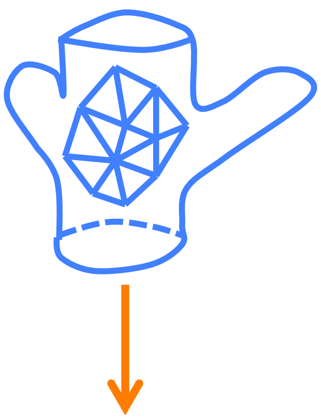
$$S_{\rm HL} = \int dt\, dx\, N\gamma \left[(1-\lambda)K^2 - 2\Lambda\right]$$

where

$$N = N(t)$$

$$\Lambda > 0 \quad \lambda < 1$$

DT

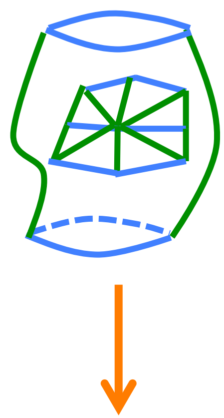


Liouville gravity

Known

continuum

CDT

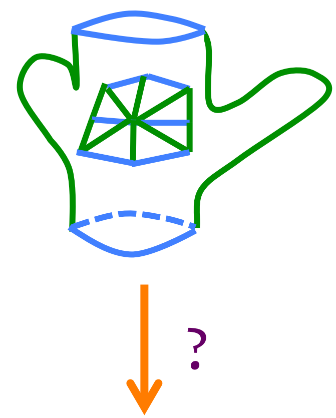


Projectable
Horava-Lifshitz gravity

Our work

continuum

Generalised CDT



Non-projectable
Horava-Lifshitz gravity?

Wild guess