Tensor network and a black hole

石原雅文

東北大学原子分子材料科学高等研究機構 (WPI-AIMR)

Collaborators: 松枝宏明　仙台高等専門学校

橋爪洋一郎　東京理科大学

H. Matsueda, M. I. and Y. Hashizume

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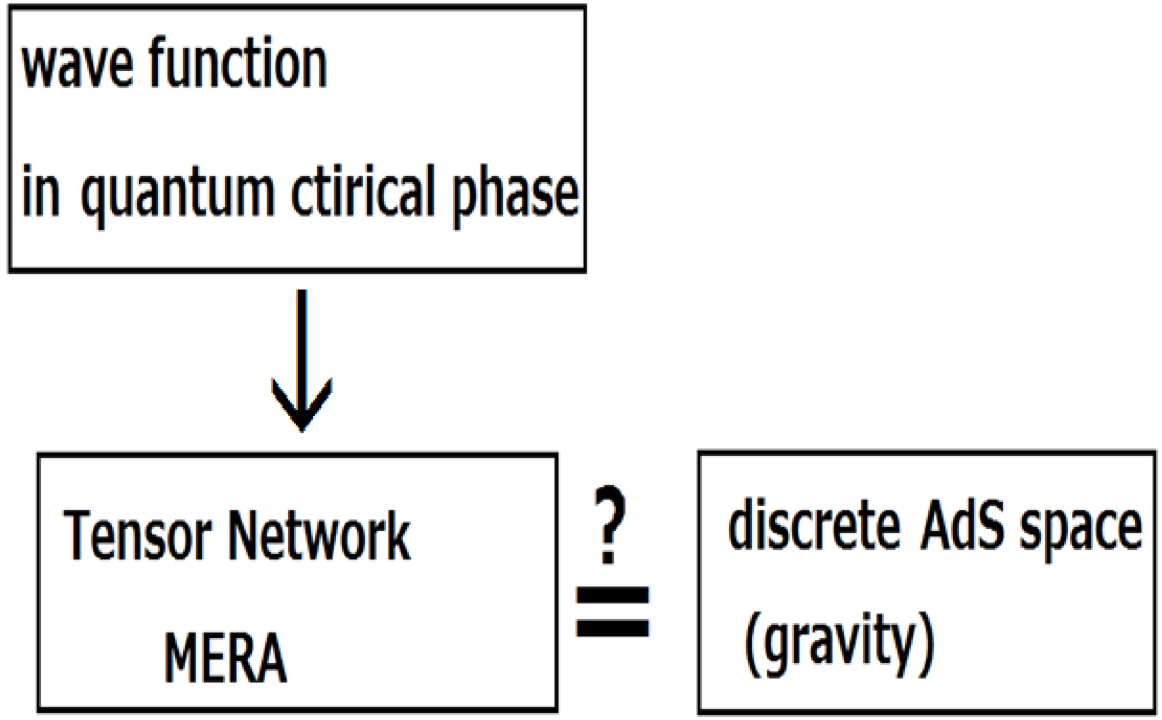
Introduction

Recently the relation between **Tensor Network of wave function in quantum critical phase** and discrete **Anti de Sitter (AdS) space** has been suggested. *(B.Swingle '2009)*



MERA: (Multiscale Entanglement Renormalization Ansatz):

tensor network describing the wave function in critical phase

Outline

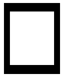
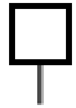
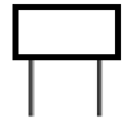
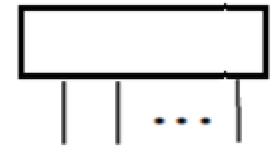
1 Tensor Network (MPS, MERA)

2 Tensor Network and Entanglement Entropy

3 AdS/CFT and MERA

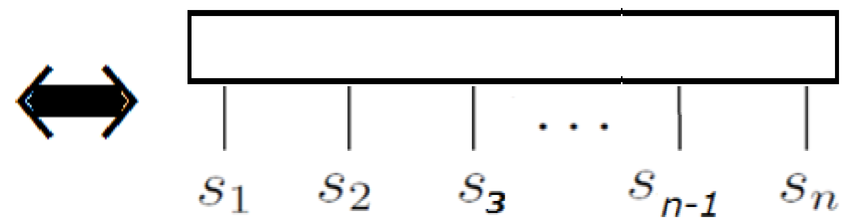
4 MERA at thermal system and AdS black hole

Tensor Network

Scalar  Vector  Matrix  Tensor 

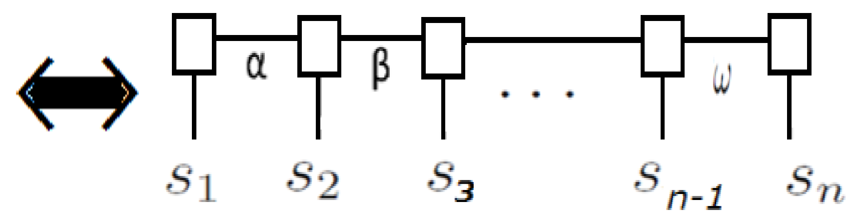
Tensor Network: graphical representation of the wave function of quantum many-body system

$$|\psi\rangle = \sum_{\{s_j\}} c^{s_1 s_2 \cdots s_n} |s_1 s_2 \cdots s_n\rangle$$



Matrix Product States (MPS)

$$|\psi\rangle = \sum_{\{s_j\}} c^{s_1}_{\alpha} c^{s_2}_{\alpha\beta} c^{s_3}_{\beta\gamma} \cdots c^{s_{n-1}}_{\psi\omega} c^{s_n}_{\omega} |s_1 s_2 \cdots s_n\rangle$$



External line: spin indices Internal line: inner product

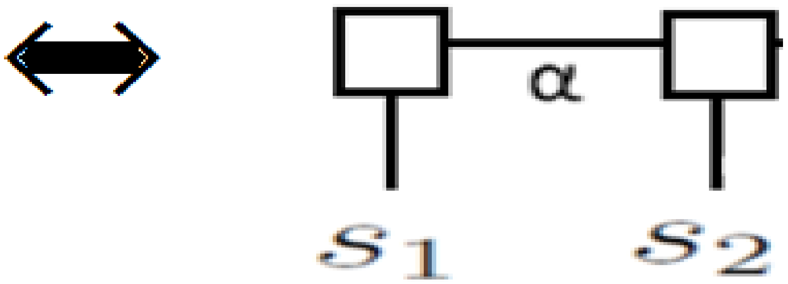
E.g. Tensor Network in 2-site Heisenberg model

2-site anti-ferromagnetic Heisenberg model.

$$\boldsymbol{H} = \boldsymbol{J}\overrightarrow{\boldsymbol{S}_1} \cdot \overrightarrow{\boldsymbol{S}_2} = \frac{1}{2}\mathbf{J}(\mathbf{S}_1^+ \boldsymbol{S}_2^-) + \boldsymbol{J}\boldsymbol{S}_1^z \boldsymbol{S}_2^z$$

Ground state and its MPS representation

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) = \frac{1}{\sqrt{2}}\sum_{s_1,s_2=\uparrow,\downarrow} A_\alpha^{s_1} B_\alpha^{s_2} |s_1 s_2\rangle$$



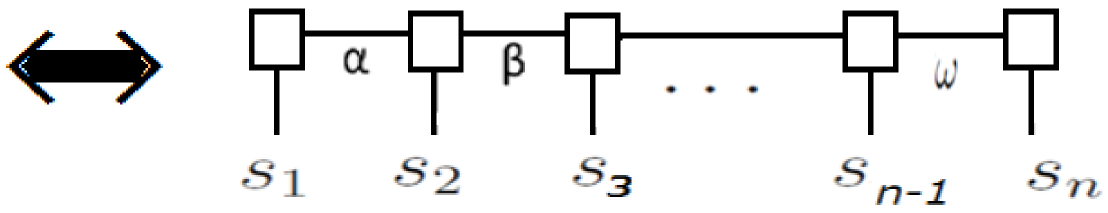
A, B are the following vectors. (dimension: ***m=2***)

$$A^{\uparrow} = (1, 0),\ \ A^{\downarrow} = (0, -1),\ \ B^{\uparrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ \ B^{\downarrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

Matrix Product States

Matrix Product States: Coefficient of the wave function is the product of matrices. (gapped n-particle system)

$$|\psi\rangle = \sum_{\{s_j\}} c_{\alpha}^{s_1} c_{\alpha\beta}^{s_2} c_{\beta\gamma}^{s_3} \cdots c_{\psi\omega}^{s_{n-1}} c_{\omega}^{s_n} |s_1 s_2 \cdots s_n\rangle$$

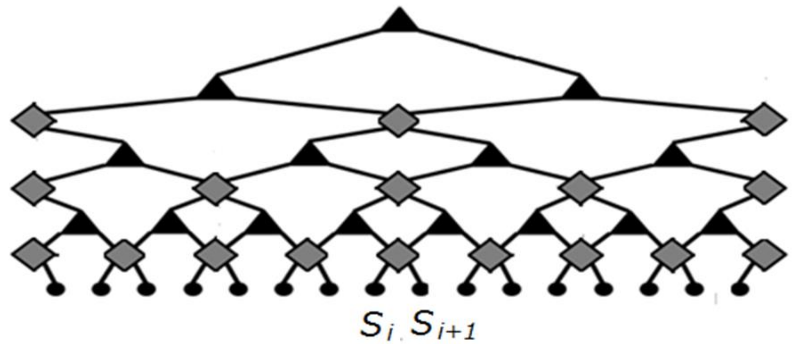


Vertex: matrix internal line: inner product external line: spin indices

MERA

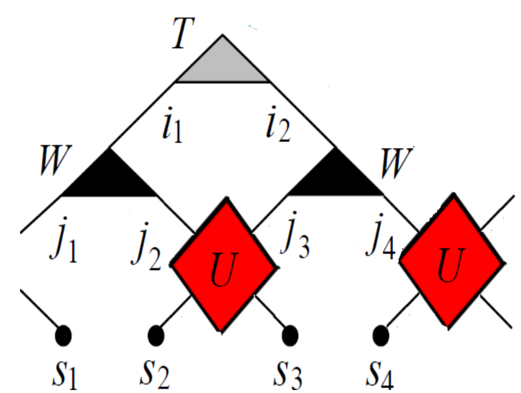
Tensor network of ground state wave-function of n-particle system in **quantum critical phase** can be written by **MERA** (Multi-scale entanglement renormalization ansatz) (G.Vidal 2007)

$|\psi\rangle \quad \Longleftrightarrow$

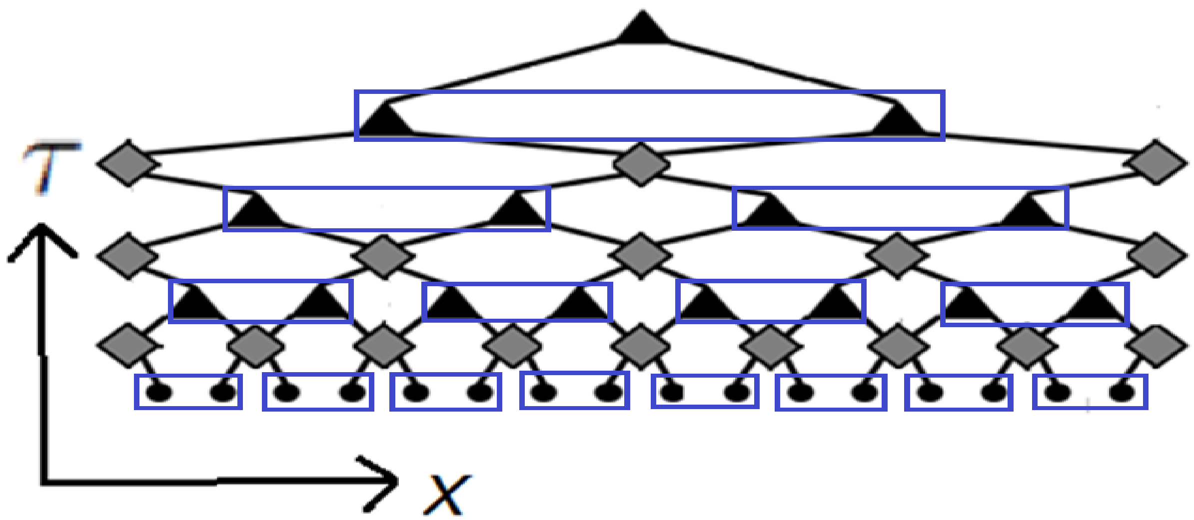


4-site case:

$$|\psi\rangle = \sum_i \sum_j \sum_s T_{i_1 i_2} W^{i_1}_{j_1 j_2} W^{i_2}_{j_3 j_4} U^{j_2 j_3}_{s_2 s_3} U^{j_4 j_1}_{s_4 s_1} |s_1 s_2 s_3 s_4\rangle \quad \Longleftrightarrow$$



MERA and RG transformation



τ direction: RG transformation (coarse-graining)

- **“projection”** along the RG transformation (3-rank tensor)
- **“disentangler”** which removes the short-range entanglement between each blocks (4-rank tensor)

Entanglement Entropy

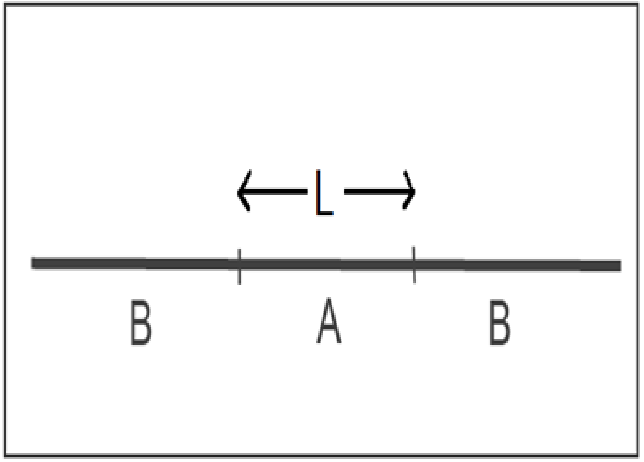
Tensor Network is useful for the calculation of the entanglement entropy

Entanglement Entropy $\boldsymbol{S_{EE}}$: Number of correlations between region *A* and region *B.*

$$\boldsymbol{S_{EE} = -Tr(\rho_A ln\, \rho_A)}$$

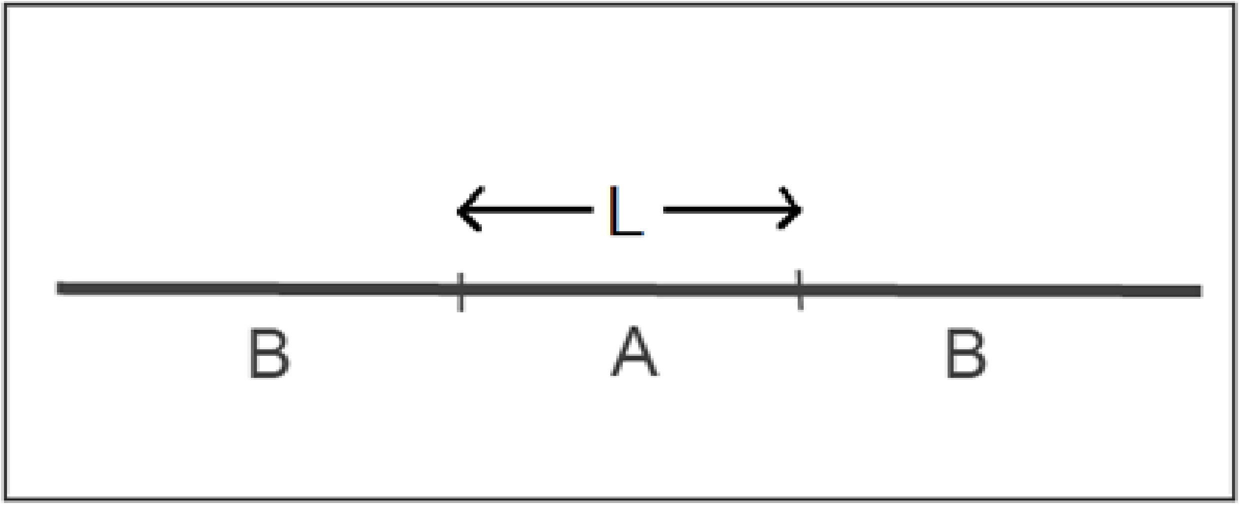
$\boldsymbol{\rho_A = Tr_B \rho_{tot}}$: *reduced density matrix*

$\boldsymbol{\rho_{tot}} = |\boldsymbol{\psi}\rangle\langle\boldsymbol{\psi}|$ *:density matrix of the total system*



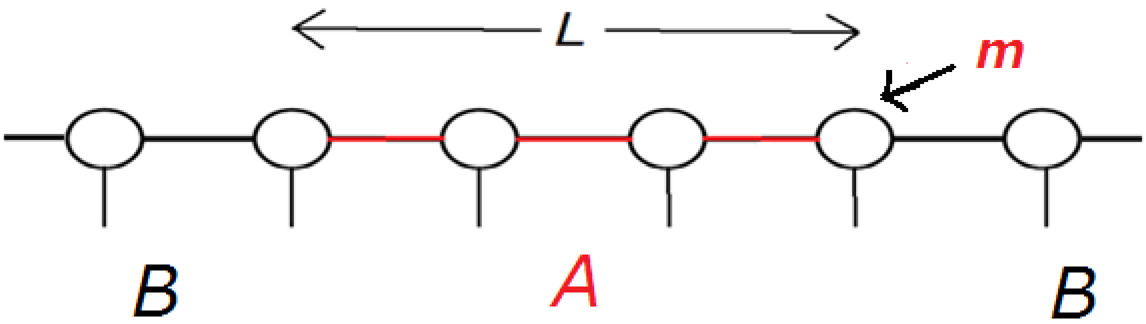
Entanglement Entropy of gapped system

Entanglement Entropy in (1+1)-dimensional gapped system is known to be constant *(independent of system size **L**).*



$$S_{EE} \sim const$$

Entanglement Entropy by Matrix Product States



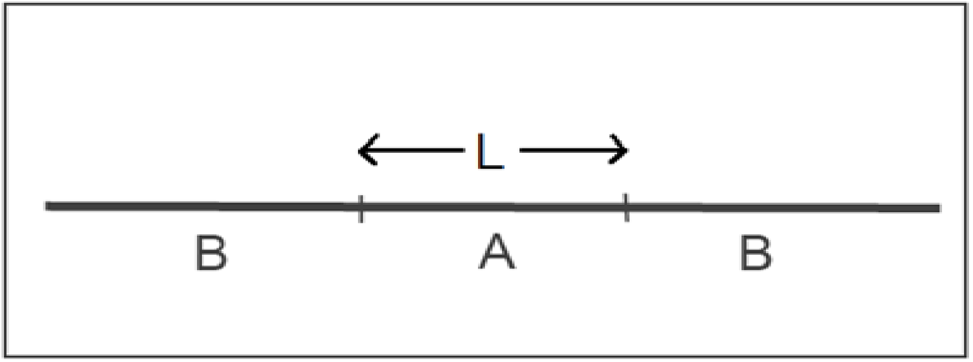
each matrix dimension: $\boldsymbol{m}$

degree of freedoms at the two boundaries $= \boldsymbol{m} \times \boldsymbol{m}$

$$\boldsymbol{S_{EE} \sim \ln m^2 \sim const}$$

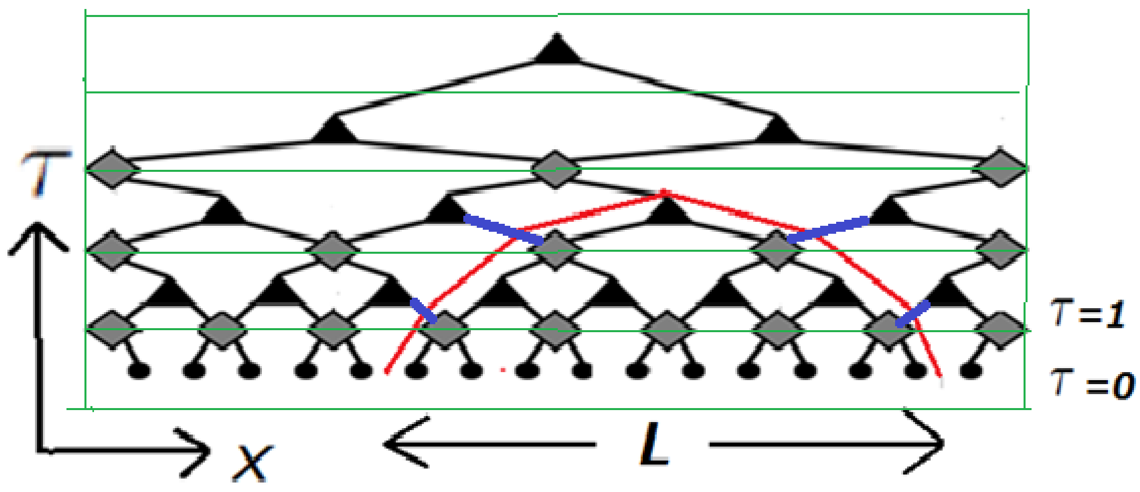
Entanglement Entropy in critical system

Entanglement entropy in (1+1)-dimensional critical system is not constant but has the $\ln L$-dependence for the long range correlation in the system.



$$S_{EE} \sim \ln L$$

Entanglement entropy in critical phase



tensor dimension : m

Number of boundary bonds (blue bonds) $\sim \ln L$

Total degree of freedoms at the boundary $\sim m^{\ln L}$

Entanglement Entropy: $S_{EE} \sim \ln m^{\ln L} \sim \ln L$

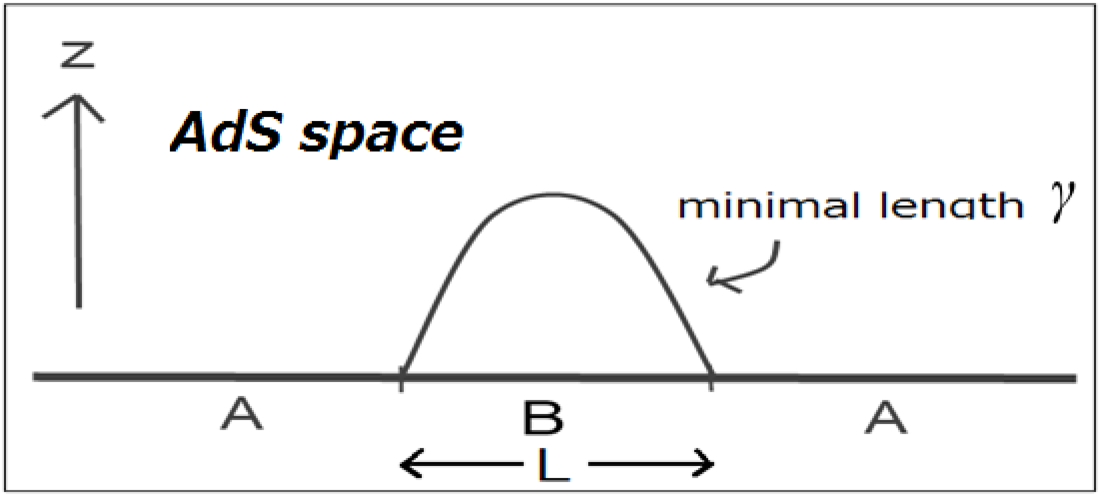
Entanglement Entropy by AdS/CFT

Entanglement Entropy by AdS/CFT *(S. Ryu and T. Takayanagi '06)*

$$S_{EE} = \frac{\gamma}{4G} \sim \ln L$$

γ : the minimal length in ***AdS3*** space

G : Newton constant



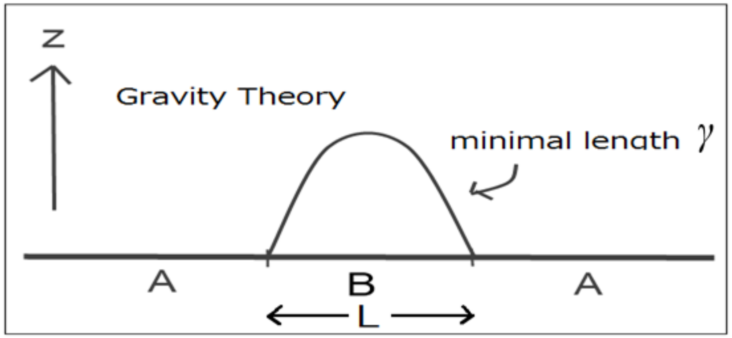
AdS space-time and MERA

MERA network $\approx$ a discrete version of AdS space

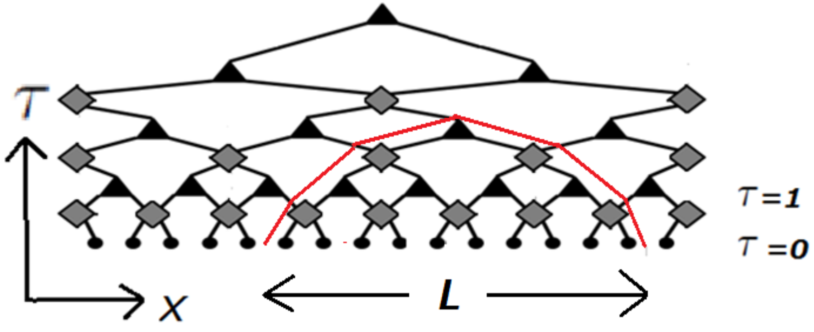
(B. Swingle, 2009)

$$z = 2^{\tau}$$

$$ds^2 = \frac{dz^2 + dx^2}{z^2} = \{d\,(\tau \ln 2)\}^2 + (2^{-\tau} dx)^2$$



$\approx$



MERA at thermal system and AdS black hole

(d+1)-dimensional Quantum field theory at thermal system	⟺	((d+1)+1)-dimensional AdS black hole

MERA at thermal system $\stackrel{?}{=}$ a discrete version of AdS black hole

Thermo Field Dynamics Formalism

(Y. Takahashi and H. Umezawa, 1975)

the thermal state for temperature $\frac{1}{\beta}$

= the products of state in Hilbert space and that of copy (tilde) space

$$|\psi(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} = e^{-\frac{\beta H}{2}}|\mathrm{I}\rangle \qquad |I\rangle = \sum_n |n\rangle|\tilde{n}\rangle = \sum_n |n\tilde{n}\rangle$$

$$H \ldots \{|n\rangle\}$$

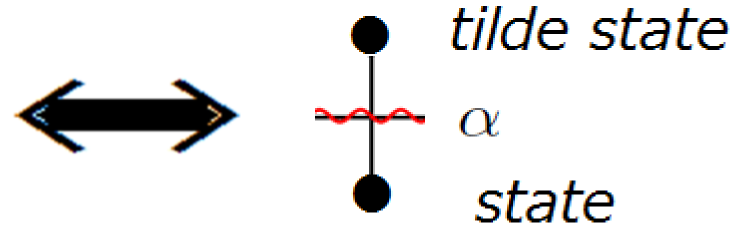
the Vacuum Expectation Value of the operator ***A***

$$\langle A\rangle_s = \langle\psi(\beta)|A|\psi(\beta)\rangle = TrA\rho(\beta)$$

Tensor Network of Thermal State in single site model

Thermal state at $T = \frac{1}{2k_B\theta}$ can be written as an Matrix Product States.

$$|O(\theta)\rangle = \left(\cos(\hbar\omega\theta) + \sin(\hbar\omega\theta)\, a^{\dagger}\tilde{a}^{\dagger}\right)|0\tilde{0}\rangle = \sum_{m,\tilde{n}=0,1} A_{\alpha}^{n} A_{\alpha}^{\tilde{n}} |n\tilde{n}\rangle$$



$$A^{0} = \left(\sqrt{\cos(\hbar\omega\theta)}, 0\right) \qquad A^{1} = \left(0, \sqrt{\sin(\hbar\omega\theta}\right)$$

$$A^{\tilde{0}} = \begin{pmatrix} \sqrt{\cos(\hbar\omega\theta)} \\ 0 \end{pmatrix} \qquad A^{\tilde{1}} = \begin{pmatrix} 0 \\ \sqrt{\sin(\hbar\omega\theta)^{*}} \end{pmatrix}$$

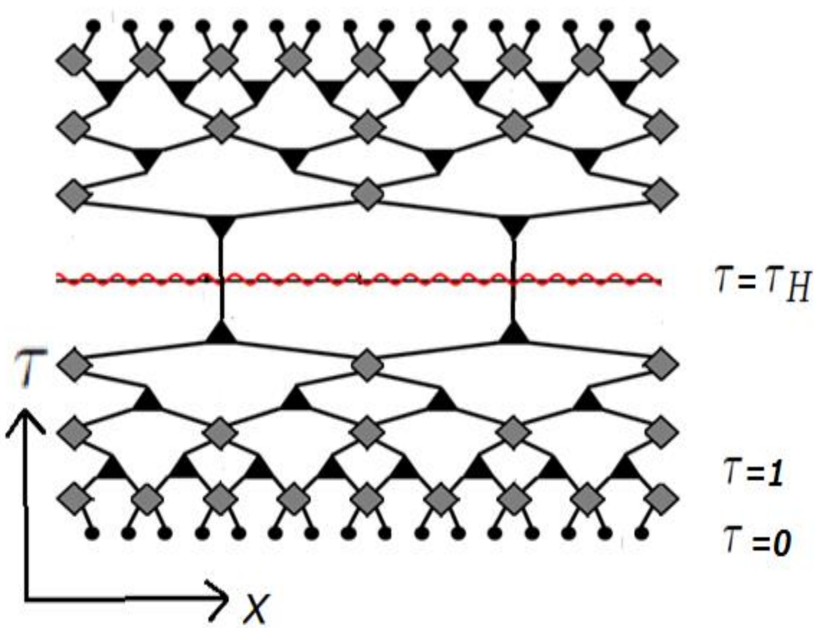
MERA at thermal system

By using Thermo Field Dynamics *(Y. Takahashi and H. Umezawa, 1975)*

, we suggest the following MERA at thermal system and the

interface (red line) corresponds to the AdS black hole horizon.

$$|\psi\rangle = \sum_{\{n_j\}} \sum_{\{\tilde{n}_k\}} A^{n_1 n_2 \cdots n_L} A^{\tilde{n}_1 \tilde{n}_2 \cdots \tilde{n}_L} |n_1 n_2 \cdots n_L \tilde{n}_1 \tilde{n}_2 \cdots \tilde{n}_L\rangle$$



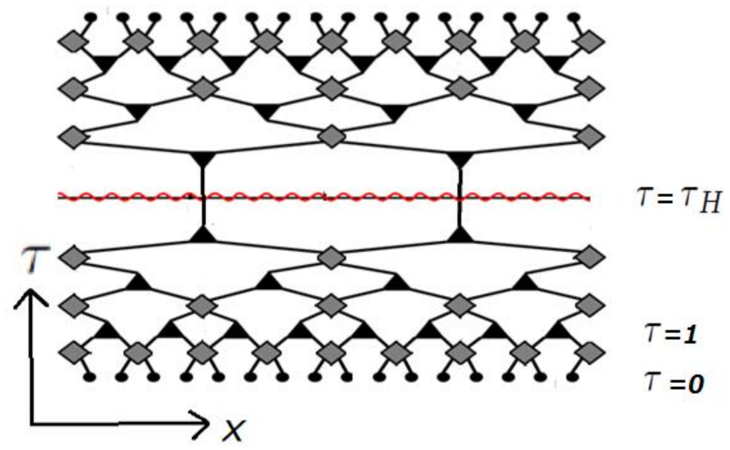
Entanglement Entropy by thermal MERA

the degree of freedoms χ at the interface (red line)

$$\chi = m^A$$

A: number of bonds at the interface

m: tensor dimensions



the entanglement entropy S_{EE} at the interface

L: system size

$$\frac{L}{2^{\tau_H}} = A \qquad S_{EE} = \ln \chi = \frac{L}{z_H} \ln m$$

$$z = 2^{\tau}$$

$$z_H = 2^{\tau_H}$$

Hawking Temperature

Entanglement Entropy in (1+1)-dimensional CFT at finite temperature,

$$\boldsymbol{S_{EE} = \frac{c}{3}\ln\left(\frac{\beta}{\pi\epsilon}\sinh\left(\frac{\pi L}{\beta}\right)\right) \simeq \frac{c}{3}\ln\left(\frac{\beta}{2\pi\epsilon}\right) + \frac{c}{3}\frac{\pi L}{\beta}}$$

$\boldsymbol{\epsilon}$: UV-cutoff *(P. Calabrese and J. Cardy and J.S. Mech, 2004)*

By comparing these two formula, we can find that

$$\boldsymbol{k_B T = \left(\frac{3}{c\pi} ln\, m\right)\frac{1}{z_H}}$$

Estimation of the *tensor dimension*

Hawking Temperature in AdS-black hole

$$k_B T = \frac{1}{2\pi z_H}$$

$$ds^2_{BH} = \frac{L^2}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2\right) \qquad f(z) = 1 - \left(\frac{z}{z_H}\right)^2$$

By comparing the Hawking temperature with the temperature given by MERA , we can get the relation between **central charge** and **tensor dimension** near the interface

$$\frac{3}{c\pi}\ln m = \frac{1}{2\pi} \quad \Longleftrightarrow \quad m = e^{\frac{c}{6}}$$

Summary

Recently, the relation between AdS/CFT and MERA becomes interesting topic.

We consider that how the black hole horizon appears in the MERA network by using thermo filed dynamics (TFD) formalism and it is appeared as the interface between MERA and tilde-MERA network.

By comparing the Hawking temperature with the temperature given by MERA , we can get the relation between **central charge** and **tensor dimension** near the interface

Future Works

Relation between two-dimensional MERA and AdS/CFT

(triangular lattice, square lattice…)

Relation between D-brane and top tensor of MERA *(M. Nozaki, S.Ryu and T. Takayanagi, 2012).*