From the Berkovits formulation to the Witten formulation in open superstring field theory

鳥居真吾

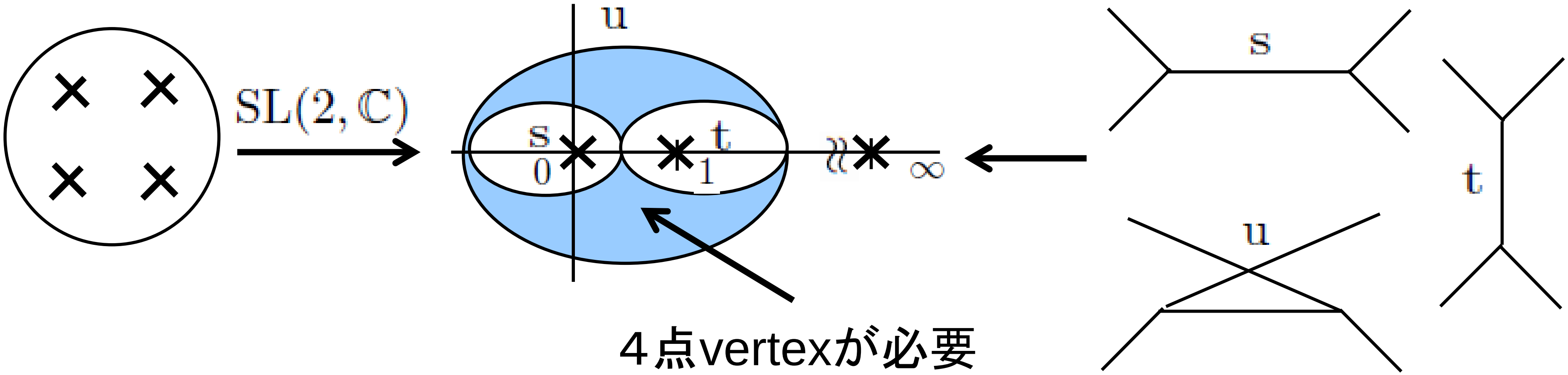
Collaborators: 飯森悠樹, 野海俊文, 大川祐司

研究の背景：String Field Theory (SFT) と moduli

bosonic SFT

理論のゲージ不変性 ⟺ Riemann面のmoduliのcovering

例：**closed** bosonic SFT [Zwiebach]における４点tree振幅



SuperSFT

ゲージ不変性 ⟺？ super-Riemann面の supermoduliのcovering

2 formulations of Open SuperSFT (NS-sector)

Witten formulation

action は Chern-Simons 型
(cubic action)

4点on-shell tree振幅は発散.
ゲージ対称性がsingular.

picture-changing operator
(PCO) あり。

Berkovits formulation

Wess-Zumino-Witten 型
(non-polynomial action)

正しい4点on-shell tree振幅を再現.
ゲージ対称性もOK.

PCO なし。

※注意: 一般には、以下のreductionが起きない。[Witten], [Donagi-Witten]

supermoduli $\not\Rightarrow$ bosonic moduli + PCO insertion

fermionic moduli の積分

2 formulations of Open SuperSFT (NS-sector)

Berkovits formulation

Wess-Zumino-Witten 型
(non-polynomial action)

higher-point vertices が重要 ⇒ 正しい４点on-shell tree振幅を再現.
ゲージ対称性もOK.

Berkovits理論のhigher-point vertices

bosonic moduli の covering への寄与 : measure ゼロ

⇒ higher-point vertices は supermoduli の covering に関係あり？

２つのformulations を比較することで、higher-point vertices の役割を明らかにできるのではないか？

本研究では何をしたか？

２つのformulations を比較することで、higher-point vertices の役割を調べた。(supermoduli との詳しい関係は future work.)

How?

ある特別なゲージで、Berkovits 理論の partial gauge fixing をする。

結果

1. partially gauge-fixed action
   ≒ Witten action + counterterms

2. residual gauge transf.
   ＝ regularized version of the Witten gauge transf.

Plan

1. Two Formulations of Open SuperSFT
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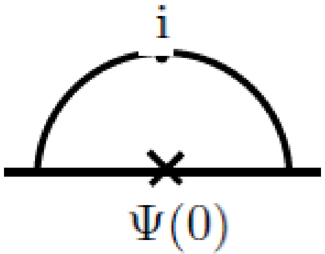
1. Two Formulations of Open SuperSFT

string field (例) bosonic string field

$$\Psi_{\text{bosonic}} = \int \frac{d^{26}k}{(2\pi)^{26}} \left(\phi(k) c_1 |0;k\rangle + A_\mu(k) \alpha^\mu_{-1} c_1 |0;k\rangle + \cdots \right) \Leftrightarrow$$



Hilbert space of the superstring

- spanned by $X^\mu, \psi^\mu, b, c, \beta, \gamma$ ⇒ fermionize: $\beta = e^{-\phi}\underline{\partial\xi}\,, \ \gamma = \eta e^{\phi}$

微分なしの ξ は不要(ξ_0 は不要)

$$\xi(z) = \sum_n \xi_n z^{-n}$$

$\boxed{\text{Without } \xi_0} \longrightarrow$ small Hilbert space $\mathscr{H}_{\text{small}}$

$$\eta_0 \mathscr{H}_{\text{small}} = 0\,, \quad \{\xi_0, \eta_0\} = 1\,.$$

$\boxed{\text{With } \xi_0} \longrightarrow$ large Hilbert space $\mathscr{H}_{\text{large}}$

$$\mathscr{H}_{\text{large}} = \mathscr{H}_{\text{small}} \oplus \xi_0 \mathscr{H}_{\text{small}}$$

$$\underset{\text{small}}{\langle\langle (\cdots) \rangle\rangle} = \mathrm{i}\, \underset{\text{large}}{\langle \xi_0 (\cdots) \rangle} \quad \text{for} \quad (\cdots) \in \mathscr{H}_{\text{small}}$$

The Witten formulation

$$S^{\mathrm{W}} = -\left\langle\!\!\left\langle \frac{1}{2}\Psi * (Q\Psi) + \frac{g}{3}\underline{X(\mathrm{i})}\Psi * \Psi * \Psi \right\rangle\!\!\right\rangle \qquad \Psi \in \mathscr{H}_{\underline{\mathrm{small}}}$$

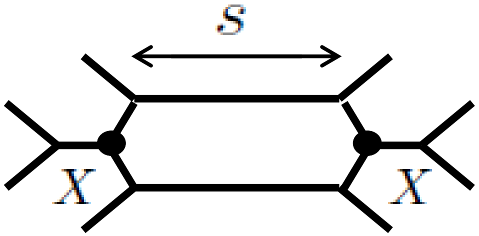
picture-changing operator (PCO)

※以下、積の記号 ” ＊ ” は省略
することあり。

$$X(z) \equiv \{Q, \xi(z)\}$$

PCO が引き起こす問題

1. 振幅の発散



$$X(z_1)X(z_2) \sim \mathcal{O}\left((z_1 - z_2)^{-2}\right)$$

OPE が singular なので、s → 0 で発散

2. ゲージ変換が singular

$$\delta\Psi = Q\Lambda + g\underline{X(\mathrm{i})}[\Psi, \Lambda]$$ ➡ cubic term の変分が発散

The Berkovits formulation

$$S^{\mathrm{B}} = \frac{\mathrm{i}}{2g^2}\Big\langle G^{-1}(QG)G^{-1}(\eta_0 G) - \int_0^1 \mathrm{d}t\,(\hat{G}^{-1}\partial_t\hat{G})\Big\{\hat{G}^{-1}(Q\hat{G}),\hat{G}^{-1}(\eta_0\hat{G})\Big\}\Big\rangle$$

$$G = \exp\big(g\Phi\big)\,,\quad \hat{G} = \exp\big(tg\Phi\big)\,.\qquad \Phi \in \underline{\mathscr{H}_{\mathrm{large}}}$$

$$S^{\mathrm{B}} = -\frac{\mathrm{i}}{2}\Big\langle \Phi\,(Q\eta_0\Phi)\Big\rangle - \frac{\mathrm{i}}{3!}\,g\Big\langle \Phi\{Q\Phi,\eta_0\Phi\}\Big\rangle - \frac{\mathrm{i}}{4!}\,g^2\Big\langle [\Phi,Q\Phi]\,[\Phi,\eta_0\Phi]\Big\rangle + \cdots$$

Relation to the WZW action: $g \to G\,,\ \partial \to Q\,,\ \bar{\partial} \to \eta_0$

PCO なし ➡ 発散なし　正しい4点振幅を再現 [Berkovits-Echevarria]

gauge transf. $\delta\,\mathrm{e}^{g\Phi} = g\Big[\big(Q\epsilon_Q\big)\,\mathrm{e}^{g\Phi} + \underline{\mathrm{e}^{g\Phi}\big(\eta_0\,\epsilon_\eta\big)}\Big]$

partial gauge fixing する。

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The basic idea of partial gauge fixing

Witten formulation

Berkovits formulation

free EOM

$$Q\Psi = 0$$

$$Q\eta_0\Phi = 0$$

$$(\Psi \in \mathscr{H}_{\mathrm{small}} \iff \eta_0\Psi = 0)$$

The basic idea of partial gauge fixing

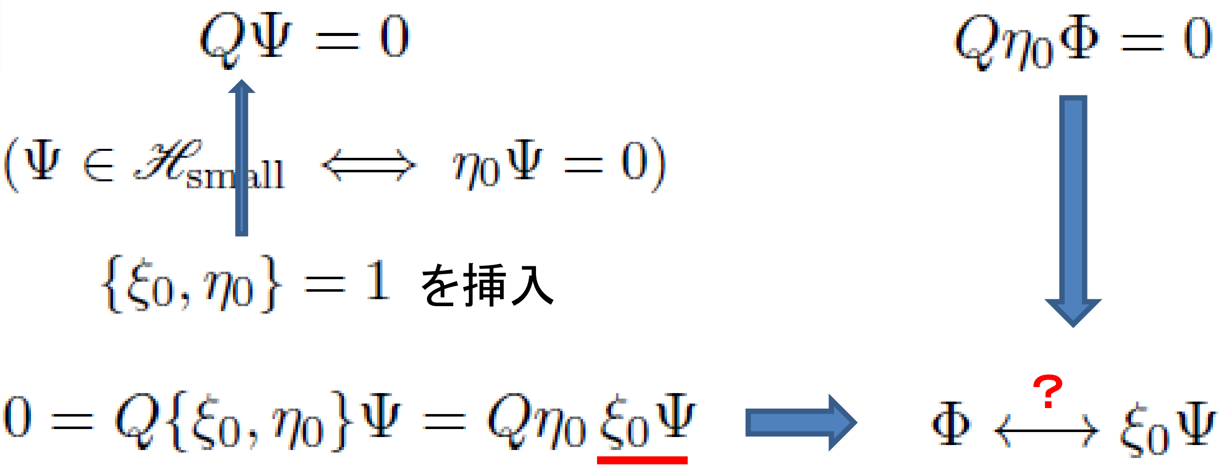
	Witten formulation	**Berkovits** formulation
free EOM	$Q\Psi = 0$	$Q\eta_0\Phi = 0$

$$(\Psi \in \mathscr{H}_{\text{small}} \iff \eta_0\Psi = 0)$$

$\{\xi_0, \eta_0\} = 1$ を挿入

$$0 = Q\{\xi_0, \eta_0\}\Psi = Q\eta_0\,\underline{\xi_0\Psi}$$

The basic idea of partial gauge fixing



free EOM

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Berkovits formulation

$Q\eta_0\Phi = 0$

$\Phi \overset{?}{\longleftrightarrow} \xi_0\Psi$

The basic idea of partial gauge fixing

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Berkovits formulation:

$$Q\eta_0\Phi = 0$$

$$\Phi \overset{?}{\longleftrightarrow} \xi_0\Psi$$

Free gauge transf. $\delta\Phi = Q\epsilon_Q + \underline{\eta_0\epsilon_\eta}$ を使う。

$$\Rightarrow \quad \underline{\xi_0\Phi = 0} \iff \Phi = \xi_0\Psi\,,\ \exists\Psi \in \mathscr{H}_{\text{small}}$$

partial gauge fixing

The basic idea of partial gauge fixing

Witten formulation

Berkovits formulation

free action

$$S^{\mathrm{W}} = -\frac{1}{2}\langle\!\langle \Psi * (Q\Psi) \rangle\!\rangle$$

$$S^{\mathrm{B}} = -\frac{\mathrm{i}}{2}\langle \Phi * (Q\eta_0\Phi) \rangle$$

$$\Big\downarrow \quad \Phi = \xi_0\Psi\,, \quad \exists\Psi \in \mathscr{H}_{\mathrm{small}}$$

$$S^{\mathrm{B}} = -\frac{\mathrm{i}}{2}\langle (\xi_0\Psi)\,(Q\underline{\eta_0\xi_0\Psi}) \rangle$$

$$\Big\downarrow \quad \eta_0\xi_0\Psi = (1-\xi_0\eta_0)\Psi = \Psi$$

$$\langle\!\langle (\cdots) \rangle\!\rangle = \mathrm{i}\,\langle \xi_0\,(\cdots) \rangle$$

$$S^{\mathrm{B}} = -\frac{1}{2}\langle\!\langle \Psi\,(Q\Psi)\, \rangle\!\rangle$$

※free の場合の議論は ξ_0 でなくても成り立つ。

$\Xi^2 = 0\,,\ \{\Xi, \eta_0\} = 1$ という Ξ を用いて $\Xi\Phi = 0$ でもOK だが・・・

Interacting theory で2つのformulation の関係を見るには、Ξ の選び方が重要

Witten action の cubic term

$$-\frac{g}{3}\langle\langle \underline{X(\mathrm{i})}\,\Psi^3 \rangle\rangle$$

$X(z) \equiv \{Q, \xi(z)\}$

Berkovits action の cubic term

$$\sim g\,\langle \Phi\,(Q\Phi)\,(\eta_0\Phi) \rangle$$

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$$\Xi\overset{??}{=}\xi(\mathrm{i})$$

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$$X(z) \equiv \{Q, \xi(z)\}$$

gauge を regularize できれば、
Witten action と等価！

$$\langle\langle (\cdots) \rangle\rangle = \mathrm{i}\,\langle \Xi\,(\cdots) \rangle$$

Berkovits action の cubic term

$$\sim g\,\langle \Phi\,(Q\Phi)\,(\eta_0\Phi) \rangle$$

??

$$\Xi = \xi(\mathrm{i})$$

$$\xi(\mathrm{i})\Phi = 0$$

$$(\Leftrightarrow \quad \Phi = \xi(\mathrm{i})\Psi)$$

$$g\,\langle \underline{\xi(\mathrm{i})X(\mathrm{i})}\,\Psi^3 \rangle$$

← not well-defined

$$\xi(z)X(0) = O(z^{-2})$$

One-parameter family of regular gauges

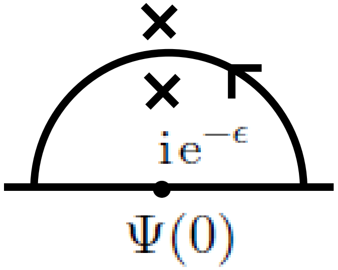
$$\boxed{\Xi_\epsilon \Phi = 0\,, \quad \Xi_\epsilon = \oint \frac{\mathrm{d}z}{2\pi \mathrm{i}} u_\epsilon(z)\xi(z) \quad (0 < \epsilon < \infty)\,.}$$

$u_\epsilon(z)$: midpoint z = i 付近にsimple pole を持つ関数の族

$$\boxed{u_\epsilon(z) = \frac{1}{z - \mathrm{i}e^{-\epsilon}} - \frac{1}{z - \mathrm{i}e^{\epsilon}}}$$

$$\Xi_\epsilon \Phi = 0 \iff \Phi = \Xi_\epsilon \Psi$$

$$\Xi_\epsilon \Psi(0) =$$



$$= \xi(\mathrm{i}e^{-\epsilon})\Psi(0) + \underline{\mathcal{O}(\epsilon)}$$

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$$\Phi = \Xi_\epsilon\Psi$$
$$\mathcal{X}_\epsilon \equiv \{Q,\Xi_\epsilon\}$$

cubic

$$\mathrm{i}\langle\Phi\left(Q\Phi\right)\left(\eta_0\Phi\right)\rangle = \mathrm{i}\langle\Xi_\epsilon\Psi\left(\mathcal{X}_\epsilon-\Xi_\epsilon Q\right)\Psi\Psi\rangle$$
$$= \underline{\langle\!\langle\Psi\left(\mathcal{X}_\epsilon\Psi\right)\Psi\rangle\!\rangle} - \mathrm{i}\underline{\langle\Xi_\epsilon\Psi\left(\Xi_\epsilon Q\Psi\right)\Psi\rangle}$$

$\Big\downarrow\ \epsilon\to 0$ $\qquad\qquad$ $\Big\downarrow\ \epsilon\to 0$

Witten型 $\langle\!\langle X(\mathrm{i})\Psi^3\rangle\!\rangle$ $\qquad$ vanishes $\because\ \xi(\mathrm{i})\xi(\mathrm{i})=0$

quartic

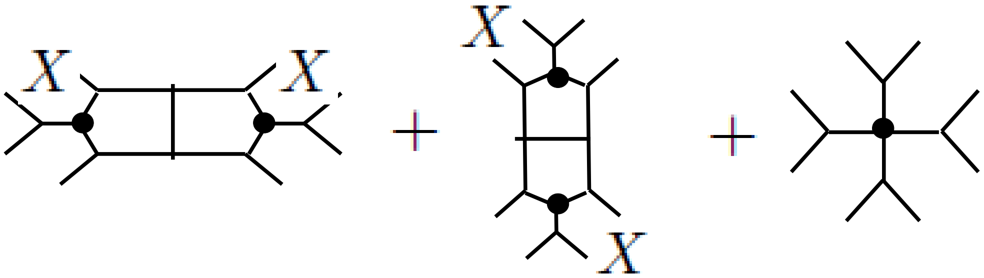
$$\mathrm{i}\langle\Phi\left(Q\Phi\right)\Phi\left(\eta_0\Phi\right)\rangle = \mathrm{i}\langle\Xi_\epsilon\Psi\left(\mathcal{X}_\epsilon-\Xi_\epsilon Q\right)\Psi\left(\Xi_\epsilon\Psi\right)\Psi\rangle$$
$$= \mathrm{i}\underline{\langle\Xi_\epsilon\Psi\left(\mathcal{X}_\epsilon\Psi\right)\left(\Xi_\epsilon\Psi\right)\Psi\rangle} - \mathrm{i}\underline{\langle\Xi_\epsilon\Psi\left(\Xi_\epsilon Q\Psi\right)\left(\Xi_\epsilon\Psi\right)\Psi\rangle}$$

$\Big\downarrow\ \epsilon\to 0$ $\qquad\qquad$ $\Big\downarrow\ \epsilon\to 0$

diverges $\qquad\qquad$ vanishes

$$\because\ X(\mathrm{i})\xi(\mathrm{i})\xi(\mathrm{i})=\infty$$

quartic terms = counterterm



= correct on-shell amp.

quintic term   finite in the $\epsilon \to 0$ limit.

n-pt terms (n≧6)   vanish in the $\epsilon \to 0$ limit.

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ゲージ変換

$$\delta\, e^{g\Phi} = g\Big[(Q\,\epsilon_Q)\, e^{g\Phi} + e^{g\Phi}\,\underline{(\eta_0\,\epsilon_\eta)} \Big]$$

$$\Downarrow$$

$$\delta\Phi = (Q\epsilon_Q + \underline{\eta_0\epsilon_\eta}) - \frac{g}{2}[\Phi, Q\epsilon_Q - \underline{\eta_0\epsilon_\eta}] + O(g^2)$$

constraint

$$\Xi_\epsilon\, \delta\Phi = 0 \Longleftrightarrow \Xi_\epsilon \Big(Q\,\epsilon_Q + \eta_0\,\epsilon_\eta - \cdots \Big) = 0$$

$\eta_0\,\epsilon_\eta$ は $Q\,\epsilon_Q$ に関して解ける

$$\Downarrow$$

residual gauge transf. のパラメータは1つだけ。

Ψ を用いてresidual gauge transf. を表すと…

$$\Phi = \Xi_\epsilon \Psi \Longleftrightarrow \Psi = \eta_0 \Phi$$

Witten の string field に対応

$$\delta\Psi = \delta_0\Psi + g\,\delta_1\Psi + O(g^2)$$

with

$$\delta_0\Psi = -Q\eta_0\epsilon_Q\,,$$

$$\delta_1\Psi = -\frac{1}{2}\Big(\big[\Psi, (1+\eta_0\Xi_\lambda)\,Q\epsilon_Q\big] - \big[\Xi_\lambda\Psi, Q\eta_0\epsilon_Q\big]\Big).$$

ϵ_Q を分解する

$$\epsilon_Q = \Xi_\epsilon\eta_0\,\epsilon_Q + \eta_0\,\Xi_\epsilon\,\epsilon_Q \quad \big(\,\{\Xi_\epsilon, \eta_0\} = 1\,\big)$$

$$\equiv \underline{-\Xi_\epsilon\Lambda} + \Lambda' \;\in\; \Xi_\epsilon\mathscr{H}_{\text{small}} \oplus \mathscr{H}_{\text{small}}$$

実は、この component だけで、全てのゲージ変換を実現できる。

Ψ を用いてresidual gauge transf. を表すと…

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ϵ_Q を分解する

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実は、この component だけで、全てのゲージ変換を実現できる。

$$\delta\Psi = Q\Lambda + g\Big([\Psi, \mathcal{X}_\epsilon\Lambda] - \frac{1}{2}\left([\Psi, \Xi_\epsilon Q\Lambda] + [\Xi_\epsilon\Psi, Q\Lambda]\right)\Big) + \mathcal{O}(g^2)$$

naively $\xrightarrow{\epsilon\to 0} Q\Lambda + g\Big(X(\mathrm{i})[\Psi,\Lambda] - \frac{1}{2}\underset{=0}{\underline{\left(-\xi(\mathrm{i})\{\Psi, Q\Lambda\} + \xi(\mathrm{i})\{\Psi, Q\Lambda\}\right)}}\Big) + \mathcal{O}(g^2)$

$$= \underline{Q\Lambda + gX(\mathrm{i})[\Psi,\Lambda]} + \underline{\mathcal{O}(g^2)}$$

Witten型

divergent

$$\boxed{\begin{aligned} S &= S_0 + gS_1 + g^2S_2 + \mathcal{O}(g^3) \\ \delta\Psi &= \delta_0\Psi + g\,\delta_1\Psi + g^2\delta_2\Psi + O(g^3) \end{aligned}}$$

$\epsilon \to 0$ で $\delta_1 S_1$ はWitten 理論と同様の発散を含むが、

全ての発散はキャンセルされる:　$\delta_2 S_0 + \delta_1 S_1 + \delta_0 S_2 = 0$

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Summary

- partial gauge fixing を行うことでBerkovits 理論におけるhigher-point interation terms の果たす役割を明らかにした。
(Witten 理論で存在した発散に対するcounterterm として働く)

- $\epsilon \to 0$ の極限で…
  quartic term: singular
  quintic term: finite
  n-point term (n≧6): vanishes

- residual gauge transf. は ϵ_Q の $\Xi_\epsilon \mathscr{H}_{\mathrm{small}}$ 成分で生成され、
$\epsilon \to 0$ の極限で、Witten の gauge transf. と対応付く。

- $\epsilon > 0$ のpartially gauge-fixed Berkovits theory は Witten theory のregularized version とみなせる。