Holographic entanglement temperature for low thermal excited states

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Wu-zhong Guo, SH, Jun Tao, JHEP 1308 (2013) 050
SH, Danning Li, Jun-Bao Wu, JHEP 1310 (2013) 142

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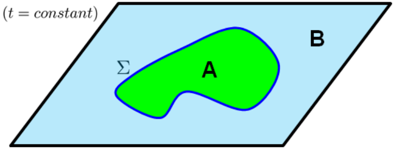
Outline

- General background of Entanglement Entropy (EE).
- Holographic Entanglement Entropy (HEE).
- HE temperature in higher derivative gravities.
- HE temperature in nonconformal cases.
- Summary.

Basics of Entanglement Entropy

- General diagnostic: divide quantum system into two parts and use entropy as measure of correlations between subsystems
- In QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions.
- Integrate out degrees of freedom in outside region. Remaining dof are described by a density matrix ρ_A.
- Calculate von Neumann entropy: $S_{EE} = -\mathrm{Tr}(\rho_A \log \rho_A)$.



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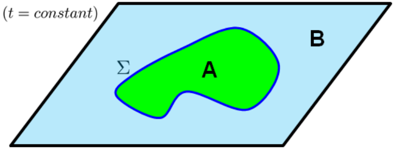

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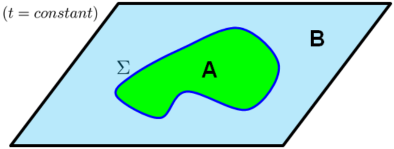

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  3. Subadditivity: $S_{A+B} \leq S_A + S_B$.

Replica to calculate EE in QFT

- One can follow replica approach to calculate VEE.
- Firstly, one should introduce the Renyi entropy as following

$$S_A^n = -\frac{\log tr_A \rho_A^n}{n-1}.$$

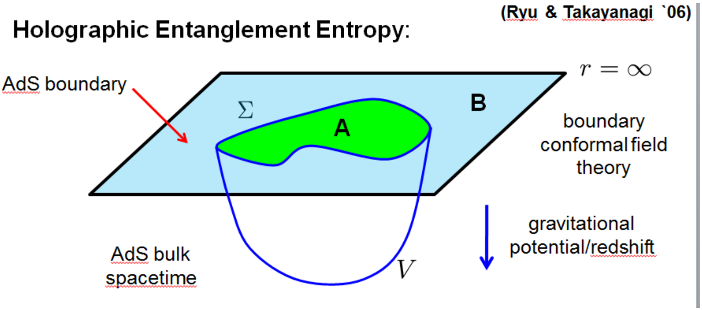
Where the $\rho_A^n = Pe^{-\int_0^{2\pi n} d\tau H_{b,n}(\tau)}$.

- It is easy to see that the entanglement entropy and the Renyi entropy are related by.

$$S_A = \lim_{n\to 1} S_A^n$$

- The relation provides a practical way to compute EE in field theory.
- Normally, it is difficult to calculate EE even in free field theory.

Holographic Entanglement Entropy



The holographic entanglement entropy of a subsystem A on the boundary is given by the area of the ($t = const$) bulk minimal surface γ_A

$$S_A = \frac{Area(\gamma_A)}{4G}, \quad \partial\gamma_A = \partial A$$

Extensive ways to check HEE

- leading contribution yields area law $S_{EE} \sim \frac{\text{Area}}{\text{cut off}^{d-2}}$
- recover known results for d=2 CFT (Holzhey, Larsen and Wilczek; Calabrese and Cardy) : $S_{EE} = \frac{c}{3}\log(\frac{C}{\pi\delta}\sin(\frac{\pi l}{C}))$.
- $S_A = S_{\bar{A}}$ in a pure state, where the A and $\bar{A}$ share the same entangled surface.
- strong sub-additivity (Headrick and Takayanagi): $S_{A+B} \leq S_A + S_B$
- for even d, connection of universal/logarithmic contribution in S_{EE} to central charges of boundary CFT, eg, in $d = 4$
- New proof given by (Lewkowycz and Maldacena)
- Generalization of Euclidean path integral calc's for S_{BH}, extended to "periodic" bulk solutions without Killing vector. Where breaking the $U(1)$ Isometry time direction.
- For AdS/CFT, just translates replica trick for boundary CFT to bulk and then

$$\Delta\tau = 2\pi \to 2\pi n \longrightarrow \log Z(n) = \log \mathrm{Tr}\left[\rho^n\right] = -I_{grav}(n)$$
$$\longrightarrow S = -n\partial_n \left[\log Z(n) - n\log Z(1)\right]\Big|_{n=1}$$

- at n=1, linearized gravity eom demand: induced curvature vanishing. The Euclidean time circle shrinks to zero on an extremal surface in bulk.

Motivation: 'First Law'

- First law of thermodynamics: $TdS = dE$. Just start from this formula.
- In a general quantum system, can we find the analogous relation between the EE (information) and energy of A:

$$T_{ent} dS_A = dE_A \quad ?$$

- The first study in field theory in (F. C. Alcaraz, M. I. Berganza, G. Sierra, PRL 106, 201601)
- First holographic studied in (Jyotirmoy Bhattacharya, Masahiro Nozaki, Tadashi Takayanagi, Tomonori Ugajin, PRL 110, 091602)

General Perturbed Background

- For a given asymptotically AdS$_{d+1}$ metric as the ground state

$$ds^2_{(0)} = \frac{R^2}{z^2}\left(\frac{1}{f(z)}dz^2 - f(z)dt^2 + d\vec{x}^2_{d-1}\right)$$

consider linear perturbations in the Fefferman-Graham gauge

$$ds^2_{(1)} = \frac{R^2}{z^2}\left[h_{\mu\nu}(z,t,\vec{x})dx^\mu dx^\nu\right]$$

- In terms of HEE formula, the variation of the area A may arise from two sources:
  1. $\delta x^a(\zeta^\alpha)$, i.e. the variation of the shape of the surface.
  2. δg_{ab}, the variation of the bulk geometry.

Variation of $A(\gamma_A)$

- More explicitly

$$\delta A = \int d^{d-1}\zeta \frac{1}{2}\sqrt{h}h^{\alpha\beta}\delta\left(\frac{\partial x^a}{\partial \zeta^\alpha}\frac{\partial x^b}{\partial \zeta^\beta}g_{ab}\right) = \delta_x A + \delta_g A$$

which has two contributions

$$\delta_x A = \int d^{d-1}\zeta \frac{1}{2}\sqrt{h}h^{\alpha\beta} 2\frac{\partial \delta x^a}{\partial \zeta^\alpha}\frac{\partial x^b}{\partial \zeta^\beta}g_{ab}$$

$$\delta_g A = \int d^{d-1}\zeta \frac{1}{2}\sqrt{h}h^{\alpha\beta}\frac{\partial x^a}{\partial \zeta^\alpha}\frac{\partial x^b}{\partial \zeta^\beta}\delta g_{ab}$$

So, the minimal surface condition as a constraint is trivial at the linear order; one can simply use

$$\delta_g S = \frac{1}{4G}\int d^{d-1}\zeta \frac{1}{2}\sqrt{h^{(0)}}h^{(0)\alpha\beta}h^{(1)}_{\alpha\beta}$$

- If we generalize to high derivative gravity, the functional for minimal surface will be modified due to high derivative gravities' appear. We will see in the coming examples.

HEE temperature in higher derivative gravities

HEE for 4-dimensional CFT

- The 5-dimensional Lovelock gravity can be realized by adding the Gauss-Bonnet term to pure Einstein gravity theory [David Lovelock,J. Math. Phys. 12 (1971)498].

$$I = \frac{1}{2{\ell_p}^3} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + \frac{\lambda_5 L^2}{2} L_4 \right], \tag{1}$$

with

$$L_4 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2, \tag{2}$$

and λ_5 denote the coupling of Gauss-Bonnet gravity and L stands for the Radius of AdS background.

- Vacuum state

$$ds^2 = \frac{\tilde{L}^2}{z^2}(-dt^2 + dz^2 + dx_1^2 + dx_2^2 + dx_3^2) \tag{3}$$

$\tilde{L}$ is the effective AdS radius in Gauss-Bonnet gravity and is defined by $\tilde{L}^2 = \frac{L^2}{f_\infty}$ with

$$f_\infty = \frac{1 - \sqrt{1 - 4\lambda_5}}{2\lambda_5}. \tag{4}$$

- The low excited state

$$ds^2_{BB} = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + \frac{dz^2}{f(z)}\right],$$

$$f(z) = \frac{1}{2\lambda_5}\left(1 - \sqrt{1 - 4\lambda_5\left(1 - \frac{z^4}{z_h^4}\right)}\right),$$

where z_h is the horizon of the black brane.

$$ds^2 = \frac{\tilde{L}^2}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{g(z)} + dx_1^2 + dx_2^2 + dx_3^2\right), \quad (5)$$

where $f(z) \simeq g(z) = 1 - mz^4$. Where m corresponds to thermal excitation.

- The holographic EE should be modified as following form

$$S_A = \frac{2\pi}{\ell_p^3}\int_M d^3x\sqrt{h}\left[1 + \lambda_5 L^2 \mathcal{R}\right] + \frac{4\pi}{\ell_p^3}\int_{\partial M} d^2x\sqrt{h}\lambda_5 L^2 \mathcal{K}, \quad (6)$$

where the integral is evaluated on the bulk surface M, whose boundary is A, $\mathcal{R}$ is the Ricci scalar for the intrinsic geometry of M, and $\mathcal{K}$ is the trace of the extrinsic curvature of the boundary of M, h is the determinant of the induced metric on M. The second term in the first integral is presented due to higher derivative gravity in the background.

- The variation of entanglement entropy in subsystem with a round ball configuration.

$$\begin{aligned}\Delta S_A &= \frac{8\pi^2\tilde{L}^3}{\ell_p^3} m R_0^4 \int_{\epsilon/R_0}^{\frac{\pi}{2}} dx\left(\frac{1}{2}\sin x\cos^4 x - \lambda_5\frac{L^2}{\tilde{L}^2}\sin x\cos^4 x\right) \\ &= \frac{8\pi^2\tilde{L}^3}{\ell_p^3}\left(\frac{1}{10}-\frac{1}{5}\lambda_5 f_\infty\right) m R_0^4. \end{aligned} \tag{7}$$

The a and c are equal at the limit $\lambda_5 \to 0$.

$$c = \pi^2\frac{\tilde{L}^3}{l_p^3}\left(1-2\lambda_5 f_\infty\right), \qquad a = \pi^2\frac{\tilde{L}^3}{l_p^3}\left(1-6\lambda_5 f_\infty\right). \tag{8}$$

Where a and c are different types of central charges in dual field theory.

- In terms of standard Dictionary (S. de Haro, S. N. Solodukhin and K. Skenderis ('00), K. Skenderis ('02)), the energy momentum tensor (energy density of subsystem) can be

$$T_{tt} = \frac{3m\tilde{L}^3(1-2\lambda_5 f_\infty)}{2\ell_p^3} \tag{9}$$

- The entanglement temperature for roll ball

$$\frac{1}{T_{ent}} = \frac{\Delta S}{\Delta E} = \frac{2\pi}{5}R_0. \tag{10}$$

[Wu-zhong Guo, SH, Jun Tao, JHEP 1308 (2013) 050]

- Similarly, the variation of entanglement entropy in subsystem with stripe configuration

$$\begin{aligned} \Delta S_A &= \frac{2m\tilde{L}^3\pi l_0^2 z_*^2}{(1+2\lambda f_\infty)\ell_p^3}\int_{\epsilon/z_*}^1 du u\sqrt{1-u^6} \\ &= \frac{m\tilde{L}^3\sqrt{\pi} l_0^2 l^2}{20(1+2\lambda f_\infty)^3\ell_p^3}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})^2}{\Gamma(\frac{2}{3})^2\Gamma(\frac{5}{6})} \\ &\simeq \frac{a}{2\pi^2}\frac{m\sqrt{\pi}l_0^2 l^2}{10}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})^2}{\Gamma(\frac{2}{3})^2\Gamma(\frac{5}{6})}, \end{aligned} \tag{11}$$

- The energy

$$\Delta E = \frac{2\pi m(1 - 2f_\infty \lambda_5)\tilde{L}^3 R_0^3}{\ell_p^3}. \tag{12}$$

- The entanglement temperature for strip

$$\frac{1}{T_{ent}} = \frac{\Delta S}{\Delta E} = \frac{a}{c}\frac{\sqrt{\pi}}{30}\frac{\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})^2}{\Gamma(\frac{2}{3})^2\Gamma(\frac{5}{6})}l. \tag{13}$$

- Entanglement temperature can also studied in D=6 CFT with dual 7-dimensional Lovelock gravity with same way. The result is similar as what I have shown here.

HEE temperature in non-conformal cases

Potential reconstruction

- The action in string frame [SH, Danning Li, Jun-Bao Wu, JHEP 1310 (2013) 142]

$$S_{5D} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^S}e^{-2\phi}\Big(R^S + 4\partial_\mu\phi\partial^\mu\phi - V_S(\phi) - \frac{Z(\phi)}{4g_g^2}e^{\frac{-4\phi}{3}}F_{\mu\nu}F^{\mu\nu}\Big), \tag{14}$$

where the action (14) is written in string frame, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwell field.

- The background ansatz in Einstein frame

$$ds_E^2 = \frac{L^2e^{2A_e}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i\right), = \frac{L^2e^{2A_s-\frac{4\phi}{3}}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i\right), \tag{15}$$

with $A_e = A_s - 2\phi/3$.

- The general background solution

$$\phi(z) = \int_0^z \frac{e^{2A_s(x)}\left(\frac{3}{2}\int_0^x y^2 e^{-2A_s(y)} A_s(y)'^2\, dy + \phi_1\right)}{x^2}\, dx \tag{16}$$

$$+\frac{3A_s(z)}{2} + \phi_0, \tag{17}$$

$$A_0(z) = A_{00} + A_{01}\left(\int_0^z \frac{y e^{\frac{2\phi(y)}{3} - A_s(y)}}{Z(\phi(y))}\, dy\right), \tag{18}$$

$$f(z) = \int_0^z x^3 e^{2\phi(x) - 3A_s(x)}\left(\frac{A_{01}{}^2\left(\int_0^x \frac{y e^{\frac{2\phi(y)}{3} - A_s(y)}}{Z(\phi(y))}\, dy\right)}{g_g^2 L^2} + f_1\right) dx \tag{19}$$

$$+f_0, \tag{20}$$

$$V_E(z) = \frac{e^{-2A_s(z) + \frac{4\phi(z)}{3}} z^2 f(z)}{L^2} 2\Bigg(-\frac{e^{-2A_s(z)+\frac{4\phi(z)}{3}} Z(\phi(z)) z^2 A_0'(z)^2}{4 g_g^2 L^2 f(z)}$$
$$- \frac{2\left(3 + 3z^2 A_s(z)'^2 + 4z\phi'(z) + z^2\phi'(z)^2 - 2zA_s(z)'\left(3 + 2z\phi'(z)\right)\right)}{z^2}$$
$$- \frac{f'(z)\left(-3 + 3zA_s(z)' - 2z\phi'(z)\right)}{2zf(z)}\Bigg), \tag{21}$$

where the $\phi_0, A_{00}, A_{01}, f_0, f_1$ are all integration constants and can be determined by suitable UV and IR boundary conditions.

The first zero temperature background

- We have already figured out systematical algorithm to obtain general gravity solutions with dilaton potential like $\phi^2, \phi^3, \phi^4, \phi^6, \ldots$ in EDM system (SH, Danning Li, work in progress).
- The first analytical zero temperature solution

$$
\begin{aligned}
A_{et1}(z) &= \log\left(\frac{z}{z_0 \sinh(\frac{z}{z_0})}\right), \\
f_{t1}(z) &= 1, \\
\phi_{t1}(z) &= \frac{3z}{2z_0}, \\
V_{Et1}(\phi) &= -\frac{12 + 9\sinh^2\left(\frac{2\phi_{t1}}{3}\right)}{L^2}.
\end{aligned} \tag{22}
$$

To simplify following analysis, we have set $p_1 = \frac{3}{2z_0}$. We have checked that this solution is $N = 1$ BPS solution in paper [SH, Ya-Peng Hu, Jian-Hui Zhang,JHEP 1112 (2011) 078].

- We just only show the two new nontrivial gravity solutions here. The series expansion of the first black hole solution

$$\begin{aligned}
\phi_{b1}(z) &= p_1 z + p_3 z^3 + \frac{((405 f_{41} p_1 + 612 p_1^2 p_3) z^5)}{3240} \\
&+ \frac{(8100 f_{41} p1^3 + 229635 f_{41} p_3 + 10944 p_1^4 p_3 + 133164 p_1 p_3^2) z^7}{612360} \\
&+ O(z^7) \\
f_{b1}(z) &= 1 - f_{41} z^4 - \frac{4}{27} f_{41} p_1^2 z^6 + \frac{-13 f_{41} p_1^4}{1215} - \frac{f_{41} p_1 p_3}{5}) z^8 \\
&+ \frac{(-10935 f_{41}^2 p_1^2 - 328 f_{41} p_1^6 - 37908 f_{41} p_1^3 p_3 - 78732 f_{41} p_3^2) z^{10}}{688905} \\
&+ O(z^{10}) \\
A_{eb1}(z) &= -(2/27) p_1^2 z^2 + (\frac{(4 p_1^4)}{3645} - \frac{(2 p_1 p_3)}{15}) z^4 \\
&+ \frac{(-54675 f_{41} p_1^2 - 128 p_1^6 - 67068 p_1^3 p_3 - 393660 p_3^2) z^6)}{4133430} \\
&+ \frac{((-50625 f_{41} p_1^4 + 64 p_1^8 - 3444525 f_{41} p_1 p_3 - 74952 p_1^5 p_3 - 2943}{62001450} \\
&+ O(z^8).
\end{aligned}$$

The second zero temperature background

- The second analytical zero temperature solution

$$A_{et2}(z) = -\log\left(1+\frac{z}{z_0}\right), \tag{24}$$

$$f_{t2}(z) = 1, \tag{25}$$

$$\phi_{t2}(z) = 3\sqrt{2}\sinh^{-1}\left(\sqrt{\frac{z}{z_0}}\right), \tag{26}$$

$$V_{Et2}(\phi_{t2}) = -\frac{12}{L^2} - \frac{42\sinh^4\left(\frac{\phi_{t2}}{3\sqrt{2}}\right)}{L^2} - \frac{42\sinh^2\left(\frac{\phi_{t2}}{3\sqrt{2}}\right)}{L^2}. \tag{26}$$

Which is so called the second zero temperature solution.

- The series expansion of the second black hole solution

$$\begin{aligned}
\phi_{b2}(z) &= p_{\frac{1}{2}}\sqrt{z}-\frac{1}{108}p_{\frac{1}{2}}^{3}z^{3/2}+\frac{p_{\frac{1}{2}}^{5}z^{5/2}}{4320}+\frac{\left(653184p_{\frac{7}{2}}-5p_{\frac{1}{2}}^{7}\right)z^{7/2}}{653184}\\
&+\frac{z^{9/2}\left(18895680f_{42}p_{\frac{1}{2}}+515p_{\frac{1}{2}}^{9}+\frac{171}{2}\left(653184p_{\frac{7}{2}}-5p_{\frac{1}{2}}^{7}\right)p_{\frac{1}{2}}^{2}\right)}{302330880}\\
&+\frac{z^{11/2}\left(69284160f_{42}p_{\frac{1}{2}}^{3}+1135p_{\frac{1}{2}}^{11}+\frac{517}{2}\left(653184p_{\frac{7}{2}}-5p_{\frac{1}{2}}^{7}\right)p_{\frac{1}{2}}^{4}\right)}{13302558720}\\
&+O(z^{\frac{11}{2}}),\\
f_{b2}(z) &= 1-f_{42}z^{4}-\frac{1}{15}2f_{42}p_{\frac{1}{2}}^{2}z^{5}-\frac{1}{162}f_{42}p_{\frac{1}{2}}^{4}z^{6}-\frac{f_{42}p_{\frac{1}{2}}^{6}z^{7}}{10206}\\
&+z^{8}\left(-\frac{f_{42}p_{\frac{1}{2}}^{8}}{1119744}-\frac{f_{42}\left(653184p_{\frac{7}{2}}-5p_{\frac{1}{2}}^{7}\right)p_{\frac{1}{2}}}{5598720}\right)\\
&+\frac{z^{9}\left(-35f_{42}p_{\frac{1}{2}}^{10}-7f_{42}\left(653184p_{\frac{7}{2}}-5p_{\frac{1}{2}}^{7}\right)p_{\frac{1}{2}}^{3}-839808f_{42}^{2}p_{\frac{1}{2}}^{2}\right)}{151165440}\\
&+O(z^{10}), \qquad (27
\end{aligned}$$

- $$\begin{aligned} A_{eb2}(z) &= -\frac{1}{18}p_{\frac{1}{2}}^2 z + \frac{1}{648}p_{\frac{1}{2}}^4 z^2 - \frac{p_{\frac{1}{2}}^6 z^3}{17496} + \frac{p_{\frac{1}{2}}^8}{559872} z^4 \\ &- \frac{p_{\frac{1}{2}}\left(653184 p_{\frac{7}{2}} - 5 p_{\frac{1}{2}}^7\right)}{8398080} z^4 \\ &+ \frac{z^5\left(-2519424 f_{42} p_{\frac{1}{2}}^2 - 109 p_{\frac{1}{2}}^{10} - 9\left(653184 p_{\frac{7}{2}} - 5 p_{\frac{1}{2}}^7\right) p_{\frac{1}{2}}^3\right)}{604661760} \\ &+ \frac{z^6\left(-37791360 f_{42} p_{\frac{1}{2}}^4 + 325 p_{\frac{1}{2}}^{12} - 159\left(653184 p_{\frac{7}{2}} - 5 p_{\frac{1}{2}}^7\right) p_{\frac{1}{2}}^5\right)}{228562145280} \\ &+ O(z^6). \end{aligned} \tag{28}$$

Construct boundary energy momentum tensor

- The total action

$$\begin{aligned} I_{\rm ren} &= S_{\rm 5D} + S_{\rm GH} + S_{\rm count} \\ &= \frac{1}{16\pi G_5}\int_M d^5x\sqrt{-g^E}\left(R - \frac{4}{3}\partial_\mu\phi\partial^\mu\phi - V_E(\phi) - \frac{Z(\phi)}{4g_g^2}F_{\mu\nu}F^{\mu\nu}\right) \\ &- \frac{1}{16\pi G_5}\int_{\partial M} d^4x\sqrt{-\gamma}\left[2K - \frac{6}{L} + \frac{8\lambda_2\phi^2}{3L} + \frac{64\lambda_4\phi^4}{9L^2} + \frac{512\lambda_6\phi^6}{81L^3}\right], \end{aligned} \tag{29}$$

with $\lambda_2, \lambda_4, \lambda_6$ are coefficients of count terms ϕ^2, ϕ^4, ϕ^6 introduced here.

- In terms of on-shell action, we can confirm that the black hole solutions is thermal excitation of zero temperature solutions in these two groups of solutions.

Boundary energy momentum tensor

- Boundary terms introduced in first group of solution [SH, Danning Li, Jun-Bao Wu, JHEP 1310 (2013) 142]

$$\lambda_2 = \frac{1}{4}, \lambda_4 = 0, = \lambda_6 = 0. \tag{30}$$

The energy momentum tensor of the first solution

$$T_{tt} = \frac{L^3}{16\pi G_5}(\frac{3}{2}f_{41} - p_1(\frac{2p_1^3}{81} + \frac{2}{3}p_3)). \tag{31}$$

- Boundary terms introduced in second group of solution

$$\lambda_2 = \frac{1}{8}, \lambda_4 = \frac{L}{1152}, \lambda_6 = \frac{L^2}{414720}. \tag{32}$$

The energy momentum tensor of the second solution

$$T_{tt} = \frac{L^3}{16\pi G_5}\left(\frac{3f_{42}}{2} - \frac{p_{\frac{1}{2}}^8}{5511240} - \frac{p_{\frac{1}{2}}p_{\frac{7}{2}}}{2}\right). \tag{33}$$

Entanglement temperature in these two solutions

- Entanglement temperature in first group of solution

$$\frac{1}{T_{ent}} = \frac{\Delta S_{fst}}{\Delta E_{fst}} = \frac{(0.350546 f_{41} - 0.409903 p_1 p_3) \frac{L_s \Gamma(\frac{1}{6})^2}{\pi^2 \Gamma(\frac{2}{3})^2}}{(\frac{3}{2} f_{41} - \frac{2}{3} p_1 p_3)}. \quad (34)$$

One can find that the $T_{ent} \sim [E]$ through dimensional analysis with $[f_{41}] = [E^4], [p_1] = [E^1], [p_3] = [E^3], [L_s] = [E^{-1}]$.

- Entanglement temperature in second group of solution

$$\frac{1}{T_{ent}} = \frac{\Delta S_{snd}}{\Delta E_{snd}} = \frac{(0.350546 f_{42} - 0.23911 p_{\frac{7}{2}} p_{\frac{1}{2}})}{\left(\frac{3 f_{42}}{2} - \frac{p_{\frac{1}{2}} p_{\frac{7}{2}}}{2}\right)} \frac{L_s \Gamma(\frac{1}{6})^2}{\pi^2 \Gamma(\frac{2}{3})^2} \quad (35)$$

- In the first group of solutions, the parameter p_3 is related to condensation of the dimension 3 operator $\mathcal{O}_3$ which holographically dual to scalar ϕ_{b1} at special temperature. In this case, the temperature is determined by the f_{41} with fixing non-vanishing source p_1.
- In the second group of solutions, $\frac{1}{653184}\left(653184 p_{\frac{7}{2}} - 5 p_{\frac{1}{2}}^7\right)$ corresponds to condensation of operator $\mathcal{O}_{\frac{7}{2}}$ with dimension $\frac{7}{2}$ living on the boundary. Where the condensation is induced by the source $p_{\frac{1}{2}}$.
- One should note that there should be two ways quantize $\phi_{b1,b2}$ by imposing Dirichlet or Neumann conditions at the aAdS boundary, which are often called standard and alternative quantization respectively, and lead to two different QFTs.
- Non-conformal entanglement temperatures do not only depend on geometric data of the subsystem but also data of boundary gauge theory.

Summary and future direction

Summary and Future Plans

- From two kinds of gravity theories, we confirm that there is first law like theorem about HEE in low excitation states.
- HE temperature is also helpful to understand Covariant Entropy bound.
- HEE in higher spin theory, definition and thermal dynamical properties in higher dimensional cases.
- Using localization technique to study the SUSY version of S modified EE.
- ...

Thanks for your attention!