Emergent bubbling geometries in gauge theories with SU(2|4) symmetry

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Introduction

- In gauge/gravity correspondence

It is not clear how 10D (11D) background geometry in string theory is realized in corresponding gauge theory

10D(11D) geometry should be emergent in gauge theories

Motivation

- A nice example of emergent geometry was given by LLM geometry and chiral primary operators in N=4 SYM

[Lin-Lunin-Maldacena, Berenstein, Takayama-Tsuchiya]

- What about other gauge theories ???

- Generic description of 10D geometry in terms of gauge theory DOF is not known yet.

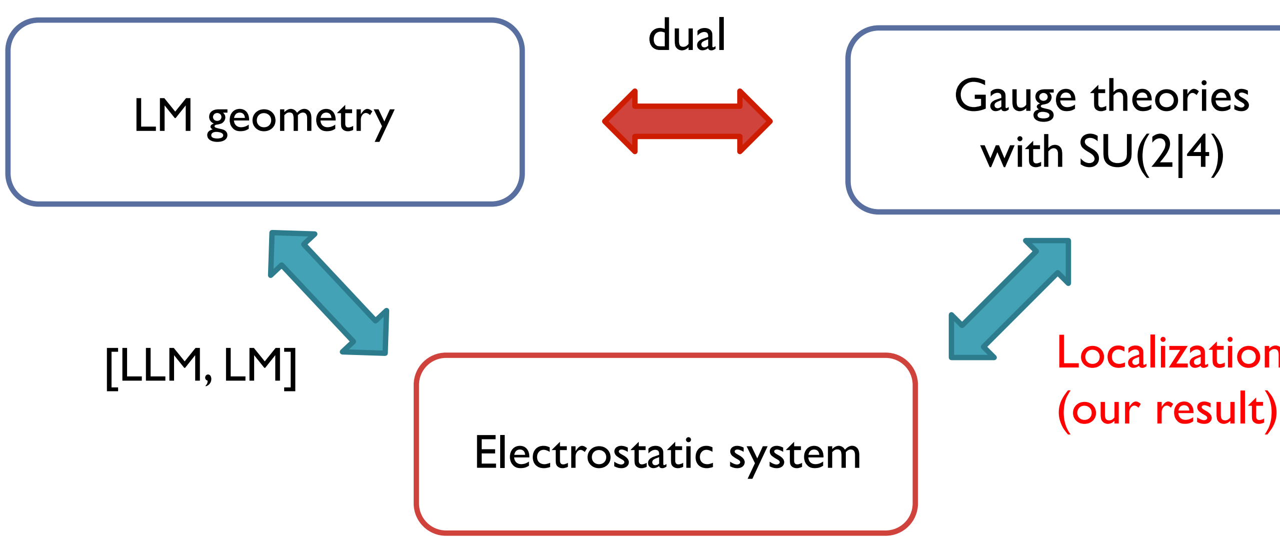
- We need to construct more examples to find a general principle for gauge theoretic description of geometry.

Our setup and result

- We consider gauge theories with SU(2|4) symmetry.

Gauge theories with SU(2|4) sym
- N=4 SYM on $R\times S^3/Z_k$
- N=8 SYM on $R\times S^2$
- Plane wave (BMN) matrix model

- Dual geometries for these theories were constructed by Lin-Maldacena
  LM geometry is characterized by a certain electrostatic system.



- Applying localization, we find ¼ BPS sector of gauge theories are also described by the same electrostatic system as the gravity side

Contents

2. Gauge theories with SU(2|4) symmetry

Gauge theories with SU(2|4) symmetry

4D N=4 SYM on $R\times S^3$ → Truncation of KK modes on S^3

N=4 SYM on $R\times S^3/Z_k$
N=8 SYM on $R\times S^2$
Plane wave matrix model (PWMM)

◆ Common features

- Massive
- SU(2|4) (16 SUSY)
- Many discrete vacua — Holonomy, Monopoles, Fuzzy spheres
- Gravity dual for theory around each vacuum [Lin-Maldacena]

PWMM

[Berenstein-Maldacena-Nastase]

$$\begin{cases} M,N = 1,\cdots,9 \\ i,j,k = 1,2,3 \\ a,b = 4,\cdots,9 \end{cases}$$

$$S_{\mathrm{PWMM}} = \frac{1}{g^2}\int dt\,\mathrm{tr}\left[\frac{1}{2}(D_tX_M)^2 - \frac{1}{4}[X_M,X_N]^2 + \frac{1}{2}\Psi^\dagger D_t\Psi - \frac{1}{2}\Psi^\dagger\gamma_M[X_M,\Psi] + \frac{\mu^2}{2}(X_i)^2 + \frac{\mu^2}{8}(X_a)^2 + i\mu\epsilon_{ijk}X_iX_jX_k + i\frac{3\mu}{8}\Psi^\dagger\gamma_{123}\Psi\right],$$

- Mass deformation of BFSS matrix model
- $SU(2|4)$ symmetry = 16 SUSY + $\underset{t}{R} \times \underset{X_i}{SO(3)} \times \underset{X_a}{SO(6)}$
- Vacua：fuzzy sphere (representation of SU(2) generators)

$N_5^{(s)}$ dim irrep

multiplicity

$$X_i = \mu L_i = \mu \bigoplus_{s=1}^{\Lambda}\left(L_i^{[N_5^{(s)}]} \otimes \mathbf{1}_{N_2^{(s)}}\right) \qquad [L_i, L_j] = i\epsilon_{ijk}L_k$$

Irreducible decomposition

Labelled by $\{(N_2^{(s)}, N_5^{(s)})\}$ & Λ

4. Lin-Maldacena geometry

Lin-Maldacena geometry

- SU(2|4) symmetric solution in IIA SUGRA

$$ds_{10}^2 = \left(\frac{\ddot{V} - 2\dot{V}}{-V''}\right)^{1/2} \left\{-4\frac{\ddot{V}}{\ddot{V} - 2\dot{V}}dt^2 - 2\frac{V''}{\dot{V}}(dr^2 + dz^2) + 4d\Omega_5^2 + 2\frac{V''\dot{V}}{\Delta}d\Omega_2^2\right\}$$

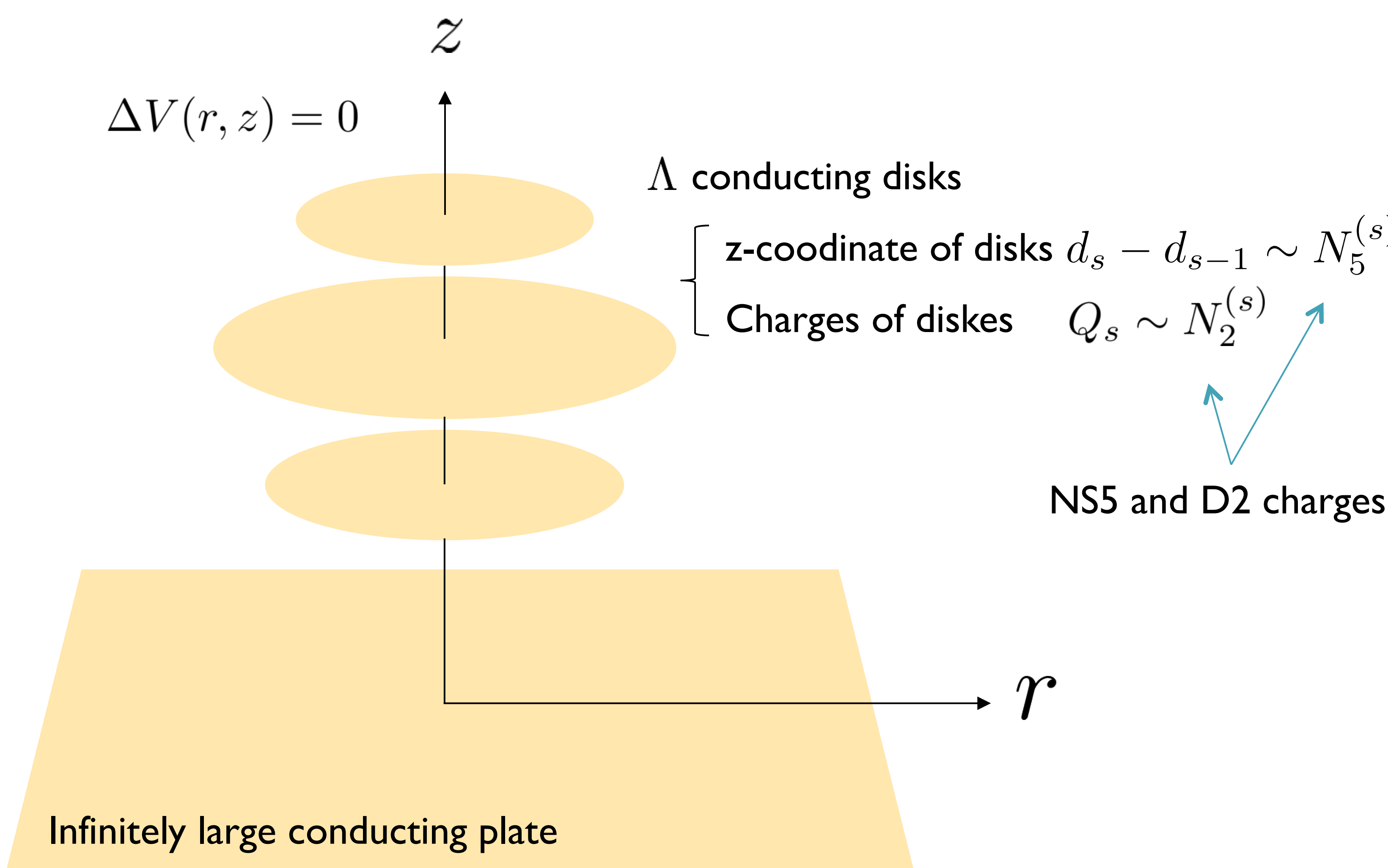
$$C_1 = -\frac{(\dot{V}^2)'}{\ddot{V} - 2\dot{V}}dt, \quad C_3 = -4\frac{\dot{V}^2 V''}{\Delta}dt \wedge d\Omega_2,$$

$$B_2 = \left(\frac{(\dot{V}^2)'}{\Delta} + 2z\right)d\Omega_2, \quad e^{4\Phi} = \frac{4(\ddot{V} - 2\dot{V})^3}{-V''\dot{V}^2\Delta^2}, \qquad \Delta := (\ddot{V} - 2\dot{V})V'' - (\dot{V}')^2$$

- Solution depends only on a single function $V(r, z)$
- EOM ⇒ $V(r, z)$ satisfies the Laplace equation in a certain axially symmetric electrostatic system

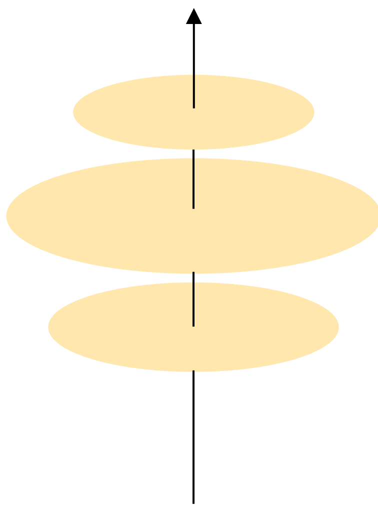
Electro static system for LM geometry

- Dual of PWMM is determined by solving Laplace eq of following system



- Geometry is labbled by $\{(N_2^{(s)}, N_5^{(s)})\}$ Λ & 1:1 with vacua of PWMM

Disk configurations for the other gauge theories

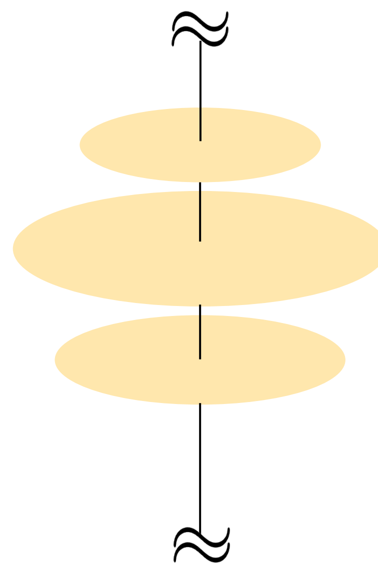


(I) $V \to 0 \ (z \to \pm\infty)$

D2-brane solution

⇕

SYM on R×S^2



(II) Periodic B.C.

D2-brane + T-dual

⇕

SYM on R×S^3/Z_k

(III) Two infinite plates

NS5-brane solution

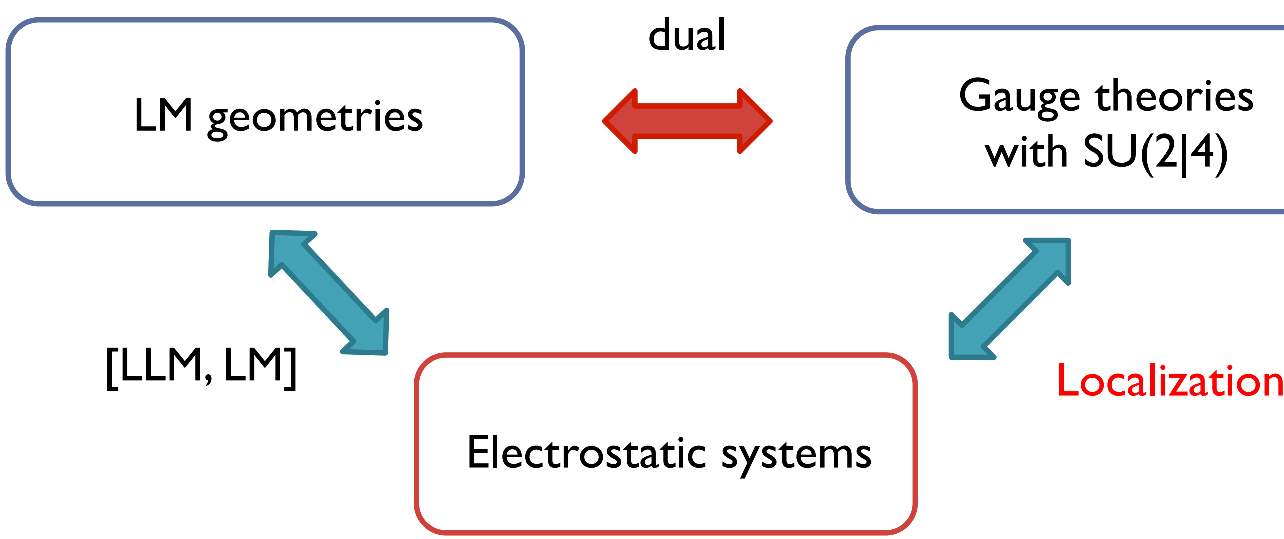
⇕

Little string theory on R×S^5

B.C ⇔ Theory

Disk config ⇔ Vacuum

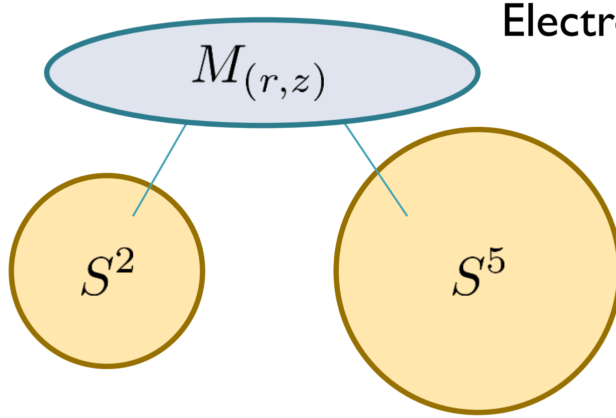
4. Localization in gauge theories and emergent LM geometry



The sector we considered

- LM geometry is locally $R \times S^2 \times S^5 \times M_{(r,z)}$

Electrostatic problem is defined here



- In PWMM, we consider $\phi(t) := X_3(t) + iX_9(t)$

SO(3) scalar SO(6) scalar

From symmetry, we expect $\phi(t)$ describes $M_{(r,z)}$

(Actually we considered $\phi(t) = X_3(t) + i(X_9(t)\cos t + X_8(t)\sin t)$
to preserve ¼ supersymmetries.)

- We consider sector made of only ϕ . $\langle \mathrm{Tr}(\phi^n) \cdots \rangle$

Localization on R×S^D

- Usually, people consider completely compact space like S^D to perform the localization computation. (to have finite moduli integral)

- However, localization is also useful for theories on R×S^d and can be done in almost same way as theories on S^d

  In our case, (I) construct SUSY s.t. $Q\phi = 0$, [Pestun]
  (II) add $-tQV$ to the action, where $V = Q\bar{\Psi}\Psi$,
  (III) path integral is dominated by the saddle of V .

- Only difference ⇒ Need to fix B.C. for the R direction

  Our boundary condition :

  All fields approaches to vacuum configuration

  Path integral with this B.C. defines theory around fixed vacuum.

Result of Localization (for PWMM)

$$\langle \mathrm{Tr}(\phi^n)\cdots\rangle = \langle \mathrm{Tr}(L_3+iM)^n\cdots\rangle_{MM}$$

VEV of PWMM around a fixed vacuum

VEV of the following matrix integral and

$$M=\bigoplus_s \left(\mathbf{1}_{N_5^{(s)}}\otimes M_s\right)$$

$N_2^{(s)}\times N_2^{(s)}$ Hermitian matrix

$$Z=\int\prod_{s=1}^{\Lambda}\prod_{i=1}^{N_2^{(s)}} dq_{si} Z_{1-\mathrm{loop}} e^{-\frac{2}{g^2}\sum_{s,i}N_5^{(s)}q_{si}^2}$$

$$Z_{1-\mathrm{loop}}=\prod_{s,t=1}^{\Lambda}\prod_{J}\prod_{i=1}^{N_2^{(s)}}\prod_{j=1}^{N_2^{(t)}}\left[\frac{\{(2J+2)^2+(q_{si}-q_{tj})^2\}\{(2J)^2+(q_{si}-q_{tj})^2\}}{\{(2J+1)^2+(q_{si}-q_{tj})^2\}^2}\right]^{\frac{1}{2}}$$

q_{si} : eigenvalues of M_s

Multi matrix model with Λ matrices

Saddle point approximation

In appropriate large-N limit where SUGRA approximation is good,
the matrix integral can be evaluated by the saddle point approximation

The matrix integral is described as a classical theory defined by

$$S = \sum_{s=1}^{\Lambda} \frac{2N_5^{(s)}}{g^2} \int dx x^2 \rho^{(s)}(x) - \frac{1}{2} \sum_{s=1}^{\Lambda} \int dx dy \log \tanh^2 \frac{\pi(x-y)}{2} \rho^{(s)}(x) \rho^{(s)}(y)$$

$$-\frac{1}{2} \sum_{s,t=1}^{\Lambda} \int dx dy \left[\frac{N_5^{(s)} + N_5^{(t)}}{(N_5^{(s)} + N_5^{(t)})^2 + (x-y)^2} - \frac{|N_5^{(s)} - N_5^{(t)}|}{(N_5^{(s)} - N_5^{(t)})^2 + (x-y)^2} \right] \rho^{(s)}(x) \rho^{(t)}(y)$$

$$\rho^{(s)}(x) := \frac{1}{N_2^{(s)}} \sum_{i=1}^{N_2^{(s)}} \delta(q_{si} - x)$$: Eigenvalue density for each s

Claim : this theory is equivalent to the electrostatic system on gravity side

Classical action for the electrostatic system

constant

charge density

$$S = \int \frac{1}{2}(\partial_i V)^2 + \sum_s \int_{\mathrm{s-th\ disk}} (V - c_s)\rho_s$$

Variation of V $\Rightarrow$ $\Delta V = \sum_s \rho_s \delta(\mathrm{s-th\ disk})$

Variation of ρ_s $\Rightarrow$ $V = c_s$ (on s-th disk)

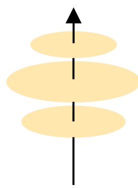
Eliminating V using EOM, we obtain $S(\rho) = \sum_{s,t} \int \rho_s \Delta^{-1} \rho_t + \cdots$

In fact, this action coincides with the action of matrix integral !!!

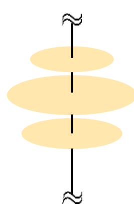
charge densities ⇔ eigenvalue densities

For the other gauge theories

Eliminating V, we can obtain EOM for ρ for gravity duals of the other gauge theories.

(I)  (dual of SYM on R×S^2)

$$f_s(x) + \frac{1}{\pi}\sum_{t=1}^{\Lambda}\int_{-R_t}^{R_t} du \frac{|d_s-d_t|}{(d_s-d_t)^2+(x-u)^2} f_t(u) = \frac{1}{\pi}\left(\Delta'_s + 2W_0 d_s^2 - 2W_0 x^2\right),$$

(II)  (dual of SYM on R×S^3/Z$_k$)

$$f'_\alpha(x) + \sum_{\beta\in K}\int_{-R_\beta}^{R_\beta} du\, K_k\left(\frac{\alpha-\beta}{k}, x, u\right) f'_\beta(u) = -\frac{4}{\pi}W_0 x,$$

$$K_k(\nu,x,u) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp \frac{\cosh\left\{\frac{\pi k}{2}p\left(|\nu|-\frac{1}{2}\right)\right\}}{\sinh\frac{\pi k}{4}|p|}\left(e^{ip(x-u)} - e^{ip(x+u)}\right)$$

Exactly same EOM are obtained from the matrix integral on gauge theory side

Summary

- By applying localization to gauge theories with SU(2|4) symmetry, we obtained multi-matrix integrals
- We found that eigenvalue density = charge density in LM geometry
- LM geometry can be reconstructed from eigenvalues in gauge theories

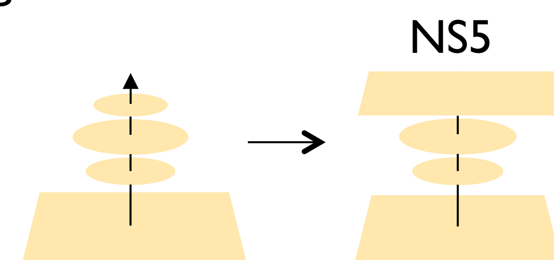
Emergent geometry !

Outlook

- So far, we have studied only saddle point configuration (vacuum states)

Excitation in matrix integral ⇔ gravitons ?

- Double scaling limit ? PWMM → Little string



[Ling-Mohazab-Shieh-Anders-Raamsdonk]